Exercise 1: SOLUTIONS

Response by: **SOLUTIONS**

Exercise 1-1: Induction Practice

To show:

$$n! < n^n$$

We will show this using a proof by induction.

Base case: $n = 2$

$$2! < 2^2$$
$$2 < 4$$

So the base holds

Inductive Hypothesis:

Assume $k! < k^k$ for arbitrary $k \geq 2$

Inductive Step: Show $k! < k^k \rightarrow (k+1)! < (k+1)^{k+1}$

To begin, consider $(k+1)!$:

$$(k+1)! = (k+1) \cdot k!$$
$$(k+1) \cdot k! < (k+1)k^k$$
$$(k+1)k^k < (k+1)(k+1)^k$$
$$(k+1)(k+1)^k = (k+1)^{k+1}$$

and so $(k+1)! < (k+1)^{k+1}$