

Exercise 1: SOLUTIONS

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Exercise 1-1: Induction Practice

To show:

$$n! < n^n$$

We will show this using a proof by induction.

Base case: $n = 2$

$$2! < 2^2$$

$$2 < 4$$

So the base holds

Inductive Hypothesis:

Assume $k! < k^k$ for arbitrary $k \geq 2$

Inductive Step: Show $k! < k^k \rightarrow (k+1)! < (k+1)^{k+1}$

To begin, consider $(k+1)!$:

$$(k+1)! = (k+1) \cdot k!$$

$$(k+1) \cdot k! < (k+1)k^k$$

$$(k+1)k^k < (k+1)(k+1)^k$$

$$(k+1)(k+1)^k = (k+1)^{k+1}$$

and so $(k+1)! < (k+1)^{k+1}$