Retake of Quiz 6: Tractability

Collaboration: This quiz is open note but individual. You may use any resources that were provided to you by the course, or that you had recorded as part of your own notes prior to when you first viewed this quiz. You may not discuss this quiz with your cohort-mates, class-mates, tutor, friends, family, or anyone else except the course staff (who will answer clarification questions only).

Problem 1: P vs NP

Suppose that we have problems A, B and C where A reduces to B in polynomial time and B reduces to C in polynomial time.

For each of the following scenarios for the complexity class memberships of A, B, and C indicate whether we could conclude:

– $\mathsf{P} = \mathsf{NP}$,

– $\mathsf{P} \subset \mathsf{NP}$,

– or *Inconclusive* (meaning that scenario alone does not resolve the question).

Select one option and provide a short justification to support your answer.

a) $A \in \mathsf{NP}$, $B \in \mathsf{P}$, and $C \in \mathsf{NP\text{-}Hard}$

b) $A \in \mathsf{NP\text{-}Hard}$, $B \in \mathsf{P}$, and $C \in \mathsf{NP}$

c) $A \in \mathsf{NP\text{-}Hard}$, $B \in \mathsf{EXP}$, and $C \in \mathsf{P}$

Problem 2: MajSAT

Recall that 3-SAT is in NP-Hard, where the 3-SAT problem is to determine whether there exists at least one way to assign Boolean values to each variable in a 3-CNF formula so that the formula evaluates to True. Since the clauses are combined with AND operations, for the formula to evaluate to True *all* clauses must be satisfied by the assignment.

We will define MajSAT as a "relaxation" of 3-SAT. Instead of requiring that *all* clauses are made True, we just require for a majority (more than half) of the clauses to be made True.

Here are some examples:

The formula: $(x \vee y \vee z) \wedge (x \vee y \vee \overline{z}) \wedge (x \vee \overline{y} \vee z) \wedge (x \vee \overline{y} \vee \overline{z}) \vee (\overline{x} \vee y \vee z) \wedge (\overline{x} \vee y \vee \overline{z}) \wedge (\overline{x} \vee \overline{y} \vee z) \wedge (\overline{x} \vee \overline{y} \vee \overline{z})$ does not belong to 3-SAT since there are no satisfying assignments, however the assignment $x = True$, $y = True$, $z = True$ satisfies 7 of the 8 clauses (which is more than the 5 of 8 required to be a majority), and so the formula belongs to MajSAT.

The formula: $(x \vee y \vee z) \wedge (x \vee y \vee \overline{z}) \wedge (x \vee \overline{y} \vee z) \wedge (x \vee \overline{y} \vee \overline{z}) \vee (\overline{x} \vee y \vee z) \wedge (\overline{x} \vee y \vee \overline{z}) \wedge (\overline{x} \vee \overline{y} \vee z)$ belongs to MajSAT because there is an assignment (specifically $x = True, y = True, z = True$) which satisfies all 7 of 7 clauses.

The formula: $(\overline{x} \vee \overline{x} \vee \overline{x}) \wedge (\overline{x} \vee \overline{x} \vee \overline{x}) \wedge (\overline{x} \vee \overline{x} \vee \overline{x}) \wedge (x \vee x \vee x) \wedge (x \vee x \vee x) \wedge (x \vee x \vee x)$ does not belong to MajSAT since no assignment satisfies more than 3 of the 6 clauses.

Using a proof by reduction, demonstrate that MajSAT is in NP-Hard.

