PS1: Representation

Collaboration: You should work on the problems yourself, before discussing with others, including your cohorts at your cohort meeting. In addition to discussing with your cohortmates, you may discuss the problems with any other current CS3102 students you want, and use any resources you want except for any materials from previous offerings of this course or complete solutions that might be available on the web, which are not permitted. Sharing and subsequently submitting any text, code, images, figures, etc. constitutes plagiarism, so make sure all submitted materials are created exclusively by members of your cohort.

Problem 1: Hall of Fame

For this problem you're going to be sharing contributions made by notable people in computing. There's no need to do extensive research or anything, but (with your cohort and prior to your preparation meeting) identify several contributions made by a notable person (selected according to the directions below). Don't forget to mention who your contributions are about!

If you're in a cohort: Each cohort has been named after someone who is (at least loosely) connected to theoretical computer science. You can find your cohort namesake here: Cohort Namesakes. Identify a few important contributions made by your cohort namesake.

If you're working alone: Select either an individual who appears on a Notable Women in Computing Playing Card, or else a Turing Award Winner. Please do not select anyone who is also a cohort namesake.

As an additional step for this problem, pick one interesting fact and a URL to some work done by person (e.g. a publication, software, artwork, novel, lecture recording, youtube video, etc.), then post a message in #ps1 on discord to share this with the rest of the class (only one post per cohort is necessary.

Problem 2: Induction Practice

Use induction to show that $n! < n^n$ for all $n \in \mathbb{N}$ where $n > 2$

Problem 3: More Induction

Prove, using induction, that the largest number that can be represented by a binary string of length n is $2^n - 1$, for all $n \in \mathbb{N}$. To do this, show (remember, by induction) that the string consisting of n 1s in a row represents the value $2^n - 1$. Note that the empty string represents the value 0. Use the standard binary number representation we discussed in class.

Problem 4: Leaf me alone. I'm bushed.

You can consider a list to be a data structure where each item is followed by at most one other item. A binary tree is a data structure is like a list, except that each item may have up to two next items. In other words, a binary tree is like a branching list (the "branching" nature is what motivates the vocab term "tree", since each split could could make up to two branches it's called a "binary tree").

Vocabulary of trees borrows from the vocabulary of family trees, where if node A branches to nodes B and C (see example below), we say that A is the parent of B (and C), B is a child of A, and B and C are siblings. In computer science (like in Disney movies) all parents are single parents.

The "root" of a tree is the node with no parents. A "leaf" of a tree is a node with no children. The "height" of a tree is the length of the longest path from the root to a leaf (the length of a path is the number of edges it crosses). See the examples below for illustrations.

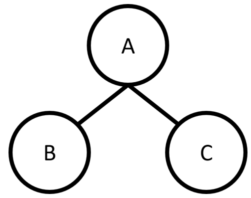

Figure 1: A Tree of height 1. Its root is A, its leaves are B and C.

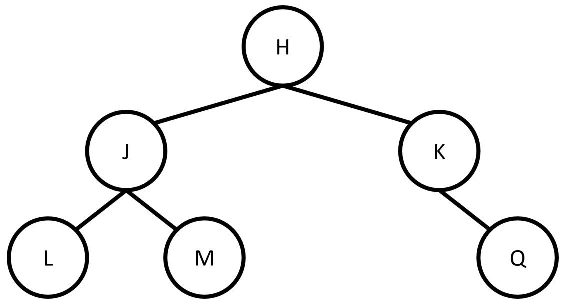

Figure 2: A Tree of height 2. Its root is H, its leaves are L, M, and Q.

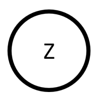

Figure 3: A Tree of height 0. Its root is Z, its only leaf is also Z.

Show that, for any $h \in \mathbb{Z}^+$, the maximum number of leaves that a binary tree of height h could have is 2^h by describing a bijection between the set of leaves in a height-h binary tree and the set $\{0,1\}^h$.

Problem 5: Busjections

The UTS buses have LED displays that give information about route, service, etc. These displays are 97 pixels wide and 17 pixels tall. Each pixel could have 1 of 2 colors: orange (on) or black (off). Answer these questions about the length of binary strings necessary to represent the bus display.

1. We will store the contents of the display as a binary string. Assuming all configurations are represented with the same number of bits, what is the minimum number of bits required to do so? Justify your answer by describing a bijection between all strings of the length you indicate, and all possible **configurations** of the display.
2. Suppose, to limit the amount of light pollution caused by buses, we require that no more than half of the pixels could be on at a time. Assuming they are all represented with the same number of bits, what is the minimum number of bits required to represent all allowable **configurations** of the display, now? *Hint: Begin with a bijection between configurations of the display which are majority orange vs. those that are majority black, noting that there are an odd number of total pixels. What does the existence of such a bijection tell you about the total number of configurations? What impact does that have on the number of bits you need?*
3. ($\star$) Using your answer from the previous part (which must be correct to earn the bonus), describe a succinct bijection between binary strings of the length you identified and configurations of the display. (Note: problems labelled with $\star$ are optional and worth bonus points)