Searching for a more Global Class of RC Curves

Problems with KISS:

- End points always change if rate & curve are not properly increased & decreased in inverse proportions to each other.
- Still lacks intuitive parameters one needs to properly tune without scripting or grabbing a calculator

Problems with BetaFlight:

- Little more intuitive but sacrifices the degree of observability and feel between changes.
- Still needs a script to keep the center-stick sensitivity slope the same.

Must have properties:

- Constant End Points that DOES NOT change.
- Tunable center-stick slope that DOES NOT change end points.
- Center-stick linear width that DOES NOT change end points or center-stick slope.

Does it exist? Yes of course it does with a little mathematical theory of course!

First things first normalize the range of the input:

$$\hat{x} = \frac{x - (\max(x) - \min(x))/2}{\max(x) - \min(x)}, \qquad -1 \le \hat{x} \le 1$$

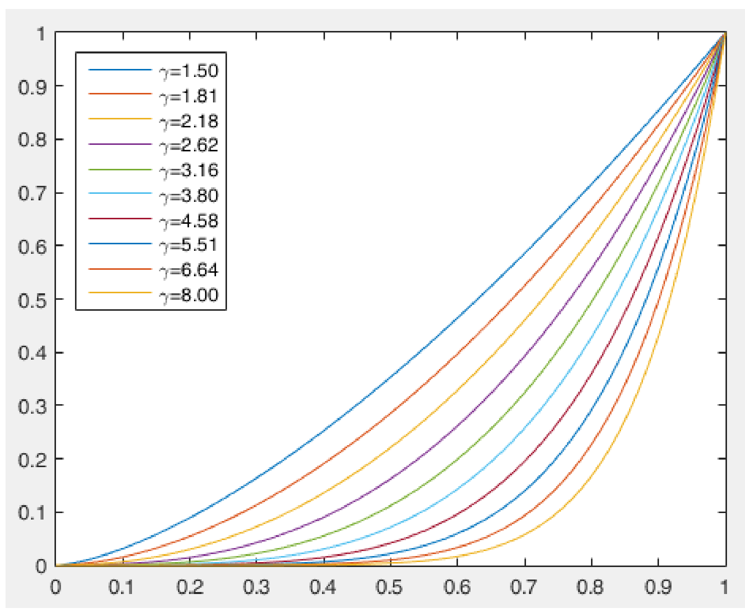
Further we know the curve is going to be ODD that is

If $f(x)$ *is ODD then by definition*: $f(-x) = -f(x)$, we can focus on the curve on the range $0 \le \hat{x} \le 1$

This helps defining the curve's "linear width" using a gamma curve which also has the added benefit of preserving its endpoint.

$$g(\hat{x};\, \gamma) = \hat{x}^{\gamma}$$

$$\frac{d}{d\hat{x}} g(\hat{x};\, \gamma) = \gamma \cdot \hat{x}^{\gamma - 1},$$



One issue you may say is that it's maximally flat at zero when gamma is above a certain amount, which is not good for trying to hover in acro-mode. So to change that we add another class of function whose derivative is the function itself the exponential function. The exponential function gives slope at center stick since any non-zero number to the zero-th power is 1 the derivative is 1, consequently though this now requires an offset of -1 so zero stick input still reads zero rotational velocity.

$$h(\hat{x}, \rho) = e^{\hat{x}/\rho} - 1$$

Lastly this function needs to be normalized by the output of itself at 1, with the parameter rho ρ and scaled by the amount of center stick sensitivity s_c giving:

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$$s_c \cdot \hat{h}(\hat{x},\rho) = s_c \cdot \frac{h(\hat{x},\rho)}{h(1,\rho)}$$

When added to the gamma function to maintain normalization and constant end-points the entire function needs renormalized by itself at 1.

$$\hat{f}(\hat{x};\gamma,\rho,s_c) = \frac{f(\hat{x};\gamma,\rho,s_c)}{f(1;\gamma,\rho,s_c)} = \frac{1}{1+s_c}\left[\hat{x}^{\gamma} + s_c \cdot \left(\frac{e^{\hat{x}/\rho}-1}{e^{1/\rho}-1}\right)\right]$$

Multiplying by the max rotational velocity ω_{Max} the end points are always preserved

$$\omega(\hat{x},\gamma,\rho,s_c.\omega_{MAX}) = \begin{cases} -\omega_{MAX}\cdot f(-\hat{x}) & \hat{x}<0 \\ +\omega_{MAX}\cdot f(+\hat{x}) & \hat{x}\geq 0 \end{cases} = \begin{cases} -\omega_{MAX}\cdot f(-\hat{x}) & \hat{x}<0 \\ +\omega_{MAX}\cdot f(+\hat{x}) & \hat{x}\geq 0 \end{cases}$$

Furthermore there should exist relations for calculating γ & ρ from s_c & some width parameter w which defines when the gamma curve $g(\hat{x},s_c)$ starts to become nonlinear from center stick. **TBD**