PS3: Gates and Straightline

Collaboration: You should work on the problems yourself, before discussing with others, including your cohorts at your cohort meeting. In addition to discussing with your cohortmates, you may discuss the problems with any other current CS3102 students you want, and use any resources you want except for any materials from previous offerings of this course or complete solutions that might be available on the web, which are not permitted. Sharing and subsequently submitting any text, code, images, figures, etc. constitutes plagiarism, so make sure all submitted materials are created exclusively by members of your cohort.

Problem 1: Straightline Programming

For this problem, you will be asked to implement several straightline programs in Python. You must adhere to the rules of straightline programs. In particular, make sure:

– Variables contain only bits.

– Function parameters are only the characters `'0'` and `'1'`.

– Each line may only have a single function and a variable assignment (no nested functions, no conditionals, no loops).

– Variable names may not be reused within the same function.

We provide test cases for all functions. If you followed the above rules, and you pass all tests, you can be confident that your implementation is correct.

To complete this problem, download the straightline.py program. The comments in that file guide you through the assignment, I recommend reading through all the comments linearly. Everything you must implement is marked with a comment containing "`TODO`". Specifically, you will be implementing three functions: IMPL, XOR_nand, and COMP_2.

Problem 2: *MAJ,NOT,1* equals *AON* (based on Exercise 3.6 in TCS book)

Let $MAJ : \{0,1\}^3 \to \{0,1\}$ and $NOT : \{0,1\} \to \{0,1\}$ be defined as in lecture. Let $1 : \{0,1\}^0 \to \{0,1\}$ simply give the constant value *1* Prove that $\{MAJ, NOT, 1\}$ is equivalent to *AND, OR, NOT*. In other words, show that any function that can be computed by *AND,OR,NOT* can also be computed using combinations of $\{MAJ, NOT, 1\}$, and vice-versa.

Problem 3: *NOR* is equivalent to *AND,OR,NOT* (Exercise 3.7 in TCS book)

Let $NOR : \{0,1\}^2 \to \{0,1\}$ defined as $NOR(a,b) = NOT(OR(a,b))$. Prove that circuits using just the gate $\{NOR\}$ is a equivalent to those using the set of gates *AND,OR,NOT*.

Problem 4: XOR is not equivalent to AON (based on Exercise 3.5 in TCS book)

Prove that circuits using the gate set $\{XOR, 0, 1\}$ is not equivalent to $\{AND, OR, NOT\}$. In particular, show that there are functions that can be computed using $\{AND, OR, NOT\}$ that cannot be computed using $\{XOR, 0, 1\}$. (*Hint: if we could compute all of AND, OR, and NOT using XOR,0,1 then they would be equivalent.*)

