Cournot Competition with Idiosyncratic Tax

Nate Dwyer and Sandro Lera

February 4, 2019

1 Model Definition

We simulate a basic economy of buyers and sellers by basing it on the Cournot competition model.

The 'geography' of our model is one-dimensional with periodic boundaries, i.e. a topological circle. We denote the length of that circle by L. We measure physical distances with the metric $min(|x-y|, L-|y-x|)$ for any two points ('agents') x,y on L. Without loss of generality, we set $L = 1$.

1.1 Buyers

On that circle we place some $|B|$ buyers randomly distributed. The buyers all have an identical demand curve $P = a - Q$, and each buyer $b \in B$ buys a quantity $\vec{b}$, where b_i is the amount b buys from the ith seller. Each buyer tries to buy as much total quantity as they can while staying under the curve, i.e. they try to maximize $sum(\vec{b})$.

1.2 Sellers

On that circle we will also find some $|S|$ sellers, who all offer the same product, but at potentially different prices and in potentially different quantities. We denote the prices that are offered by a vector $\vec{p}$ and the quantities by the vector $\vec{q}$, where the ith seller offers q_i amount of the product at a price p_i. Each seller chooses q_i and p_i simultaneously in an attempt to maximize profit.

1.3 Idiosyncratic tax

For any buyer b and seller s, we define a product-distance $D = D(b, s)$ such that the seller s that is physically closest to b has distance $D(b, s) = 1$, the next closest has distance $D = 2$, and so on.

If buyer b buys from seller s, the total price, including transaction cost (tax), is given by $p(b, s) = p_s * D(b, s)^\gamma$ where gamma is a scaling coefficient.

1.4 Profit Function

The profit function is as follows: $\Pi_i = p_i * q_i^s - c * q_i$, where c is the price to produce each unit of product, and q_i^s is the quantity the ith seller sells.

2 Setup

We construct a set of buyers B and a set of sellers S, and transaction cost function $t : (B \times S) \to \mathbb{R}$, which has subfunctions $t_b : S \to \mathbb{R}$ s.t. for all $s \in S$, $t(b, s) = t_b(s)$.

Also, we construct an demand curve for all $b \in B$, namely $P = a - Q$.

3 Function to put into nashpy

We construct a function $\Pi : (\mathbb{R} \times \mathbb{R})^{|S|} \to \mathbb{R}^{|S|}$ s.t. $\Pi(\vec{s}) = \vec{\pi}$, where the elements of $\vec{s}$ are the tuples (q_i, p_i) listing the quantity produced and the price offered by the ith seller, and the elements of $\vec{\pi}$ are π_i, the profits of the ith seller.

Given $\vec{s}$, $\vec{\pi}$ is calculated by first calculating $\vec{q}^s$, the vector of total quantity sold, then using it to calculate $\vec{\pi}$ using the formula $\pi_i = p_i q_i^s$.

$q_i^s = \sum_{b \in B} b_i$, where b_i is the amount seller b buys from the ith seller.

Each buyer b chooses a vector $\vec{b}$, where each element of the vector b_i is the amount b bought from the ith buyer.

Set $\vec{q}^{unsold} = \vec{q}$

For each $b_i \in \vec{b}$, have it maximize the total quantity bought by b while remaining under the individual demand curve $P = a - Q$. The resulting vector that lists the amount b buys from each seller is $\vec{b}$. Subtract $\vec{b}$ from $\vec{q}^{unsold}$ to get a new $\vec{q}^{unsold}$.

3.1 Algorithm to determining what a buyer purchases

To determine $\vec{b}$, do the following:

Calculate b's perceived prices, $t_b(s_i) * p_i = \vec{p}^b$, then turn it into a $|S|$ by 2 matrix M with the first column being the price, $\vec{p}$ the second column being the remaining quantity, $\vec{q}^{unsold}$.

Sort M's rows by the perceived price column, and re-index the rows
$0, 1, \cdots, j, \cdots, |S|$.

Find the maximal index j and quantity $q_j^* \leq q_j^{unsold}$ such that $p_j + q_j^* + \sum_{k=0}^{j} q_k \leq a$. Now we have a vector of amount bought by b as follows: $< q_0, q_1, \cdots q_{j-1}, q_j^*, 0, 0, \cdots, 0 >$.

Finally, re-index this new vector using the i-index instead of the j index to get $\vec{b}$.