Algorithms for NP-Hard Problems
Sample Exam for Search and Inference

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This is a sample exam for the search and inference part of the course. You are expected to spend around 30-45 minutes on this part.

1 Exhaustive Search

In the *travelling salesperson problem*, given a graph and a starting node, the goal is to find a cycle in a graph such that all nodes are visited exactly once and the sum of weights of the edge is minimised. You task is to execute an exhaustive search algorithm for this problem.

Define a problem representation and simulate the first six variable selections (not counting propagations!) of an exhaustive search algorithm with pruning on the graph given in Figure 1 (see next page) using these fixed rules:

- Start by considering the node D (or one of its edges, depending on your representation).
- The variable selection strategy says that the next selected variable must be such that the current partially constructed path/cycle is always connected, and variables are selected in a greedy fashion, e.g., select the next node by looking for the edge with minimum weight.
- The sum of edges in the cycle must be less than 30.

Note: Even though propagations are not counted, you still need to show them (if any)!

2 Modelling with Symmetry Breaking

In a *machine scheduling* problem, you have given:

- A set of five machines, m_1, m_2, m_3, m_4.
- Resource capacities for each machine $C(m_i)$: $C(m_1) = 100, C(m_2) = 60, C(m_3) = 70, C(m_4) = 100$.
- A set of tasks t_i, where each task requires $R(t_i)$ resources.

The goal of the problem is to find an assignment of tasks to machines such that the resource requirement for each machine does not exceed the capacity of the machine.

Model this problem and propose one symmetry breaking constraint. Estimate the search space size with and without the symmetry breaking constraint (make sure to show your derivation).

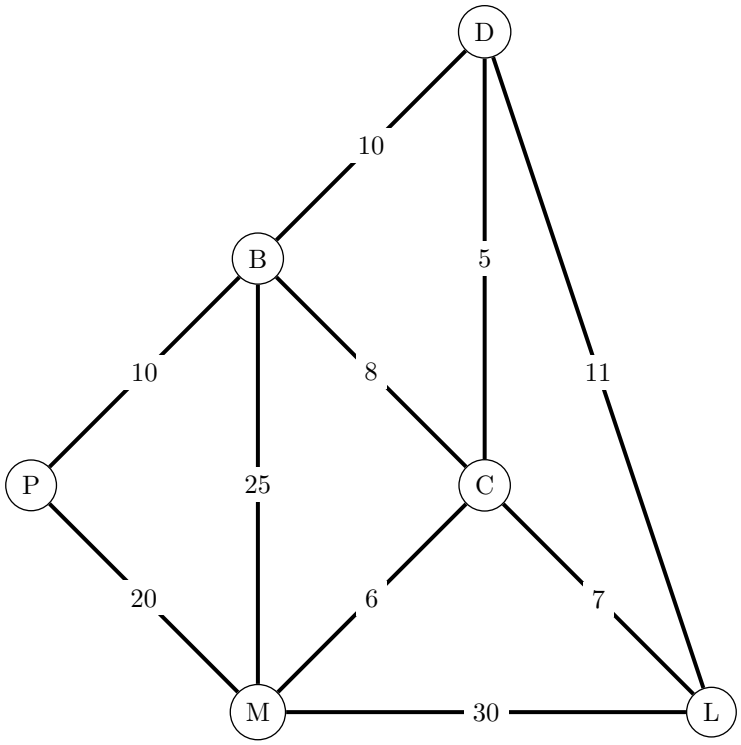

Figure 1: Graph for Q1

3 Propagation

Consider the constraint of the form $x + 5 = 2y^2$, where x and y are integer variables which may take both positive and negative values.

Select a representation for integer variables, and describe an arc consistency algorithm, discussing its advantages and disadvantages.

4 Look-Ahead

Recall the graph colouring problem: you are given a graph and the task is to colour its nodes using at most k colours such that no two neighbouring nodes have the same colour.

Propose one look-ahead strategy for this problem and discuss its advantage and disadvantages. Be precise when possible!

5 Relaxation

Say you are solving a Sudoku problem, but apart from merely finding a solution, you need the solution that maximises the sum of numbers in one of the two main diagonals (you can choose which one).

Propose one relaxation for this problem and discuss its advantages and disadvantages when used as part of an exhaustive search algorithm.