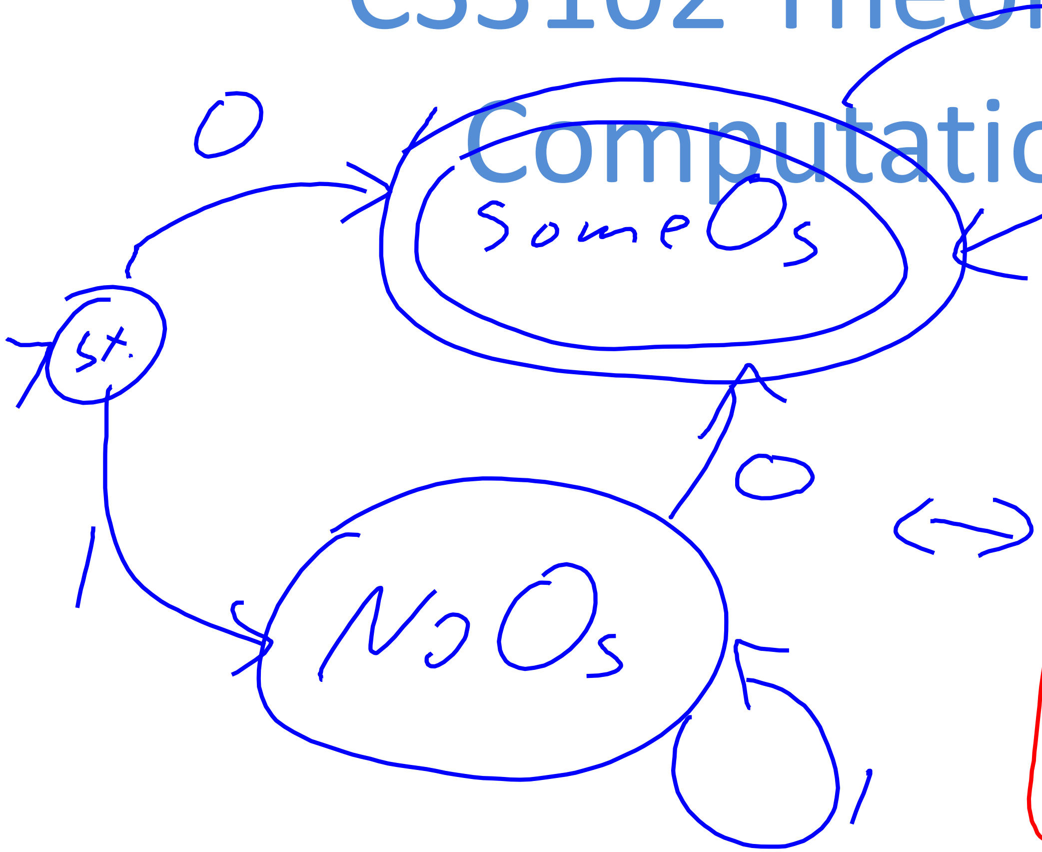
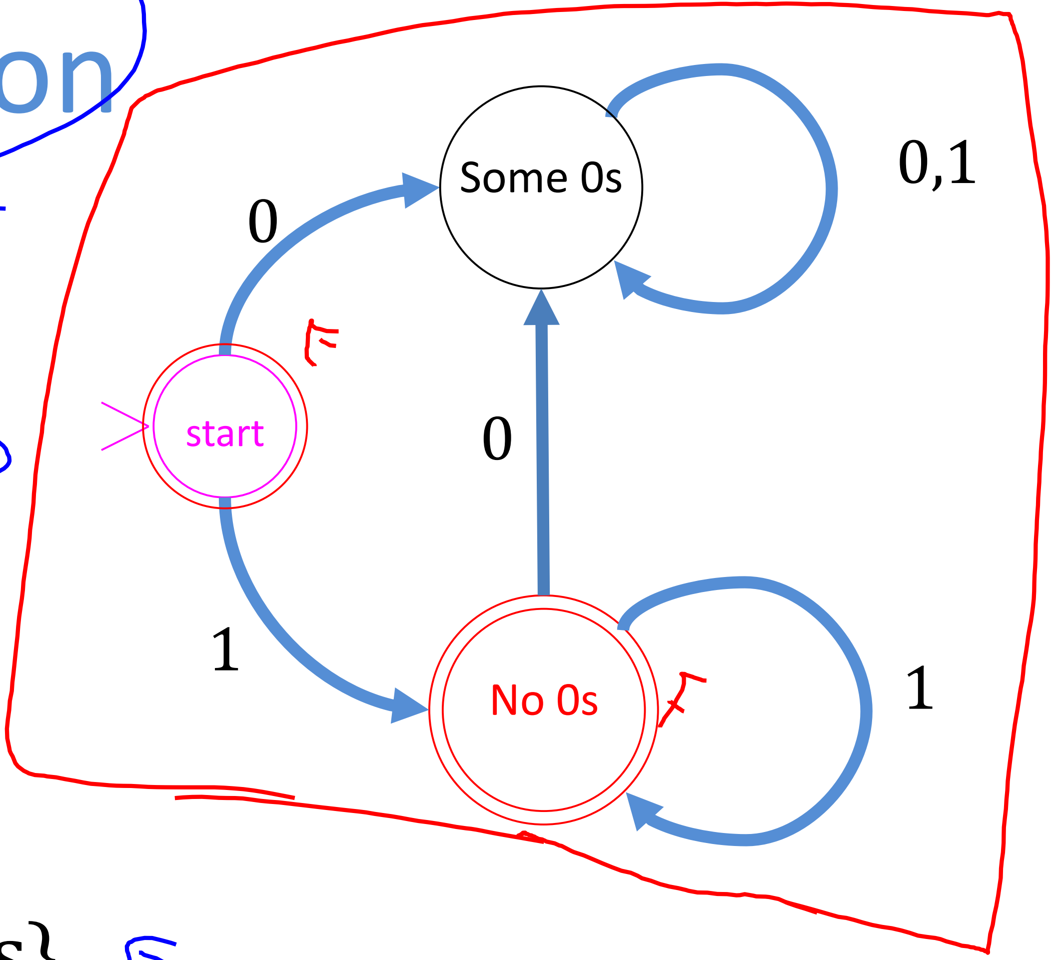
hello?

# CS3102 Theory of Computation




Warm up:

This automaton computes infinite AND:

$$AND = \{x \in \{0,1\}^* | x \text{ has no 0s}\}$$

Show how to compute infinite NAND:

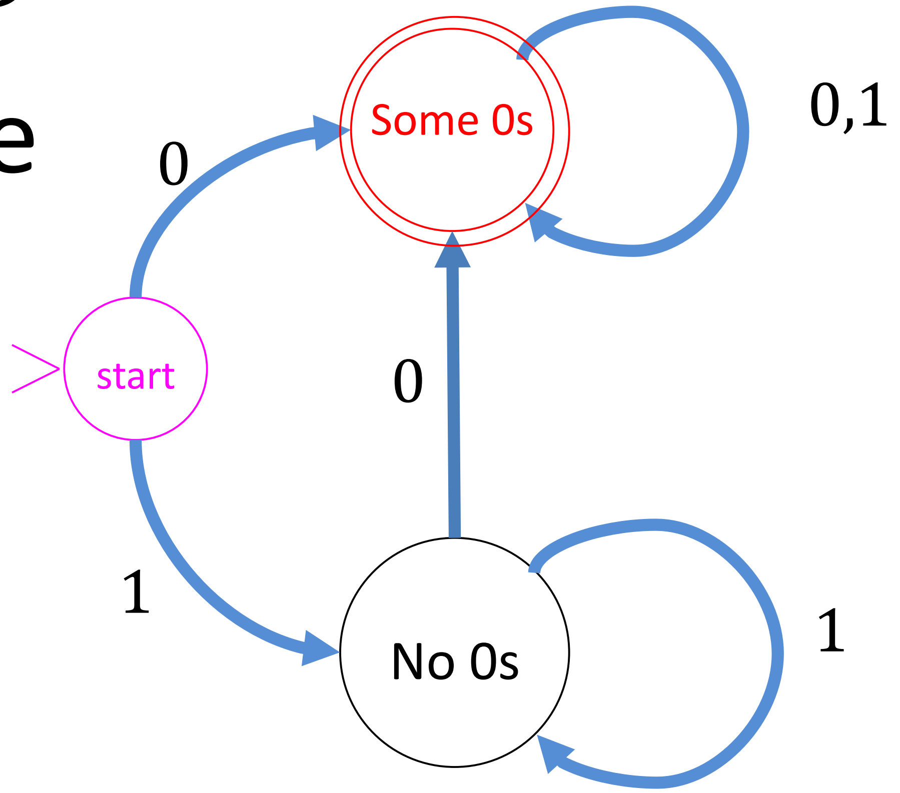
$$NAND = \{x \in \{0,1\}^* | x \text{ has a 0}\}$$

complements $\{0,1\}^*$

# Infinite NAND Automaton

all strings $\{0,1\}^*$

- Observation: $AND^c = NAND$
- NAND should do the opposite of ~~N~~AND
- Switch final states and non-final states!

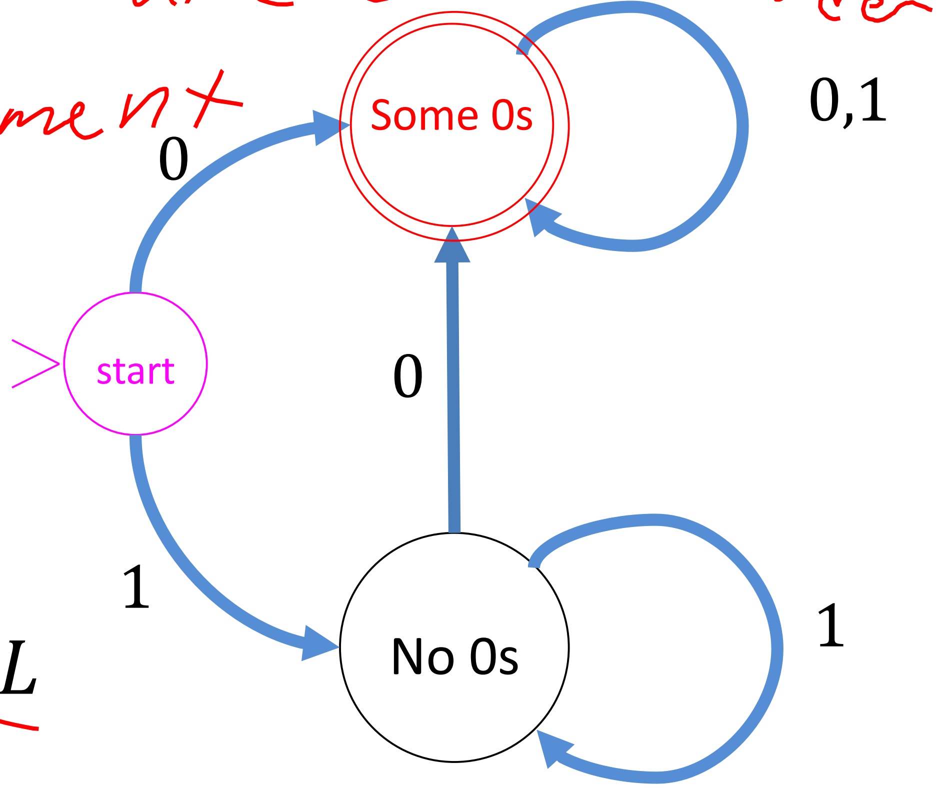


# AND to NAND

Languages computable by FSA are closed under complement

- AND:
  - $Q = \{start, No0s, Some0s\}$
  - $q_0 = start$
  - $\boxed{F = \{start, No0s\}}$
  - $\delta$ defined as the arrows
- NAND:
  - $Q, q_0, \delta$ don't change
  - $\boxed{F = Q - F}$
- In general, If we can compute a language $L$ with a FSA, we can compute $L^c$ as well

closure



# Logistics

3

- Exercise 4 (partially) released
  - First part is on creating finite automata
  - The rest released today, due Friday (March 6)
- Quiz will be released Thursday, due Tuesday

Hamming distance

2 nate

note

# Last Time

- Languages and decision problems
  - A different way of thinking about functions
- Introducing Finite State Automata

  non

  - DFA: *Deterministic* finite state automaton
  - Language of a FSA: The set of strings for which that automaton returns 1

# FSA are strictly more powerful than NAND circuits

- How can we show this?
  - Show that there is at least one function we can do with FSA but not NAND-CIRC
    - Done! (infinite XOR)
  - Show anything we can do with NAND-CIRC can also be done with FSA
    - How?
    - We need to be able to compute any finite function

• Convert a Circuit into an FSA

FSA $\geq$ NAND

• 1 lang we can't do w/ NAND

FSA > NAND

# Computing any finite function with NAND-CIRC

- Summary:
  - "Manually Precompute" the output for every (finitely-many) possible input
  - When we receive the actual input, do a "lookup"
- Our proof before:
  - Make a variable to represent each possible input, assigning its value to match the correct output
  - Use LOOKUP to return the proper variable for the given input

# Straightline Code for *f*

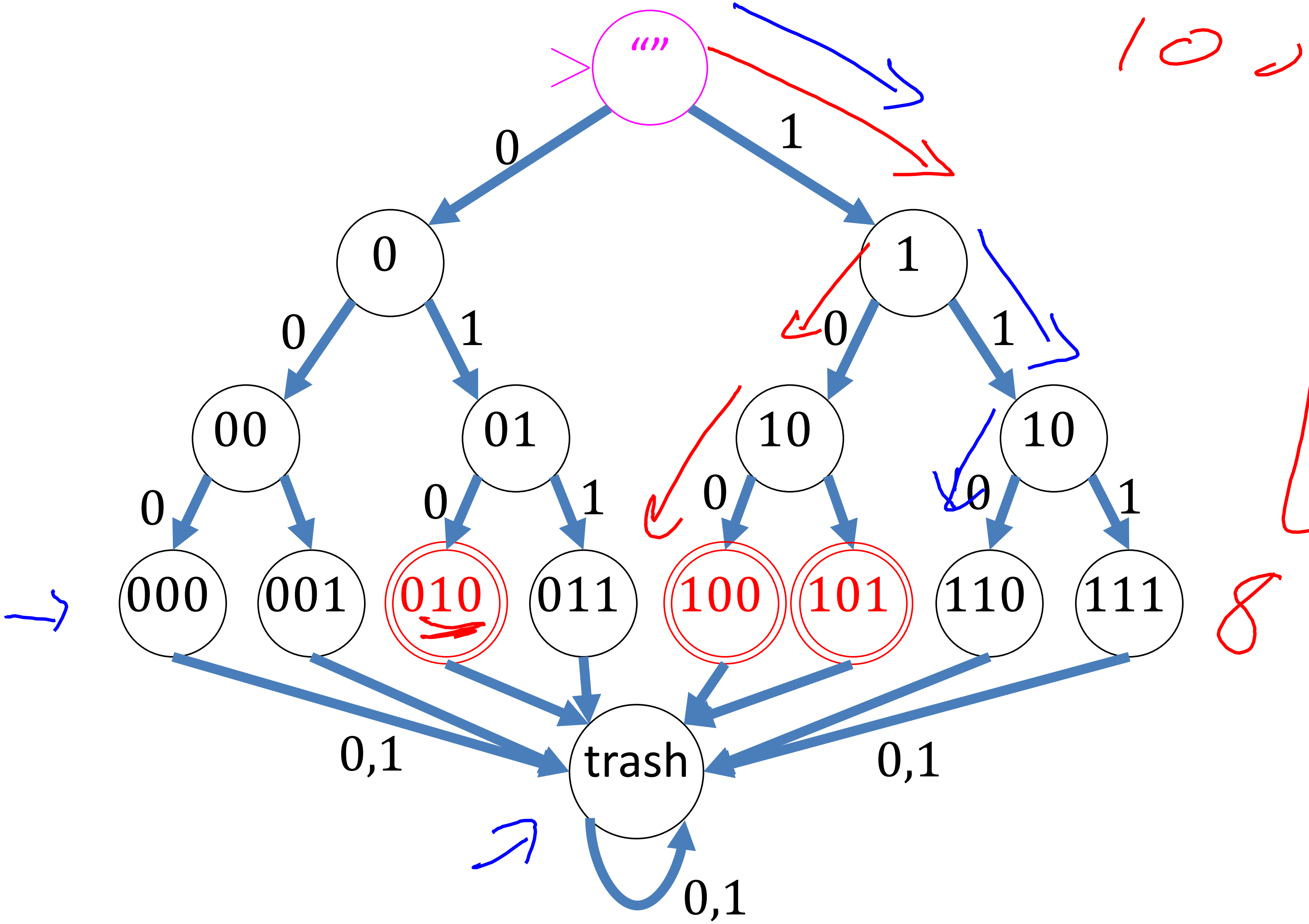
| Input | Output |
|---|---|
| 000 | 0 |
| 001 | 0 |
| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 1 |
| 110 | 0 |
| 111 | 0 |

```
def F(x0,x1,x2):
    F000=0
    F001=0
    F010=1
    F011=0
    F100=1
    F101=1
    F110=0
    F111=1
    return LOOKUP3(F000,F001,F010,F011,F100,F101,F110,F111,x0,x1,x2)
```

# Computing finite functions with FSA

- Summary:
  - "Manually Precompute" the output for every (finitely-many) possible input
  - When we receive the actual input, do a "lookup" Tree
- Same idea, but with Automata:
  - Make a state for every possible input, determining whether or not it is final depending on the correct output
  - Do a "binary tree traversal" with the given input to navigate to its correct output

# FSA for $f$




| Input | Output |
| --- | --- |
| 000 | 0 |
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| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 1 |
| 110 | 0 |
| 111 | 0 |

# Regular Expressions

| Name | Decision Problem | Function | Language |
|---|---|---|---|
| Regex | Does this string match this pattern? | $f(b) = \begin{cases} 1 & \text{the string matches} \\ 0 & \text{the string doesn't} \end{cases}$ | $\{b \in \Sigma^* \mid b \text{ matches the pattern}\}$ |

- A way of describing a language
- Give a "pattern" of the strings, every string matching that pattern is in the language
- Examples:
  - $(a|b)c$ matches : $ac$ and $bc$
  - $(a|b)^*c$ matches : $c, ac, bc, aac, abc, bac, bbc, \ldots$

# "Pieces" of a Regex

$\epsilon$

- Empty String:
  - Matches just the string of length 0
  - Notation: $\varepsilon$ or ""

\varepsilon

- Literal Character
  - Matches a specific string of length 1
  - Example: the regex $a$ will match just the string $a$
- Alternation/Union /or
  - Matches strings that match at least one of the two parts
  - Example: the regex $a|b$ will match $a$ and $b$
- Concatenation
  - Matches strings that can be dividing into 2 parts to match the things concatenated
  - Example: the regex $(a|b)c$ will match the strings $ac$ and $bc$
- Kleene Star
  - Matches strings that are 0 or more copies of the thing starred
  - Example: $(a|b)c^*$ will match $a$, $b$, or either followed by any number of $c$'s

Note: The compents here are the minimal necessary. In practice, regexes have other components as well, those are just "syntactic sugar".

a|b d cd

a b e f cef

(ab|c)(d|ef)

[abc]

(a|b|c)

[a|b]

(a|b)*

""

abbbba

(a|b)*.c*

""

a

b

c

abbacc

(ab*|cd)*

acdacd

# Regex for UVA computing IDs

- A UVA computing id is formatted as:
  - 2-3 letters a, b
  - A digit 2, 3
  - 1-3 letters a, b

$(a|b)(a|b)(\varepsilon|a|b)$

$(2|3)$

$(a|b)(\varepsilon|a|b)(\varepsilon|a|b)$

# AND as a Regex

- $AND = \{x \in \{0,1\}^* | x \text{ has no 0s}\}$

$1^*$

# NAND as a Regex

- $NAND = \{x \in \{0,1\}^* | x \text{ has a } 0\}$

$(0|1)^* 0 (0|1)^*$

$1^* 0 (0|1)^*$

# XOR as a Regex

- $XOR = \{x \in \{0,1\}^* | x \text{ has an odd number of 1s}\}$

$$\left(0^* \; 1 \left(0^* 1 0^* 1\right)\right)^*$$

# FSA = Regex

- Finite state Automata and Regular Expressions are equivalent models of computing
- Any language I can represent as a FSA I can also represent as a Regex (and vice versa)
- How would I show this?

# Showing FSA $\leq$ Regex

- Show how to convert any FSA into a Regex for the same language
- We're going to skip this:
  - It's tedious, and people virtually never go this direction in practice, but you can do it (see textbook theorem 9.12)

# Showing Regex $\leq$ FSA

- Show how to convert any regex into a FSA for the same language
- Idea: show how to build each “piece” of a regex using FSA

# “Pieces” of a Regex

- Empty String:
  - Matches just the string of length 0
  - Notation: $\varepsilon$ or “”
- Literal Character
  - Matches a specific string of length 1
  - Example: the regex $a$ will match just the string $a$
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# FSA for the empty string

# FSA for a literal character

# FSA for Alternation/Union

- Tricky…
- What does it need to do?