Computation Theory (CS 170), Section 01, Summer 2023
Assignment 01; due 11:59 PM Eastern, Wednesday, 7 June 2023

Answer each of the following three questions to the best of your ability. **Make sure that your submission follows the formatting guidelines given at the end of this document.**

You can find instructions for how to submit your work to Gradescope on the class website:

https://canvas.tufts.edu/courses/48012/pages/how-to-submit-homework

---

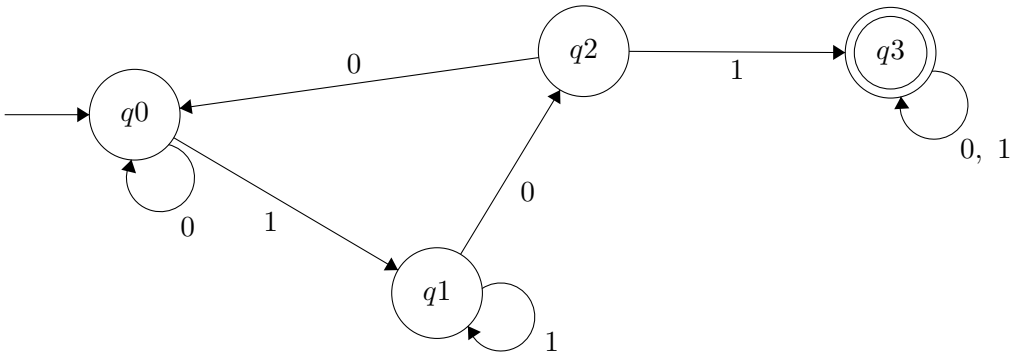
In the following, we will assume (unless otherwise indicated) that we are using alphabet $\Sigma = \{0, 1\}$.

[**1**] (*5 pts.*) Show that the following language is regular by constructing a DFA for it.

$$L_1 = \{w \mid w \text{ contains the substring } 101\}$$

Note: when you answer this question, provide both: (*a*) the state diagram of the DFA, and (*b*) the full formal specification of the machine, as given by Definition 1.5 (Sipser, 3rd ed., p. 35).

(*a*)



(*b*)
$\Sigma = \{0, 1\}$
$Q = \{q0, q1, q2, q3\}$
$q0 = \{q0\}$
$F = \{q3\}$

δ	**0**	**1**
q0	q0	q1
q1	q2	q1
q2	q0	q3
q3	q3	q3

[**2**] (*5 pts.*) The following regular language is formed of a combination of two simpler regular languages. Identify those two languages, and give state diagrams for DFA's that recognize each.

Then, combine those DFA's into one for the more complex language, using the explicit procedure described in the Sipser text (3rd ed., p. 46).

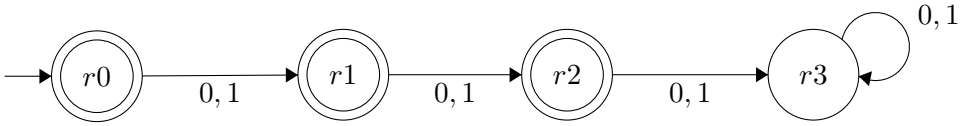
$$L_2 = \{w \mid |w| \leq 2 \text{ and } w \text{ contains at least one } \mathtt{0}\}$$

Note: when you design the two simpler machines, use different letters (like q and r) for the states of each, so it is more obvious how you have combined them into the final machine. Also note that, unlike the prior question, you ***do not*** need a formal specification, only a diagram, for each machine.

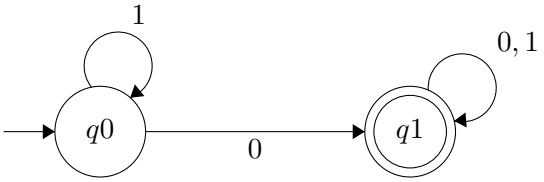
$$A = \{w \mid |w| \leq 2\}$$

$$B = \{w \mid w \text{ contains at least one } \mathtt{0}\}$$

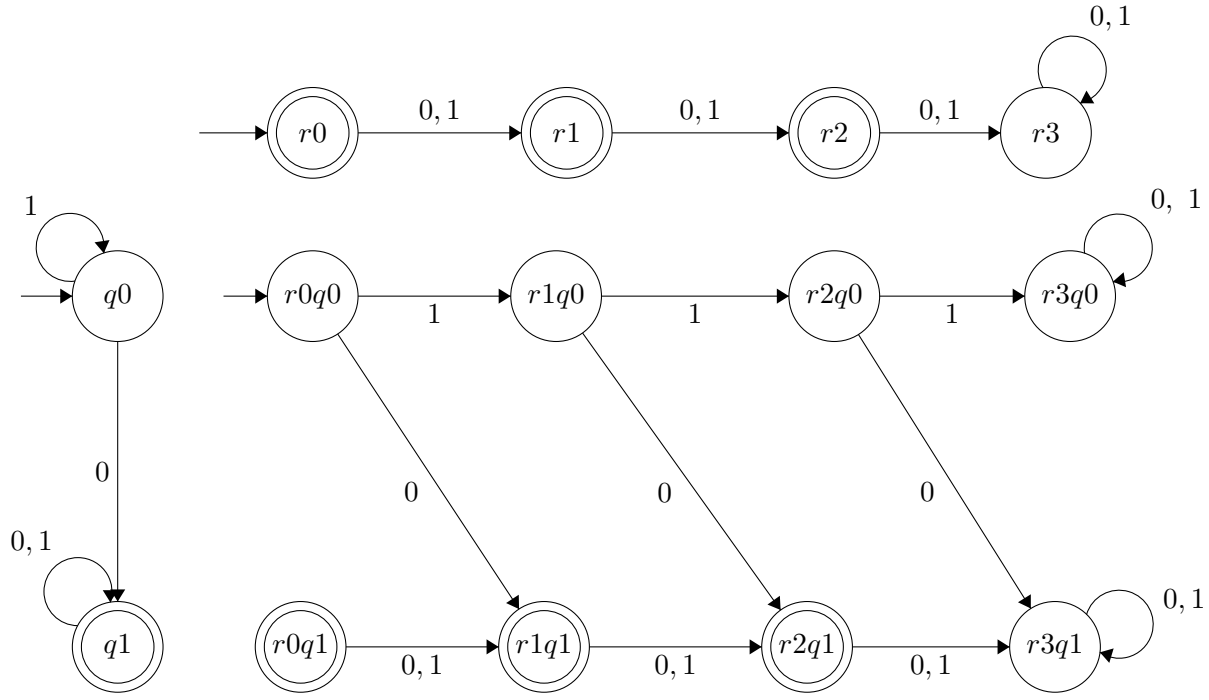
D(A):



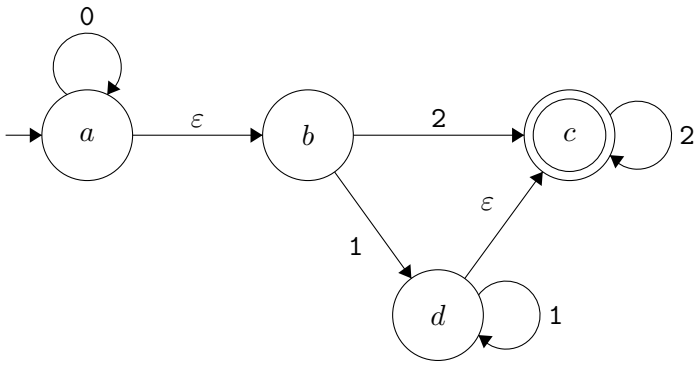
D(B):



D(A ∩ B):



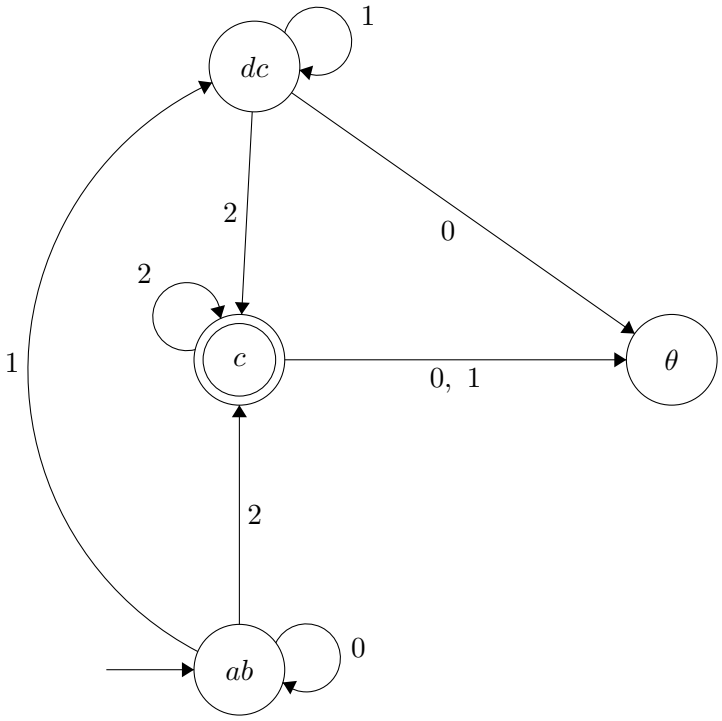
[3] (*5 pts.*) Consider the following NFA, on the 3-character alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}, \mathtt{2}\}$:



Give a precise definition of the language of this machine. Then, use the procedure of Theorem 1.39, (Sipser, 3rd ed., p. 55f) to convert it to a corresponding DFA. Give the final DFA diagram.

Note: here, the DFA can get quite large, and involves a number of unreachable states. For full points, all such states should be ***excluded*** from the diagram, to keep it simpler.

$$L = \{w \mid w \text{ ends with 2 and the only 0's are leading}\}$$



Formatting instructions appear on the following page.

Format requirements: work should correspond to the following guidelines:

- Work must be in type-written format, with any diagrams rendered using software to produce professional-looking results. No hand-written or hand-drawn work will be graded.
- Work must be submitted in PDF format to Gradescope.

You can find links to information about using LaTeX to produce type-written mathematical work,[1] on the class website:

https://canvas.tufts.edu/courses/48012/pages/latex-document-preparation

While you can use any drawing/layout program to produce images, there is a handy web-based tool for drawing finite-state diagrams for DFA/NFA that produces nice results (the image in this assignment was generated using that tool, exporting the code for the image to LaTeX for inclusion):

http://madebyevan.com/fsm/

[1] LaTeX was used to produce this document.