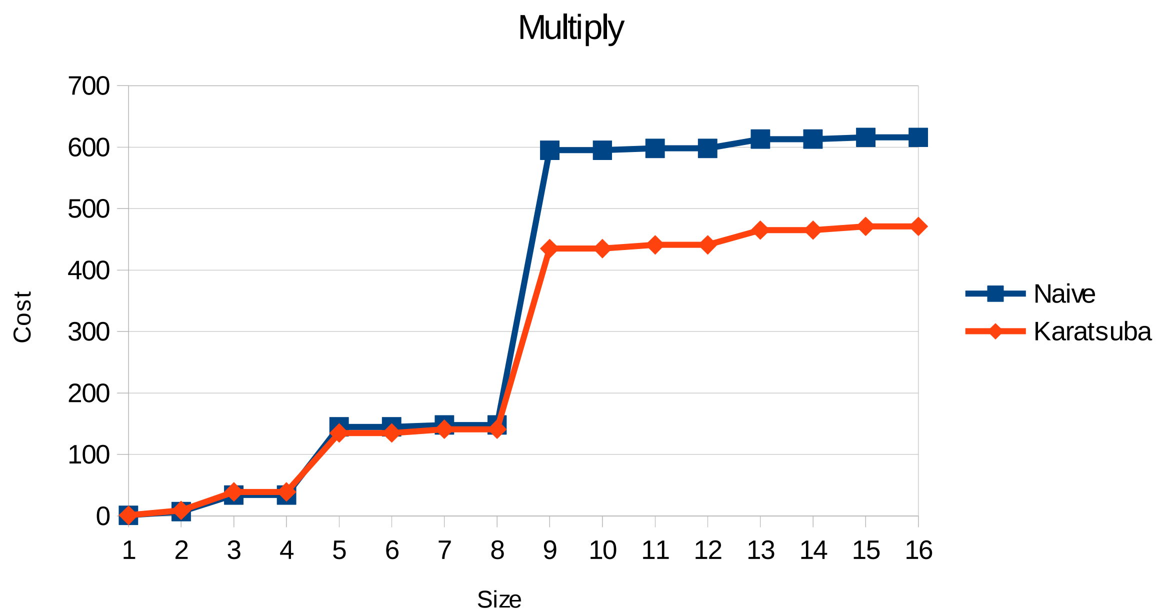
Nathalie Raffray
260682940



The cost of the naive algorithm is slightly lower at the beginning when the size is low (i.e. 1-4). This is because when the size is low, there are little recursions and as the cost of each recursion is 3, compared to 6 for Karatsuba, the overall cost is less. However, we can see the cost of the naive algorithm rising very quickly in comparison to Karatsuba for bigger values. The recursions have increased and therefore the cost of Karatsuba, which only requires 3 recursions at each call instead of the 4 recursions for naive, is asymptotically lower.

PART 2

2.

a) $T(n) = 25 * T(\frac{n}{5}) + n$

$a = 25 \quad b = 5$

$\left.\begin{array}{l}\log_5 25 = 2 \\ \varepsilon = 1\end{array}\right\} O(n^{\log_b a - \varepsilon})$

$\Rightarrow 1 > 0$ so by case 1 $T(n) = \Theta(n^2)$

b) $T(n) = 2T(\frac{n}{3}) + n\log(n)$

$a = 2$; $b = 3$; $n\log n \in \Omega(n)$

$\log_3 2 + (1 - \log_3 2) = 1$

$n\log n \in \Omega(n^{\log_3 2 + (1 - \log_3 2)}) = \Omega(n)$

$\Rightarrow 1 - \log_3 2 > 0$

and

we know

$\log(\frac{n}{3}) \leq \log(n)$

$2(\frac{n}{3}\log(\frac{n}{3})) = \frac{2}{3} n\log(\frac{n}{3}) \leq \frac{2}{3} n\log(n)$

so by case 3 $T(n) \in \Theta(n\log(n))$

c) $T(n) = T\left(\frac{3n}{4}\right) + 1$

$\log_{3/4} 1 = 0$

$1 \in \Theta(1) = \Theta(\log(n)^{1-1})$

by case 2 $T(n) \in \Theta(\log n)$

d) $T(n) = 7T\left(\frac{n}{3}\right) + n^3$

$a = 7 \quad b = 3$

$\log_3 7 < 2$

$\Rightarrow \varepsilon = 2 - \log_3 7 > 0$

$n^3 \in \Omega\left(n^{\log_3 7 + (2 - \log_3 7)}\right) = \Omega(n^2)$

We know: $n > 1$, $\frac{4}{5} < 1$

$7\left(\frac{n}{3}\right)^3 = \frac{7}{27} n^3 \leq \frac{1}{2} n^3$

by case 3 $T(n) \in \Theta(n^3)$

e) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1 \quad b = 2 \quad \log_2 1 = 0$

$1 \leq 2 - \cos(n) \leq 2 + 1 = 3$

$\Downarrow$ $\qquad\qquad \Theta(n) = \Theta(n \log^{1-1}(n))$

$n \leq (2 - \cos(n))n \leq 3n$

$\rightarrow n(2 - \cos(n)) \in \Theta(n \log^{1-1}(n))$

$\in \Omega(n^{0+1})$

$\frac{n}{2}\left(2 - \cos\left(\frac{n}{2}\right)\right) \leq cf(n) \quad \forall n > n_0$ with $c < 1$

There is no $c < 1$ where this is true! Master theorem does not app

PART 2

3. $T_A(n) = 7\,T_A\left(\frac{n}{2}\right) + n^2$

$T_B(n) = \alpha\,T_B\left(\frac{n}{4}\right) + n^2$

$\log_2 7 > 2$

$\log_2 7 - 2 > 0 \qquad O\left(n^{\log_2 7 - (\log_2 7 - 2)}\right) = O(n^2)$

$n^2 \in O(n^2) \quad \Rightarrow T_A(n) \in O\left(n^{\log_2 7}\right)$

$\log_2 7 = \frac{\ln 7}{\ln 2}$

$\frac{\ln \alpha}{\ln 4} < \frac{\ln 7}{\ln 2}$

$\frac{\ln \alpha}{\ln 7} < \frac{\ln 4}{\ln 2}$

$\log_7 \alpha < 2$

$7^2 = 49$ so the largest integer value of α can be 48.