Cournot Competition with Idiosyncratic Taxation

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1 Model Definition

We simulate an economy of buyers and sellers as an extension of the Cournot competition model, with additional buyer-seller specific ('idiosyncratic') transaction cost ('tax'). The goal of this tax is to avoid market dominance by one single seller based on initially only small differences in quality. Instead, the model is aimed at striking a 'fair' balance between market competition and seller protection.

The 'geography' of our model is one-dimensional with periodic boundaries, i.e. a topological circle, of length L. Due to the periodic boundary conditions, we measure physical distances with the metric $\min(|x-y|, L-|y-x|)$ for any two points ('agents') x,y on L. Without loss of generality, we set $L=1$. Our model does not rely on the implemented geography, and any other, potentially more realistic topology could be considered.

1.1 Buyers

On that circle we place some $|B|$ buyers randomly distributed. The buyers all have an identical demand curve $Q+P=a$, and each buyer $b \in B$ buys a quantity $\vec{b}$, where b_i is the amount b buys from the ith seller. Each buyer tries to buy as much total quantity as they can while keeping $P+Q<a$ i.e. they try to maximize $\sum_{i=0}^{|S|} b_i$.

1.2 Sellers

On that circle we will also find some $|S|$ sellers, who all offer the same product, but at potentially different prices and in potentially different quantities. We denote the prices that are offered by a vector $\vec{p}$ and the quantities by the vector $\vec{q}$, where the ith seller offers q_i amount of the product at a price p_i. Each seller chooses q_i and p_i simultaneously in an attempt to maximize profit.

1.3 Idiosyncratic tax

For any buyer b and seller s, we define a product-distance $D = D(b,s)$ such that the seller s that is physically closest to b has distance $D(b,s) = 1$, the next closest has distance $D=2$, and so on.

If buyer b buys from seller s, the total price, including transaction cost (tax), is given by $p(b,s) = p_s \cdot D(b,s)^\gamma$ where γ is a scaling coefficient.

1.4 Profit Function

The profit function for seller i reads

$$\Pi_i = p_i q_i^s - c q_i, \tag{1}$$

where c is the price to produce each unit of product, and q_i^s is the quantity the ith seller sells (which may be less but never more than the quantity q_i that was produced). Below, we then introduce the profit vector $\vec{\pi} = (\pi_1, \ldots, \pi_{|S|})$ to find the Nash-equilibrium based on the profit functions of all sellers.

2 Mathematical Representation

We construct a set of buyers B and a set of sellers S, and transaction cost function $t : (B \times S) \rightarrow \mathbb{R}$, which has subfunctions $t_b : S \rightarrow \mathbb{R}$ s.t. for all $s \in S$, $t(b, s) = t_b(s)$. I don't see what are these sub-functions for. The way I see it, a specific customer-seller tax is a mapping from $B \times S$ to $\mathbb{R}$. There is no subfunction.

Also, we construct an demand curve for all $b \in B$, namely $P = a - Q$.

3 Function to put into Gambit

We construct a function $\Pi : (\mathbb{R} \times \mathbb{R})^{|S|} \rightarrow \mathbb{R}^{|S|}$ s.t. $\Pi(\vec{s}) = \vec{\pi}$, where the elements of $\vec{s}$ are the tuples (q_i, p_i) listing the quantity produced and the price offered by the ith seller, and the elements of $\vec{\pi}$ are π_i, the profits of the ith seller. Write from seller s, here and everywhere else.

Given $\vec{s}$, $\vec{\pi}$ is calculated by first calculating $\vec{q}^s$, the vector of total quantity sold, then using it to calculate $\vec{\pi}$ using equation (1).

(don't start a sentence like that) $q_i^s = \sum_{b \in B} b_i$, where b_i is the amount seller b buys from the ith seller. Turn that thing into vectorial notation. You have two matrices, Q and Q^s, and the same for prices. Then you have Q_{sb} instead of Q_{ji}

Each buyer b chooses a vector $\vec{b}$, where each element b_i is the amount b bought from the ith buyer.

Set $\vec{q}^{unsold} = \vec{q}$

For each $b_i \in \vec{b}$, have it maximize the total quantity bought by b while remaining under the individual demand curve $P = a - Q$. The resulting vector that lists the amount b buys from each seller is $\vec{b}$. Subtract $\vec{b}$ from $\vec{q}^{unsold}$ to get a new $\vec{q}^{unsold}$.

3.1 Algorithm to determining what a buyer purchases

To determine $\vec{b}$, do the following: Use a proper enumeration environment

1. bla
2. bla

Calculate b's perceived prices, $t_b(s_i) * p_i = \vec{p}^b$, then turn it into a $|S|$ by 2 matrix M with the first column being the price, $\vec{p}$ the second column being the remaining quantity, $\vec{q}^{unsold}$.

Sort M's rows by the perceived price column, and re-index the rows
$0, 1, \cdots, j, \cdots, |S|$.

Find the maximal index j and quantity $q_j^* \leq q_j^{unsold}$ such that $p_j + q_j^* + \sum_{k=0}^{j} q_k \leq a$. Now we have a vector of amount bought by b as follows: $< q_0, q_1, \cdots q_{j-1}, q_j^*, 0, 0, \cdots, 0 >$.

Finally, re-index this new vector using the i-index instead of the j index to get $\vec{b}$.

4 Open Questions

- With this model, we want to demonstrate that initially small differences in quality do not lead to proportional feedback mechanisms that lead to an unequal output distribution So it might be questionable if the current Cournot model is the model of our interests, because it assumes all buyers set the same price.