

Quiz 3: Universality

Collaboration: This quiz is open note but individual. You may use any resources that were provided to you by the course, or that you had recorded as part of your own notes prior to when you first viewed this quiz. You may not discuss this quiz with your cohort-mates, class-mates, tutor, friends, family, or anyone else except the course staff (who will answer clarification questions only).

Problem 1: Asymptotic Operators

Let $f(n) = 2^n n^2$ and $g(n) = 2^n n$, which of the following are true? Support your answer to each part with a convincing argument that uses the definitions of O , Ω , Θ given in lecture.

- (a) $f \in O(g)$
- (b) $f \in \Omega(g)$
- (c) $f \in \Theta(g)$

Problem 2: Circuit Complexity

Show that $\text{XOR} \in \text{SIZE}(5) \cap \text{SIZE}^{\text{AON}}(5)$.

Problem 3: FSA and Regex

For each of the following languages, give both a regular expression and a (deterministic) finite state automaton that computes the language.

1. $\{x \in \{0, 1\}^* \mid x \neq \varepsilon\}$
2. $\{x \in \{0, 1\}^* \mid x \text{ starts with } 0 \text{ and ends with } 000 \text{ or else starts with } 1 \text{ and ends with } 1\}$