Practice Quiz 1

Autumn 2025

Name ______________________________

Net ID ____________________ (@uw.edu)

Academic Integrity: You may not use any resources on this quiz except for your one-page (front and back) reference sheet, writing instruments, your own brain, and the exam packet itself. This quiz is otherwise closed notes, closed neighbor, closed electronic devices, etc.. The last two pages of this exam serve as a reference sheet as well as scratch work (respectively). Please detach those last two pages from the exam packet. No markings on these last two pages will be graded. Your answer for each question must fit in the answer box provided.

Instructions: Before you begin, **Put your name and UW Net ID at the top of this page.** Make sure that your name and ID are LEGIBLE. Please ensure that all of your answers appear within the boxed area provided.

(1 ESNU)**Question 1: Valley Finder - Divide and Conquer**

For this problem, you will write an algorithm that takes as input an array of doubles that's length n where $n > 4$, where this array has the following properties:

– index 0 of the array contains the value 0

– index 1 of the array contains the value -1

– index $n-1$ of the array contains the value 0

– index $n-2$ of the array contains the value -1

– no two consecutive indices of the array contain the same value.

Your goal is to write an algorithm FindValley that outputs an index i such that i is a local minimum (meaning that the value at index $i-1$ is smaller than the value at index i, which is larger than the value at index $i+1$).

Note that the properties of the input array guarantee that there is a local minimum at some index.

a) Design an algorithm for FindValley that performs only $O(\log n)$ comparison operations (i.e. a $>, <, \geq, \leq$ operation on elements in the array).

b) Write out the recurrence that describes its number of comparisons and indicate how that yields the required bound on the number of tests.

(1 ESNU)**Question 2: Asymptotic Analysis**

a) Give a value for n_0 and c that shows $3n^3 - 2n$ belongs to $O(n^3)$. Justify why it works. (Hint: see the definition of O in the reference sheet at the end of the quiz.)

b) What is the running time of a recursive algorithm for a problem that reduces the problem on inputs of size n to 8 subproblems of the same type of size $n/2$ plus $\Theta(n^3)$ nonrecursive work? First express the running time as a recurrence relation, then give an asymptotic bound using the master theorem.

(1 ESNU)**Question 3: Algorithm Correctness**

In this problem you will use loop invariants to show that that the following algorithm correctly finds the sum of the largest two elements in a list.

```
sumLargestTwo(list):
    if(list.length == 0):
        return 0
    if(list.length == 1):
        return list[0]
    temp0 = list[0]
    temp1 = list[1]
    first = max(temp0, temp1)
    second = min(temp0, temp1)
    for(i = 0; i < list.length; i++):
        if(list[i]>first):
            second = first
            first = list[i]
        else if(list[i]>second):
            second = list[i]
    return first + second
```

a) Provide the loop invariant that you will use to demonstrate correctness

b) Show that your loop invariant holds

c) Show that your loop invariant ensures that the algorithm returns the correct value.

d) What other two things need to be shown in order to guarantee algorithm correctness (you do not need to prove these, just name or describe them)?

(1 ESNU)**Question 4: Stable Matchings**

Consider the preference lists below for groups A and B.

Group A	Group B
a_1 :b_1, b_2, b_3, b_4	b_1 :a_2, a_3, a_1, a_4
a_2 :b_2, b_3, b_1, b_4	b_2 :a_3, a_1, a_2, a_4
a_3 :b_3, b_1, b_2, b_4	b_3 :a_1, a_3, a_2, a_4
a_4 :b_1, b_2, b_4, b_3	b_4 :a_1, a_2, a_3, a_4

1. Give the stable matching produced by the Gale-Shapley algorithm when group A are the proposers.

2. Give the stable matching produced by the Gale-Shapley algorithm when group B are the proposers.

3. Justify that EVERY stable matching matches a_4 with b_4. (Hint: consider the proposer optimality and receiver pessimality properties of Gale-Shapley.)

Reference

Nothing written on this page will be graded.

Logs

$$x^{\log_x(n)} = n$$
$$\log_a(b^c) = c\log_a(b)$$
$$a^{\log_b(c)} = c^{log_b(a)}$$
$$\log_b(a) = \frac{\log_d(a)}{\log_d(b)}$$

Asymptotic Notation

$f(n)$ is $O(g(n))$ provided that after some input size n_0, $f(n) \geq c \cdot g(n)$ for some constant c.

$f(n)$ is $\Omega(g(n))$ provided that after some input size n_0, $f(n) \leq c \cdot g(n)$ for some constant c.

$f(n)$ is $\Theta(g(n))$ provided that $f(n)$ is $O(g(n))$ and $f(n)$ is $\Omega(g(n))$

Master Theorem

Suppose that $T(n) = aT(\frac{n}{b}) + O(n^k)$ for $n > b$. Then:

– if $a < b^k$ then $T(n)$ is $O(n^k)$

– if $a = b^k$ then $T(n)$ is $O(n^k \log n)$

– if $a > b^k$ then $T(n)$ is $O(n^{\log_b a})$

Proposer Optimality / Receiver Pessimality

A pair (p, r) is a valid pair if there is some stable matching where they are matched together

Proposer Optimality: Every proposer is matched with their most preferred valid pair.

Receiver Pessimality: Every receiver is matched with their least preferred valid pair.

Scratch Work

Nothing written on this page will be graded.