Although the original theory of has been developed for the more general case of vector-valued functions, we restrict this presentation to the case of univariate functions. Let $x(t)$ to be a positive continuous function defined on the unit inverval $\mathbb{I} = [0, 1]$. Let $\|\cdot\|$ be a norm on the function and we can assume $\|x(t)\| = 1$, that is, the function has been normalized by taking $x(t)/\|x(t)\|$. Given some family of approximation methods $\mathcal{F}$, let $\hat{x}(\cdot)$ be an approximation of $x(t)$ reconstructed from regularly samples spaced at intervals h. Then the absolute recover error function is defined

$$\delta^{\mathcal{F}}(h) = \inf_{\hat{x}\in\mathcal{F}} \sup |x(t) - \hat{x}(t)| \tag{0.1}$$

$$h_x^*(\varepsilon, \mathcal{F}) = \begin{cases} \inf\{h \leq 1 : \delta^{\mathcal{F}}(h) > \varepsilon\} & \text{if} \{h : \delta^{\mathcal{F}}(h) > \varepsilon\} > 0 \\ 1, & \text{if no such } \hat{x}(t) \in \mathcal{F} \quad \text{exists.} \end{cases}$$

In words, the function $h^*(\varepsilon, \mathcal{F})$ is the minimum grid spacing h, or sample density, needed to approximate $x(t)$ within ε.

Definition 0.1. Epsilon-complexity. The number

$$\mathbb{S}_x(\varepsilon, \mathcal{F}) = -\log h_x^*(\varepsilon)\mathcal{F}) \tag{0.2}$$

is the $(\varepsilon, \mathcal{F})-$complexity, or simply, $\varepsilon-$complexity of the function $x(\cdot)$.

We will be referring variously to the Hölder condition or Hölder continuity of a function which we now define:

Definition 0.2. Hölder Continuity. A function $x(t)$ is said to be Hölder continuous if there exists non-negative constants, C, α such that

$$|x(t) - x(s)| \leq C \|x - s\|^{\alpha}$$

for all t and s. We refer to this as the Hölder condition with Hölder exponent α or say a function is Hölder α.

Darkhovsky and Pirytinska have show that for almost any individual Hölder class function and given a rich enough family of approximation methods then the epsilon complexity has an approximately linear relationship to $\log \varepsilon$[**?**]

$$-\log h_x^*(\varepsilon, \mathcal{F}) \approx \mathbb{A} + \mathbb{B} \log \varepsilon. \tag{0.3}$$

The relationship serves the basis, after rearrangement, for the estimation of ε-complexity for discrete time series. In practial applications any function or time series is acquired as some discrete set of samples, which we will assume to be taken at regular intervals. We consider this discrete set to be a continuous function restricted to a uniform grid. In this case, $h * (\epsilon, \mathcal{F})$, which we simplify here to $h^*(\epsilon)$, is the proportion of sampled points and $\frac{1}{h^*(\varepsilon)}$ is the number of points needed to approximate some function $x(t)$ within ε. Let n be the total number of points in the initial sampling, then we denote the proportion of points needed for reconstruction with ε

$$S(\cdot) \stackrel{\text{def}}{=} \frac{n}{h^*(\varepsilon)}. \tag{0.4}$$

Definition 0.3. The $\varepsilon-$complexity of a function given represented by a set of values sampled on a uniform grid is $-\log S(\cdot)$.

It follows from defintion 0.1 that

Proposition 0.4.

$$-\log S(\varepsilon, \mathcal{F}) \to \mathbb{S}_x(\varepsilon, \mathcal{F})$$

as $n \to \infty$.

For the discrete case, rearrangement of the relation 0.3 gives and substitution of $\log S$ for $\mathbb{S}_x(\varepsilon, \mathcal{F})$ gives us the relation

$$\log \varepsilon \approx A + B \log S. \tag{0.5}$$

For practical computations, some set number of grid spacings h determines the set $\{S_h\}$ which are used to estimate the linear relation in 0.5. Unlike for the continuous case, we find the minimum errors corresponding to the set $\{S_h\}$. The parameters of this linear relationship, A, B, are the $\varepsilon-$complexity coefficients or simply the complexity coefficients.