Test Yourself

These problems are for additional practice on topics that will be on Exam 1. They will not be graded and you do not need to turn in anything for these problems. But, if you feel less confident on the topics covered in these questions, you should find it useful to do these problems to prepare for Exam 1. We are happy to answer questions about these problems during office hours.

Note: One of the practice problem sub-parts asks you to prove something that is not true! Try to figure out which one it is!

Practice 1 *Relation Properties*

Considered the relation, $\leq$ (less than or equal to, with the standard meaning), with the domain set, $\mathbb{N}$ and codomain set $\mathbb{N}$. Which of these properties does the $\leq$ relation have: function, total, injective, surjective, bijective?

Practice 2 *Set Cardinality*

a. Assume $R : A \to B$ is an *total injective* function between A and B. What must be true about the relationship between $|A|$ and $|B|$?

b. Assume $R : A \to B$ is an *total surjective* function between A and B. What must be true about the relationship between $|A|$ and $|B|$?

c. Assume $R : A \to B$ is a (not necessarily total) *surjective* function between A and B. What must be true about the relationship between $|A|$ and $|B|$?

Practice 3 *Countable and Uncountable Infinities*

a. Prove that the integers, i.e., $\dots, -2, -1, 0, 1, 2, \dots$, are countably infinite.

b. Prove that the number of total injective functions between $\mathbb{N}$ and $\mathbb{N}$ is countable.

c. Prove that the number of different chess positions is countable. (A chess position is defined by the locations of pieces on an 8×8 board, where each square on the board can be either empty, or contain a piece from $\{\mathrm{Pawn}, \mathrm{Knight}, \mathrm{Bishop}, \mathrm{Castle}, \mathrm{Queen}, \mathrm{King}\}$ of one of two possible colors.)

d. Prove that number of Ziggy Pig ice cream dishes is uncountable. A Ziggy Pig ice cream can contain any number of scoops ($scoops \in \mathbb{N}$), and each scoop can be of any flavor, where distinct flavors are identified by $v \in \mathbb{N}$.

Practice 4 *Vacuous Fish*

Proof that all fish who have eaten Ziggy Pig ice creams (as find in Practice 3) with an infinite number of scoops are Coho Salmon.

Practice 5 *Induction Practice 1*

Prove by induction that every natural number less than 2^{k+1} can be written as $a_0 \cdot 2^0 + a_1 \cdot 2^1 + a_2 \cdot 2^2 + \cdots + a_k \cdot 2^k$ where all the a_i values are either 0 or 1.

Practice 6 *Induction Practice 2*

Prove by induction that every finite non-empty subset of the natural numbers contains a *greatest* element, where an element $x \in S$ is defined as the *greatest* element if $\forall z \in S - \{x\}.\ x > z$.

Practice 7 *Circuit Evaluation*

A "good" Boolean circuit always eventually evaluates to a value using the definition of circuit evaluation. Prove that a Boolean circuit where there is a cycle on a path between an input and an output will never produce a value for that output.