Developmental change in the acuity of the "Number Sense": The approximate number system in 3-, 4-, 5-, 6-year-olds and adults

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Abstract

Behavioral, neuropsychological, and brain imaging research points to a dedicated system for processing number that is shared across development and across species. This foundational Approximate Number System operates over multiple modalities, forming representations of the number of objects, sounds, or events in a scene. Importantly, it is imprecise, and hence differs from exact counting. Evidence suggests that the resolution of the Approximate Number System, as specified by a Weber Fraction, increases with age such that adults can discriminate numerosities that infants cannot. However, the Weber Fraction has yet to be determined for subjects of any age between 9 months and adulthood, leaving its developmental trajectory unclear. Here we identify the Weber Fraction of the Approximate Number System in 3-, 4-, 5-, and 6-year-old children, and adults. We show that the resolution of this system continues to increase throughout childhood, with adult-like levels of acuity attained surprisingly late in development.

The ability to nonverbally represent number is shared across species and across development (for reviews see Dehaene, 1998; Feigenson et al., 2004). The foundational Approximate Number System that underlies this ability (henceforth, the ANS) produces abstract number representations (Barth, Kanwisher, & Spelke, 2003) that support arithmetic computation across the lifespan (Barth et al., 2003; Barth, LaMont, Lipton, Dehaene, Kanwisher, & Spelke, 2006; McCrink & Wynn, 2004). The ANS is activated when adults perform symbolic number tasks (e.g., Dehaene, Piazza, Pinel, & Cohen, 2003; Piazza, Pinel, LeBihan, & Dehaene, 2007), and may even provide a foundation for more sophisticated mathematics (Gilmore, McCarthy, & Spelke, 2007). However, it represents number only approximately (Dehaene, 1998; Gallistel & Gelman, 2000) and the imprecision of its numerical representations is radically greater in infants than in adults (e.g., Pica, Lemer, Izard, & Dehaene, 2004; Xu & Spelke, 2001). Given that the ANS is thought to play an important role in math learning (Booth & Siegler, 2006; Jordan, Kaplan, Locuniak, & Ramineni, 2007), it is surprising that no research to date has explored the full developmental trajectory of its representational acuity. Here we test ANS acuity in 3-, 4-, 5-, and 6-year-old children and adults, using psychophysical modeling to determine the finest numerical discriminations possible at each age. Our findings reveal that the Approximate Number System does not attain full acuity until quite late in development, long after children have begun formal instruction in mathematics.

The Approximate Number System differs from counting in that it produces inexact number representations. A hallmark of the ANS is that the imprecision of its representations grows with the target numerosity, such that the ability to nonverbally discriminate two quantities depends on their ratio (Moyer & Landauer, 1967). This ratio-dependence is observed when adults estimate numbers of items (Halberda et al., 2006; Whalen et al., 1999), produce target numbers of actions (Cordes et al., 2001; Whalen et al., 1999), judge the more numerous of two arrays (Barth et al., 2003), and estimate the results of arithmetic events (Pica, Lemer, Izard, & Dehaene, 2004). Because of the inexactness of ANS representations, two quantities cannot be distinguished when the distance between them is too small. The finest numerical ratio that adults can consistently discriminate has been identified as 7:8. This limit can also be stated as a Weber Fraction that measures the smallest numerical change to a stimulus that can be reliably detected. The Weber Fraction is equal to the difference between the two numbers divided by the smaller number (e.g., 7:8 -> (8-7)/7 = .14). When asked to indicate the more numerous of two simultaneously-presented arrays containing 20-80 dots, French adults' Weber Fraction is .12 and Amazonian adults' Weber Fraction is .17; thus on average these adults could discriminate ratios differing by about 7:8 (Pica et al., 2004).

Ratio-dependent numerical performance also reveals that preverbal infants use the ANS, albeit with drastically less acuity than adults. Six-month old infants discriminate arrays of 4 vs. 8, 8 vs. 16, and 16 vs. 32 dots, all of which instantiate a 1:2 ratio, but fail to discriminate 8 vs. 12 and 16 vs. 24, which instantiate a 2:3 ratio (Xu & Spelke, 2000; Xu, 2003; Xu et al., 2005). The same pattern obtains in audition: 6-month olds discriminate 4 vs. 8 and 8 vs. 16 but not 4 vs. 6 or 8 vs. 12 tones (Lipton & Spelke, 2003; 2004). Infants' discrimination threshold changes with development: 9-month olds succeed with the 2:3 ratios with which 6-month olds fail, in both the auditory and visual domains (Lipton & Spelke, 2003; Wood & Spelke, 2005; Xu & Arriaga, 2007). The identical increase in acuity across sensory modalities suggests that it is the ANS itself, and not the visual or auditory perception systems, that is improving.

The above data show that ANS acuity increases over the lifespan, with subjects exhibiting a Weber Fraction of 1.0 at 6 months, 0.5 at 9 months, and 0.14 in adulthood. However, these studies leave a large gap in our understanding of the development of nonverbal enumeration. It is known that preschool- and early school-aged children (the ages during which formal instruction in mathematics typically begins) show performance controlled by numerical ratio, making more errors with close than with distant numerical comparisons (Barth et al., 2005; Huntley-Fenner, 2001; Huntley-Fenner & Cannon, 2000; Starkey & Cooper, 1995; Temple & Posner, 1998). Yet no study to date has identified the finest numerical discriminations children can make, as none has used psychophysical methods similar to those used with adults to identify children's Weber Fraction. For this reason, the developmental trajectory of changes in ANS acuity between 9 months and adulthood remains entirely undescribed. For example, it is not known whether numerical acuity rapidly asymptotes during the first year of life (similar to stereoacuity; Held, Birch, & Gwiazda, 1980), gradually increases throughout early childhood (similar to executive function, Diamond, 2002; Zelazo, Craik, & Booth, 2004), or shows a discontinuous change when children master verbal counting at around age 4 (similar to a vocabulary spurt; Carey, 1978; Goldfield & Reznick, 1990). Furthermore, changes in ANS acuity have implications for formal instruction in mathematics. Many math curricula aim to tap children's intuitions regarding "possible" or "impossible" solutions to quantitative problems, encouraging children to estimate numerical magnitudes before arriving at an exact answer (Johnson, 1979; Levin, 1981). Given the widespread nature of such teaching tools, it is surprising that children's ANS acuity during the early years of mathematics instruction has not been determined.

We presented 3-, 4-, 5-, and 6-year-old children and adults with a nonverbal number discrimination task that did not permit counting, with controls for continuous variables that often correlate with number. Varying the numerical ratio between stimulus arrays allowed us to determine the Weber Fraction of the ANS for each age group, and thereby to describe the function by which the ANS reaches the adult-state of representational precision.

Method

Participants

We tested 5 age groups with 16 participants per group: 3-, 4-, 5-, 6-year-olds and adults. Summaries of mean age, age range and sex appear in Table 1. Adults were undergraduate and graduate students at the Johns Hopkins University. Children were from the Baltimore area and were recruited via mail and telephone. Each child was tested only once. Thirty-one additional children were tested but not included in the final analysis due to parental interference (2), failure to complete the task (21), or equipment failure (8).

Materials

On each trial of the numerical discrimination task, two arrays of between 1 and 14 items appeared side-by-side on a large video screen (Figure 1). Items varied in size but were otherwise identical, and on each trial were randomly chosen from 46 possible images of familiar objects (Table 2). Each array appeared within a background frame demarcating “Big Bird’s Xs” on the left side of the screen and “Grover’s Xs” on the right.

On each trial, subjects heard a recorded voice prompting them to select the greater of the two quantities (e.g., “Who has more [pieces of pizza]?”). Labels and the carrier phrase “Who has more” were recorded by a female native English speaker in a child-friendly voice and were played over a centralized computer speaker.

Procedure

Subjects sat at a table approximately 150 cm from the screen (viewing area 120 x 90 cm). The average object subtended 2° of visual angle from this distance. The study began with the recorded female voice saying, “Let’s play a game,” followed by four practice trials. On practice trials the computer first displayed Big Bird’s items for 2000 ms accompanied by the labeling phrase, “Here are Big Bird’s [pieces of pizza].” Next the computer displayed Grover’s items for 2000 ms accompanied by the labeling phrase, “Here are Grover’s [pieces of pizza].” Finally, both arrays appeared simultaneously for 2000 ms accompanied by the carrier phrase, “Who has more [pieces of pizza].” The carrier portion of the phrase began before the items appeared; label onset was synchronized to the items’ visual onset. Subjects indicated which character had the greater number of items via a color-coded keyboard, pressing a red key to begin each trial, a yellow key to indicate that Big Bird had more, and a blue key to indicate that Grover had more. The experimenter and, for children, a caregiver accompanied subjects into the testing room and sat approximately 100 cm behind them so as not to influence performance. To maintain subjects’ motivation, the computer provided auditory feedback on every trial (e.g., “That’s right!” for correct responses; “Oh, that’s not right” for incorrect responses). Item type and item position within the background frames were free to vary randomly throughout practice and test trials. The 4 practice trials were followed by 66 test trials, which were identical to the simultaneous portion of the practice trials.

Displays were controlled either for average item size (Area Correlated trials) or summed continuous extent (Area Anti-Correlated trials). For each ratio presented, on half of the trials the larger numerosity had more total surface area (Area Correlated trials), and on the other half the smaller numerosity had more total surface area (Area Anti-Correlated trials). Area Anti-Correlated trials controlled for the total summed perimeter of the items and their total surface area, two dimensions of continuous extent to which infants have shown sensitivity (Clearfield & Mix, 1999; Feigenson, Carey, & Spelke, 2002). These trials equated the total summed horizontal and total summed vertical extent of the items in Big Bird's and Grover's arrays. This procedure equated perimeter and anti-correlated surface area such that the array with the smaller number of items had more total surface area. For example, on an Area Anti-Correlated trial on which Big Bird had 6 items and Grover had 12, Grover would be the numerically correct choice by a ratio of 2 (12/6 = 2), but Big Bird would be the correct choice in total area by an equal and opposite ratio (20943pixels/10471pixels = 2). Because the difference in total surface area varied with the numerical ratio of the two arrays (as summarized in Table 3), the easier the numerical ratio was to discriminate, the more wrong surface area became. This, combined with consistent feedback, was included to discourage the use of continuous variables as a cue. Finally, on both Area Correlated and Area Anti-Correlated trials, individual item size varied to ensure that items in the less numerous array were not all larger than those in the more numerous array (Figure 1).

Unequal numbers of trials were presented from each ratio bin in order to focus on more difficult ratios (Ratio bin 1:2 = 2 trials, 2:3 = 2, 3:4 = 2, 4:5 = 2, 5:6 = 10, 6:7 = 10, 7:8 = 10, 8:9 = 14, 9:10 = 14). These numbers were initially chosen for adult subjects and so focus more heavily on difficult ratios (e.g. 5:6 – 9:10). To ensure consistency, we used the same ratio distribution for all ages.

Display time was adjusted for each age group through pilot testing, and was chosen to be long enough to allow subjects to view both arrays but short enough to prevent serial counting[1]. Display time was 2500 ms for 3-year olds, 1200 ms for 4-, 5-, and 6-year olds, and 750 ms for adults. The winning side (Big Bird or Grover), ratio presented, trial type (Area Correlated, Area Anti-Correlated), item type, and absolute number of items presented varied randomly across trials.

Results

To ask whether performance differed across age groups or depended on numerical ratio, we entered each subject's percent correct for each ratio bin (e.g., 1:2, 2:3, etc.) into a 5 (Age Group) X 2 (Sex) X 2 (Trial Type: Area Correlated, Area Anti-Correlated) X 9 (Ratio) repeated measures ANOVA. These data are presented in Figure 2, which plots percent correct for each age group as a function of Ratio (numerosity of larger set / numerosity of smaller set). There was a significant Age Group effect, with subjects performing better with increasing age, $F(4, 70) = 28.891$, $p < .001$, a significant Ratio effect, with subjects performing better with increasing Ratio, $F(8, 560) = 20.199$, $p < .001$, and a marginally significant effect of Sex with females performing slightly better than males overall, $F(1, 70) = 3.122$, $p = .082$.

[1]Children's verbal counting ability was assessed for the purposes of a separate study, and revealed that none of the 3-year old children understood the Cardinal Word Principle (Gelman & Gallistel, 1978) and so could not have used counting to determine the numerosities of the sets.

Regarding the effect of total area on subjects' numerical discriminations, we observed a significant Trial Type effect, $F(1, 70) = 6.138$, $p < .05$. We investigated this effect via planned t-tests comparing each age group's percent correct on Area Correlated and Area Anti-Correlated trials to chance (50%). Collapsing across all Ratios revealed that all age groups performed significantly above chance on both trial types, as summarized in Table 4, but performed slightly better on Area Correlated trials. All age groups based their responses on number not area, though area had some effect on judgments. We also observed a significant Trial Type X Ratio interaction, as the difference in performance on Area Correlated and Area Anti-Correlated trials was larger for easier Ratios than for harder Ratios, $F(8, 560) = 2.62$, $p < .01$. A significant Age Group X Ratio X Trial Type interaction revealed that older children and adults showed less differentiation in performance between Area Correlated and Area Anti-Correlated trials as a function of Ratio than did younger children. In particular 3-, 4- and 5-year-olds performed better on Area Correlated trials than Area Anti-Correlated trials with 1:2 ratio comparisons, whereas 6-year-olds and adults showed little or no difference in performance with this comparison ratio, $F(32, 560) = 1.575$, $p < .05$.

Subjects' performance also varied as a function of Ratio (Figure 2). If subjects were using the Approximate Number System (ANS) to determine the more numerous array, then percent correct as a function of Ratio (collapsed across Trial Type) should be well fit by a computational model of the ANS. Pica et al. (2004) examined performance on a task similar to ours in adults and children from both a developed country (France) and from an indigenous culture (the Munduruku of Amazonia) whose language lacks exact large number words (e.g., "seven"). Pica and colleagues found that both groups' performance was well fit by a psychophysical model (Green & Swets, 1966) that models ANS representations as "noisy"

Gaussian random variables, and numerical discrimination as the subtraction of the two Gaussian random variables that represent the numerosities of the two arrays[2]. This model has a single free parameter, the Weber Fraction (*w*), which determines the increase in percent correct with increasing ratio. We rely on this same psychophysical model and on curve-fitting via Levenberg-Marquardt non-linear least squares fit in order to determine the Weber Fraction for each age group we tested.

As has been observed in previous numerical discrimination tasks (Pica et al, 2004), subjects' performance can fail to reach 100% correct because of a tendency to guess randomly on some trials. As seen in Figure 2, this was the case for our 3-, 4- and 5-year-old children. The simplest way to account for this tendency computationally is to include a parameter that is a constant probability of guessing randomly on any particular trial (for additional discussion of this approach see the supporting online materials of Pica et al, 2004). This parameter lowers the model's asymptotic performance while retaining an accurate estimate of the Weber Fraction (*w*). We included this parameter in our model as follows: where p_{guess} is the probability of guessing randomly, p_{error} is the probability of being incorrect given the model, and chance is .5 multiplied by 100 to return a percentage.

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[2] In the psychophysics model, each numerosity is represented as a Gaussian random variable (i.e., X2 & X1) with means n2 & n1 and standard deviations equal to the critical Weber Fraction (*w*) * n. Subtracting the Gaussian for the smaller set from that for the larger set returns a new Gaussian that has a mean of n2-n1 and a standard deviation of $w\sqrt{n1^2+n2^2}$ (simply the difference of two Gaussian random variables). Percent correct is then equal to 1-error rate, where error rate is defined as the area under the tail of the resulting Gaussian curve computed as:

$$= \frac{1}{2}\operatorname{erfc}\left(\frac{|n_1 - n_2|}{\sqrt{2}w\sqrt{n_1^2 + n_2^2}}\right)$$

$$\text{Percent Correct} = [(1-p_{\text{guess}})(1-p_{\text{error}}) + p_{\text{guess}}*.5\ ] *100$$

As seen in Figure 3a, 3b, and Table 5, the psychophysical model provides an accurate fit to our data from 6-year-olds and adults (adult R^2 value = .93). Performance for 3-, 4-, and 5-year-olds deviates from this model, however, because younger children were at chance for more difficult ratios. Figures 3d and 3e show that data from 3- and 4-year-olds exhibit a sigmoidal shape, as children ceased to rely on ANS representations for the hardest ratios and instead guessed randomly (e.g., 3-year-olds asymptote at 53% for comparisons of ratio 1.25 and lower). These data points artificially pull the performance of the psychophysical model down as it attempts to reduce the error between the fit and these data points, resulting in a Weber Fraction that likely underestimates the actual acuity of these subjects (witness the low R^2 values for these fits in Table 5).

To gain further accuracy in estimating the Weber Fraction at each age (especially for younger children), we also modeled performance for each age group using a sigmoidal function fit by the Levenberg-Marquardt algorithm. We relied on the sigmoidal equation:

$$y = \text{lower} + \frac{-\text{lower} + \text{upper}}{1 + e^{\frac{\text{inflection} - x}{\text{rate}}}}$$

From the sigmoidal equation for each age group, the Weber Fraction (*w*) can be estimated as the inflection point of the sigmoid (psychologically equivalent to the midpoint between successful discrimination (upper asymptote) and subjective-equality (lower asymptote)). As seen in Figures 3d, 3e and Table 5, the sigmoidal function provides an accurate fit to our data from younger children (e.g. 4-year-olds R^2 value = .98). Table 5 lists the estimated Weber Fraction for each age group from both the psychophysics model and the sigmoid model along with the R^2-values for the fit. These numbers are also translated into a nearest whole number ratio (e.g., Weber Fraction of .25 = 4:5 ratio). As Table 5 shows, estimated Weber Fraction decreased with age, confirming that ANS acuity increased across the age groups we tested. Whereas 3-year-olds can accurately discriminate numerosities differing by a 3:4 ratio, 6-year-olds have sufficient acuity to discriminate numerosities differing by a 5:6 ratio, and adult-acuity in our sample was as high as 10:11. Using a numerical discrimination task similar to ours, Pica et al (2004) estimated untrained French adults' Weber Fraction to be .12 or a 9:10 ratio. The extent of individual differences in Weber Fraction in adults and children remains to be determined, but our estimate and that of Pica et al (2004) suggest that the average acuity in adults from educated numerate cultures is in the range of 9:10 or 10:11.

In Figure 4 we plot estimates from both models alongside estimates from the developmental literature on infants' numerical acuity (Xu & Spelke, 2000; Lipton & Spelke, 2003). These estimates have been modeled by least squares fit to determine the developmental trajectory of the increasing acuity of the ANS (i.e., decreasing Weber Fraction). These suggest a logarithmic decrease in Weber Fraction throughout childhood[3], with adult-like levels of numerical acuity being attained sometime during the pre-teen years.

General Discussion

Although children between 3 and 6 years old have already begun formal instruction in mathematics, the present results show that the acuity of the Approximate Number System is still developing during this time. Indeed, the sharpening of the ANS does not appear to be complete

[3] Or, a power function with an exponent of -.55.

until early adolescence. Given the central role this system plays in supporting mathematical intuitions, this protracted period of development highlights the importance of coming to understand the effects of changes in ANS acuity on math learning and achievement (Booth & Siegler, 2006; Jordan et al, 2007).

What causes the ANS to increase in acuity? Although some of the sharpening of this system may be due to simple maturation of the neural circuitry subserving the ANS, recent evidence suggests that experience can also affect its development. Practice at numerical discrimination appears to increase acuity in children with math learning disabilities (Wilson, Revkin, Cohen, Cohen & Dehaene, 2006). After approximately 8 hours of practice spanning 5 weeks on a computer game designed to engage the ANS, 7- to 9-year-old children showed improvement in both symbolic and nonsymbolic numerical tasks. Continued engagement in numerical discrimination throughout childhood may therefore contribute to the increase in acuity we observed in normally-developing children. This suggests the intriguing possibility that subpopulations that do not frequently engage in numerical discrimination might show increasingly reduced acuity with age relative to individuals undergoing formal education. Existing data are consistent with this hypothesis. Although Pica et al (2004) stressed the similarity in the numerical discrimination abilities of their French and Munduruku subjects, Munduruku adults' Weber Fraction was estimated to be .17 (approximately 6:7) whereas French adults' was estimated to be .12 (approximately 9:10). This divergence may be the result of differences in day-to-day engagement in numerical discrimination. Training studies in normally developing children and adults will be helpful in determining whether practice with numerical discrimination leads to increases in acuity.

Changes in working memory capacity and executive functioning may also affect numerical discrimination. There are improvements in both spatial and verbal working memory in the preschool and early elementary school years (for review see Cowan, 1997), as well as in aspects of executive functioning such as inhibition and cognitive flexibility (Espy, Kaufmann, McDiarmid & Glisky, 1999; Happaney, Zelazo & Stuss, 2004). These factors correlate with math achievement in tasks that focus on symbolic and explicit mathematical reasoning (Espy, McDiarmid, Cwik, Stalets, Hamby & Senn, 2004; Gathercole & Pickering, 2000; McClelland, Cameron, Connor, Farris, Jewkes & Morrison, 2007; McClelland, Acock & Morrison, 2006). In our task, although working memory and executive function were likely needed to maintain multiple arrays in memory and to focus on the dimension relevant to the task (i.e., number rather than area), differences in these factors was minimized by lengthening the display times for younger children. Nonetheless, an important future direction is to determine how developmental changes in these non-numerical abilities affect deployment of the ANS.

Orthogonal to changes in ANS acuity, we also found that 3- and 4-year-old children exhibited an increased likelihood of guessing randomly, especially on harder numerical comparisons. Developmental change in executive function is a likely contributor to this difference across age groups (Happaney et al, 2004). In particular, 6-year-olds remained vigilant and attempted to respond correctly even on the most difficult comparisons such as 9:10, even though the acuity of their ANS supported a level of accuracy that only attained 62% correct. That they remained vigilant is seen in Figure 3, where percent correct on difficult ratios for this age group did not deviate from the predictions of the psychophysics model. Future studies may tailor the ratios presented to focus on a level of difficulty more appropriate for each child (e.g., between1:2 and 5:6 for 3-year-olds). The present results provide developmental milestones of ANS acuity that can guide such future research.

In summary, the ability to nonverbally approximate number plays a role in quantitative reasoning throughout the human lifespan, even after the ability to represent exact integers is attained. Cognitive psychology and cognitive neuroscience have made much recent progress in understanding this ability. However, little research has addressed the development of the Approximate Number System after infancy. Here we show that the acuity of the ANS continues to increase between ages 3 and 6 years, and does not reach adult-like levels until sometime during the preteen years. The protracted nature of ANS development, spanning the period when symbolic mathematical instruction begins, has implications both for math education and for our understanding of the interplay between individual experience and the "number sense".

References

Barth, H., Kanwisher, N., & Spelke, E.S. (2003). The construction of large number representations in adults. *Cognition, 86*, 201-221.

Barth, H., LaMont, K., Lipton, J., Dehaene, S., Kanwisher, N., & Spelke, E.S. (2006). Non-symbolic arithmetic in adults and young children. *Cognition, 98*, 199-222.

Booth, J.L. & Siegler, R.S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology, 41*, 189-201.

Carey, S. (1978). The child as a word learner. In M. Halle, J. Bresnan & G.A. Miller (Eds.), *Linguistic theory and psychological reality* (pp. 264-293). Cambridge, MA: MIT Press.

Clearfield, M.W. and Mix, K.S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science, 10*, 408–411.

Cordes, S., Gelman, R., & Gallistel, C.R. (2001). Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. *Psychonomic Bulletin and Review*, *8*, 698-707.

Cowan, N. (1997). The development of working memory. In *The development of memory in childhood* (pp. 163-199). N. Cowan (Ed.) Hove, East Sussex UK: Psychology Press.

Dehaene, S. (2001). Substracting pigeons: Logarithmic or linear? *Psychological Science, 12,* 244-246.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20*, 487-506.

Dehaene, S., Molko, N., Cohen, L. & Wilson, A.J. (2004). Arithmetic and the brain. *Current Opinion in Neurobiology, 14*, 1-7.

Diamond, A. (2002). Normal development of prefrontal cortex from birth to young adulthood: Cognitive functions, anatomy, and biochemistry. In D.T. Stuss & R.T. Knight (Eds.), *Principles of frontal lobe function* (pp. 466-503). London, UK: Oxford University Press.

Espy, K. A., McDiarmid, M. M., Cwik, M. F., Stalets, M. M., Hamby, A. & Senn, T. E. (2004). The contribution of executive functions to emergent mathematic skills in preschool children. *Developmental Neuropsychology, 26*, 465-486.

Espy, K. A., Kaufmann, P. M., McDiarmid, M. D., & Glisky, M. L. (1999). Executive functioning in preschool children: Performance on A-not-B and other delayed response format tasks. *Brain and Cognition, 41,* 178-199.

Feigenson, L., Carey, S. and Spelke, E. (2002). Infants' discrimination of number vs. continuous extent. *Cognitive Psychology, 44,* 33–66.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8,* 307-314.

Gallistel, C.R. & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences, 4,* 59-65.

Gathercole, S. E., & Pickering, S. J. (2000). Working memory deficits in children with low achievements in the national curriculum at 7 years of age. *British Journal of Educational Psychology, 70*, 177–194.

Gelman, R. & Gallistel, C. R. (1978). The child's understanding of number. Cambridge, MA: Harvard University Press.

Gilmore, C.K., McCarthey, S.E., & Spelke, E.S. (2007). Symbolic arithmetic knowledge without instruction. *Nature, 447*, 589-591.

Goldfield, B.A. & Reznick, J.S. (1990). Early lexical acquisition: Rate, content, and the vocabulary spurt. *Journal of Child Language*, 17, 171-183.

Halberda, J., Sires, S. F. & Feigenson, L. (2006). Multiple spatially overlapping sets can be enumerated in parallel. *Psychological Science, 17*, 572-576.

Happaney, K., Zelazo, P. D., & Stuss, D. T. (2004). Development of orbitofrontal function: Current themes and future directions. *Brain and Cognition, 55*, 1–10.

Held, R., Birch, E., & Gwiazda, J. (1980). Stereoacuity of human infants. *Proceedings of the National Academy of Sciences*, 77, 5572-574.

Huntley-Fenner, G. (2001). Children's understanding of number is similar to adults' and rats': Numerical estimation by 5-7 year olds. *Cognition, 78*, B27-B40.

Huntley-Fenner, G. & Cannon, E. (2000). Preschoolers' magnitude comparisons are mediated by a preverbal analog magnitude mechanism. *Psychological Science, 11*, 147-152.

Johnson, D.C. (1979). Teaching estimation and reasonableness of results. *Arithmetic Teacher*, *27*, 34-35.

Jordan, N.C., Kaplan, D., Locuniak, M.N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research and Practice*, 23, 36-46.

Levin, J.A. (1981). Estimation technique for arithmetic: Every day math and mathematics instruction. *Educational Studies in Mathematics, 12*, 421-431.

Lipton, J.S. & Spelke, E.S. (2003). Origins of number sense: Large number discrimination in human infants. *Psychological Science, 14*, 396-401.

Lipton, J.S. & Spelke, E.S. (2004). Discrimination of large and small numerosities by human infants. *Infancy*, 5, 271-290.

McCrink, K. & Wynn, K. (2004). Large-number addition and subtraction by 9-month old infants. *Psychological Science, 15*, 776-781.

McClelland, M. M.; Cameron, C. E.; Connor, C. M., Farris, C. L., Jewkes, A. M. & Morrison, F. J. (2007). Links between behavioral regulation and preschoolers' literacy, vocabulary, and math skills.. *Developmental Psychology, 43*, 947-959.

McClelland, M. M., Acock, A. C., & Morrison, F. J. (2006). The impact of kindergarten learning-related skills on academic trajectories at the end of elementary school. *Early Childhood Research Quarterly, 21*, 471– 490.

Meck, W.H. & Church, R.M. (1983) A mode control model of counting and timing processes. *Journal of Experimental Psychology: Animal Behavior Processes, 9*, 320-334.

Moyer, R.S. & Landauer, T.K. (1967). Time required for judgments of numerical inequality. *Nature, 215*, 1519-1520.

Piazza, M., Pinel, P., LeBihan, D., & Dehaene, S. (2007). A magnitude code common to numerosities and number symbols in human intraparietal cortex. *Neuron, 53*, 293-305.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, 306, 499-503.

Starkey, P. & Cooper, R.G. (1995). The development of subitizing in young children. *British Journal of Developmental Psychology, 13*, 399-420.

Whalen, J., Gallistel, C.R. & Gelman, R. (1999). Non-verbal counting in humans: The psychophysics of number representation. *Psychological Science, 10*, 130-137.

Wilson, A. J., Revkin, S. K., Cohen, D., Cohen, L. & Dehaene, S. (2006). An open trial assessment of "The Number Race", an adabptive computer game for remediation of dyscalculia. *Behavioral and Brain Functions, 2:20.*

Wood, J.N. & Spelke, E.S. (2005). Infants' enumeration of actions: Numerical discrimination and its signature limits. *Developmental Science, 8*, 173-181.

Xu, F. & Spelke, E. (2000). Large number discrimination in 6-month-old infants. *Cogntion, 74,* B1-B11.

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15-B25.

Xu, F., Spelke, E.S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science, 8,* 88-101.

Xu, F. & Arriaga, R. (2007). Number discrimination in 10-month old infants. *British Journal of Developmental Psychology, 25*, 103-108.

Zelazo, P.D., Craik, F.I.M., & Booth, L. (2004). Executive function across the life span. *Acta Psychologica, 115*, 167-183.

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Rights of participants

Guidelines for testing human research subjects were followed as certified by the Johns Hopkins University Institutional Review Boards. Participants' rights were protected throughout.

Table 1

Description of participants

Group	Mean Age (yrs; months)	Range	Sex (M, F)
3-Years	3; 9	3; 4 – 3; 11	10, 6
4-Years	4; 4	4; 0 – 4; 11	10, 6
5-Years	5; 5	5; 0 – 5; 9	9, 7
6-Years	6; 1	6; 0 – 8; 3	10, 6
Adults	20; 2	18; 9 – 32; 3	6, 10

Table 2

Objects used in Experiment 1, reported according to the label used

Bagels	Chairs	Hats	Shovels
Balls	Cherries	Hearts	Squirrels
Bananas	Cookies	Keys	Strawberries
Bicycles	Cows	Ladybugs	Teddy Bears
Blocks	Crayons	Leaves	Tractors
Boots	Cupcakes	Pieces of Pizza	Trains
Bottles	Doggies	Pigs	Trucks
Bows	Dollies	Rings	Umbrellas
Buckets	Donkeys	Sailboats	Violins
Bunnies	Elephants	Sandwiches	Wagons
Butterflies	Flags	School Busses	
Cars	Hammers	Seashells	

Table 3

Controls for continuous extent on Area Anti-Correlated trials

Whole-Number Ratio	Numeric Ratio (N2/N1)	Total Perimeter	Ratio of Total Area (N1/N2)
1:2	2	equated	2
2:3	1.5	equated	1.5
3:4	1.33	equated	1.33
4:5	1.25	equated	1.25
5:6	1.2	equated	1.2
6:7	1.16	equated	1.16
7:8	1.14	equated	1.14
8:9	1.13	equated	1.13
9:10	1.11	equated	1.11

Table 4

Percent correct compared to chance (50%) across all trials for each age group

	Area Correlated			Area Anti-Correlated		
Age Group	Percent Correct Mean (SE)	*t* df=15	*p* <	Percent Correct Mean (SE)	*t* df=15	*p* <
3-Years	61.7% (3.1)	3.741	.005	60.1% (3.2)	3.178	.01
4-Years	64.4% (3.0)	4.838	.001	58.5% (2.6)	3.256	.005
5-Years	70.3% (2.7)	7.525	.001	63.7% (3.8)	3.596	.005
6-Years	79.8% (3.5)	8.537	.001	76.1% (2.0)	13.280	.001
Adults	87.7% (1.2)	32.039	.001	87.2% (1.2)	31.099	.001

Table 5

Estimated Weber Fraction (*w*)

Age Group	Psychophysics Model			Sigmoid Model		
	w	R^2	Nearest Whole Number Fraction	*w*	R^2	Nearest Whole Number Fraction
3-Years	.525	.502	2:3	.333	.829	3:4
4-Years	.383	.632	3:4	.240	.976	4:5
5-Years	.229	.785	4:5	.225	.938	5:6
6-Years	.179	.846	6:7	.199	.940	5:6
Adults	.108	.926	9:10	.097	.988	10:11

Note. R^2 values represent the agreement between the modeled-fit and the data for the entire function, see Figure 3.

Figure Captions

Figure 1. Sample trial from the "Who Has More" task.

Figure 2. Scatterplots of percent correct (±SE) on the Who Has More task as a function of ratio and trial type for each group (chance = 50%). (a) adults, (b) 6-year-olds, (c) 5-year-olds, (d) 4-year-olds, and (e) 3-year-olds.

Figure 3. Scatterplots of percent correct (±SE) on the Who Has More task, combined across trial type, as a function of ratio, presented with fits from a psychophysics model and a sigmoid model. (a) adults, (b) 6-year-olds, (c) 5-year-olds, (d) 4-year-olds, and (e) 3-year-olds.

Figure 4. Scatterplot of the estimated Weber Fraction for each age group from both the psychophysics model and the sigmoid model, combined with two estimates from the literature on infant Weber Fraction displayed as a function of age. The estimated developmental curves are logarithmic least squares fits.

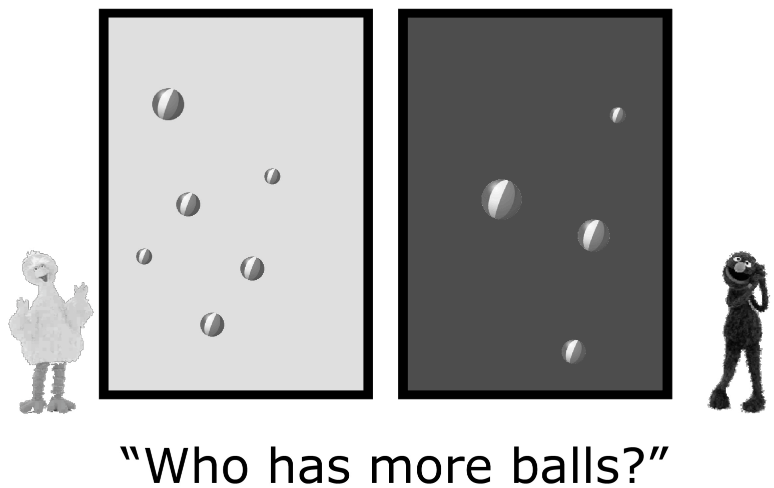
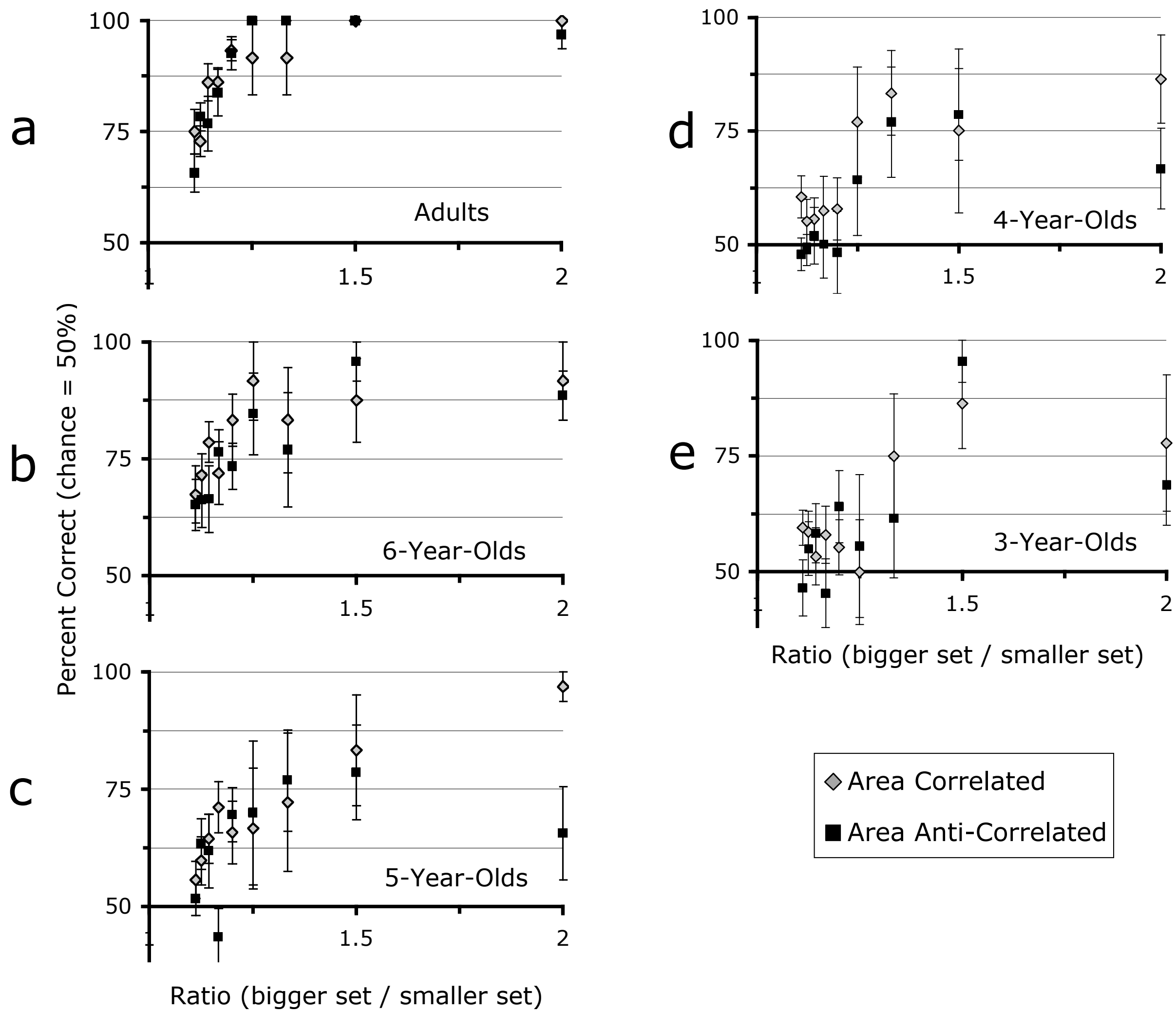


Figure 1



a
Adults
b
6-Year-Olds
c
5-Year-Olds
d
4-Year-Olds
e
3-Year-Olds
Percent Correct (chance = 50%)
Ratio (bigger set / smaller set)
Area Correlated
Area Anti-Correlated

Figure 2

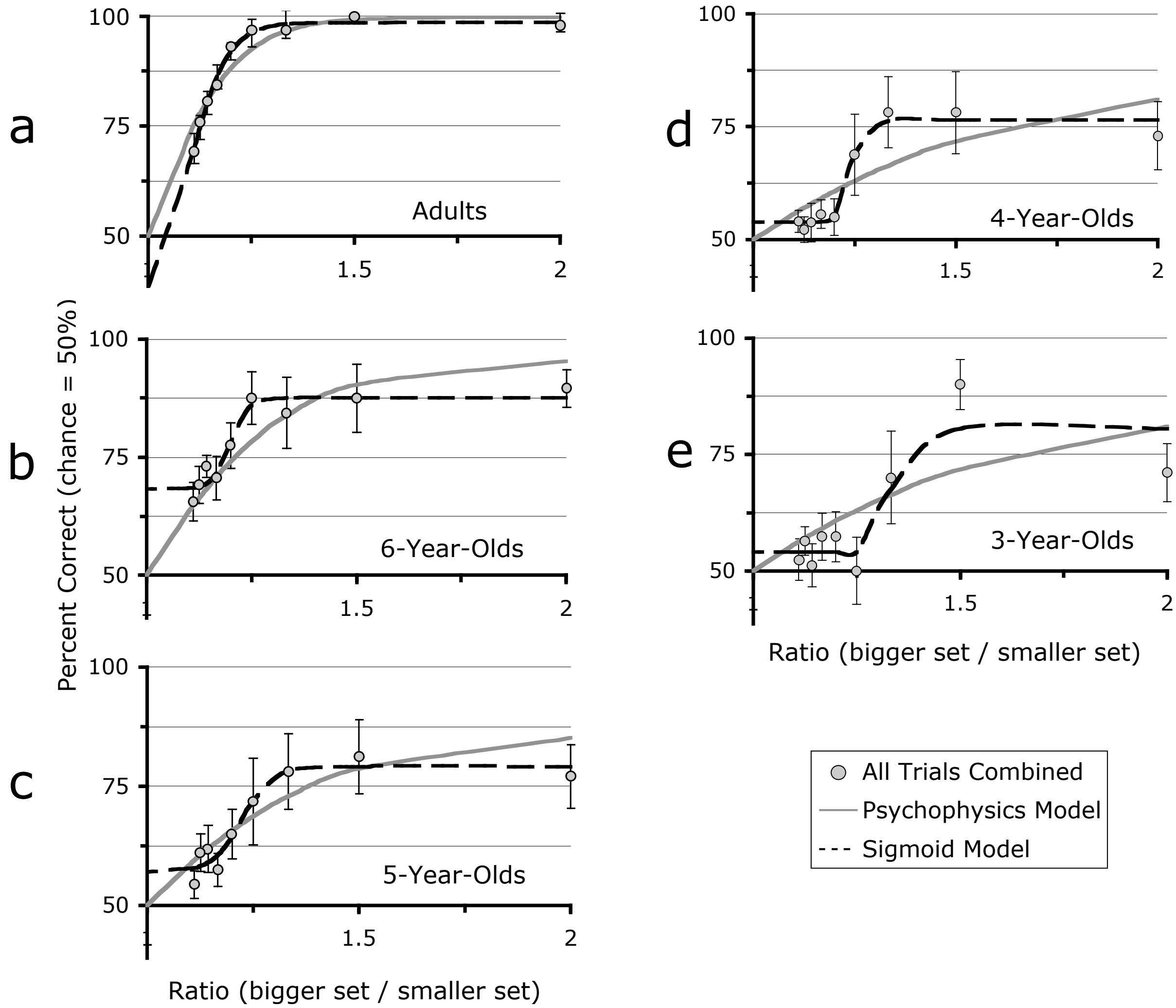
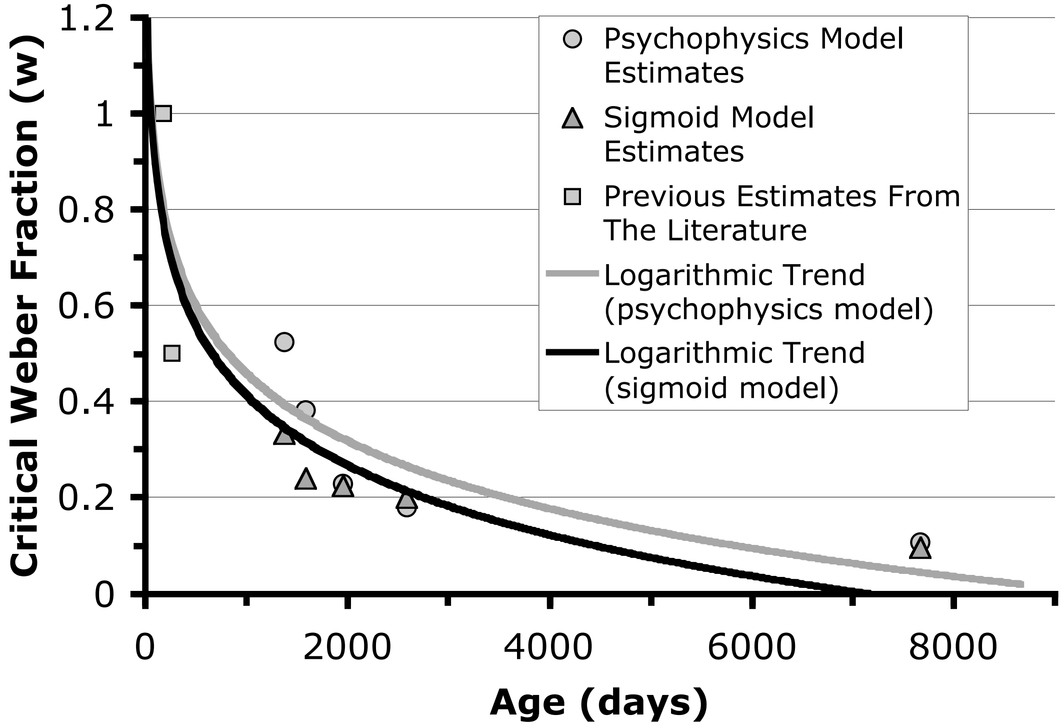

a
b
c
d
e
Percent Correct (chance = 50%)
100
75
50
1.5
2
Adults
6-Year-Olds
5-Year-Olds
4-Year-Olds
3-Year-Olds
Ratio (bigger set / smaller set)
All Trials Combined
Psychophysics Model
Sigmoid Model

Figure 3



Critical Weber Fraction (w)
1.2
1
0.8
0.6
0.4
0.2
0
0
2000
4000
6000
8000
Age (days)
Psychophysics Model Estimates
Sigmoid Model Estimates
Previous Estimates From The Literature
Logarithmic Trend (psychophysics model)
Logarithmic Trend (sigmoid model)

Figure 4