Quiz 6: Tractability

Collaboration: This quiz is open note but individual. You may use any resources that were provided to you by the course, or that you had recorded as part of your own notes prior to when you first viewed this quiz. You may not discuss this quiz with your cohort-mates, class-mates, tutor, friends, family, or anyone else except the course staff (who will answer clarification questions only).

Problem 1: P vs NP

Suppose that we have problems A and C where A reduces to C in polynomial time.

For each of the following scenarios for the complexity class memberships of A and C indicate whether we could conclude:

– $\mathsf{P} = \mathsf{NP}$,

– $\mathsf{P} \subset \mathsf{NP}$,

– or *Inconclusive* (meaning that scenario alone does not resolve the question).

Select one option and provide a short justification to support your answer.

a) $A \in \mathsf{P}$ and $C \in \mathsf{NP\text{-}Hard}$

b) $A \in \mathsf{NP\text{-}Complete}$ and $C \in \mathsf{P}$

c) $A \in \mathsf{NP}$ and $C \in \mathsf{EXP}$

Problem 2: DoubleSAT

Recall that 3-SAT is in NP-Hard, where the 3-SAT problem is to determine whether there exists at least one way to assign Boolean values to each variable in a 3-CNF formula so that the formula evaluates to True.

We will define DoubleSAT as the problem of determining whether there exists at least *two* different ways to assign Boolean values to each variable in a 3-CNF formula so that the formula evaluates to True. Here are some examples:

The formula: $(x \vee y \vee z) \wedge (x \vee y \vee \overline{z}) \wedge (x \vee \overline{y} \vee z) \wedge (x \vee \overline{y} \vee \overline{z}) \vee (\overline{x} \vee y \vee z) \wedge (\overline{x} \vee y \vee \overline{z}) \wedge (\overline{x} \vee \overline{y} \vee z) \wedge (\overline{x} \vee \overline{y} \vee \overline{z})$ does not belong to DoubleSAT since there are no satisfying assignments.

The formula: $(x \vee y \vee z) \wedge (x \vee y \vee \overline{z}) \wedge (x \vee \overline{y} \vee z) \wedge (x \vee \overline{y} \vee \overline{z}) \vee (\overline{x} \vee y \vee z) \wedge (\overline{x} \vee y \vee \overline{z}) \wedge (\overline{x} \vee \overline{y} \vee z)$ does not belong to DoubleSAT since there is only one satisfying assignment (specifically $x = True, y = True, z = True$ is the only satisfying assignment).

The formula: $(x \vee y \vee z) \wedge (x \vee y \vee \overline{z}) \wedge (x \vee \overline{y} \vee z) \wedge (x \vee \overline{y} \vee \overline{z}) \vee (\overline{x} \vee y \vee z) \wedge (\overline{x} \vee y \vee \overline{z})$ does belong to DoubleSAT since there are two satisfying assignments (specifically $x = True, y = True, z = True$ and $x = True, y = True, z = False$ both satisfy).

Using a proof by reduction, demonstrate that DoubleSAT is in NP-Hard.

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