Searching for a more Global Class of RC Curves

Problems with KISS:

- End points always change if rate & curve are not properly increased & decreased in inverse proportions to each other.
- Still lacks intuitive parameters one needs to properly tune without scripting or grabbing a calculator

Problems with BetaFlight:

- Little more intuitive but sacrifices the degree of observability and feel between changes.
- Still needs a script to keep the center-stick sensitivity slope the same.

Must have properties:

- Constant End Points that DOES NOT change.
- Tunable center-stick slope that DOES NOT change end points.
- Center-stick linear width that DOES NOT change end points or center-stick slope.

Does it exist? Yes of course it does with a little mathematical theory of course!

First things first normalize the range of the input:

$$\hat{x} = \frac{x - (\max(x) - \min(x))/2}{\max(x) - \min(x)}, \qquad -1 \le \hat{x} \le 1$$

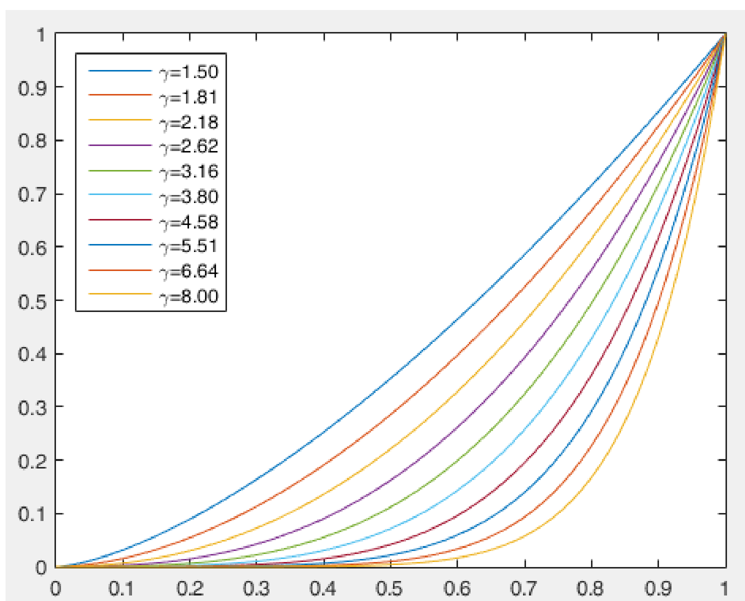
Further we know the curve is going to be ODD that is

If $f(x)$ *is ODD then by definition*: $f(-x) = -f(x)$, we can focus on the curve on the range $0 \le \hat{x} \le 1$

This helps defining the curve's "linear width" using a gamma curve which also has the added benefit of preserving its endpoint.

$$g(\hat{x};\, \gamma) = \hat{x}^{\gamma}$$

$$\frac{d}{d\hat{x}} g(\hat{x};\, \gamma) = \gamma \cdot \hat{x}^{\gamma-1},$$



One issue you may say is that it's maximally flat at zero when gamma is above a certain amount, which is not good for trying to hover in acro-mode. Its also not good for controlling the slope near zero, one simple fix is to add a linear function with a slope ρ giving an overall function after normalization:

$$f(\hat{x}, \gamma, \rho) = \frac{1}{1+\rho}[\hat{x}^{\gamma} + \rho \cdot \hat{x}], \qquad \frac{d}{d\hat{x}} f(\hat{x}; \gamma, \rho) = \frac{1}{1+\rho}[\gamma \cdot \hat{x}^{\gamma-1} + \rho],$$

$$\frac{d}{d\hat{x}} f(0; \gamma, \rho) = m_0 = \frac{\rho}{1+\rho}, \quad \rho(m_0) = \frac{m_0}{1-m_0}, \quad f(\hat{x}; \gamma, m_0) = \frac{1}{1 - m_0/(1-m_0)}\left[\hat{x}^{\gamma} + \left(\frac{m_0}{1-m_0}\right) \cdot \hat{x}\right]$$

$$f(\hat{x}; \gamma, m_0) = (1-m_0)\left[\hat{x}^{\gamma} + \left(\frac{m_0}{1-m_0}\right) \cdot \hat{x}\right] = (1-m_0) \cdot \hat{x}^{\gamma} + m_0 \cdot \hat{x}$$

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~~So to change that we add another class of function whose derivative is the function itself the exponential function. The exponential function gives slope at center stick since any non-zero number to the zero-th power is 1 the derivative is 1, consequently though this now requires an offset of -1 so zero stick input still reads zero rotational velocity.~~

$$\cancel{h(\hat{x},\rho) = e^{\hat{x}/\rho} - 1}$$

~~Lastly this function needs to be normalized by the output of itself at 1, with the parameter rho ρ and scaled by the amount of center stick sensitivity s_{ϵ} giving:~~

$$\cancel{\varphi \cdot \hat{h}(\hat{x},\rho) = \varphi \cdot \frac{h(\hat{x},\rho)}{h(1,\rho)}}$$

~~When added to the gamma function to maintain normalization and constant end-points the entire function needs renormalized by itself at 1.~~

$$\cancel{\hat{f}(\hat{x};\gamma,\rho,\varphi) = \frac{f(\hat{x};\gamma,\rho,\varphi)}{f(1;\gamma,\rho,\varphi)} = \frac{1}{1+\varphi}\left[\hat{x}^{\gamma} + \varphi \cdot \left(\frac{e^{\hat{x}/\rho} - 1}{e^{1/\rho} - 1}\right)\right]}$$

~~Taking the derivative and finding φ in terms of the center-stick slope m_0 and rho :~~

$$\cancel{\frac{d}{d\hat{x}}\hat{f}(\hat{x};\gamma,\rho,\varphi) = \hat{f}'(\hat{x};\gamma,\rho,\varphi) = \frac{1}{1+\varphi}\left[\gamma \cdot \hat{x}^{\gamma-1} + \left(\frac{\varphi}{\rho \cdot (e^{1/\rho} - 1)}\right)e^{\hat{x}/\rho}\right], \quad let\ A_{\rho} := \rho \cdot \left(e^{1/\rho} - 1\right)}$$

$$\cancel{\hat{f}'(\hat{x};\gamma,\rho,\varphi) = \frac{1}{1+\varphi}\left[\gamma \cdot \hat{x}^{\gamma-1} + \left(\frac{\varphi}{A_{\rho}}\right)e^{\hat{x}/\rho}\right], \quad \hat{f}'(0;\gamma,\rho,\varphi) = \frac{1}{1+\varphi}\left[0 + \left(\frac{\varphi}{A_{\rho}}\right) \cdot 1\right] = \frac{\varphi}{A_{\rho}(1+\varphi)} = m_0}$$

$$\cancel{\varphi = m_0 A_{\rho}(1+\varphi), \quad \varphi(\rho, m_0) = \frac{m_0 A_{\rho}}{1 - m_0 A_{\rho}}}$$

Multiplying by the max rotational velocity ω_{Max} the end points are always preserved

$$\omega(\hat{x},\gamma,\rho,\varphi(m_0,\rho),\omega_{MAX}) = \begin{cases} -\omega_{MAX} \cdot f\left(-\hat{x};\gamma,\rho,\varphi(m_0,\rho)\right) & \hat{x} < 0 \\ \omega_{MAX} \cdot f\left(\hat{x};\gamma,\rho,\varphi(m_0,\rho)\right) & \hat{x} \geq 0 \end{cases} = \begin{cases} -\omega_{MAX} \cdot f(-\hat{x}) & \hat{x} < 0 \\ \omega_{MAX} \cdot f(\hat{x}) & \hat{x} \geq 0 \end{cases}$$

Furthermore by varying the coefficients for gamma $\boldsymbol{\gamma}$ and rho ρ, the center-stick linearity width can be altered while keeping both the center-stick slope and end-points constant.

One straight forward relationship to calculate rho $\boldsymbol{\rho}$ from gamma $\boldsymbol{\gamma}$, can be made by examining how

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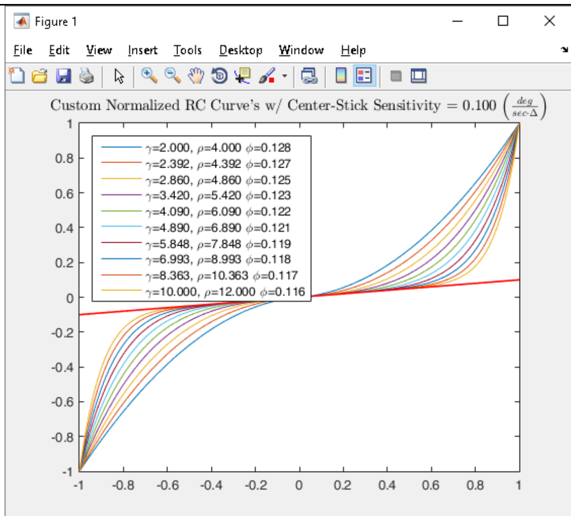

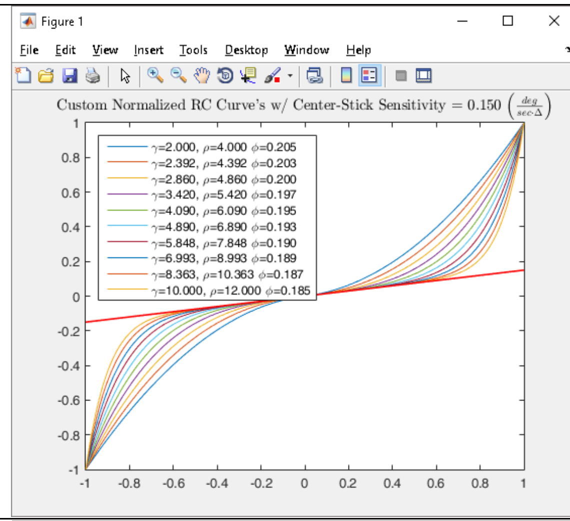

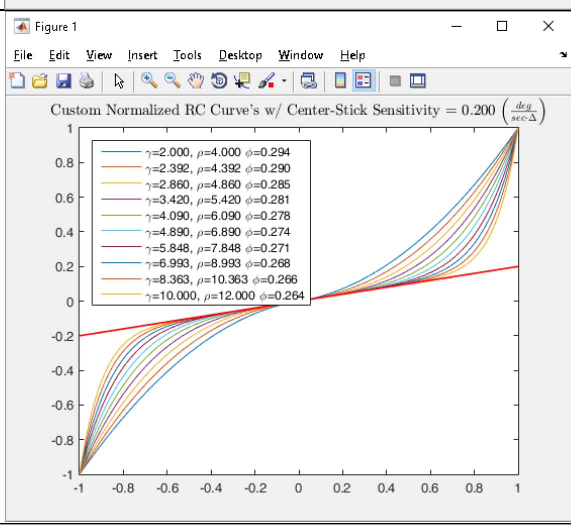

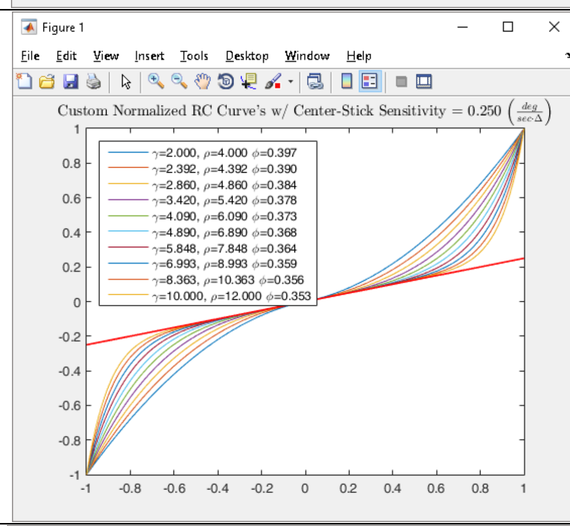

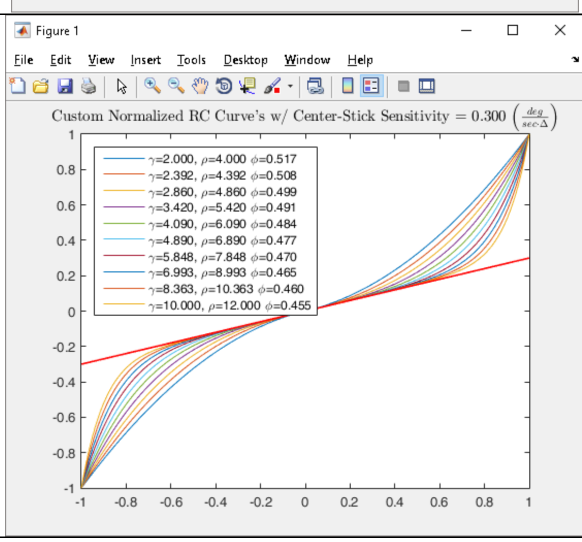

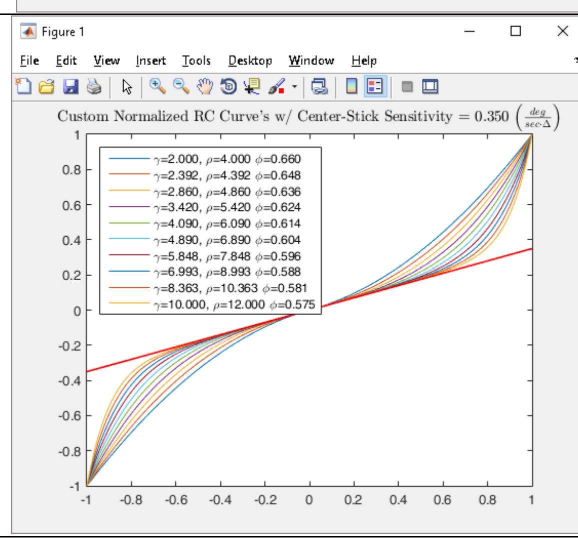

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