Exercise 0-2: Getting Started with LaTeX

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> **Collaborators and Resources:** I collaborated with David Evans on the pdf style, I consulted https://www.overleaf.com/learn/latex/Aligning%20equations%20with%20amsmath for help with the align environment.

Problem 1 *The Finest Gambit*

Proof.

For a rational number $\frac{a}{b}$, without loss of generality we may suppose that a and b are integers which share no common factors, as otherwise we could remove any common factors (i.e. suppose $\frac{a}{b}$ is in simplest terms). To say $\sqrt{2}$ is irrational is equivalent to stating that 2 cannot be expressed in the form $(\frac{a}{b})^2$. Equivalently, this says that there are no integer values for a and b satisfying

$$a^2 = 2b^2 \tag{1}$$

We argue by *reductio ad absurdum* (proof by contradiction). Assume toward reaching a contradiction that Equation 1 holds for a and b being integers without any common factor between them. It must be that a^2 is even, since $2b^2$ is divisible by 2, therefore a is even. If a is even, then for some integer c

$$a = 2c$$
$$a^2 = (2c)^2$$
$$2b^2 = 4c^2$$
$$b^2 = 2c^2$$

therefore, b is even. This implies that a and b are both even, and thus share a common factor of 2. This contradicts our hypothesis, therefore our hypothesis is false.

$\square$

Problem 2 *Vanity*