Exam 2

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```
library(R2jags)
library(dplyr)
library(brms)
library(loo)
library(rstan)
```

For the first set of questions we will be using the data file **influent.dat**. You should have received that data file with the exam. Water flows into the Mississippi river from a number of streams and rivers. These sources carry nitrogen into the river. In the data set, there are six sources of the nitrogen, which are a random sample of the many hundreds of streams and rivers that flow into the Mississippi. These sources have been classified by type. The three types are: (1) no farm land in watershed, (2) less than 50% farm land in watershed, and (3) more than 50% farmland in watershed. These three types are the only types that we seek to analyze. In the data file, the three columns are river source, nitrogen, and type.

1. First read in the data and print out the first six rows of the data.

```
influent <- read.table("influent.dat")
colnames(influent) <- c("source", "nitrogen", "type")</pre>
```

2. Ignoring the source, use brm to write a model to find the differences in type. Use priors of normal(0,100) for the type effects, and gamma(2,.1) for σ_{error} . Remember that the kind of variable you are working with will make a difference. What is the level of nitrogen estimated for type 3.

Level of nitrogen estimated for type 3 is 36.36

```
#control = list(adapt delta = 0.98),
            save pars = save pars(all = TRUE))
summary(fit1)
  Family: gaussian
   Links: mu = identity; sigma = identity
 Formula: nitrogen ~ −1 + vtype
    Data: influent (Number of observations: 37)
 Samples: 4 chains, each with iter = 5000; warmup = 1000; thin = 1;
          total post-warmup samples = 16000
 Population-Level Effects:
        Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk ESS Tail ESS
           15.61
                      2.39
                              10.93
                                        20.43 1.00
                                                      16978
                                                               11131
 vtype1
                              16.82
 vtype2
           19.92
                      1.59
                                        23.08 1.00
                                                      16060
                                                               11019
                      3.33
                              29.83
                                       42.94 1.00
 vtype3
           36.32
                                                      16383
                                                               11428
 Family Specific Parameters:
       Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk ESS Tail ESS
                     0.95
                              5.86
                                       9.57 1.00
                                                     15101
                                                              12282
 sigma
           7.46
 Samples were drawn using sampling(NUTS). For each parameter, Bulk_ESS
 and Tail ESS are effective sample size measures, and Rhat is the potential
 scale reduction factor on split chains (at convergence, Rhat = 1).
chains <- as.matrix(fit1)</pre>
dim(chains)
head(chains)
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
```

3. What is the looic for this model?

The looic is 255.6

```
All Pareto k estimates are good (k < 0.5). See help('pareto-k-diagnostic') for details.
```

4. Now run the same model using JAGS. Use the same priors for the type parameters (remember, normal priors in JAGS use precision), but use a gamma(5,.1) for σ_{error}^2 . What is the level of nitrogen estimated for type 3?

```
md1 <- "
model {
  for(i in 1:37){
    nitrogen[i] ~ dnorm(mu[i], 1/s2error)
    mu[i] <- beta[type[i]]</pre>
  # Priors
  for(i in 1:3){
    beta[i] ~ dnorm(0, 0.0001)
  s2error \sim dgamma(5, 0.1)
}
writeLines(mdl, 'fit2.txt')
source = influent$source
nitrogen = influent$nitrogen
type = influent$type
data.jags <- c('nitrogen', 'type')</pre>
parms <- c('beta' ,'s2error')</pre>
fit2 <- jags(data= data.jags, parameters.to.save = parms,</pre>
                   model.file = 'fit2.txt', inits = NULL,
                   n.iter = 20000, n.thin = 5, n.burnin = 2000,
                   n.chains = 5)
 module glm loaded
 Compiling model graph
    Resolving undeclared variables
    Allocating nodes
 Graph information:
    Observed stochastic nodes: 37
    Unobserved stochastic nodes: 4
    Total graph size: 84
```

```
Initializing model
fit2
Inference for Bugs model at "fit2.txt", fit using jags,
 5 chains, each with 20000 iterations (first 2000 discarded), n.thin = 5
 n.sims = 18000 iterations saved
          mu.vect sd.vect
                             2.5%
                                      25%
                                              50%
                                                      75%
                                                            97.5% Rhat n.eff
                   2.305 11.022 14.053 15.560 17.118 20.080 1.001 18000
beta[1]
          15.570
                   1.538 16.861 18.879 19.866 20.891 22.955 1.001 18000
beta[2]
          19.883
          36.352 3.217 29.982 34.208 36.346 38.468 42.730 1.001 18000
beta[3]
          52.802 11.465 34.535 44.578 51.389 59.398 78.966 1.001 18000
s2error
deviance 251.400
                   2.736 247.987 249.403 250.754 252.749 258.306 1.001 14000
For each parameter, n.eff is a crude measure of effective sample size,
 and Rhat is the potential scale reduction factor (at convergence, Rhat=1).
DIC info (using the rule, pD = var(deviance)/2)
pD = 3.7 and DIC = 255.1
DIC is an estimate of expected predictive error (lower deviance is better).
sims <- as.mcmc(fit2)</pre>
chains <- as.matrix(sims)</pre>
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
```

5. What is the DIC of the above model?

DIC is 255.3

6. Now we want to account for the variance in sources to make inference. Redo the model in brm, but now put source in the model appropriately. Use a gamma(2,.1) prior for σ_{source} . The other priors can stay the same. Now what is the estimate for the type 3 mean?

Estimate for type 3 mean is 12.24. This plus whatever the effect for whatever source it is.

summary(fit3)

Family: gaussian

Links: mu = identity; sigma = identity

Formula: nitrogen ~ -1 + vtype + (1 | source)
Data: influent (Number of observations: 37)

Samples: 4 chains, each with iter = 20000; warmup = 1000; thin = 10;

total post-warmup samples = 7600

Group-Level Effects:

~source (Number of levels: 6)

Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS sd(Intercept) 16.13 7.57 4.57 34.13 1.00 6961 6295

Population-Level Effects:

Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk ESS Tail ESS -9.51 21.51 1.00 7534 7448 vtype1 7.34 7.91 11.38 7.71 -5.39 24.20 1.00 6803 6943 vtype2 vtype3 12.01 10.71 -9.27 32.01 1.00 7091 6080

Family Specific Parameters:

Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS sigma 6.85 0.92 5.32 8.90 1.00 6895 6900

Samples were drawn using sampling(NUTS). For each parameter, Bulk_ESS and Tail_ESS are effective sample size measures, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat = 1).

fit3**\$**fit

Inference for Stan model: 68305c8400f0c8b7efc70543152eb5c9. 4 chains, each with iter=20000; warmup=1000; thin=10; post-warmup draws per chain=1900, total post-warmup draws=7600.

	mean	se_mean	sd	2.5%	25%	50%	75%
b_vtype1	7.34	0.09	7.91	-9.51	2.34	7.90	12.89
b_vtype2	11.38	0.09	7.71	-5.39	6.57	12.13	16.99
b_vtype3	12.01	0.13	10.71	-9.27	4.70	11.95	19.57
sd_sourceIntercept	16.13	0.09	7.57	4.57	10.86	15.03	20.22
sigma	6.85	0.01	0.92	5.32	6.20	6.75	7.40
r_source[1,Intercept]	9.93	0.10	7.97	-3.52	4.23	9.14	14.85
<pre>r_source[2,Intercept]</pre>	2.52	0.09	7.77	-10.97	-2.95	1.73	7.36
<pre>r_source[3,Intercept]</pre>	9.00	0.10	8.29	-5.58	3.10	8.51	14.21
<pre>r_source[4,Intercept]</pre>	12.63	0.10	8.20	-1.17	6.70	11.97	17.73
<pre>r_source[5,Intercept]</pre>	6.77	0.09	8.20	-7.98	0.92	6.13	12.11
<pre>r_source[6,Intercept]</pre>	23.18	0.13	11.28	1.95	15.33	23.08	31.09
lp	-145.89	0.04	3.43	-153.63	-147.89	-145.43	-143.40
z_1[1,1]	0.63	0.01	0.48	-0.32	0.33	0.62	0.93
z_1[1,2]	0.06	0.01	0.55	-1.26	-0.24	0.12	0.42
z_1[1,3]	0.57	0.01	0.54	-0.49	0.23	0.57	0.91

```
z 1[1,4]
                           0.83
                                   0.01
                                         0.50
                                                 -0.10
                                                          0.50
                                                                  0.80
                                                                           1.14
 z_{1}[1,5]
                           0.41
                                   0.01 0.54
                                                 -0.72
                                                          0.07
                                                                  0.41
                                                                           0.75
                           1.51
                                   0.01
                                         0.66
                                                 0.25
                                                          1.08
                                                                  1.47
                                                                          1.91
 z_1[1,6]
                          97.5% n eff Rhat
 b_vtype1
                          21.51
                                7481
 b_vtype2
                          24.20
                                 6609
                                         1
 b vtype3
                          32.01
                                7035
                                         1
 sd source Intercept
                          34.13
                                         1
                                6854
 sigma
                           8.90 6962
                                         1
                         27.34 6688
 r source[1,Intercept]
                                         1
 r_source[2,Intercept]
                         19.38 6703
                                         1
 r source[3,Intercept]
                         26.63
                                7372
                                         1
 r source[4,Intercept]
                         30.16 6859
                                         1
 r_source[5,Intercept]
                         24.21 7533
                                         1
 r_source[6,Intercept]
                         45.46 7070
                                         1
                        -140.49 6972
                                         1
 lp
 z_1[1,1]
                           1.58 6914
                                         1
                                         1
 z 1[1,2]
                           0.98 7031
 z 1[1,3]
                           1.64 7635
                                         1
 z_{1}[1,4]
                           1.87 7469
                                         1
 z_1[1,5]
                           1.45 7832
                                         1
                           2.91 7097
                                         1
 z_1[1,6]
 Samples were drawn using NUTS(diag_e) at Mon Mar 22 15:21:30 2021.
 For each parameter, n eff is a crude measure of effective sample size,
 and Rhat is the potential scale reduction factor on split chains (at
 convergence, Rhat=1).
# Diagnostics Look good
chains <- as.matrix(fit3)</pre>
dim(chains)
 [1] 7600
            18
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
 Quantile (q) = 0.025
 Accuracy (r) = +/- 0.005
 Probability (s) = 0.95
                                                      Dependence
                         Burn-in
                                  Total Lower bound
                         (M)
                                  (N)
                                                      factor (I)
                                        (Nmin)
  b vtype1
                        2
                                  3924
                                        3746
                                                      1.050
  b vtype2
                        4
                                  8206
                                        3746
                                                      2.190
                                                      1.120
  b_vtype3
                         3
                                  4188
                                        3746
  sd_source__Intercept
                        2
                                  3759
                                        3746
                                                      1.000
  sigma
                                  8690
                                        3746
                                                      2.320
  r_source[1,Intercept] 2
                                  3759 3746
                                                      1.000
```

```
r source[2,Intercept] 2
                                   3840
                                         3746
                                                      1.030
  r_source[3,Intercept] 2
                                   3600 3746
                                                      0.961
  r_source[4,Intercept] 2
                                                      0.993
                                   3718
                                        3746
  r_source[5,Intercept] 2
                                   3718
                                        3746
                                                      0.993
  r_source[6,Intercept] 2
                                                      0.993
                                   3718
                                        3746
                                   3639
                                         3746
                                                      0.971
  1p_
                         2
                         2
  z_{1}[1,1]
                                   3882 3746
                                                      1.040
  z_{1}[1,2]
                         2
                                   3924
                                         3746
                                                      1.050
                         2
  z_1[1,3]
                                   3639
                                        3746
                                                      0.971
                         2
                                                      0.993
  z_1[1,4]
                                   3718 3746
                         2
                                   3600 3746
                                                      0.961
  z_1[1,5]
                         2
                                   3718 3746
                                                      0.993
  z 1[1,6]
effectiveSize(sims)
              b vtype1
                                      b vtype2
                                                             b vtype3
                                      6858.278
                                                             7175.409
              7600.000
  sd_source__Intercept
                                         sigma r_source[1,Intercept]
              6811.934
                                      7600.000
                                                             7093.651
 r_source[2,Intercept] r_source[3,Intercept] r_source[4,Intercept]
                                      7600.000
                                                             6970.358
              6905.563
 r_source[5,Intercept] r_source[6,Intercept]
                                                                 1p
              7600.000
                                      7200.955
                                                             6966.401
              z 1[1,1]
                                      z 1[1,2]
                                                             z 1[1,3]
              6668.253
                                      7143.009
                                                             7600.000
              z_1[1,4]
                                      z_1[1,5]
                                                             z_1[1,6]
              7600.000
                                      7600.000
                                                             7220.784
```

7. What is the looic for this model?

looic is 252.7

```
loo3 <- loo(fit3)
```

8. Which model (problem 2 or problem 6 is better? Why?

The model for problem 6 is better. It has a looic of 252.7 compared to 255.3. Clearly accounting for source is important.

9. Now do the model with the variance for sources accounted for in JAGS. Use a gamma(5,.1) for the variance component σ_{source}^2 . What is the estimate for the type 3 mean?

Estimate for type 3 mean is 36.24.

```
mdl <- "
model {
    for(i in 1:37){
        nitrogen[i] ~ dnorm(mu[i], 1/s2error)</pre>
```

```
mu[i] <- beta[type[i]] + u[source[i]]</pre>
  }
  # Priors
  for(i in 1:3){
    beta[i] ~ dnorm(0, 0.0001)
  for(i in 1:6){
    u[i] ~ dnorm(0,1/s_source)
  s2error \sim dgamma(5, 0.1)
  s_source ~ dgamma(5,0.1)
}
writeLines(mdl, 'fit4.txt')
source = influent$source
nitrogen = influent$nitrogen
type = influent$type
data.jags <- c('nitrogen', 'type', 'source')</pre>
parms <- c('beta' , 'u', 's2error')</pre>
fit4 <- jags(data= data.jags, parameters.to.save = parms,</pre>
                  model.file = 'fit4.txt', inits = NULL,
                  n.iter = 20000, n.thin = 5, n.burnin = 2000,
                  n.chains = 5)
 Compiling model graph
    Resolving undeclared variables
    Allocating nodes
 Graph information:
    Observed stochastic nodes: 37
    Unobserved stochastic nodes: 11
    Total graph size: 135
 Initializing model
(fit4)
 Inference for Bugs model at "fit4.txt", fit using jags,
  5 chains, each with 20000 iterations (first 2000 discarded), n.thin = 5
  n.sims = 18000 iterations saved
          mu.vect sd.vect
                             2.5%
                                       25%
                                               50%
                                                       75%
                                                             97.5% Rhat n.eff
 beta[1]
          15.593
                            5.419 12.300 15.621 18.843 25.856 1.001 18000
                    5.122
 beta[2]
           19.917
                    4.056 11.835 17.324 19.906 22.505 27.959 1.001 18000
 beta[3] 36.243 7.261 21.861 31.563 36.218 40.889 50.659 1.001 18000
```

```
s2error
           45.742
                   10.521 29.092
                                   38.265 44.339 51.735 70.205 1.001 18000
                                            1.456
 u[1]
                    4.176 -6.915 -1.212
                                                    4.164
                                                             9.708 1.001 18000
           1.460
                    4.260 -13.857
                                          -5.108
                                                  -2.388
                                                             3.021 1.001 18000
 u[2]
           -5.191
                                   -7.930
                    5.040
           0.961
                          -8.933
                                  -2.290
                                                    4.204 11.108 1.001 18000
 u[3]
                                            0.941
                    4.291 -4.691
 u[4]
            3.790
                                    1.000
                                            3.767
                                                    6.552
                                                           12.443 1.001 13000
                    5.042 -11.102
                                   -4.198
                                           -0.978
                                                    2.284
                                                             8.979 1.001 18000
 u[5]
           -0.971
 u[6]
            0.106
                    6.582 -13.033 -3.980
                                            0.121
                                                    4.253 13.152 1.001 18000
                    3.832 239.069 241.797 244.001 246.727 253.897 1.001 18000
 deviance 244.616
 For each parameter, n.eff is a crude measure of effective sample size,
 and Rhat is the potential scale reduction factor (at convergence, Rhat=1).
 DIC info (using the rule, pD = var(deviance)/2)
 pD = 7.3 and DIC = 252.0
 DIC is an estimate of expected predictive error (lower deviance is better).
sims <- as.mcmc(fit4)</pre>
chains <- as.matrix(sims)</pre>
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
```

10. What is the DIC for this model?

DIC is 252.0

11. Which model (problem 4 or problem 9) is better? Why?

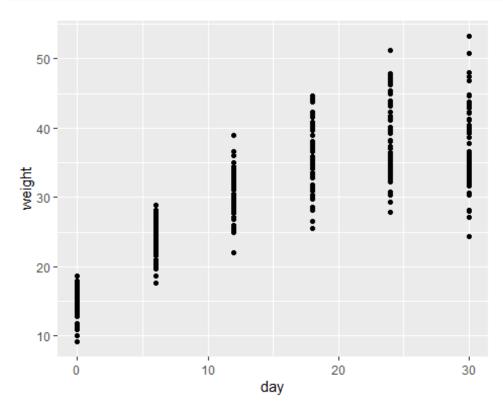
DIC for model 9 is better, it is lower. Therefore there is value in accounting for the which source it comes from. We can now make inference for sources.

We will be using the data file **pig.dat** for the the next set of questions. You should have received that data file. The data set is from a feeding experiment comparing 3 treatments (basically how much of a particular additive was included in the food ration) on young hogs over the first 30 days of life. The measurements were taken every 6 days. The pig number identifies the pig within a treatment. That is, pig 1 in trt 1 is not the same pig as pig 1 in trt 2.

12. Plot the weight on the y-axis and the day on the x-axis, ignoring the trt and the pig.

```
pig <- read.table("pig.dat", header = TRUE)</pre>
head(pig)
   Obs trt day pig weight
 1
              0
                      14.0
     1
         1
                  1
 2
                      22.1
     2
         1
              6
                  1
                      27.7
 3
     3
         1 12
                  1
 4
     4
         1 18
                  1
                      31.8
 5
     5
         1 24
                  1
                      35.3
 6
     6
         1
            30
                      32.6
```

```
ggplot(data = pig, mapping = aes(x = day, y = weight)) +
geom_point()
```



13. You will note from your plot that weight gain is fairly linear over the first 15 days or so, and then tails off. To account for this, we are going to add a quadratic term for day. That is, include a column for day^2 in the data set. Print the first 6 rows of the new data set.

```
pig$day2 = (pig$day)^2
head(pig)
   Obs trt day pig weight day2
 1
          1
              0
                  1
                       14.0
 2
     2
          1
              6
                  1
                       22.1
                               36
 3
     3
         1
            12
                  1
                       27.7
                             144
 4
                       31.8
     4
          1
             18
                  1
                              324
 5
     5
                  1
          1
             24
                       35.3
                              576
 6
            30
                       32.6
                             900
```

14. Using JAGS, run a model with an intercept, linear term, and quadratic term for the overall data. You are essentially fitting the data you plotted, with no concern about treatments or the multiple observations on each pig. Use dnorm(0,0.0001) priors for the coefficients of the model (the β 's), and a dgamma(1.1,.1) as the prior for σ_{error}^2 . Print a summary of your model. What is the DIC of the model?

```
md1 <- "
  model {
  for (i in 1:360){
    weight[i] ~ dnorm(mu[i], 1/vv)
    mu[i] \leftarrow b0 + b1*day[i] + b2*day2[i]
  }
  b0 \sim dnorm(0, 0.0001)
  b1 \sim dnorm(0, 0.0001)
  b2 \sim dnorm(0, 0.0001)
  vv \sim dgamma(1.1, 0.1)
  }
day = pig$day
day2 = pig$day2
weight = pig$weight
writeLines(mdl, 'fit5.txt')
data.jags <- c('weight', 'day', 'day2')</pre>
parms <- c('b0', 'b1', 'b2', 'vv')
fit5 <- jags(data= data.jags, parameters.to.save = parms,</pre>
                  model.file = 'fit5.txt', inits = NULL,
                  n.iter = 12000, n.thin = 5, n.burnin = 2000,
                  n.chains = 5)
 Compiling model graph
    Resolving undeclared variables
    Allocating nodes
 Graph information:
    Observed stochastic nodes: 360
    Unobserved stochastic nodes: 4
    Total graph size: 1108
 Initializing model
(fit5)
 Inference for Bugs model at "fit5.txt", fit using jags,
  5 chains, each with 12000 iterations (first 2000 discarded), n.thin = 5
  n.sims = 10000 iterations saved
           mu.vect sd.vect
                                                    50%
                                                             75%
                                                                    97.5% Rhat
                                2.5%
                                          25%
 b0
            14.738
                     0.503
                                       14.393
                                                14.740
                                                          15.080
                                                                   15.719 1.002
                              13.771
                     0.078
                              1.564
                                       1.665
                                                 1.718
                                                          1.771
                                                                    1.872 1.001
 b1
             1.719
 b2
            -0.031
                     0.002
                             -0.036
                                       -0.033
                                                -0.031
                                                          -0.030
                                                                   -0.027 1.001
            18.331
                     1.385 15.860
                                       17.366 18.254
                                                          19.222
                                                                   21.276 1.001
 ٧V
```

```
deviance 2068.906
                      2.874 2065.356 2066.790 2068.267 2070.282 2076.240 1.001
          n.eff
           2600
 b0
 b1
           6000
 h2
          10000
          10000
 deviance 10000
 For each parameter, n.eff is a crude measure of effective sample size,
 and Rhat is the potential scale reduction factor (at convergence, Rhat=1).
 DIC info (using the rule, pD = var(deviance)/2)
 pD = 4.1 and DIC = 2073.0
 DIC is an estimate of expected predictive error (lower deviance is better).
sims <- as.mcmc(fit5)</pre>
chains <- as.matrix(sims)</pre>
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
```

15. Run the same model using brm. Use the same priors for the β 's (which will now be normal(0,100) since brm works in standard deviations), and a gamma(1.1,.5) prior for the standard deviation ($sigma_{error}$). Print a summary of your model. What is the looic of the model?

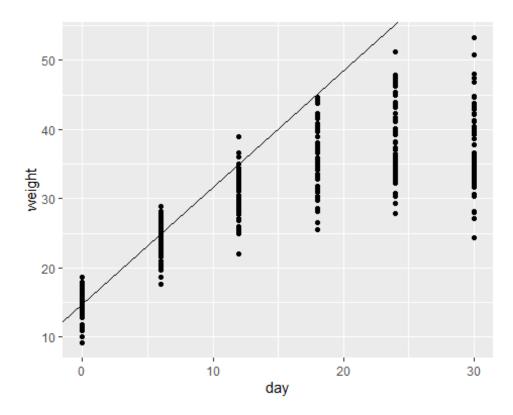
looic is 2073.6

```
fit6 <- brm(formula = weight ~ day + day2, data = pig,
            family = "gaussian",
            prior = c(set_prior("normal(0,100)", class = "b"),
                      #set_prior("normal(0,100)", class = "a"),
                      set_prior("gamma(1.1,0.5)", class = "sigma")),
            warmup = 1000, iter = 5000, chains = 4,
            #control = list(adapt delta = 0.98),
            save pars = save pars(all = TRUE), silent = TRUE)
(fit6)
  Family: gaussian
   Links: mu = identity; sigma = identity
 Formula: weight ~ day + day2
    Data: pig (Number of observations: 360)
 Samples: 4 chains, each with iter = 5000; warmup = 1000; thin = 1;
          total post-warmup samples = 16000
 Population-Level Effects:
```

```
Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk ESS Tail ESS
                          0.50
 Intercept
              14.74
                                  13.76
                                           15.71 1.00
                                                          10405
                                                                    9927
               1.72
                          0.08
                                   1.57
                                            1.87 1.00
                                                           8871
                                                                    8980
 day
 day2
              -0.03
                          0.00
                                  -0.04
                                           -0.03 1.00
                                                           9148
                                                                    9265
 Family Specific Parameters:
       Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
 sigma
           4.28
                     0.16
                               3.98
                                        4.61 1.00
                                                      10447
                                                                9294
 Samples were drawn using sampling(NUTS). For each parameter, Bulk_ESS
 and Tail_ESS are effective sample size measures, and Rhat is the potential
 scale reduction factor on split chains (at convergence, Rhat = 1).
# Convergence and ESS are good
chains <- as.matrix(fit6)</pre>
dim(chains)
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
# Looic
loo(fit6)
```

16. Using the estimated coefficients from either the JAGS or the brm model, add a best fit line to the plot you made in problem 12.

```
ggplot(data = pig, mapping = aes(x = day, y = weight)) +
geom_point() +
geom_abline(intercept = 14.73, slope = 1.69)
```



17. Using JAGS, run a model with an intercept, linear slope, and quadratic slope for each of the trt's. That is, you will be computing 9 β 's, 3 for each trt. Assume all the data are independent (that is, we are not worried about the different pigs at this point). Use dnorm(0,0.0001) priors for the coefficients of the model (the β 's), and a dgamma(1.1,.1) as the prior for σ_{error}^2 . While you have now accounted for the different treatments, you are still ignoring that there are multiple measurements on each animal.

What is the DIC for this model?

DIC is 1977.4

```
mdl <- "
  model {

for (i in 1:360){
    weight[i] ~ dnorm(mu[i], 1/vv)
    mu[i] <- b0[trt[i]] + b1[trt[i]]*day[i] + b2[trt[i]]*day2[i]
}

for(i in 1:3){
    b0[i] ~ dnorm(0, 0.0001)
    b1[i] ~ dnorm(0, 0.0001)
    b2[i] ~ dnorm(0, 0.0001)
}

vv ~ dgamma(1.1, 0.1)</pre>
```

```
trt = pig$trt
day = pig$day
day2 = pig$day2
weight = pig$weight
writeLines(mdl, 'fit7.txt')
data.jags <- c('weight', 'day', 'day2', 'trt')</pre>
parms <- c('b0' , 'b1', 'b2', 'vv')
fit7 <- jags(data= data.jags, parameters.to.save = parms,</pre>
                  model.file = 'fit7.txt', inits = NULL,
                  n.iter = 12000, n.thin = 5, n.burnin = 2000,
                  n.chains = 5
 Compiling model graph
    Resolving undeclared variables
    Allocating nodes
 Graph information:
    Observed stochastic nodes: 360
    Unobserved stochastic nodes: 10
    Total graph size: 1510
 Initializing model
fit7
 Inference for Bugs model at "fit7.txt", fit using jags,
  5 chains, each with 12000 iterations (first 2000 discarded), n.thin = 5
  n.sims = 10000 iterations saved
           mu.vect sd.vect
                                          25%
                                                   50%
                                                             75%
                                                                    97.5% Rhat
                                2.5%
                                       14.384
 b0[1]
            14.902
                     0.763
                              13.410
                                                14.899
                                                          15.413
                                                                   16.410 1.001
            14.954
                     0.750
                                       14.448
                                                14.953
                                                          15.454
                                                                   16.436 1.001
 b0[2]
                             13.495
            14.360
                     0.759
                                       13.841
                                                14.362
                                                                   15.840 1.001
 b0[3]
                             12.903
                                                          14.868
             1.723
                     0.119
                             1.487
                                       1.643
                                                 1.725
                                                          1.804
                                                                    1.958 1.001
 b1[1]
             2.011
                     0.118
                             1.783
                                       1.930
                                                 2.012
                                                          2.092
                                                                    2.240 1.001
 b1[2]
 b1[3]
             1.424
                     0.119
                             1.192
                                        1.343
                                                 1.423
                                                          1.506
                                                                    1.653 1.001
                     0.004
 b2[1]
            -0.029
                             -0.036
                                       -0.031
                                                -0.029
                                                          -0.026
                                                                   -0.021 1.001
 b2[2]
            -0.040
                     0.004
                              -0.048
                                       -0.043
                                                -0.040
                                                          -0.038
                                                                   -0.033 1.001
 b2[3]
            -0.025
                     0.004
                              -0.033
                                       -0.028
                                                -0.025
                                                          -0.023
                                                                   -0.018 1.001
 ٧٧
            13.847
                     1.037
                              11.948
                                       13.130
                                                13.800
                                                          14.527
                                                                   15.990 1.001
 deviance 1967.114
                     4.509 1960.282 1963.829 1966.427 1969.665 1977.698 1.001
          n.eff
 b0[1]
          10000
           5200
 b0[2]
 b0[3]
          10000
```

```
b1[1]
          10000
 b1[2]
           5400
 b1[3]
          10000
 b2[1]
          10000
 b2[2]
           8800
 b2[3]
          10000
           9300
 VV
 deviance 6700
 For each parameter, n.eff is a crude measure of effective sample size,
 and Rhat is the potential scale reduction factor (at convergence, Rhat=1).
 DIC info (using the rule, pD = var(deviance)/2)
 pD = 10.2 and DIC = 1977.3
 DIC is an estimate of expected predictive error (lower deviance is better).
# Diagnostics are good
sims <- as.mcmc(fit7)</pre>
chains <- as.matrix(sims)</pre>
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
```

18. Repeat this analysis using brm. You will need to create a factor variable from trt (that is, let a variable be as.factor(pig\$trt)). Use the same priors for the β 's and a gamma(1.1,.5) as the prior for σ_{error}^2 . Print a summary of the model. What is the looic?

looic is 1978.6

```
pig$trtf = as.factor(pig$trt)
fit8 <- brm(formula = weight ~ -1 + trtf + trtf:day + trtf:day2 , data =
pig,
            family = "gaussian",
            prior = c(set_prior("normal(0,100)", class = "b"),
                      set_prior("gamma(1.1,0.5)", class = "sigma")),
            warmup = 1000, iter = 5000, chains = 4,
            #control = list(adapt delta = 0.98),
            save pars = save pars(all = TRUE))
summary(fit8)
  Family: gaussian
   Links: mu = identity; sigma = identity
 Formula: weight ~ -1 + trtf + trtf:day + trtf:day2
    Data: pig (Number of observations: 360)
 Samples: 4 chains, each with iter = 5000; warmup = 1000; thin = 1;
          total post-warmup samples = 16000
```

```
Population-Level Effects:
            Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk ESS Tail ESS
               14.91
                          0.75
                                   13.46
                                            16.38 1.00
                                                            9090
 trtf1
                                                                    10205
 trtf2
               14.97
                          0.76
                                   13.48
                                            16.45 1.00
                                                            9245
                                                                    10474
               14.36
                          0.75
                                   12.88
 trtf3
                                            15.84 1.00
                                                            8936
                                                                    10113
 trtf1:day
                1.72
                          0.12
                                    1.49
                                             1.95 1.00
                                                                     9099
                                                            8112
 trtf2:day
                2.01
                          0.12
                                    1.78
                                             2.24 1.00
                                                            7920
                                                                     9231
 trtf3:day
                                    1.19
                                             1.66 1.00
                                                                     8821
                1.42
                          0.12
                                                            7623
 trtf1:day2
               -0.03
                          0.00
                                   -0.04
                                            -0.02 1.00
                                                            8999
                                                                     9997
                                            -0.03 1.00
 trtf2:day2
               -0.04
                          0.00
                                   -0.05
                                                            8704
                                                                    10182
 trtf3:day2
                          0.00
                                            -0.02 1.00
                                                                     9337
               -0.03
                                   -0.03
                                                            8326
 Family Specific Parameters:
       Estimate Est.Error 1-95% CI u-95% CI Rhat Bulk ESS Tail ESS
                     0.14
           3.72
                               3.45
                                        4.00 1.00
                                                      13182
                                                               10252
 sigma
 Samples were drawn using sampling(NUTS). For each parameter, Bulk ESS
 and Tail ESS are effective sample size measures, and Rhat is the potential
 scale reduction factor on split chains (at convergence, Rhat = 1).
# Convergence and ESS are good
chains <- as.matrix(fit8)</pre>
dim(chains)
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
# Looic
loo(fit8)
 Computed from 16000 by 360 log-likelihood matrix
          Estimate
                     SE
 elpd loo
            -989.4 15.5
 p_loo
              11.2 1.3
            1978.7 30.9
 looic
 Monte Carlo SE of elpd_loo is 0.0.
 All Pareto k estimates are good (k < 0.5).
 See help('pareto-k-diagnostic') for details.
```

19. Now we are going to take into account that there are multiple measures on each animal to properly account for this information when we make inference. Using JAGS, create a term for a random deviation from the intercept for the intercepts only (this is sometimes referred to as a random coefficients approach). That is, we are assuming each pig is a random draw from the population of all pigs. You will need to account for the numbering of the pigs in the data set, since pig 1 in trt1 is not the same as pig 1 in trt 2. In this model you will be estimating 60 intercepts (20 pigs in each of the three

treatments). Use a gamma(1.1,.1) prior for $\sigma_{intercepts}^2$. You will also need a prior for all the terms that are deviations from the overall intercept. Use a normal with a mean of 0, and a precision that is $1/\sigma_{intercepts}^2$. What is the DIC?

DIC is 1696.2

```
pig$newpig <- rep(1:60, each = 6)
mdl <- "
  model {
  for (i in 1:360){
    weight[i] ~ dnorm(mu[i], 1/vv)
    mu[i] <- b0[trt[i]] + b1[trt[i]]*day[i] + b2[trt[i]]*day2[i] +</pre>
u0[newpig[i]]
  for(i in 1:3){
    b0[i] \sim dnorm(0, 0.0001)
    b1[i] ~ dnorm(0, 0.0001)
    b2[i] \sim dnorm(0, 0.0001)
  }
  for(i in 1:60){
   u0[i] ~ dnorm(0,1/vvint)
  }
  vvint ~ dgamma(1.1, 0.1)
  vv \sim dgamma(1.1, 0.1)
}
newpig <- pig$newpig
trt = pig$trt
day = pig$day
day2 = pig$day2
weight = pig$weight
writeLines(mdl, 'fit9.txt')
data.jags <- c('weight', 'day', 'day2', 'trt', 'newpig')</pre>
parms <- c('b0', 'b1', 'b2', 'u0', 'vv', 'vvint')
fit9 <- jags(data= data.jags, parameters.to.save = parms,</pre>
                   model.file = 'fit9.txt', inits = NULL,
                   n.iter = 8000, n.thin = 5, n.burnin = 2000,
                   n.chains = 5)
```

Compiling model graph Resolving undeclared variables Allocating nodes Graph information:

Observed stochastic nodes: 360 Unobserved stochastic nodes: 71

Total graph size: 2274

Initializing model fit9 Inference for Bugs model at "fit9.txt", fit using jags, 5 chains, each with 8000 iterations (first 2000 discarded), n.thin = 5 n.sims = 6000 iterations saved 25% 50% 75% 97.5% mu.vect sd.vect 2.5% Rhat b0[1] 14.898 0.821 13.278 14.353 14.894 15.461 16.519 1.001 b0[2] 14.968 0.820 13.363 14.414 14.981 15.518 16.566 1.001 b0[3] 14.365 0.801 12.787 13.815 14.386 14.914 15.910 1.001 1.724 0.073 1.583 1.674 1.725 1.774 b1[1] 1.868 1.001 2.011 2.152 1.002 2.011 0.073 b1[2] 1.871 1.960 2.061 b1[3] 1.423 0.073 1.283 1.374 1.425 1.472 1.566 1.001 -0.029 0.002 -0.034 -0.031 -0.029 -0.027 -0.024 1.001 b2[1] b2[2] -0.040 0.002 -0.045 -0.042 -0.040 -0.039 -0.036 1.002 -0.030 0.002 -0.027 -0.025 -0.024 b2[3] -0.025 -0.021 1.001 u0[1] -3.576 1.094 -5.708 -4.315 -3.566 -2.838 -1.444 1.001 u0[2] 1.733 1.117 -0.469 0.995 1.723 2.480 3.952 1.001 u0[3] -4.090 1.099 -6.218 -4.817 -4.104 -3.375 -1.949 1.001 1.692 1.099 -0.488 0.953 1.692 2.421 3.790 1.001 u0[4] u0[5] 5.016 1.114 4.260 5.027 5.788 7.197 1.002 2.817 u0[6] 0.967 1.105 -1.159 0.217 0.963 1.722 3.106 1.002 u0[7] 0.298 1.110 -1.901 -0.435 0.302 1.038 2.510 1.001 -3.952 u0[8] **-1.753** 1.108 -2.504 -1.748-1.012 0.420 1.001 -0.930 1.001 u0[9] -2.993 1.088 -5.200 -3.713 -2.989 -2.245 u0[10] -1.328 1.095 -3.429 -2.069 -1.333 -0.586 0.817 1.001 1.103 -4.109 -2.729 -1.981 u0[11] -1.979 -1.241 0.209 1.001 1.527 1.110 -0.632 0.786 1.516 2.254 3.743 1.001 u0[12] 1.276 1.110 -0.869 0.526 1.284 2.014 3.463 1.001 u0[13] -2.243 1.108 -4.467 -2.962 -2.243 -1.502 -0.110 1.001 u0[14] 4.099 1.095 1.977 3.362 4.091 4.828 6.264 1.001 u0[15] 1.126 u0[16] 6.148 4.001 5.367 6.127 6.898 8.371 1.001 1.117 -2.986 -1.525 -0.748 -0.006 1.332 1.001 u0[17] -0.767 -2.547 1.107 -4.738 -3.285 -2.562 -1.801 -0.338 1.001 u0[18] -1.922 u0[19] 0.237 1.109 -0.502 0.224 0.962 2.432 1.001 1.095 -3.841 -2.458 -1.747-0.980 u0[20] -1.726 0.390 1.001 1.593 1.082 -0.560 0.867 1.592 2.312 3.711 1.001 u0[21] u0[22] -2.463 1.099 -4.634 -3.184 -2.460 -1.739 -0.300 1.001 -3.352 1.108 -5.514 -4.092 -3.354 -2.590 -1.177 1.001 u0[23] u0[24] 0.643 1.094 -1.507 -0.096 0.653 1.364 2.779 1.001 4.049 5.461 6.982 u0[25] 6.219 1.108 6.208 8.377 1.001

```
u0[26]
            -0.426
                      1.095
                               -2.610
                                         -1.149
                                                   -0.433
                                                              0.317
                                                                        1.713 1.001
             4.053
                      1.109
                                1.890
                                                    4.048
                                                              4.784
                                                                        6.236 1.001
u0[27]
                                          3.303
             1.416
                                                    1.406
                                                              2.156
u0[28]
                      1.091
                               -0.717
                                          0.681
                                                                        3.570 1.001
            -3.591
                      1.107
                               -5.729
                                         -4.337
                                                   -3.586
                                                             -2.845
                                                                       -1.388 1.001
u0[29]
                                                   -2.646
u0[30]
            -2.642
                      1.107
                               -4.757
                                         -3.404
                                                             -1.883
                                                                       -0.482 1.001
             1.797
                      1.101
                                                    1.782
                                                                        3.971 1.001
u0[31]
                               -0.317
                                          1.042
                                                              2.539
u0[32]
             3.324
                      1.121
                                1.166
                                          2.563
                                                    3.327
                                                              4.076
                                                                        5.525 1.001
                      1.093
                                                    1.957
                                                                        4.130 1.001
u0[33]
             1.965
                               -0.117
                                          1.208
                                                              2.705
            -2.076
                      1.105
                               -4.229
                                                   -2.058
                                                                        0.034 1.002
u0[34]
                                         -2.826
                                                             -1.321
u0[35]
             2.674
                      1.106
                                0.512
                                          1.946
                                                    2.662
                                                              3.429
                                                                        4.853 1.001
                                                             -2.968
u0[36]
            -3.705
                      1.100
                               -5.841
                                         -4.456
                                                   -3.700
                                                                       -1.527 1.001
            -1.880
                      1.099
                               -3.996
                                         -2.623
                                                   -1.882
                                                             -1.150
                                                                        0.276 1.001
u0[37]
u0[38]
            -3.518
                      1.102
                               -5.682
                                         -4.275
                                                   -3.512
                                                             -2.767
                                                                       -1.397 1.001
u0[39]
             2.672
                      1.093
                                0.508
                                          1.948
                                                    2.660
                                                              3.397
                                                                        4.800 1.001
u0[40]
            -2.796
                      1.107
                               -4.960
                                         -3.545
                                                   -2.812
                                                             -2.050
                                                                       -0.596 1.001
u0[41]
             0.647
                      1.092
                               -1.473
                                         -0.082
                                                    0.654
                                                              1.376
                                                                        2.834 1.001
u0[42]
            -1.767
                      1.096
                               -3.932
                                         -2.495
                                                   -1.758
                                                             -1.016
                                                                        0.369 1.001
u0[43]
            -4.827
                      1.096
                               -6.974
                                         -5.587
                                                   -4.825
                                                             -4.064
                                                                       -2.713 1.001
                                                   -3.788
u0[44]
            -3.760
                      1.093
                               -5.862
                                         -4.511
                                                             -3.029
                                                                       -1.576 1.001
                                         -0.132
u0[45]
             0.597
                      1.097
                               -1.587
                                                    0.594
                                                              1.337
                                                                        2.715 1.001
                                                    4.999
u0[46]
             5.018
                      1.090
                                2.897
                                          4.276
                                                              5.737
                                                                        7.147 1.001
                      1.090
                               -1.455
                                                                        2.791 1.001
u0[47]
             0.688
                                         -0.041
                                                    0.698
                                                              1.445
            -2.065
                      1.073
                               -4.181
                                         -2.774
                                                   -2.056
                                                             -1.353
                                                                        0.013 1.002
u0[48]
u0[49]
             0.957
                      1.089
                               -1.181
                                          0.232
                                                    0.945
                                                              1.691
                                                                        3.093 1.001
                      1.096
                               -0.859
                                          0.566
                                                    1.313
                                                              2.052
                                                                        3.454 1.001
u0[50]
             1.305
u0[51]
            -3.020
                      1.109
                               -5.163
                                         -3.784
                                                   -3.017
                                                             -2.255
                                                                       -0.813 1.001
                      1.096
                                                    1.584
                                                              2.347
                                                                        3.787 1.001
u0[52]
             1.603
                               -0.546
                                          0.869
                               -2.155
            -0.017
                      1.089
                                                   -0.014
                                                              0.723
                                                                        2.063 1.001
u0[53]
                                         -0.764
             3.511
                      1.078
                                1.354
                                                    3.526
                                                              4.227
                                                                        5.649 1.001
u0[54]
                                          2.777
                      1.085
u0[55]
             2.233
                                0.147
                                          1.496
                                                    2.227
                                                              2.963
                                                                        4.388 1.001
u0[56]
            -2.155
                      1.101
                               -4.302
                                                   -2.143
                                                             -1.403
                                                                       -0.011 1.001
                                         -2.895
u0[57]
            -2.569
                      1.097
                               -4.739
                                         -3.285
                                                   -2.581
                                                             -1.836
                                                                       -0.401 1.001
u0[58]
            -0.649
                      1.104
                               -2.804
                                         -1.406
                                                   -0.637
                                                              0.099
                                                                        1.515 1.001
                      1.096
                                          3.553
                                                    4.284
                                                                        6.481 1.001
u0[59]
             4.303
                                2.187
                                                              5.059
            -0.296
                      1.105
u0[60]
                               -2.398
                                         -1.066
                                                   -0.301
                                                              0.442
                                                                        1.887 1.001
                      0.437
             5.243
                                4.444
                                          4.935
                                                    5.221
                                                              5.527
                                                                        6.147 1.001
٧V
             9.174
                      1.907
                                          7.827
                                                    8.947
                                                             10.220
vvint
                                6.139
                                                                       13.607 1.001
deviance 1615.463
                     12.810 1592.802 1606.477 1614.755 1623.356 1642.846 1.001
          n.eff
b0[1]
           6000
           5700
b0[2]
b0[3]
           6000
b1[1]
           6000
           2500
b1[2]
           6000
b1[3]
b2[1]
           6000
b2[2]
           2800
           6000
b2[3]
u0[1]
           3300
           4300
u0[2]
```

u0[3]	6000
u0[4]	6000
u0[5]	1700
u0[6]	2900
u0[7]	6000
	5800
u0[8]	
u0[9]	5500
u0[10]	3500
u0[11]	4500
u0[12]	6000
u0[13]	3300
u0[14]	6000
u0[15]	6000
u0[16]	6000
u0[17]	6000
u0[18]	6000
u0[19]	6000
u0[20]	6000
u0[21]	4800
u0[22]	6000
u0[23]	6000
u0[24]	4900
u0[25]	5500
u0[26]	6000
u0[27]	5900
u0[28]	6000
u0[29]	6000
u0[30]	6000
u0[31]	6000
u0[32]	4200
u0[33]	6000
u0[34]	2800
u0[35]	6000
u0[36]	3800
u0[37]	6000
u0[38]	6000
u0[39]	6000
u0[40]	6000
u0[41]	4900
u0[42]	6000
u0[43]	6000
u0[44]	4400
u0[45]	6000
u0[46]	6000
u0[47]	6000
u0[47] u0[48]	2200
u0[40] u0[49]	6000
u0[50]	6000
u0[50] u0[51]	3300
u0[51] u0[52]	6000
u0[J2]	

```
u0[53]
           5300
 u0[54]
           6000
 u0[55]
           4100
           6000
 u0[56]
 u0[57]
           4200
 u0[58]
           6000
 u0[59]
           6000
 u0[60]
           5800
 VV
           6000
 vvint
           6000
 deviance 5500
 For each parameter, n.eff is a crude measure of effective sample size,
 and Rhat is the potential scale reduction factor (at convergence, Rhat=1).
 DIC info (using the rule, pD = var(deviance)/2)
 pD = 82.0 and DIC = 1697.5
 DIC is an estimate of expected predictive error (lower deviance is better).
# Diagnostics are good
sims <- as.mcmc(fit9)</pre>
chains <- as.matrix(sims)</pre>
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
```

20. Continuing in JAGS, add a random coefficient term for the deviations from the linear term in the model. You will now have random coefficients for both the intercepts and the linear term (slopes). You can use a gamma(1.1,.1) prior for σ_{linear}^2 . You will also need a prior for all the terms that are deviations from the overall linear term (slopes). Use a normal with a mean of 0, and a precision that is $1/\sigma_{linear}^2$. What is the DIC?

DIC is 1451.0

```
mdl <- "
    model {

    for (i in 1:360){
        weight[i] ~ dnorm(mu[i], 1/vv)
        mu[i] <- b0[trt[i]] + b1[trt[i]]*day[i] + b2[trt[i]]*day2[i] +

    u0[newpig[i]] + u1[newpig[i]]*day[i]
    }

    for(i in 1:3){
        b0[i] ~ dnorm(0, 0.0001)
        b1[i] ~ dnorm(0, 0.0001)
        b2[i] ~ dnorm(0, 0.0001)
    }

    for(i in 1:60){</pre>
```

```
u0[i] \sim dnorm(0,1/vvint)
    u1[i] \sim dnorm(0, 1/vvslp)
  }
  vvslp \sim dgamma(1.1, 0.1)
  vvint ~ dgamma(1.1, 0.1)
  vv \sim dgamma(1.1, 0.1)
newpig <- pig$newpig
trt = pig$trt
day = pig$day
day2 = pig$day2
weight = pig$weight
writeLines(mdl, 'fit10.txt')
data.jags <- c('weight', 'day', 'day2', 'trt', 'newpig')</pre>
parms <- c('b0' , 'b1', 'b2', 'u0', 'u1', 'vv', 'vvint', 'vvslp')</pre>
fit10 <- jags(data= data.jags, parameters.to.save = parms,</pre>
                  model.file = 'fit10.txt', inits = NULL,
                  n.iter = 20000, n.thin = 5, n.burnin = 2000,
                  n.chains = 5)
 Compiling model graph
    Resolving undeclared variables
    Allocating nodes
 Graph information:
    Observed stochastic nodes: 360
    Unobserved stochastic nodes: 132
    Total graph size: 2696
 Initializing model
fit10
 Inference for Bugs model at "fit10.txt", fit using jags,
  5 chains, each with 20000 iterations (first 2000 discarded), n.thin = 5
  n.sims = 18000 iterations saved
           mu.vect sd.vect
                                          25%
                                                    50%
                                                             75%
                                                                    97.5% Rhat
                                2.5%
            14.899
                     0.386
                              14.148
                                       14.639
                                                14.898
                                                          15.156
                                                                   15.677 1.001
 b0[1]
 b0[2]
            14.954
                     0.385
                              14.205
                                       14.698
                                                14.952
                                                          15.211
                                                                   15.711 1.001
                                                                   15.123 1.001
                                       14.098
                                                14.355
 b0[3]
            14.356
                     0.387
                              13.602
                                                          14.616
 b1[1]
             1.725
                     0.061
                              1.603
                                        1.684
                                                 1.724
                                                           1.766
                                                                    1.844 1.001
 b1[2]
             2.010
                     0.061
                               1.889
                                        1.969
                                                 2.011
                                                           2.051
                                                                    2.131 1.001
 b1[3]
             1.424
                     0.062
                               1.303
                                        1.383
                                                 1.424
                                                           1.465
                                                                    1.546 1.001
            -0.029
                     0.001 -0.032
                                       -0.030
                                                 -0.029
                                                          -0.028
                                                                   -0.026 1.001
 b2[1]
```

December December	h2[2]	0.040	0 002	0.042	0 041	0.040	0 020	0 027 1 001
uelij -0.624 0.773 -2.206 -1.134 -0.610 -0.180 0.851 1.001 uelij -0.712 0.772 -2.245 -1.223 -0.707 -0.193 0.788 1.001 uelij 0.967 0.790 -0.538 0.422 0.954 1.494 2.568 1.001 uelij 0.948 0.781 -0.538 0.413 0.936 1.462 2.516 1.001 uelij 0.388 0.768 -1.108 0.918 0.972 1.021 1.029 1.001 uelij 0.168 0.168 0.766 -1.294 -0.337 -1.166 -0.629 0.325 1.001 uelij 0.066 0.766 -1.443 -0.441 0.071 0.568 1.584 1.001 uelij 0.066 0.766 -1.443 -0.417 0.092 0.661 1.594 0.041 0.092 0.661 1.594 0.041 0.092 0.061 1.596 0.041 0.149<	b2[2]	-0.040	0.002	-0.043	-0.041	-0.040	-0.039	-0.037 1.001
UB 2								
μθ[3] -0.712 0.772 -2.245 -1.223 -0.707 -0.193 0.788 1.001 μθ[5] 0.948 0.780 -0.545 0.422 0.954 1.462 2.516 1.001 μθ[6] 0.398 0.768 -1.108 -0.118 0.396 0.162 1.929 1.001 μθ[7] -1.181 0.799 -2.798 -1.713 -1.166 -0.629 0.325 1.001 μθ[1] 0.066 0.766 -1.443 -0.441 0.071 0.568 1.584 1.001 μθ[11] 0.094 0.764 -1.421 -0.417 0.022 0.601 1.596 1.001 μθ[11] 0.094 0.764 -1.421 -0.417 0.092 0.601 1.596 1.001 μθ[11] -0.094 0.761 -1.693 -0.687 -0.177 0.249 0.717 1.001 μθ[13] -0.273 0.761 -1.893 -0.687 -0.177 0.324 1.325 <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>								
uθ[4] 0.967 0.790 -0.545 0.422 0.954 1.494 2.568 1.001 uθ[6] 0.948 0.781 -0.538 0.413 0.936 1.462 2.516 1.001 uθ[6] 0.398 0.768 -1.108 -0.118 0.396 0.912 1.292 1.001 uθ[8] 0.168 0.757 -1.294 -0.337 0.167 0.668 1.654 1.001 uθ[1] 0.066 0.766 -1.431 -0.441 0.071 0.568 1.584 1.001 uθ[11] 0.094 0.764 -1.421 -0.417 0.092 0.601 1.596 1.001 uθ[13] 0.287 0.764 -1.185 -0.237 0.283 0.794 1.865 1.001 uθ[14] -0.183 0.763 -1.702 -0.687 -0.177 0.324 1.355 1.001 uθ[15] 1.279 0.761 -1.775 -0.762 -0.277 0.225 1.273 1.001 </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
ue[5] 0.948 0.781 -0.538 0.413 0.936 1.462 2.516 1.001 ue[7] -1.181 0.799 -2.798 -1.713 -1.166 -0.629 0.325 1.001 ue[8] 0.168 0.757 -1.294 -0.337 0.167 0.668 1.584 1.001 ue[10] -0.773 0.777 -2.334 -1.285 -0.764 -0.249 0.717 1.001 ue[11] 0.094 0.764 -1.421 -0.417 0.092 0.601 1.596 1.001 ue[13] 0.287 0.764 -1.421 -0.417 0.092 0.601 1.596 1.001 ue[13] 0.287 0.764 -1.487 -0.237 0.682 -0.160 0.333 1.395 1.001 ue[13] 0.287 0.763 -1.702 -0.687 -0.177 0.324 1.325 1.001 ue[14] -0.183 0.763 -1.792 -0.687 -0.177 0.324 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>								
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u0[20] -1.206 0.801 -2.822 -1.733 -1.190 -0.660 0.345 1.001 u0[21] 0.364 0.762 -1.127 -0.148 0.363 0.868 1.859 1.001 u0[22] 0.400 0.765 -1.082 -0.106 0.385 0.904 1.932 1.001 u0[23] -0.592 0.770 -2.131 -1.101 -0.586 -0.065 0.877 1.001 u0[24] 0.029 0.761 -1.481 -0.475 0.627 0.531 1.534 1.001 u0[25] 1.336 0.807 -0.187 0.783 1.319 1.862 2.981 1.001 u0[26] -0.716 0.773 -2.278 -1.227 -0.707 -0.196 0.767 1.001 u0[27] 0.511 0.771 -0.996 -0.007 0.593 1.021 2.042 1.001 u0[28] 0.340 0.763 -1.140 -0.171 0.329 0.388 1.870								
u0[21] 0.364 0.762 -1.127 -0.148 0.363 0.868 1.859 1.001 u0[22] 0.400 0.765 -1.082 -0.106 0.385 0.904 1.932 1.001 u0[23] -0.592 0.770 -2.131 -1.101 -0.586 -0.065 0.877 1.001 u0[24] 0.029 0.761 -1.481 -0.475 0.027 0.531 1.534 1.001 u0[26] -0.716 0.773 -2.278 -1.227 -0.707 -0.196 0.767 1.001 u0[27] 0.511 0.771 -0.996 -0.007 0.503 1.021 2.042 1.001 u0[28] 0.340 0.763 -1.140 -0.171 0.329 0.838 1.870 1.001 u0[29] -0.880 0.774 -2.452 -1.395 -0.867 -0.351 0.598 1.001 u0[30] -0.271 0.763 -1.787 -0.777 -0.271 0.235 1.511								
u0[22] 0.400 0.765 -1.082 -0.106 0.385 0.904 1.932 1.001 u0[23] -0.592 0.770 -2.131 -1.101 -0.586 -0.065 0.877 1.001 u0[24] 0.029 0.761 -1.481 -0.475 0.027 0.531 1.534 1.001 u0[25] 1.336 0.807 -0.187 0.783 1.319 1.862 2.981 1.001 u0[26] -0.716 0.773 -2.278 -1.227 -0.707 -0.196 0.767 1.001 u0[27] 0.511 0.771 -0.996 -0.007 0.503 1.021 2.042 1.001 u0[28] 0.340 0.763 -1.140 -0.171 0.329 0.838 1.870 1.001 u0[29] -0.880 0.774 -2.452 -1.395 -0.867 -0.351 0.598 1.001 u0[30] -0.271 0.763 -1.887 -0.777 -0.271 0.235 1.219	u0[20]		0.801	-2.822	-1.733	-1.190	-0.660	0.345 1.001
u0[23] -0.592 0.770 -2.131 -1.101 -0.586 -0.065 0.877 1.001 u0[24] 0.029 0.761 -1.481 -0.475 0.027 0.531 1.534 1.001 u0[25] 1.336 0.807 -0.187 0.783 1.319 1.862 2.981 1.001 u0[26] -0.716 0.773 -2.278 -1.227 -0.707 -0.196 0.767 1.001 u0[27] 0.511 0.771 -0.996 -0.007 0.503 1.021 2.042 1.001 u0[28] 0.340 0.763 -1.140 -0.171 0.329 0.838 1.870 1.001 u0[29] -0.880 0.774 -2.452 -1.395 -0.867 -0.351 0.598 1.001 u0[30] -0.271 0.763 -1.787 -0.777 -0.271 0.235 1.219 1.001 u0[31] -0.358 0.767 -1.888 -0.863 -0.357 0.151 1.159	u0[21]	0.364	0.762	-1.127	-0.148	0.363	0.868	1.859 1.001
u0[24] 0.029 0.761 -1.481 -0.475 0.027 0.531 1.534 1.001 u0[25] 1.336 0.807 -0.187 0.783 1.319 1.862 2.981 1.001 u0[26] -0.716 0.773 -2.278 -1.227 -0.707 -0.196 0.767 1.001 u0[27] 0.511 0.771 -0.996 -0.007 0.503 1.021 2.042 1.001 u0[28] 0.340 0.763 -1.140 -0.171 0.329 0.838 1.870 1.001 u0[29] -0.880 0.774 -2.452 -1.395 -0.867 -0.351 0.598 1.001 u0[30] -0.271 0.763 -1.787 -0.777 -0.271 0.235 1.219 1.001 u0[31] -0.358 0.767 -1.888 -0.863 -0.357 0.151 1.159 1.001 u0[32] 0.238 0.760 -1.245 -0.273 0.232 0.744 1.734	u0[22]	0.400	0.765	-1.082	-0.106	0.385	0.904	1.932 1.001
u0[25] 1.336 0.807 -0.187 0.783 1.319 1.862 2.981 1.001 u0[26] -0.716 0.773 -2.278 -1.227 -0.707 -0.196 0.767 1.001 u0[27] 0.511 0.771 -0.996 -0.007 0.503 1.021 2.042 1.001 u0[28] 0.340 0.763 -1.140 -0.171 0.329 0.838 1.870 1.001 u0[29] -0.880 0.774 -2.452 -1.395 -0.867 -0.351 0.598 1.001 u0[30] -0.271 0.763 -1.787 -0.777 -0.271 0.235 1.219 1.001 u0[31] -0.358 0.767 -1.888 -0.863 -0.357 0.151 1.159 1.001 u0[32] 0.238 0.760 -1.245 -0.273 0.232 0.744 1.734 1.001 u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104 1.001 u0[35] 0.584 0.767 -0.891 0.069 <t< td=""><td></td><td>-0.592</td><td>0.770</td><td>-2.131</td><td>-1.101</td><td>-0.586</td><td></td><td>0.877 1.001</td></t<>		-0.592	0.770	-2.131	-1.101	-0.586		0.877 1.001
u0[26] -0.716 0.773 -2.278 -1.227 -0.707 -0.196 0.767 1.001 u0[27] 0.511 0.771 -0.996 -0.007 0.503 1.021 2.042 1.001 u0[28] 0.340 0.763 -1.140 -0.171 0.329 0.838 1.870 1.001 u0[29] -0.880 0.774 -2.452 -1.395 -0.867 -0.351 0.598 1.001 u0[30] -0.271 0.763 -1.787 -0.777 -0.271 0.235 1.219 1.001 u0[31] -0.358 0.767 -1.888 -0.863 -0.357 0.151 1.159 1.001 u0[32] 0.238 0.760 -1.245 -0.273 0.232 0.744 1.734 1.001 u0[33] 0.671 0.772 -0.807 0.152 0.663 1.178 2.212 1.001 u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104	u0[24]	0.029	0.761	-1.481	-0.475	0.027	0.531	1.534 1.001
u0[27] 0.511 0.771 -0.996 -0.007 0.503 1.021 2.042 1.001 u0[28] 0.340 0.763 -1.140 -0.171 0.329 0.838 1.870 1.001 u0[29] -0.880 0.774 -2.452 -1.395 -0.867 -0.351 0.598 1.001 u0[30] -0.271 0.763 -1.787 -0.777 -0.271 0.235 1.219 1.001 u0[31] -0.358 0.767 -1.888 -0.863 -0.357 0.151 1.159 1.001 u0[32] 0.238 0.760 -1.245 -0.273 0.232 0.744 1.734 1.001 u0[33] 0.671 0.772 -0.807 0.152 0.663 1.178 2.212 1.001 u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104 1.001 u0[35] 0.584 0.767 -0.891 0.069 0.567 1.092 2.126 1.001 u0[37] 0.092 0.788 -2.493 -1.402 -	u0[25]	1.336	0.807	-0.187	0.783	1.319	1.862	2.981 1.001
u0[28] 0.340 0.763 -1.140 -0.171 0.329 0.838 1.870 1.001 u0[29] -0.880 0.774 -2.452 -1.395 -0.867 -0.351 0.598 1.001 u0[30] -0.271 0.763 -1.787 -0.777 -0.271 0.235 1.219 1.001 u0[31] -0.358 0.767 -1.888 -0.863 -0.357 0.151 1.159 1.001 u0[32] 0.238 0.760 -1.245 -0.273 0.232 0.744 1.734 1.001 u0[33] 0.671 0.772 -0.807 0.152 0.663 1.178 2.212 1.001 u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104 1.001 u0[35] 0.584 0.767 -0.891 0.069 0.567 1.092 2.126 1.001 u0[37] 0.092 0.788 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.779 -2.397 -1.333	u0[26]	-0.716	0.773	-2.278	-1.227	-0.707	-0.196	0.767 1.001
u0[29] -0.880 0.774 -2.452 -1.395 -0.867 -0.351 0.598 1.001 u0[30] -0.271 0.763 -1.787 -0.777 -0.271 0.235 1.219 1.001 u0[31] -0.358 0.767 -1.888 -0.863 -0.357 0.151 1.159 1.001 u0[32] 0.238 0.760 -1.245 -0.273 0.232 0.744 1.734 1.001 u0[33] 0.671 0.772 -0.807 0.152 0.663 1.178 2.212 1.001 u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104 1.001 u0[35] 0.584 0.767 -0.891 0.069 0.567 1.092 2.126 1.001 u0[36] -0.902 0.780 -2.493 -1.402 -0.887 -0.374 0.589 1.001 u0[37] 0.092 0.768 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.777 -0.751 0.197 <t< td=""><td>u0[27]</td><td>0.511</td><td>0.771</td><td>-0.996</td><td>-0.007</td><td>0.503</td><td>1.021</td><td>2.042 1.001</td></t<>	u0[27]	0.511	0.771	-0.996	-0.007	0.503	1.021	2.042 1.001
u0[30] -0.271 0.763 -1.787 -0.777 -0.271 0.235 1.219 1.001 u0[31] -0.358 0.767 -1.888 -0.863 -0.357 0.151 1.159 1.001 u0[32] 0.238 0.760 -1.245 -0.273 0.232 0.744 1.734 1.001 u0[33] 0.671 0.772 -0.807 0.152 0.663 1.178 2.212 1.001 u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104 1.001 u0[35] 0.584 0.767 -0.891 0.069 0.567 1.092 2.126 1.001 u0[36] -0.902 0.780 -2.493 -1.402 -0.887 -0.374 0.589 1.001 u0[37] 0.092 0.768 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.779 -2.397 -1.333 -0.801 -0.294 0.671	u0[28]	0.340	0.763	-1.140	-0.171	0.329	0.838	1.870 1.001
u0[31] -0.358 0.767 -1.888 -0.863 -0.357 0.151 1.159 1.001 u0[32] 0.238 0.760 -1.245 -0.273 0.232 0.744 1.734 1.001 u0[33] 0.671 0.772 -0.807 0.152 0.663 1.178 2.212 1.001 u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104 1.001 u0[35] 0.584 0.767 -0.891 0.069 0.567 1.092 2.126 1.001 u0[36] -0.902 0.780 -2.493 -1.402 -0.887 -0.374 0.589 1.001 u0[37] 0.092 0.768 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.779 -2.397 -1.333 -0.801 -0.294 0.671 1.001 u0[39] 0.729 0.777 -0.751 0.197 0.712 1.248 2.291 <	u0[29]	-0.880	0.774	-2.452	-1.395	-0.867	-0.351	0.598 1.001
u0[32] 0.238 0.760 -1.245 -0.273 0.232 0.744 1.734 1.001 u0[33] 0.671 0.772 -0.807 0.152 0.663 1.178 2.212 1.001 u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104 1.001 u0[35] 0.584 0.767 -0.891 0.069 0.567 1.092 2.126 1.001 u0[36] -0.902 0.780 -2.493 -1.402 -0.887 -0.374 0.589 1.001 u0[37] 0.092 0.768 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.779 -2.397 -1.333 -0.801 -0.294 0.671 1.001 u0[39] 0.729 0.777 -0.751 0.197 0.712 1.248 2.291 1.001 u0[40] -0.355 0.762 -1.865 -0.856 -0.341 0.157 1.116 1.001 u0[41] 1.141 0.789 -0.362 0.601 1.	u0[30]	-0.271	0.763	-1.787	-0.777	-0.271	0.235	1.219 1.001
u0[33] 0.671 0.772 -0.807 0.152 0.663 1.178 2.212 1.001 u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104 1.001 u0[35] 0.584 0.767 -0.891 0.069 0.567 1.092 2.126 1.001 u0[36] -0.902 0.780 -2.493 -1.402 -0.887 -0.374 0.589 1.001 u0[37] 0.092 0.768 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.779 -2.397 -1.333 -0.801 -0.294 0.671 1.001 u0[39] 0.729 0.777 -0.751 0.197 0.712 1.248 2.291 1.001 u0[40] -0.355 0.762 -1.865 -0.856 -0.341 0.157 1.116 1.001 u0[41] 1.141 0.789 -0.362 0.601 1.120 1.667 2.738 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -	u0[31]	-0.358	0.767	-1.888	-0.863	-0.357	0.151	1.159 1.001
u0[34] -0.377 0.759 -1.894 -0.883 -0.365 0.128 1.104 1.001 u0[35] 0.584 0.767 -0.891 0.069 0.567 1.092 2.126 1.001 u0[36] -0.902 0.780 -2.493 -1.402 -0.887 -0.374 0.589 1.001 u0[37] 0.092 0.768 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.779 -2.397 -1.333 -0.801 -0.294 0.671 1.001 u0[39] 0.729 0.777 -0.751 0.197 0.712 1.248 2.291 1.001 u0[40] -0.355 0.762 -1.865 -0.856 -0.341 0.157 1.116 1.001 u0[41] 1.141 0.789 -0.362 0.601 1.120 1.667 2.738 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[45] 1.211 0.795 -0.309 0.672 <td< td=""><td>u0[32]</td><td>0.238</td><td>0.760</td><td>-1.245</td><td>-0.273</td><td>0.232</td><td>0.744</td><td>1.734 1.001</td></td<>	u0[32]	0.238	0.760	-1.245	-0.273	0.232	0.744	1.734 1.001
u0[35] 0.584 0.767 -0.891 0.069 0.567 1.092 2.126 1.001 u0[36] -0.902 0.780 -2.493 -1.402 -0.887 -0.374 0.589 1.001 u0[37] 0.092 0.768 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.779 -2.397 -1.333 -0.801 -0.294 0.671 1.001 u0[39] 0.729 0.777 -0.751 0.197 0.712 1.248 2.291 1.001 u0[40] -0.355 0.762 -1.865 -0.856 -0.341 0.157 1.116 1.001 u0[41] 1.141 0.789 -0.362 0.601 1.120 1.667 2.738 1.001 u0[42] -0.702 0.775 -2.275 -1.207 -0.689 -0.175 0.781 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[45] 1.211 0.795 -0.309 0.672 <t< td=""><td>u0[33]</td><td>0.671</td><td>0.772</td><td>-0.807</td><td>0.152</td><td>0.663</td><td>1.178</td><td>2.212 1.001</td></t<>	u0[33]	0.671	0.772	-0.807	0.152	0.663	1.178	2.212 1.001
u0[36] -0.902 0.780 -2.493 -1.402 -0.887 -0.374 0.589 1.001 u0[37] 0.092 0.768 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.779 -2.397 -1.333 -0.801 -0.294 0.671 1.001 u0[39] 0.729 0.777 -0.751 0.197 0.712 1.248 2.291 1.001 u0[40] -0.355 0.762 -1.865 -0.856 -0.341 0.157 1.116 1.001 u0[41] 1.141 0.789 -0.362 0.601 1.120 1.667 2.738 1.001 u0[42] -0.702 0.775 -2.275 -1.207 -0.689 -0.175 0.781 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 <t< td=""><td>u0[34]</td><td>-0.377</td><td>0.759</td><td>-1.894</td><td>-0.883</td><td>-0.365</td><td>0.128</td><td>1.104 1.001</td></t<>	u0[34]	-0.377	0.759	-1.894	-0.883	-0.365	0.128	1.104 1.001
u0[37] 0.092 0.768 -1.395 -0.422 0.087 0.605 1.620 1.001 u0[38] -0.820 0.779 -2.397 -1.333 -0.801 -0.294 0.671 1.001 u0[39] 0.729 0.777 -0.751 0.197 0.712 1.248 2.291 1.001 u0[40] -0.355 0.762 -1.865 -0.856 -0.341 0.157 1.116 1.001 u0[41] 1.141 0.789 -0.362 0.601 1.120 1.667 2.738 1.001 u0[42] -0.702 0.775 -2.275 -1.207 -0.689 -0.175 0.781 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[44] -1.759 0.836 -3.466 -2.307 -1.741 -1.182 -0.180 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 <	u0[35]	0.584	0.767	-0.891	0.069	0.567	1.092	2.126 1.001
u0[38] -0.820 0.779 -2.397 -1.333 -0.801 -0.294 0.671 1.001 u0[39] 0.729 0.777 -0.751 0.197 0.712 1.248 2.291 1.001 u0[40] -0.355 0.762 -1.865 -0.856 -0.341 0.157 1.116 1.001 u0[41] 1.141 0.789 -0.362 0.601 1.120 1.667 2.738 1.001 u0[42] -0.702 0.775 -2.275 -1.207 -0.689 -0.175 0.781 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[44] -1.759 0.836 -3.466 -2.307 -1.741 -1.182 -0.180 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 <t< td=""><td>u0[36]</td><td>-0.902</td><td>0.780</td><td>-2.493</td><td>-1.402</td><td>-0.887</td><td>-0.374</td><td>0.589 1.001</td></t<>	u0[36]	-0.902	0.780	-2.493	-1.402	-0.887	-0.374	0.589 1.001
u0[39] 0.729 0.777 -0.751 0.197 0.712 1.248 2.291 1.001 u0[40] -0.355 0.762 -1.865 -0.856 -0.341 0.157 1.116 1.001 u0[41] 1.141 0.789 -0.362 0.601 1.120 1.667 2.738 1.001 u0[42] -0.702 0.775 -2.275 -1.207 -0.689 -0.175 0.781 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[44] -1.759 0.836 -3.466 -2.307 -1.741 -1.182 -0.180 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001	u0[37]	0.092	0.768	-1.395	-0.422	0.087	0.605	1.620 1.001
u0[40] -0.355 0.762 -1.865 -0.856 -0.341 0.157 1.116 1.001 u0[41] 1.141 0.789 -0.362 0.601 1.120 1.667 2.738 1.001 u0[42] -0.702 0.775 -2.275 -1.207 -0.689 -0.175 0.781 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[44] -1.759 0.836 -3.466 -2.307 -1.741 -1.182 -0.180 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001	u0[38]	-0.820	0.779	-2.397	-1.333	-0.801	-0.294	0.671 1.001
u0[41] 1.141 0.789 -0.362 0.601 1.120 1.667 2.738 1.001 u0[42] -0.702 0.775 -2.275 -1.207 -0.689 -0.175 0.781 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[44] -1.759 0.836 -3.466 -2.307 -1.741 -1.182 -0.180 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001	u0[39]	0.729	0.777	-0.751	0.197	0.712	1.248	2.291 1.001
u0[42] -0.702 0.775 -2.275 -1.207 -0.689 -0.175 0.781 1.001 u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[44] -1.759 0.836 -3.466 -2.307 -1.741 -1.182 -0.180 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001	u0[40]	-0.355	0.762	-1.865	-0.856	-0.341	0.157	1.116 1.001
u0[43] -1.075 0.794 -2.696 -1.597 -1.055 -0.532 0.424 1.001 u0[44] -1.759 0.836 -3.466 -2.307 -1.741 -1.182 -0.180 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001	u0[41]	1.141	0.789	-0.362	0.601	1.120	1.667	2.738 1.001
u0[44] -1.759 0.836 -3.466 -2.307 -1.741 -1.182 -0.180 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001	u0[42]	-0.702	0.775	-2.275	-1.207	-0.689	-0.175	0.781 1.001
u0[44] -1.759 0.836 -3.466 -2.307 -1.741 -1.182 -0.180 1.001 u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001	u0[43]	-1.075	0.794	-2.696	-1.597	-1.055	-0.532	0.424 1.001
u0[45] 1.211 0.795 -0.309 0.672 1.190 1.730 2.841 1.001 u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001	u0[44]	-1.759	0.836	-3.466	-2.307	-1.741	-1.182	-0.180 1.001
u0[46] 0.922 0.781 -0.559 0.388 0.903 1.436 2.511 1.001 u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001		1.211	0.795	-0.309	0.672	1.190	1.730	2.841 1.001
u0[47] 0.569 0.773 -0.924 0.052 0.555 1.076 2.118 1.001		0.922	0.781	-0.559	0.388	0.903	1.436	2.511 1.001
		0.569		-0.924				2.118 1.001
	u0[48]	-0.067	0.764	-1.573	-0.571	-0.063	0.441	1.427 1.001

u0[49]	1.041	0.795	-0.477	0.492	1.025	1.564	2.649 1.001
u0[50]	0.364	0.757	-1.109	-0.141	0.362	0.859	1.878 1.001
u0[51]	-1.124	0.791	-2.717	-1.647	-1.095	-0.587	0.376 1.001
u0[52]	0.226	0.768	-1.276	-0.290	0.221	0.734	1.758 1.001
u0[53]	-0.128	0.761	-1.611	-0.629	-0.132	0.376	1.349 1.001
u0[54]	0.490	0.774	-0.994	-0.030	0.478	0.993	2.041 1.001
u0[55]	-0.421	0.767	-1.964	-0.926	-0.416	0.091	1.091 1.001
u0[56]	-1.617	0.822	-3.289	-2.165	-1.591	-1.050	-0.084 1.001
u0[57]	0.045	0.765	-1.463	-0.455	0.045	0.555	1.545 1.001
u0[58]	-0.551	0.769	-2.078	-1.062	-0.547	-0.035	0.936 1.001
u0[59]	0.971	0.784	-0.525	0.437	0.956	1.487	2.558 1.001
u0[60]	0.426	0.768	-1.064	-0.091	0.414	0.927	1.982 1.001
u1[1]	-0.210	0.060	-0.327	-0.250	-0.209	-0.170	-0.093 1.001
u1[2]	0.129	0.060	0.012	0.088	0.129	0.170	0.246 1.001
u1[3]	-0.237	0.060	-0.354	-0.278	-0.237	-0.198	-0.118 1.001
u1[4]	0.036	0.060	-0.082	-0.004	0.037	0.077	0.153 1.001
u1[5]	0.284	0.060	0.164	0.243	0.284	0.324	0.402 1.001
u1[6]	0.037	0.060	-0.082	-0.004	0.036	0.078	0.154 1.001
u1[7]	0.125	0.060	0.002	0.084	0.124	0.165	0.243 1.001
u1[8]	-0.144	0.059	-0.261	-0.184	-0.143	-0.104	-0.027 1.001
u1[9]	-0.225	0.060	-0.342	-0.265	-0.224	-0.184	-0.108 1.001
u1[10]	-0.030	0.060	-0.147	-0.071	-0.030	0.010	0.086 1.001
u1[10]	-0.153	0.060	-0.269	-0.193	-0.153	-0.112	-0.036 1.001
u1[11]	0.126	0.059	0.012	0.086	0.126	0.166	0.242 1.001
u1[12]	0.068	0.060	-0.047	0.028	0.120	0.108	0.186 1.001
u1[13] u1[14]	-0.147	0.060	-0.264	-0.187	-0.147	-0.107	-0.030 1.001
u1[14] u1[15]	0.190	0.060	0.070	0.149	0.190	0.230	0.307 1.001
u1[15] u1[16]	0.130	0.061	0.222	0.300	0.130	0.381	0.462 1.001
u1[10] u1[17]	-0.031	0.059	-0.147	-0.071	-0.032	0.009	0.085 1.001
u1[1/] u1[18]	-0.051	0.060	-0.147	-0.194	-0.052	-0.114	-0.037 1.001
u1[18] u1[19]	0.009	0.060	-0.271	-0.134	0.009	0.049	0.126 1.001
u1[19] u1[20]	-0.020	0.061	-0.139	-0.051	-0.020	0.022	0.100 1.001
	0.084	0.060	-0.139	0.045	0.084	0.022	0.201 1.001
u1[21]	-0.214	0.060	-0.331	-0.254		-0.173	-0.097 1.001
u1[22]	-0.214 -0.194			-0.234	-0.214		
u1[23]		0.059	-0.311		-0.195	-0.155	-0.076 1.001
u1[24]	0.046	0.059	-0.070	0.006	0.046	0.086	0.162 1.001
u1[25]	0.337	0.061	0.218	0.295	0.337	0.377	0.458 1.001
u1[26]	0.031	0.060	-0.084	-0.009	0.031	0.072	0.149 1.001
u1[27]	0.253	0.060	0.137	0.212	0.252	0.293	0.370 1.001
u1[28]	0.077	0.060	-0.039	0.036	0.077	0.117	0.194 1.001
u1[29]	-0.187	0.060	-0.303	-0.227	-0.187	-0.147	-0.069 1.001
u1[30]	-0.171	0.060	-0.288	-0.211	-0.171	-0.130	-0.053 1.001
u1[31]	0.164	0.060	0.046	0.124	0.164	0.203	0.281 1.001
u1[32]	0.223	0.059	0.107	0.184	0.223	0.263	0.339 1.001
u1[33]	0.086	0.060	-0.031	0.046	0.086	0.126	0.205 1.001
u1[34]	-0.119	0.059	-0.237	-0.159	-0.119	-0.079	-0.004 1.001
u1[35]	0.146	0.060	0.030	0.106	0.146	0.186	0.263 1.001
u1[36]	-0.193	0.060	-0.309	-0.233	-0.193	-0.152	-0.076 1.001
u1[37]	-0.145	0.060	-0.261	-0.185	-0.145	-0.105	-0.029 1.001
u1[38]	-0.185	0.060	-0.302	-0.226	-0.185	-0.144	-0.068 1.001

```
u1[39]
              0.131
                       0.059
                                 0.016
                                           0.091
                                                    0.131
                                                              0.171
                                                                        0.248 1.001
 u1[40]
                       0.059
                                -0.291
                                                    -0.175
                                                                       -0.059 1.001
             -0.175
                                          -0.215
                                                              -0.135
                                                                        0.066 1.001
 u1[41]
             -0.051
                       0.060
                                -0.171
                                          -0.093
                                                    -0.051
                                                              -0.011
 u1[42]
                                -0.186
                                                    -0.066
                                                              -0.026
                                                                        0.052 1.001
             -0.066
                       0.060
                                          -0.106
                                -0.378
 u1[43]
             -0.259
                       0.061
                                          -0.300
                                                    -0.259
                                                              -0.218
                                                                       -0.140 1.001
 u1[44]
                       0.062
                                -0.242
                                          -0.162
                                                              -0.079
                                                                        0.000 1.001
             -0.121
                                                    -0.121
 u1[45]
             -0.061
                       0.061
                                -0.179
                                          -0.103
                                                    -0.061
                                                              -0.021
                                                                        0.057 1.001
                                          0.248
                                                    0.289
                                                                        0.408 1.001
 u1[46]
              0.289
                       0.060
                                0.170
                                                              0.329
                       0.060
                                -0.120
                                          -0.039
                                                     0.000
                                                              0.041
                                                                        0.120 1.001
 u1[47]
              0.000
 u1[48]
             -0.145
                       0.060
                                -0.262
                                          -0.185
                                                    -0.145
                                                              -0.105
                                                                       -0.027 1.001
 u1[49]
             -0.021
                       0.061
                                -0.140
                                          -0.062
                                                    -0.022
                                                              0.019
                                                                        0.098 1.001
                       0.060
                                -0.054
                                           0.024
                                                    0.064
                                                                        0.182 1.001
 u1[50]
              0.064
                                                              0.104
 u1[51]
             -0.122
                       0.060
                                -0.238
                                          -0.162
                                                    -0.122
                                                              -0.082
                                                                       -0.004 1.001
 u1[52]
              0.098
                       0.060
                                -0.018
                                           0.058
                                                    0.098
                                                              0.138
                                                                        0.216 1.001
 u1[53]
              0.010
                       0.060
                                -0.107
                                          -0.030
                                                    0.009
                                                              0.050
                                                                        0.128 1.001
                                                                        0.334 1.001
 u1[54]
              0.215
                       0.060
                                0.101
                                           0.175
                                                    0.215
                                                              0.255
 u1[55]
              0.202
                       0.060
                                0.085
                                          0.161
                                                    0.202
                                                              0.242
                                                                        0.320 1.001
 u1[56]
             -0.016
                       0.061
                                -0.134
                                          -0.058
                                                    -0.016
                                                              0.025
                                                                        0.108 1.001
 u1[57]
             -0.190
                       0.060
                                -0.307
                                          -0.230
                                                    -0.191
                                                              -0.150
                                                                       -0.073 1.001
 u1[58]
              0.002
                       0.060
                                -0.115
                                          -0.038
                                                     0.001
                                                              0.042
                                                                        0.121 1.001
 u1[59]
              0.230
                       0.061
                                0.113
                                          0.188
                                                    0.229
                                                              0.271
                                                                        0.349 1.001
                                                                        0.058 1.001
 u1[60]
             -0.059
                       0.060
                                -0.177
                                          -0.099
                                                    -0.059
                                                              -0.020
              2.154
                       0.193
                                 1.808
                                           2.016
                                                     2.144
                                                              2.276
                                                                        2.563 1.001
 ٧٧
 vvint
              1.233
                       0.460
                                 0.491
                                           0.907
                                                     1.180
                                                              1.498
                                                                        2.278 1.001
 vvslp
              0.031
                       0.007
                                 0.021
                                           0.027
                                                     0.031
                                                              0.035
                                                                        0.047 1.001
 deviance 1294.889
                      17.733 1262.084 1282.569 1294.252 1306.464 1331.381 1.001
 DIC info (using the rule, pD = var(deviance)/2)
 pD = 157.2 and DIC = 1452.1
 DIC is an estimate of expected predictive error (lower deviance is better).
# Diagnostics are good
sims <- as.mcmc(fit10)</pre>
chains <- as.matrix(sims)</pre>
sims <- as.mcmc(chains)</pre>
raftery.diag(sims)
effectiveSize(sims)
```

21. Continuing in JAGS, add a random coefficient term for the deviations from the quadratic term in the model. You will now have random coefficients for the intercepts, the linear term (slopes), and the quadratic term. You can use a gamma(1.1,.1) prior for $\sigma^2_{quadratic}$. You will also need a prior for all the terms that are deviations from the overall quadratic term. Use a normal with a mean of 0, and a precision that is $1/\sigma^2_{quadratic}$. What is the DIC?

```
md1 <- "
  model {
  for (i in 1:360){
    weight[i] ~ dnorm(mu[i], 1/vv)
    mu[i] <- b0[trt[i]] + b1[trt[i]]*day[i] + b2[trt[i]]*day2[i] +</pre>
u0[newpig[i]] + u1[newpig[i]]*day[i] + u2[newpig[i]]*day2[i]
  for(i in 1:3){
    b0[i] \sim dnorm(0, 0.0001)
    b1[i] \sim dnorm(0, 0.0001)
    b2[i] \sim dnorm(0, 0.0001)
  for(i in 1:60){
    u0[i] ~ dnorm(0,1/vvint)
    u1[i] ~ dnorm(0, 1/vvslp)
    u2[i] \sim dnorm(0, 1/vvslp2)
  }
  vvslp2 \sim dgamma(1.1, 0.1)
  vvslp \sim dgamma(1.1, 0.1)
  vvint ~ dgamma(1.1, 0.1)
  vv \sim dgamma(1.1, 0.1)
newpig <- pig$newpig</pre>
trt = pig$trt
day = pig$day
day2 = pig$day2
weight = pig$weight
writeLines(mdl, 'fit11.txt')
data.jags <- c('weight', 'day', 'day2', 'trt', 'newpig')</pre>
parms <- c('b0' , 'b1', 'b2', 'u0', 'u1', 'vv', 'vvint', 'vvslp')</pre>
fit11 <- jags(data= data.jags, parameters.to.save = parms,</pre>
                   model.file = 'fit11.txt', inits = NULL,
                   n.iter = 30000, n.thin = 5, n.burnin = 2000,
                   n.chains = 5)
 Compiling model graph
    Resolving undeclared variables
    Allocating nodes
 Graph information:
```

Observed stochastic nodes: 360 Unobserved stochastic nodes: 193

Total graph size: 3118

Initializing model

fit11

Inference for Bugs model at "fit11.txt", fit using jags,
5 chains, each with 30000 iterations (first 2000 discarded), n.thin = 5
n.sims = 28000 iterations saved

n.sims =	= 28000 it	erations	saved				
	mu.vect	sd.vect	2.5%	25%	50%	75%	97.5% Rhat
b0[1]	14.896	0.398	14.115	14.632	14.893	15.162	15.673 1.001
b0[2]	14.959	0.400	14.172	14.695	14.956	15.225	15.747 1.001
b0[3]	14.351	0.397	13.582	14.084	14.353	14.617	15.129 1.001
b1[1]	1.724	0.058	1.611	1.685	1.724	1.763	1.838 1.001
b1[2]	2.010	0.058	1.898	1.971	2.010	2.049	2.122 1.001
b1[3]	1.424	0.058	1.311	1.386	1.424	1.463	1.537 1.001
b2[1]	-0.029	0.002	-0.032	-0.030	-0.029	-0.028	-0.026 1.001
b2[2]	-0.040	0.002	-0.043	-0.041	-0.040	-0.039	-0.037 1.001
b2[3]	-0.025	0.002	-0.028	-0.026	-0.025	-0.024	-0.022 1.001
u0[1]	-0.794	0.809	-2.411	-1.336	-0.785	-0.252	0.783 1.001
u0[2]	0.025	0.791	-1.523	-0.506	0.027	0.555	1.583 1.001
u0[3]	-0.876	0.816	-2.503	-1.414	-0.860	-0.322	0.674 1.001
u0[4]	1.103	0.815	-0.448	0.545	1.089	1.641	2.739 1.001
u0[5]	1.195	0.831	-0.377	0.628	1.177	1.737	2.892 1.001
u0[6]	0.426	0.797	-1.104	-0.115	0.417	0.954	2.034 1.001
u0[7]	-1.233	0.814	-2.863	-1.778	-1.218	-0.676	0.329 1.001
u0[8]	0.100	0.793	-1.458	-0.434	0.097	0.629	1.668 1.001
u0[9]	-0.003	0.799	-1.583	-0.537	-0.001	0.530	1.561 1.001
u0[10]	-0.878	0.797	-2.462	-1.407	-0.868	-0.336	0.653 1.001
u0[11]	0.041	0.798	-1.540	-0.488	0.043	0.573	1.610 1.001
u0[12]	-0.108	0.797	-1.677	-0.637	-0.099	0.426	1.466 1.001
u0[13]	0.381	0.792	-1.163	-0.155	0.378	0.906	1.954 1.001
u0[14]	-0.327	0.797	-1.926	-0.855	-0.319	0.210	1.222 1.001
u0[15]	1.505	0.840	-0.099	0.920	1.492	2.068	3.171 1.001
u0[16]	1.534	0.856	-0.074	0.944	1.510	2.101	3.255 1.001
u0[17]	-0.307	0.795	-1.882	-0.833	-0.298	0.230	1.239 1.001
u0[18]	-0.473	0.802	-2.062	-1.006	-0.469	0.067	1.080 1.001
u0[19]	0.089	0.798	-1.459	-0.443	0.079	0.620	1.656 1.001
u0[20]	-1.365	0.822	-3.034	-1.906	-1.349	-0.806	0.198 1.001
u0[21]	0.439	0.798	-1.104	-0.094	0.429	0.967	2.037 1.001
u0[22]	0.292	0.799	-1.292	-0.240	0.289	0.821	1.870 1.001
u0[23]	-0.731	0.816	-2.367	-1.276	-0.715	-0.174	0.834 1.001
u0[24]	0.041	0.793	-1.507	-0.488	0.038	0.564	1.612 1.001
u0[25]	1.672	0.863	0.034	1.087	1.649	2.246	3.411 1.001
u0[26]	-0.771	0.803	-2.373	-1.303	-0.753	-0.233	0.766 1.001
u0[27]	0.707	0.808	-0.832	0.152	0.703	1.241	2.343 1.001
u0[28]	0.384	0.800	-1.176	-0.152	0.385	0.925	1.954 1.001
u0[29]	-1.078	0.821	-2.726	-1.628	-1.060	-0.515	0.484 1.001

u0[30]	-0.360	0.798	-1.950	-0.893	-0.347	0.172	1.200 1.001
u0[31]	-0.260	0.795	-1.832	-0.793	-0.259	0.275	1.298 1.001
u0[32]	0.405	0.803	-1.147	-0.140	0.395	0.935	1.995 1.001
u0[33]	0.750	0.809	-0.819	0.202	0.742	1.282	2.353 1.001
u0[34]	-0.482	0.791	-2.040	-1.014	-0.478	0.054	1.064 1.001
u0[35]	0.712	0.811	-0.857	0.158	0.700	1.256	2.359 1.001
u0[36]	-1.096	0.818	-2.722	-1.640	-1.085	-0.532	0.465 1.001
u0[37]	0.034	0.796	-1.533	-0.504	0.034	0.564	1.609 1.001
u0[38]	-1.007	0.819	-2.632	-1.560	-0.995	-0.447	0.568 1.001
u0[39]	0.879	0.811	-0.687	0.329	0.867	1.416	2.495 1.001
u0[40]	-0.486	0.798	-2.055	-1.016	-0.475	0.054	1.066 1.001
u0[41]	1.236	0.819	-0.334	0.676	1.218	1.770	2.886 1.001
u0[42]	-0.792	0.803	-2.393	-1.323	-0.782	-0.255	0.751 1.001
u0[43]	-1.276	0.830	-2.962	-1.821	-1.261	-0.710	0.313 1.001
u0[44]	-2.022	0.864	-3.760	-2.596	-2.003	-1.432	-0.385 1.001
u0[45]	1.322	0.817	-0.245	0.764	1.318	1.865	2.952 1.001
u0[46]	1.176	0.830	-0.399	0.613	1.156	1.720	2.852 1.001
u0[47]	0.563	0.807	-0.997	0.010	0.554	1.100	2.179 1.001
u0[48]	-0.174	0.798	-1.735	-0.707	-0.177	0.374	1.391 1.001
น0[้49]	1.167	0.809	-0.370	0.615	1.156	1.697	2.812 1.001
u0[50]	0.428	0.794	-1.105	-0.113	0.424	0.962	2.001 1.001
u0[51]	-1.309	0.827	-2.970	-1.857	-1.296	-0.743	0.272 1.001
u0[52]	0.390	0.799	-1.142	-0.153	0.381	0.919	1.985 1.001
u0[53]	-0.089	0.788	-1.636	-0.615	-0.089	0.443	1.463 1.001
u0[54]	0.635	0.812	-0.931	0.082	0.628	1.167	2.267 1.001
u0[55]	-0.380	0.791	-1.929	-0.906	-0.377	0.153	1.173 1.001
u0[56]	-1.812	0.842	-3.505	-2.372	-1.791	-1.229	-0.220 1.001
u0[57]	-0.070	0.800	-1.653	-0.603	-0.064	0.465	1.497 1.001
u0[58]	-0.588	0.805	-2.190	-1.120	-0.576	-0.049	0.973 1.001
u0[59]	1.187	0.824	-0.377	0.624	1.166	1.734	2.851 1.001
u0[60]	0.437	0.796	-1.124	-0.103	0.437	0.969	1.997 1.001
u1[1]	-0.171	0.081	-0.325	-0.226	-0.173	-0.118	-0.005 1.001
u1[2]	0.105	0.078	-0.051	0.053	0.106	0.158	0.254 1.001
u1[3]	-0.206	0.081	-0.361	-0.261	-0.207	-0.153	-0.043 1.001
u1[4]	0.022	0.078	-0.132	-0.030	0.023	0.075	0.173 1.001
u1[5]	0.225	0.086	0.049	0.169	0.227	0.285	0.388 1.001
u1[6]	0.038	0.077	-0.114	-0.014	0.038	0.089	0.189 1.001
u1[7]	0.097	0.080	-0.062	0.043	0.097	0.151	0.252 1.001
u1[8]	-0.108	0.079	-0.260	-0.161	-0.109	-0.056	0.051 1.001
u1[9]	-0.198	0.080	-0.351	-0.251	-0.199	-0.145	-0.035 1.001
u1[10]	-0.019	0.078	-0.171	-0.071	-0.019	0.032	0.135 1.001
u1[11]	-0.124	0.079	-0.276	-0.178	-0.126	-0.072	0.033 1.001
u1[12]	0.089	0.080	-0.071	0.037	0.092	0.143	0.244 1.001
u1[13]	0.046	0.078	-0.112	-0.005	0.047	0.098	0.198 1.001
u1[14]	-0.083	0.084	-0.241	-0.141	-0.085	-0.028	0.089 1.001
u1[15]	0.150	0.082	-0.017	0.096	0.151	0.205	0.305 1.001
u1[16]	0.277	0.090	0.092	0.218	0.281	0.339	0.442 1.002
u1[17]	-0.036	0.077	-0.190	-0.087	-0.036	0.015	0.114 1.001
u1[17]	-0.117	0.081	-0.271	-0.171	-0.118	-0.064	0.045 1.001
u1[19]	0.014	0.077	-0.139	-0.038	0.014	0.065	0.165 1.001

u1[20]	-0.002	0.078	-0.153	-0.055	-0.002	0.050	0.155	1.001
u1[21]	0.070	0.078	-0.082	0.018	0.070	0.123	0.222	1.001
u1[22]	-0.145	0.085	-0.306	-0.204	-0.148	-0.089	0.028	1.002
u1[23]	-0.173	0.080	-0.328	-0.226	-0.174	-0.119	-0.012	1.001
u1[24]	0.045	0.077	-0.108	-0.006	0.044	0.095	0.198	1.001
u1[25]	0.261	0.091	0.073	0.201	0.266	0.324	0.430	1.001
u1[26]	0.026	0.077	-0.126	-0.025	0.026	0.078	0.178	1.001
u1[27]	0.192	0.085	0.019	0.137	0.196	0.250	0.353	1.001
u1[28]	0.083	0.077	-0.066	0.031	0.082	0.135	0.237	1.001
u1[29]	-0.131	0.084	-0.290	-0.188	-0.133	-0.075		1.001
u1[30]	-0.160	0.079	-0.312	-0.212	-0.161	-0.107	-0.002	
u1[31]	0.100	0.084	-0.072	0.046	0.103	0.157	0.257	1.001
u1[32]	0.157	0.086	-0.020	0.101	0.161	0.217		1.001
u1[33]	0.088	0.078	-0.063	0.036	0.088	0.140		1.001
u1[34]	-0.088	0.078	-0.239	-0.141	-0.089	-0.037	0.069	1.001
u1[35]	0.122	0.079	-0.037	0.070	0.122	0.175		1.001
u1[36]	-0.148	0.082	-0.304	-0.203	-0.150	-0.094	0.019	1.001
u1[37]	-0.120	0.079	-0.271	-0.172	-0.120	-0.067	0.040	1.001
u1[38]	-0.145	0.082	-0.301	-0.200	-0.146	-0.091	0.017	1.001
u1[39]	0.105	0.079	-0.053	0.052	0.106	0.158	0.257	1.001
u1[40]	-0.142	0.080	-0.297	-0.197	-0.143	-0.089	0.018	1.001
u1[41]	-0.043	0.078	-0.195	-0.095	-0.044	0.009	0.109	1.001
u1[42]	-0.058	0.078	-0.209	-0.109	-0.058	-0.006	0.097	1.001
u1[43]	-0.227	0.083	-0.385	-0.283	-0.229	-0.173	-0.060	1.001
u1[44]	-0.078	0.082	-0.235	-0.134	-0.080	-0.023	0.090	1.001
u1[45]	-0.054	0.078	-0.207	-0.105	-0.054	-0.002	0.100	1.001
u1[46]	0.222	0.088	0.041	0.163	0.226	0.284	0.387	1.001
u1[47]	0.043	0.079	-0.108	-0.011	0.040	0.095	0.205	1.001
u1[48]	-0.105	0.080	-0.259	-0.159	-0.106	-0.053	0.058	1.001
u1[49]	-0.030	0.078	-0.184	-0.080	-0.030	0.022	0.125	1.001
u1[50]	0.057	0.078	-0.097	0.005	0.057	0.109	0.210	1.001
u1[51]	-0.095	0.079	-0.248	-0.148	-0.096	-0.042	0.064	1.001
u1[52]	0.023	0.085	-0.152	-0.033	0.026	0.082	0.182	1.001
u1[53]	-0.019	0.078	-0.178	-0.071	-0.018	0.034	0.132	1.001
u1[54]	0.188	0.081	0.027	0.135	0.188	0.241	0.344	1.001
u1[55]	0.180	0.079	0.022	0.128	0.180	0.232	0.336	1.001
u1[56]	0.000	0.079	-0.152	-0.054	-0.001	0.052	0.157	1.001
u1[57]	-0.135	0.084	-0.293	-0.192	-0.137	-0.080	0.038	1.001
u1[58]	-0.013	0.078	-0.169	-0.065	-0.013	0.039	0.138	1.001
u1[59]	0.188	0.083	0.020	0.133	0.189	0.244		
u1[60]	-0.044	0.077	-0.194	-0.096	-0.045	0.007	0.108	1.001
VV	2.075	0.195	1.731	1.939	2.063	2.198		1.001
vvint	1.503	0.540	0.633	1.125	1.438	1.808		1.001
vvslp	0.024	0.007	0.011	0.019	0.024	0.029	0.041	
deviance	2 1281.125	19.963	1243.238	1267.563	1280.745	1294.475	1321.159	1.001

For each parameter, n.eff is a crude measure of effective sample size, and Rhat is the potential scale reduction factor (at convergence, Rhat=1).

```
DIC info (using the rule, pD = var(deviance)/2)
pD = 199.2 and DIC = 1480.3
DIC is an estimate of expected predictive error (lower deviance is better).

sims <- as.mcmc(fit10)
chains <- as.matrix(sims)
sims <- as.mcmc(chains)
raftery.diag(sims)
effectiveSize(sims)</pre>
```

22. Using DIC, which of the models that allow inference over all pigs would you choose?

Using DIC I would choose the model from question 20. This model just has random coefficients for the intercept and the linear day term. Adding a random effect for pig on the quadratic day term does not improve the model.

Problems 23 and 24 are not required. If completed they will be worth bonus points. 10 points for problem 23, and 5 points for problem 24.

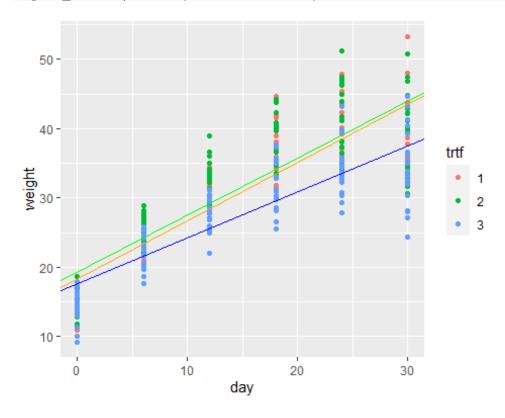
23. Since brm does not do well with these types of models, now you will do the work in Stan. Create a hierarchical model with hierarchical terms for both the intercepts and linear terms (slopes), but not for the quadratic term. This is a hierarchical model that is much like the random coefficients model in number 20. Use normal priors with means of 0 and standard deviations of 100 where appropriate, and gamma(1.1,.5) priors for the variance terms. What are means of the chains for the overall intercept in treatment 1, the overall linear term coefficient (slope) in treatment 1, and the overall quadratic term coefficient in treatment 1?

Intercept for treatment 1 is 18.5, Linear term for treatment 1 is 0.85

```
model <- "
data {
  int <lower = 1> N;
  int q;
  real weight[N];
 int day[N];
  int trt[N];
}
parameters {
  real alpha[q];
  real beta[q];
  real mu alpha;
  real mu beta;
  real <lower = 0> serr;
  real <lower = 0> salpha;
  real <lower = 0> sbeta;
}
```

```
model {
  mu alpha \sim normal(0, 100);
  mu beta \sim normal(0, 100);
  alpha ~ normal(mu_alpha, salpha);
  beta ~ normal(mu beta, sbeta);
  serr \sim gamma(1.1, 0.5);
  salpha \sim gamma(1.1, 0.5);
  sbeta \sim gamma(1.1, .5);
  for(i in 1:N){
    weight[i] ~ normal(alpha[trt[i]] + beta[trt[i]] * day[i], serr);
  }
}
generated quantities {
  vector[N] log_lik;
  real s2error;
  real s2int;
  real s2slope;
  s2error = serr*serr;
  s2int = salpha*salpha;
  s2slope = sbeta*sbeta;
 for (i in 1:N) log_lik[i] = normal_lpdf(weight[i] | alpha[trt[i]] +
beta[trt[i]]*day[i], serr);
}
writeLines(model, 'hier.stan')
trt = pig$trt
day = pig$day
weight = pig$weight
N <- 360
q < -3
hier_dat <- list(N=N, weight = weight, trt = trt, day = day, q=q)
fit12 <- stan(file = "hier.stan", data = hier_dat, iter = 11000,</pre>
            warmup = 1000, chains = 4, thin = 2)
chains <- as.matrix(fit12)</pre>
dim(chains)
 [1] 20000
              375
sims <- as.mcmc(chains)</pre>
```

```
ggplot(data = pig, mapping = aes(x = day, y = weight, col = trtf)) +
  geom_point() +
  geom_abline(intercept = 18.4, slope = 0.84, col = "orange") +
  geom_abline(intercept = 19.38, slope = 0.82, col = "green") +
  geom_abline(intercept = 17.664, slope = 0.663, col = "blue")
```



24. Compute the waic for the model in problem 23.

waic is 2145.3