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Assignment 3

Date :
Page No.

7. FFT is $O(n \log_2 n)$ algorithm:-

→ suppose for simplicity $n = 2^m$.

Then $\tilde{w}_q = \frac{1}{\sqrt{n}} \sum_{p=0}^{n-1} w_p \cdot \exp\left(-\frac{2\pi p q}{n} i\right)$

$= \frac{1}{\sqrt{n}} \sum_{p=0}^{\frac{n}{2}-1} w_{2p} \cdot \exp\left(-\frac{2\pi i q}{n} \cdot (2p)\right)$

$+ \frac{1}{\sqrt{n}} \sum_{p=0}^{\frac{n}{2}-1} w_{2p+1} \cdot \exp\left(-\frac{2i\pi q}{n} \cdot (2p+1)\right)$

$= \frac{1}{\sqrt{n}} \sum_{p=0}^{\frac{n}{2}-1} w_{2p} \exp\left(-\frac{2\pi i q p}{n/2}\right)$

$+ \frac{1}{\sqrt{n}} \exp\left(-\frac{2\pi q i}{n}\right) \sum_{p=0}^{\frac{n}{2}-1} w_{2p+1} \exp\left(-\frac{2\pi i q p}{n/2}\right)$

$= F_q^e + W^q \cdot F_q^o$

where F_q^e = DFT of even indexed nos.

F_q^o = DFT of odd indexed nos

$W^q = \exp\left(-\frac{2\pi q i}{n}\right)$

Thus given F_q^e, W^q & F_q^o, there are $2n$ steps required.

let $S(n)$ = steps for DFT of n nos.

Then $S(n) = 2 \times S\left(\frac{n}{2}\right) + 2n$
↳ addition & multiplications

Thus the recursion relation is:

$$s(n) = 2s\left(\frac{n}{2}\right) + 2n$$

with $s(1) = 1$

The term $S\left(\frac{n}{2}\right)$ reaches $S(1)$ after m iterations where $m = \log_2 m$.

For each iteration there are $O(n)$ steps.

Why?

$$s(n) = 2\left[2\left(s\left(\frac{n}{4}\right)\right) + 2\left(\frac{n}{2}\right)\right] + 2n$$

$$= 2^2 s\left(\frac{n}{4}\right) + 2n + 2n$$

$$= 2^2\left[2\left(s\left(\frac{n}{4}\right)\right) + 2\left(\frac{n}{4}\right)\right] + 2n + 2n$$

$$= 2^3 s\left(\frac{n}{4}\right) + 2n + 2n + 2n$$

$$\vdots$$

$$\underbrace{2^m s\left(\frac{n}{2^m}\right)}_{1} + 2nm \simeq n + 2n\log_2 n$$

Thus $s(n) \sim O(n\log_2 n)$