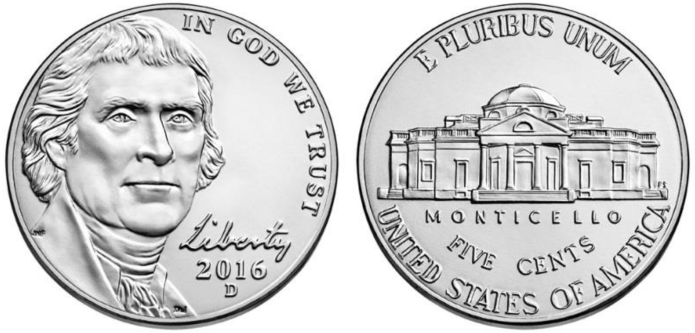
CS4102 Algorithms

Fall 2018

Warm up

Given access to unlimited quantities of pennies, nickels dimes, and quarters, (worth value 1, 5, 10, 25 respectively), provide an algorithm which gives change for a given value x using the fewest number of coins.



Change Making

43 cents

Change Making Algorithm

- Given: target value x, list of coins $C = [c_1, \ldots, c_n]$
  (in this case $C = [1,5,10,25]$)
- Repeatedly select the largest coin less than the remaining target value:

while($x > 0$)
    let $c = \max(c_i \in \{c_1, \ldots, c_n\} \mid c_i \leq x)$
    print c
    $x = x - c$

Why does this always work?

- If $x < 5$, then pennies only
  - 5 pennies can be exchanged for a nickel

Only case Greedy uses pennies!

- If $5 \leq x < 10$ we must have a nickel
  - 2 nickels can be exchanged for a dime

Only case Greedy uses nickels!

- If $10 \leq x < 25$ we must have at least 1 dime
  - 3 dimes can be exchanged for a quarter and a nickel

Only case Greedy uses dimes!

- If $x \geq 25$ we must have at least 1 quarter

Only case Greedy uses quarters!

Warm up 2

Given access to unlimited quantities of pennies, nickels dimes, kims, and quarters, (worth value 1, 5, 10, 11, 25 respectively), give 90 cents change using the fewest number of coins.

Greedy solution

90 cents

Greedy solution

90 cents

Today’s Keywords

- Greedy Algorithms
- Choice Function
- Change Making
- Interval Scheduling
- Exchange Argument

CLRS Readings

- Chapter 16

Homework

- Hw6 Due Friday November 9, 11pm
  - Dynamic Programming and Greedy
  - Written assignment

Greedy vs DP

- Dynamic Programming:
  - Require Optimal Substructure
  - Several choices for which small subproblem
- Greedy:
  - Require Optimal Substructure
  - Must only consider one choice for small subproblem

Greedy Algorithms

- Require Optimal Substructure
  - Solution to larger problem contains the solution to a smaller one
  - Only one subproblem to consider!
- Idea:
  1. Identify a greedy choice property
     - How to make a choice guaranteed to be included in some optimal solution
  2. Repeatedly apply the choice property until no subproblems remain

Change Making Choice Property

- Largest coin less than or equal to target value must be part of some optimal solution (for standard U.S. coins)

Interval Scheduling

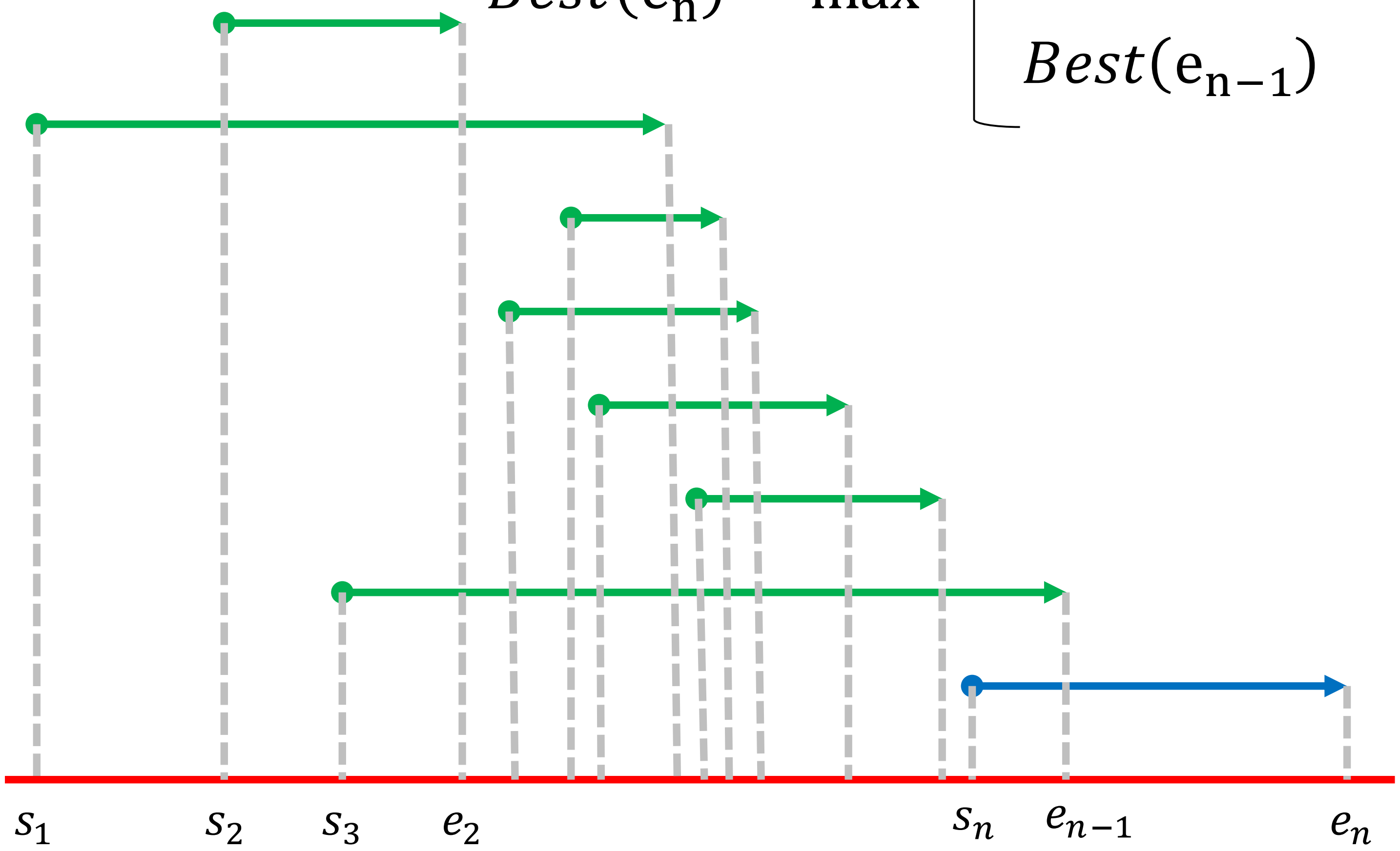
- Input: List of events with their start and end times (sorted by end time)
- Output: largest set of non-conflicting events (start time of each event is after the end time of all preceding events)

[2, 3.25]	CS4102
[1, 4]	Corn Maze Run
[3, 4]	CHS Prom
[3.5, 4.75]	DMB concert
[4, 5.25]	Bingo
[4.5, 6]	SCUBA lessons
[5, 6.5]	Roller Derby
[7, 8]	Pumpkin Carving

Interval Scheduling DP

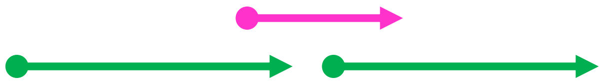
$Best(t)$ = max # events that can be scheduled before time t

$$Best(\mathrm{e_n}) = \max \begin{cases} Best(\mathrm{s_n}) + 1 & \text{Include event } n \\ Best(\mathrm{e_{n-1}}) & \text{Exclude event } n \end{cases}$$

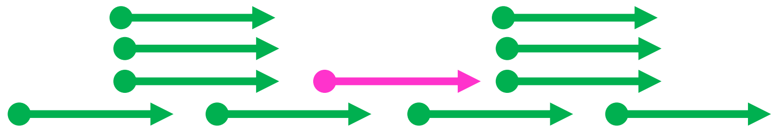


Greedy Interval Scheduling

- Step 1: Identify a greedy choice property
  - Options:
    - Shortest interval



    - Fewest conflicts



    - Earliest start

    - Earliest end

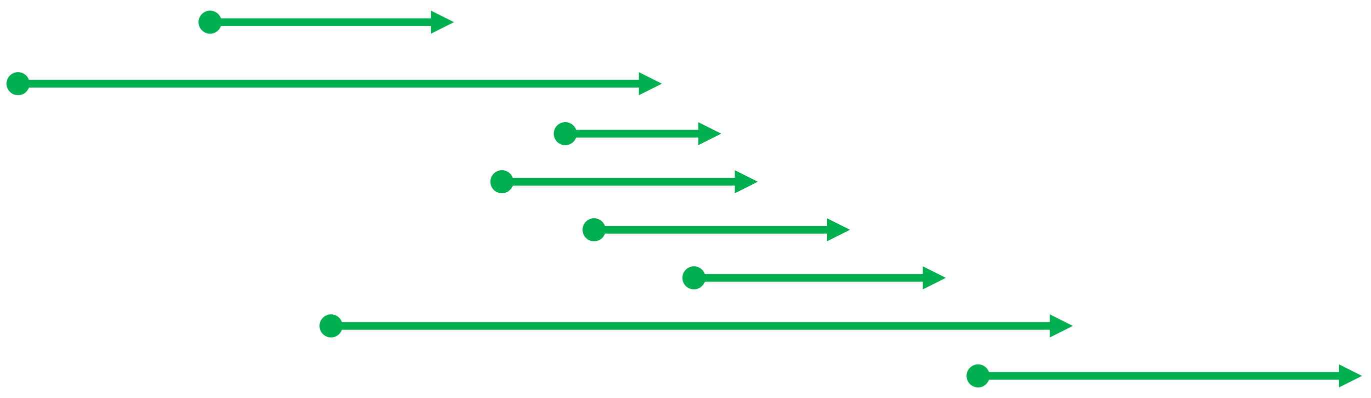
Prove using Exchange Argument

Interval Scheduling Algorithm

Find event ending earliest, add to solution,

Remove it and all conflicting events,

Repeat until all events removed, return solution



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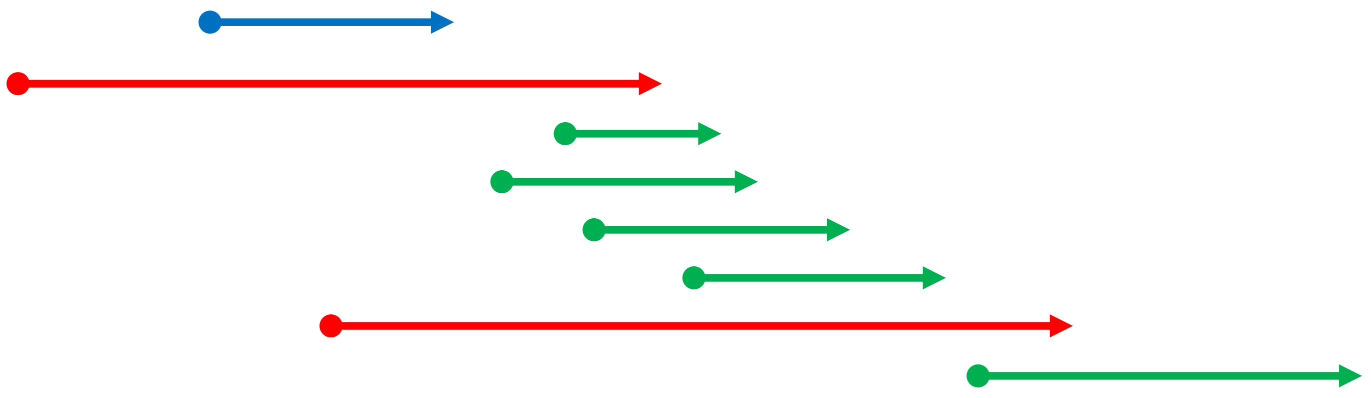

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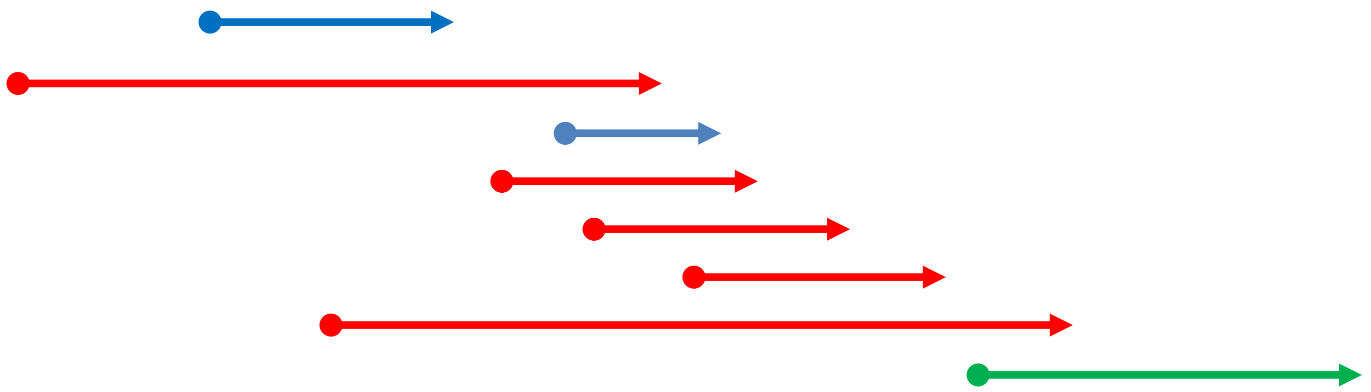

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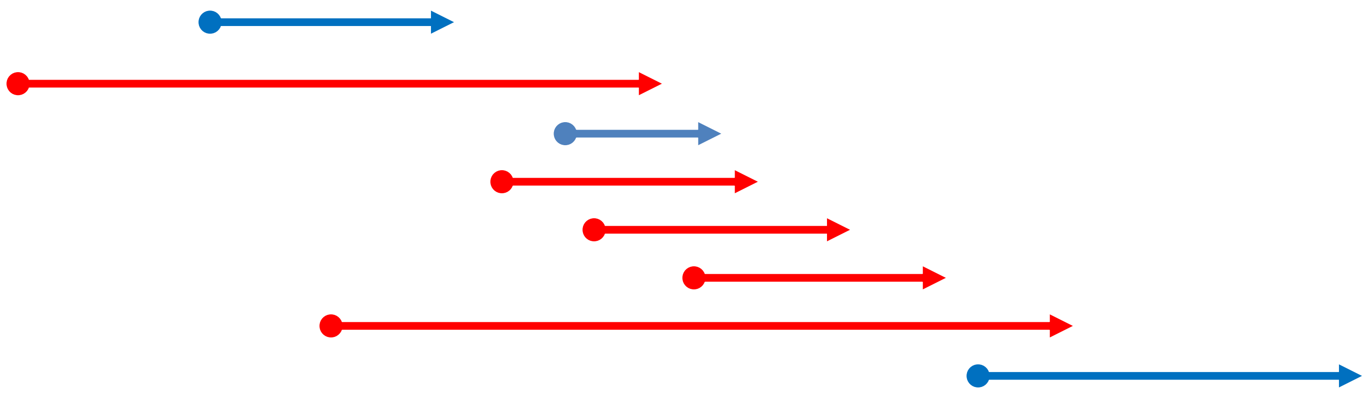

Interval Scheduling Run Time

Find event ending earliest, add to solution,

Remove it and all conflicting events,

Repeat until all events removed, return solution

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Equivalent way

StartTime = 0
For each interval (in order of finish time):          O(n)
        if end of interval < Start Time:          O(1)
                do nothing
        else:
                add interval to solution          O(1)
                StartTime = end of interval
```

Exchange argument

- Shows correctness of a greedy algorithm
- Idea:
  - Show exchanging an item from an arbitrary optimal solution with your greedy choice makes the new solution no worse
  - How to show my sandwich is at least as good as yours:
    - Show: “I can remove any item from your sandwich, and it would be no worse by replacing it with the same item from my sandwich”

Exchange Argument for Earliest End Time

- Claim: earliest ending interval is always part of some optimal solution

- Let $OPT_{i,j}$ be an optimal solution for time range $[i, j]$
- Let a^* be the first interval in $[i, j]$ to finish overall
- If $a^* \in OPT_{i,j}$ then claim holds
- Else if $a^* \notin OPT_{i,j}$, let a be the first interval to end in $OPT_{i,j}$
  – By definition a^* ends before a, and therefore does not conflict with any other events in $OPT_{i,j}$
  – Therefore $OPT_{i,j} - \{a\} + \{a^*\}$ is also an optimal solution
  – Thus claim holds

Next Time: Caching Problem

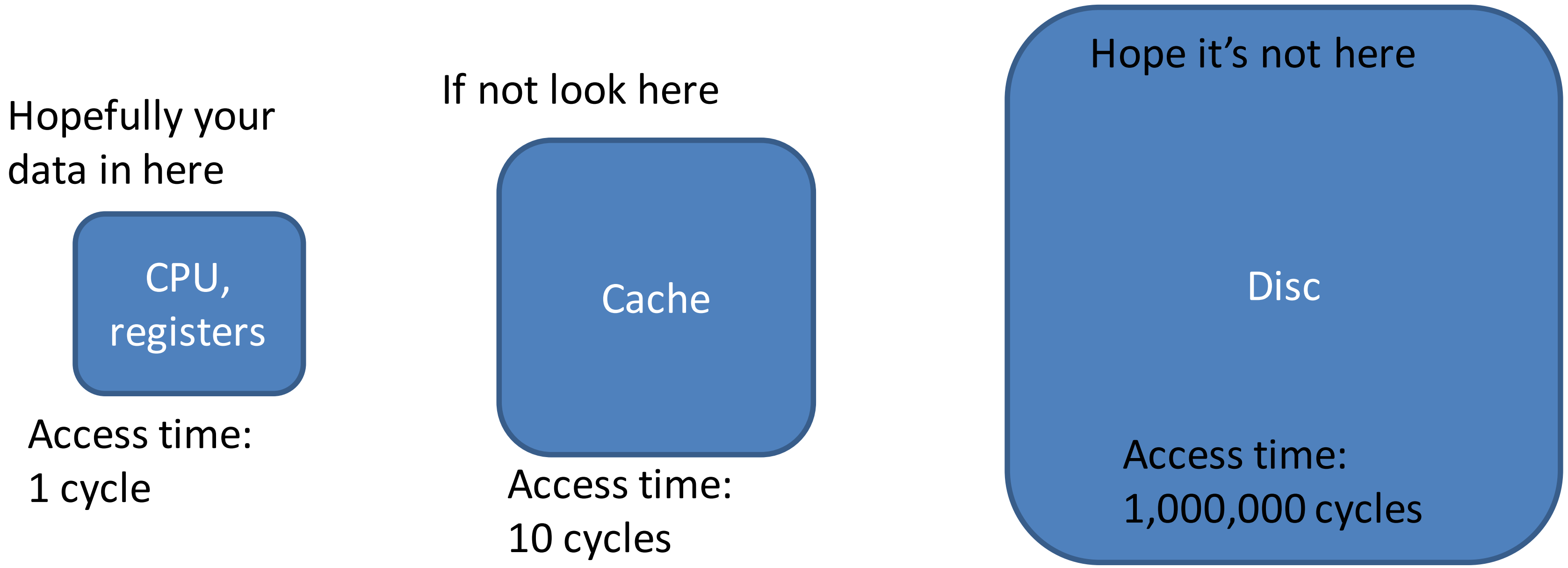
- Why is using too much memory a bad thing?

Von Neumann Bottleneck

- Named for John von Neumann
- Inventor of modern computer architecture
- Other notable influences include:
  - Mathematics
  - Physics
  - Economics
  - Computer Science

Von Neumann Bottleneck

- Reading from memory is VERY slow
- Big memory = slow memory
- Solution: hierarchical memory
- Takeaway for Algorithms: Memory is time, more memory is a lot more time



Exchange Argument for (U.S. coin) Change Making

- Claim: largest coin is always part of some optimal solution

- Consider that we had an optimal solution for target value x, call this OPT_x
- Let c^* be the largest coin in OPT_x
- If $c^* \in OPT_x$ then claim holds
- Else if $c^* \notin OPT_x$, let c be the largest coin in OPT_x
  – Using slide 4, we could exchange c for a combination of coins including c^* to have a solution that is no worse