Frequency Shifter with Raised Cosine

Say we want to move a frequency window but we don't want the shift to interfere with the band location that it's shifting into. To solve this we need to shift the band in the location the window is shifting into, but if we shift it by the same amount we're shifting the window we're trying to shift, we end up shifting the entire spectrogram. The solution to this is to incrementally shift frequencies beyond our window so that the frequencies beyond the window shift less and less until they don't shift at all. Additionally, we don't want to alter frequencies in the direction we're not trying to shift.

One way to do this is with a raised cosine window of order p.

Say we have a window of frequencies we're trying to shift. The lower and upper frequencies of this window are denoted by f_1 and f_2. The center of this band is denoted by b_c. The bins we want to shift the window is $b_{shift} = b_s$ We are using bins instead of frequencies, where b = (frequency)*(frequency resolution of the spectrogram)

$$(1)\quad b_c = \frac{b_1+b_2}{2}$$

Our raised cosine window will be defined as:

$$(2)\quad x[n] = \left\{\begin{matrix} \cos^p\big(\omega(n-b_c)\big),\ |\omega(n-b_c)| < \pi/2 \\ 0,\ if\ n \le b_1\ and\ b_s > 0 \\ 0,\ if\ n \ge b_2\ and\ b_s < 0 \\ 0,\ if\ |\omega(n-b_c)| \ge \pi/2 \end{matrix}\right\}$$

b_c is going to be our center bin of the raised cosine. Now we want the beginning and end of the window to have a certain attenuation threshold, A, preferably high, like 0.9 or 0.95.

$$(3)\quad A = \cos^p(\omega(b_c - b_1))$$

$$(4)\quad \omega = \frac{\cos^{-1} A^{1/p}}{b_c - b_1}$$

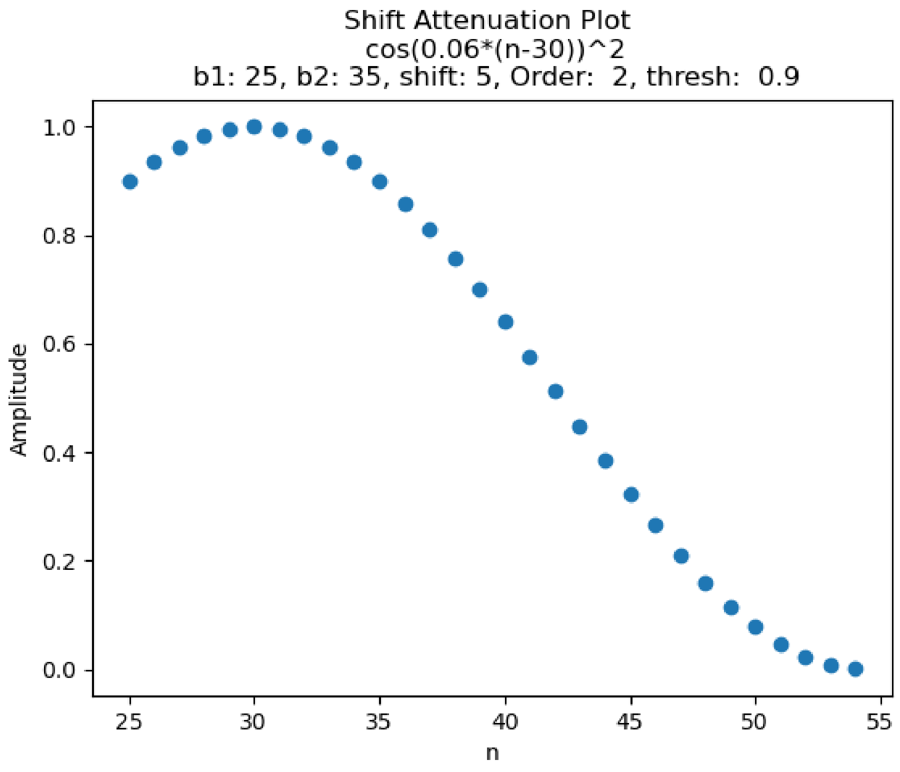
$x[n]$ determines how far indices will shift. If we have data $y[k]$ that contains frequency components we want to shift by b_c, then we will be shifting every element by $x[k]$ into a placeholder $z[k]$ and adding onto whatever was already shifted there, then copying back into the original array, $y[k]$

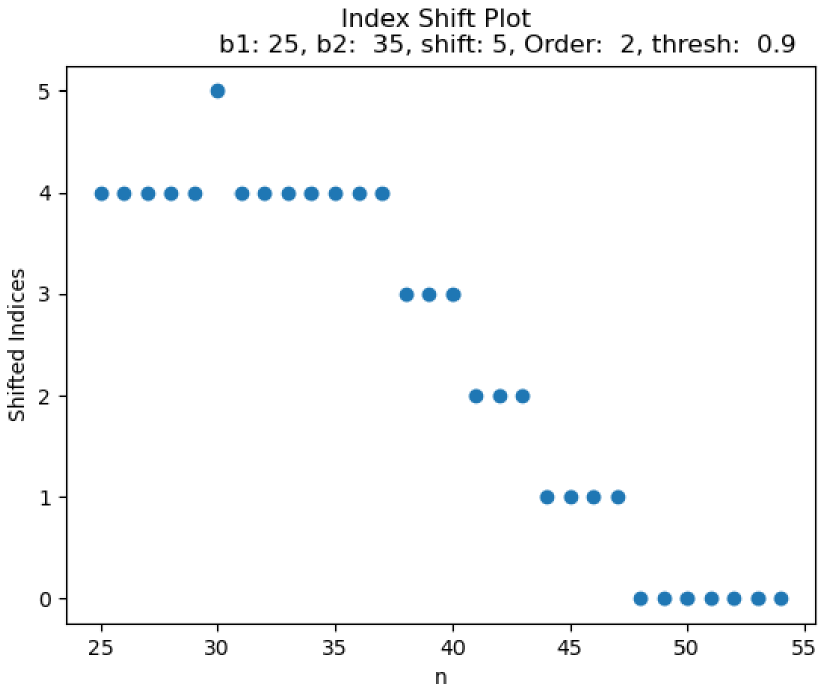
$$z[k] = 0$$

$$z[k] = z[k] + y\big[k + b_s x[k]\big]\ for\ all\ k\ in\ y$$

$$y[k] = z[k]$$

Example: f_1: 500 hz, f_2: 700 Hz. Shift frequency: 100 Hz, SR = 20480, fftsize = 1024


Shift Attenuation Plot
cos(0.06*(n-30))^2
b1: 25, b2: 35, shift: 5, Order: 2, thresh: 0.9
Amplitude
n


Index Shift Plot
b1: 25, b2: 35, shift: 5, Order: 2, thresh: 0.9
Shifted Indices
n