Computation Theory (CS 170), Summer 2023
Assignment 02; due 11:59 PM Eastern, Wednesday 14th June 2023

Answer each of the following three questions to the best of your ability. **Make sure that your submission follows the formatting guidelines given at the end of this document.**

You can find instructions for how to submit your work to Gradescope on the class website:

https://canvas.tufts.edu/courses/44798/pages/how-to-submit-homework

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[1] (*5 pts.*) Consider the following language on binary alphabet $\Sigma = \{\texttt{0}, \texttt{1}\}$:

$$L = \{w = \texttt{0}^i\texttt{1}^j \mid (j - i) \text{ is a non-negative even number}\}$$

Examples of strings in L are ε, `0111`, `00011111`, etc. Note that $\varepsilon \in L$ since $\varepsilon = \texttt{0}^0\texttt{1}^0$, and $(0-0) = 0$, a non-negative even number. Also note that any string consisting only of an even number of `1`'s is also a member of L, since if $w = \texttt{1}^j$ and j is even, so is $(j - 0)$.

Prove that this language is non-regular, using the Pumping Lemma.

Proof:

For the sake of contradiction assume L is regular so by the Pumping Lemma some pumping length p exists.

Consider s = $0^p1^p\varepsilon$ L so $|s| = 2p \geq p$

So by the Pumping Lemma we can divide s = xyz such that $|y| \geq 0$ and $|xy| \leq p$

So $y = 0^k$ for some $k > 0$

But $xy^2z = 0^{p+k}1^p = 0^{i+k}1^j$ which would produce a negative number since $(i + k) > j$ so $xy^2z \not\varepsilon L$

So by this contradiction we can conclude L is non-regular

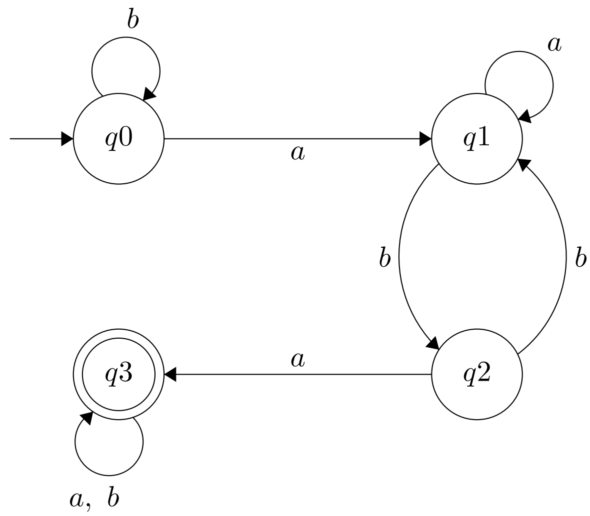
[2] (*5 pts.*) Let the alphabet $\Sigma = \{\texttt{a}, \texttt{b}\}$. Define the two following languages:

$$L_1 = \{\texttt{a}^n\texttt{b}\texttt{a}^n \mid n \geq 1\}$$
$$L_2 = \{w_1\texttt{a}^n\texttt{b}\texttt{a}^n w_2 \mid n \geq 1 \texttt{ and } w_1, w_2 \in \Sigma^\star\}$$

Examples of strings in L_1 include: `aba`, `aabaa`, `aaabaaa`, `aaaabaaaa`, etc.

Examples of strings in L_2 include: `aba`, `baabaab`, `bbaaabaaa`, `abaaaabaaaab`, etc. (since the strings w_1 and w_2 that form the front and back can be any combination of `a`'s and `b`'s).

a. Show that L_2 is regular, by giving a DFA or NFA for it.



b. Show that L_1 is not regular, using the Pumping Lemma to prove it.

Proof:
For the sake of contradiction assume L_1 is regular so by the Pumping Lemma some pumping length p exists.
Consider s $= \mathtt{a}^p b a^p \varepsilon$ L so $|s| = 2p + 1 \geq p$
So by the Pumping Lemma we can divide s = xyz such that $|y| \geq 0$ and $|xy| \leq p$
So $y = a^k$ for some $k > 0$
But $xy^2z = \mathtt{a}^{p+k} b a^p = \mathtt{a}^{n+k} b a^n$ which would be uneven so $xy^2z \not\varepsilon L$
So by this contradiction we can conclude L_1 is non-regular

[3] (*5 pts.*) Let our alphabet be $\Sigma = \{\mathtt{0}, \mathtt{1}, \mathtt{-}, \mathtt{=}\}$, and let our language be the set of strings expressing correct arithmetic differences between binary numbers. (Here, we will define a binary number as any non-empty binary string; leading zeros are allowed. Thus, for example, both `11101` and `0011101` are allowed, as each is a binary number, equal to 29.):

$$L_* = \{w_1\mathtt{=}w_2\mathtt{-}w_3 \mid w_1,\ w_2, \text{ and } w_3 \text{ are binary numbers, and } w_2 \text{ is the sum of } w_1 \text{ and } w_3\}$$

Show that this language is not regular, using the Pumping Lemma to prove it.
Proof:
For the sake of contradiction assume L_* is regular so by the Pumping Lemma some pumping length p exists.
Consider s $= (\mathtt{001}^p = 010 - 001) \varepsilon$ L so $|s| = p + 8 \geq p$
So by the Pumping Lemma we can divide s = xyz such that $|y| \geq 0$ and $|xy| \leq p$
So $y = 001^k$ for some $k > 0$
But $xy^2z = (001^2 = 010 - 001) = (001001 = 010 - 001)$ which is an incorrect expression so $xy^2z \not\varepsilon L_*$
So by this contradiction we can conclude L_* is non-regular

Format requirements: work should correspond to the following guidelines:

- Work must be in type-written format, with any diagrams rendered using software to produce professional-looking results. No hand-written or hand-drawn work will be graded.
- Work must be submitted in PDF format to Gradescope.

You can find links to information about using LaTeX to produce type-written mathematical work,[1] on the class website:

https://canvas.tufts.edu/courses/44798/pages/latex-document-preparation

While you can use any drawing/layout program to produce images, there is a handy web-based tool for drawing finite-state diagrams for DFA/NFA that produces nice results (the image in this assignment was generated using that tool, exporting the code for the image to LaTeX for inclusion):

http://madebyevan.com/fsm/

[1] LaTeX was used to produce this document.