KISS FC – Finding an Optimal RC-Stick Input Remapping Curve

Obtained code from website http://ultraesc.de/KISSFC/rates.html after html inspection:

```
function calculateDegSec(input,rate,grate,usecurve){
    var setpoint = input;
    var RPY_useRates = 1-Math.abs(input)*grate;
    var rxRAW = input*1000;
    var curve = rxRAW*rxRAW/1000000;
    setpoint = ((setpoint*curve)*usecurve+setpoint*(1-usecurve))*( rate/10);

    return Math.round(((2000*(1/RPY_useRates))*setpoint)*100)/100;
}
```

Simplifying and reducing redundant variables a more mathematical approach:

$$x = RC\ Input,\qquad r = RC\ Rate,\quad \alpha_r = Rate,\quad \rho_r = RC\ Curve,\quad y = RC\ Output$$
$$-1 \le x, y \le 1,\quad 0 \le \rho_r \le 1$$

$$c(x; r, \rho_r) = x^3\left(\frac{r}{10}\right)\cdot\rho_r + x\cdot(1-\rho_r)\left(\frac{r}{10}\right) = [\rho_r\cdot x^3 + (1-\rho_r)\cdot x]\left(\frac{r}{10}\right)$$
$$e(x;\alpha_r) = \frac{2000}{1-\alpha_r\cdot|x|}$$
$$y(x; r, \alpha_r, \rho_r) = round\big(e(x,\alpha_r)\cdot c(x, r, \rho_r)\big)/100$$
$$Y(x) = e(x,\alpha_r)\cdot c(x,r,\rho_r) = \left(\frac{200r}{1-\alpha_r\cdot|x|}\right)[\rho_r\cdot x^3 + (1-\rho_r)\cdot x],\ \ Y(0)=0,\ \ Y(1) = \frac{200r}{1-\alpha_r}$$

$$\frac{d}{dx}Y(x) = Y'(x) = 200r\cdot\frac{\overbrace{\big(3\rho_r x^2 + (1-\rho_r)\big)(1-\alpha_r\cdot x) - (-\alpha_r)(\rho_r\cdot x^3 + (1-\rho_r)\cdot x)}^{N(x)}}{(1-\alpha_r\cdot x)^2},\ \ for\ x \ge 0$$
$$\begin{aligned}N(x) &= \big(3\rho_r x^2 + (1-\rho_r)\big)(1-\alpha_r\cdot x) - (-\alpha_r)(\rho_r\cdot x^3 + (1-\rho_r)\cdot x)\\ &= \big(3\rho_r x^2 + (1-\rho_r)\big) - (\alpha_r\cdot x)\big(3\rho_r x^2 + (1-\rho_r)\big) + \alpha_r\rho_r x^3 + \alpha_r(1-\rho_r)x\\ &= \big(3\rho_r x^2 + (1-\rho_r)\big) - (3\alpha_r\rho_r x^3 + \alpha_r(1-\rho_r)x) + \alpha_r\rho_r x^3 + \alpha_r(1-\rho_r)x\\ &= -2\alpha_r\rho_r x^3 + 3\rho_r x^2 + (1-\rho_r)\end{aligned}$$

$$Y'(x) = 200r\cdot\frac{-2\alpha_r\rho_r x^3 + 3\rho_r x^2 + (1-\rho_r)}{(1-\alpha_r\cdot x)^2},\quad Y'(0) = 200r\cdot(1-\rho_r),\quad Y'(1) = 200r\cdot\frac{2\rho_r(1-\alpha_r)\ +1}{(1-\alpha_r)^2}$$
$$Y(1) = \frac{200r}{1-\alpha_r},\quad \boxed{\alpha_r = 1 - \frac{200r}{Y(1)}\ ,\qquad \alpha_r > 0,\quad r < \frac{Y(1)}{200}}$$
$$Y'(0) = 200r\cdot(1-\rho_r),\quad \boxed{\rho_r = 1 - \left(\frac{Y'(0)}{200r}\right),\quad \rho_r > 0,\quad r > \frac{Y'(0)}{200}}$$

$$Y'(x) = 200r\cdot\frac{-2\alpha_r\rho_r x^3 + 3\rho_r x^2 + (1-\rho_r)}{(1-\alpha_r\cdot x)^2} = (1+p)\cdot 200r\cdot(1-\rho_r),\quad p \ge 0$$
$$-2\alpha_r\rho_r x^3 + 3\rho_r x^2 + (1-\rho_r) = (1+p)(1-\rho_r)(1-\alpha_r\cdot x)^2,\ \ Solve\ for\ x$$
$$-2\alpha_r\rho_r x^3 + 3\rho_r x^2 + (1-\rho_r) = (1+p)(1-\rho_r)(1-2\alpha_r\cdot x + \alpha_r^2 x^2\ )$$
$$-2\alpha_r\rho_r x^3 + \big(3\rho_r - \alpha_r^2(1+p)(1-\rho_r)\big)x^2 + 2\alpha_r(1+p)(1-\rho_r)\cdot x + \big((1-\rho_r) - (1+p)(1-\rho_r)\big) = 0$$