Searching for a more Intuitive Class of RC Curves

Problems with KISS:

- End points always change if rate & curve are not properly increased & decreased in inverse proportions to each other.
- Still lacks intuitive parameters one needs to properly tune without scripting or grabbing a calculator

Problems with BetaFlight:

- Little more intuitive but sacrifices the degree of observability and feel between changes.
- Still needs a script to keep the center-stick sensitivity slope the same.

Must have properties:

- Constant End Points that DOES NOT change.
- Tunable center-stick slope that DOES NOT change end points.
- Center-stick linear width that DOES NOT change end points or center-stick slope.

Does it exist? Yes of course it does with a little math and some good old calculus!

First things first normalize the range of the input:

$$\hat{x} = \frac{x - (\max(x) - \min(x))/2}{\max(x) - \min(x)}, \qquad -1 \le \hat{x} \le 1$$

Further we know the curve is going to be ODD that is

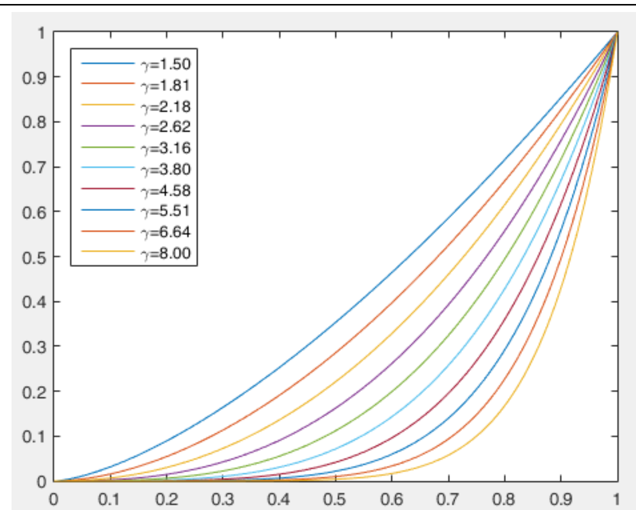
If $f(x)$ *is ODD then by definition*: $f(-x) = -f(x)$, we can focus on the curve on the range $0 \le \hat{x} \le 1$

This helps defining the curve's "linear width" using a gamma curve which also has the added benefit of preserving its endpoint (and not effecting the center point).

$$g(\hat{x};\, \gamma) = \hat{x}^{\gamma}$$

$$\frac{d}{d\hat{x}} g(\hat{x};\, \gamma) = \gamma \cdot \hat{x}^{\gamma-1}$$

$$\frac{d}{d\hat{x}} g(0; \gamma) = 0$$



One issue you may say is that it's maximally flat at zero when gamma is above a certain amount, which is not good for trying to hover in acro-mode. Its also not good for controlling the slope near zero to scale this, one simple fix is to add an addition linear function with a slope ρ giving an overall function after normalization:

$$f(\hat{x}, \gamma, \rho) = \frac{1}{1+\rho}[\hat{x}^{\gamma} + \rho \cdot \hat{x}], \qquad \frac{d}{d\hat{x}} f(\hat{x}; \gamma, \rho) = \frac{1}{1+\rho}[\gamma \cdot \hat{x}^{\gamma-1} + \rho],$$

Eliminating rho we see and replacing with the normalized center stick slope

$$\frac{d}{d\hat{x}} f(0; \gamma, \rho) = m_0 = \frac{\rho}{1+\rho}, \quad \rho(m_0) = \frac{m_0}{1 - m_0}$$

$$f(\hat{x}; \gamma, \rho(m_0)) = \frac{1}{1 + m_0/(1 - m_0)}\left[\hat{x}^{\gamma} + \left(\frac{m_0}{1 - m_0}\right) \cdot \hat{x}\right] = (1 - m_0)\left[\hat{x}^{\gamma} + \left(\frac{m_0}{1 - m_0}\right) \cdot \hat{x}\right] = (1 - m_0) \cdot \hat{x}^{\gamma} + m_0 \cdot \hat{x}$$

But wait there's more!...

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Now say we want to calculate gamma that gives us a center stick linear region of width $\boldsymbol{w}$ and slope tolerance $\boldsymbol{p}$, that is mathematically:

$$\frac{d}{d\hat{x}} f(\boldsymbol{w}; \gamma, \rho(m_0)) = (1-m_0)\cdot\gamma\cdot w^{\gamma-1} + m_0 = (1+p)\cdot m_0, \qquad where\ both\ \ 0 < w, p < 1$$

$$\gamma\cdot w^{\gamma-1} = \frac{p\cdot m_0}{(1-m_0)} \quad\Longrightarrow\quad \gamma\cdot w^{\gamma} = p\cdot\frac{m_0}{(1-m_0)}\cdot w$$

Rewriting w^{γ} as $e^{\gamma\cdot\ln(w)}$ and multiplying both sides by $\ln(w)$ gives

$$(\gamma\cdot\ln(w))\cdot e^{(\gamma\cdot\ln(w))} = p\cdot\frac{m_0}{(1-m_0)}\cdot w\cdot\ln(w)$$

Taking the lamberts W-function of both sides and solving for gamma γ in terms of width w. (NOTE: care must be taken which domain branch to use i.e. RHS < -1/e then -1, else 0) :

$$\gamma\cdot\ln(w) = W\left(p\cdot\frac{m_0}{(1-m_0)}\cdot w\cdot\ln(w)\right) \quad\Leftrightarrow\quad \gamma = W\left(p\cdot\frac{m_0}{(1-m_0)}\cdot w\cdot\ln(w)\right)/\ln(w)$$

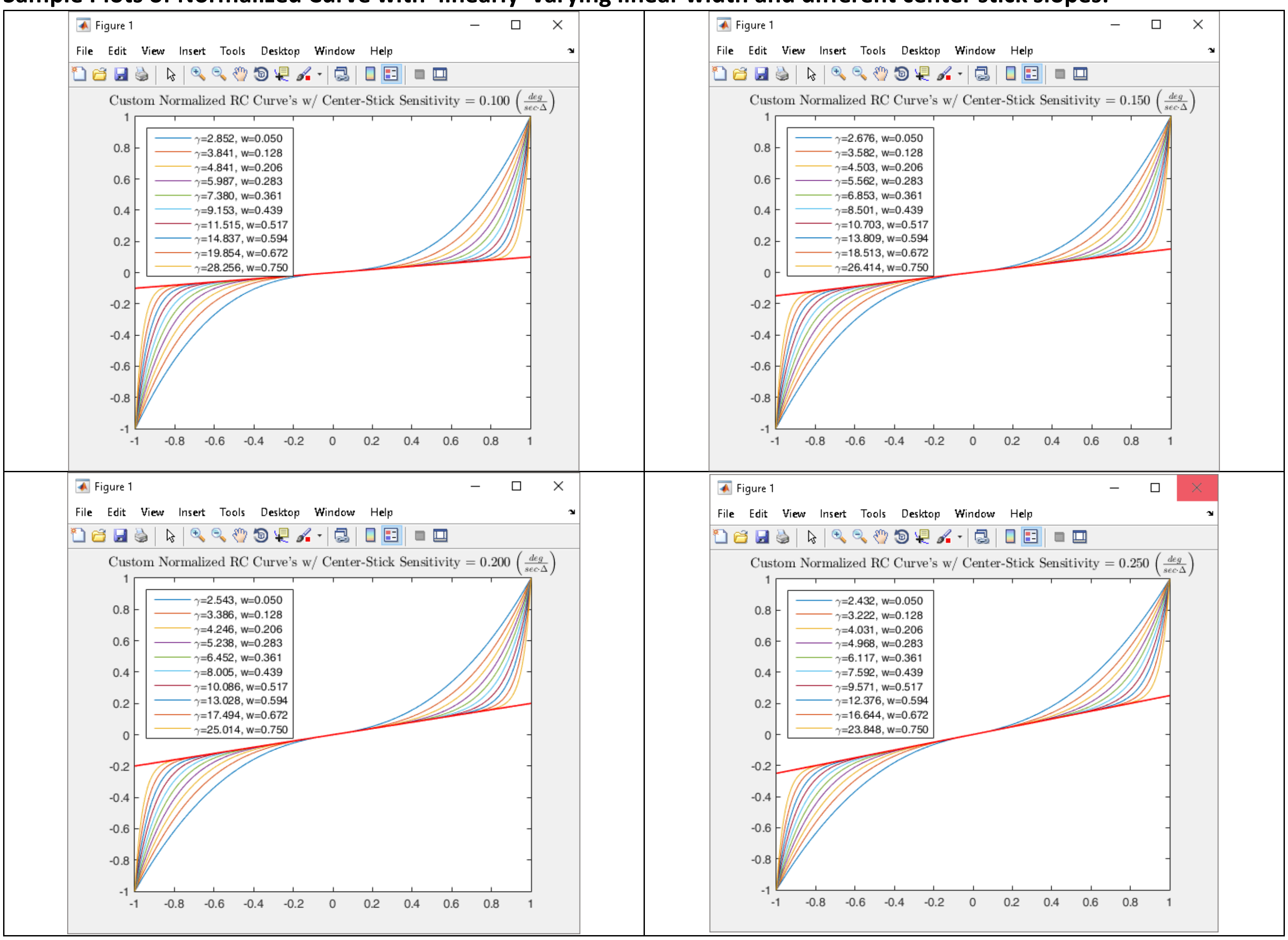
$$\gamma(w; p, m_0) \coloneqq W\left(p\frac{m_0}{(1-m_0)}\cdot w\ln(w)\right)/\ln(w)$$

Now writing the complete form of the new RC curve with the respect to the max rate ω_{MAX}, relative center stick slope m_0, and linear center stick width w (w/ a sub tolerance parameter).

$$\boxed{\omega_{RC}(\hat{x}, \omega_{MAX}, \gamma(\boldsymbol{w}, p, m_0), m_0) = sign(\hat{x})\cdot\omega_{MAX}[(1-m_0)\cdot|\hat{x}|^{\gamma} + m_0\cdot|\hat{x}|]}$$

$$\boxed{\gamma(\boldsymbol{w}, p, m_0) \coloneqq W\left(p\frac{m_0}{(1-m_0)}\cdot w\ln(w)\right)/\ln(w),\ \ where\ W(x)\ is\ defined\ as\ \ x \coloneqq W(x)\cdot e^{W(x)}}$$

Sample Plots of Normalized Curve with 'linearly' varying linear width and different center stick slopes:



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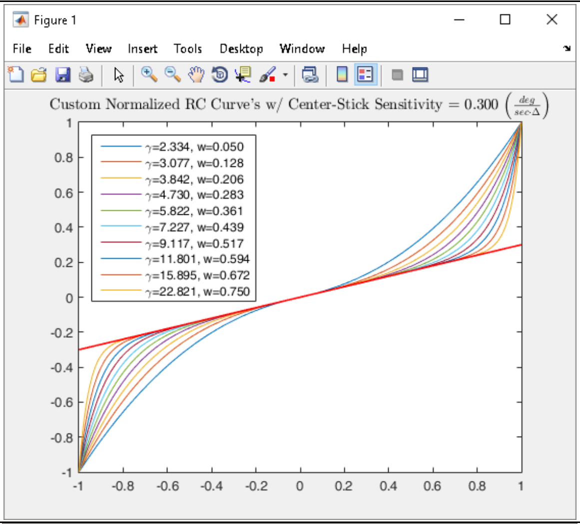
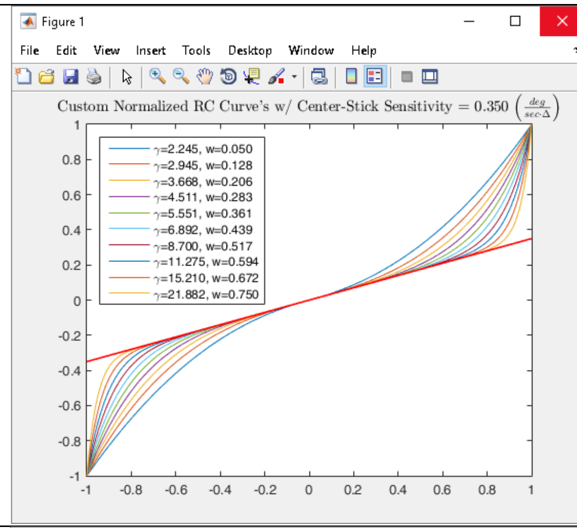
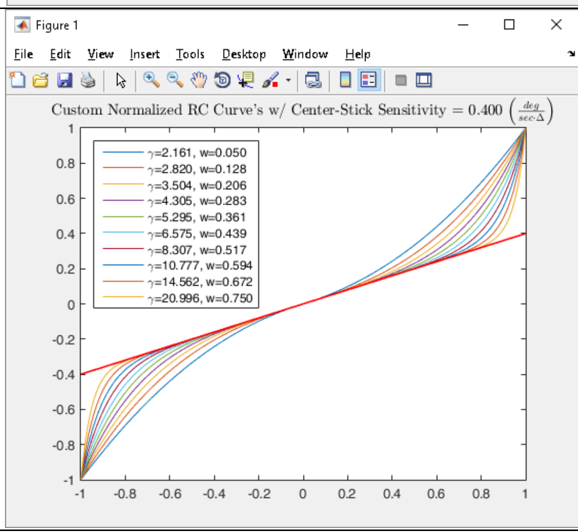
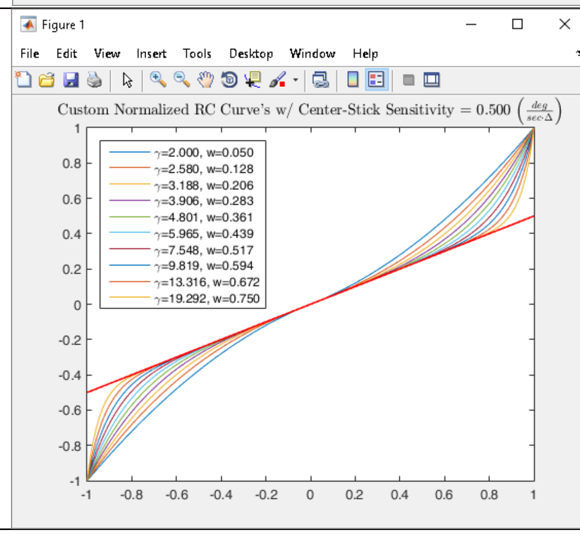
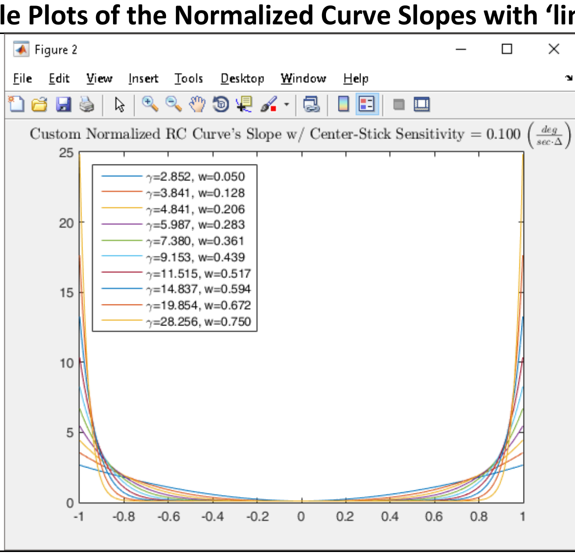


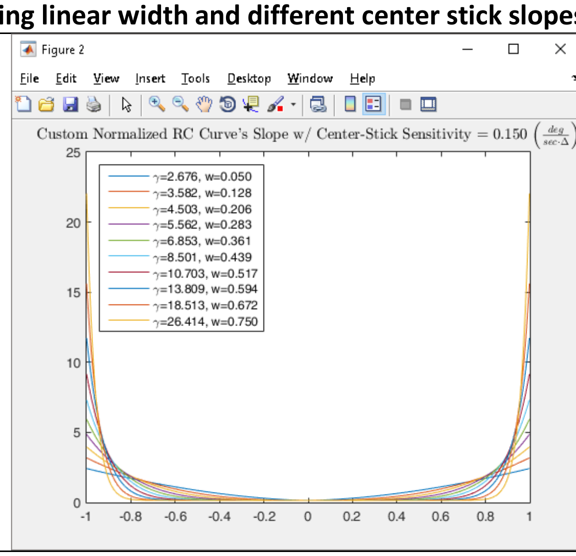






Sample Plots of the Normalized Curve Slopes with 'linearly' varying linear width and different center stick slopes:





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