Quiz 1: Induction and Uncountability

Collaboration: This quiz is open note but individual. You may use any resources that were provided to you by the course, or that you had recorded as part of your own notes prior to when you first viewed this quiz. You may not discuss this quiz with your cohort-mates, class-mates, tutor, friends, family, or anyone else except the course staff (who will answer clarification questions only).

Problem 1: Induction

Prove using induction that the number of binary strings of length n that start with a 1 is 2^{n-1}.

Problem 2: Diagonalization

Using a proof by diagonalization, prove that the following set is uncountable:

$$F = \left\{ f : \mathbb{Z}^+ \to \{0,1\}^* \middle| \forall n \in \mathbb{Z}^+ . f(n) \in \{0,1\}^n \right\}$$

In other words, this is the set of all functions that map positive integers to strings of that length. For example, if $f(3) = 101$ then f may be in F since the length of 101 is 3 (we would need for all the infinitely many other values of $\mathbb{Z}^+$ to also map to strings of the right length before concluding for certain it was a member of F). If $f(3) = 0100$ then f is not a member of F since the length of $0100 \neq 3$).