Time Series Analysis

Modeling Heteroskedasticity:
Case Study

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Exchange Rates Prediction:
Comparing Predictions

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About This Lesson

Computing Prediction

```
#Prediction of the return time series and the volatility sigma
    nfore = length(data.test)
    fore.series.1 = NULL
    fore.sigma.1 = NULL
    for(f in 1: nfore){
        data = data.train
        if(f>2)
            data = c(data.train,data.test[1:(f-1)])
            final.model.1 = ugarchfit(spec.1, data, solver = 'hybrid')
            fore = ugarchforecast(final.model.1, n.ahead=1)
            fore.series.1 = c(fore.series.1, fore@forecast$seriesFor)
            fore.sigma.1 = c(fore.sigma.1, fore@forecast$sigmaFor)
    }
```

Loop through all the time points and predict one day at a time

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Prediction Accuracy (USD/EUR)

```
> #Mean Squared Prediction Error (MSPE)
> mean((fore.series.1 - data.test)^2)
[1] 7.351851e-06
> mean((fore.series.2 - data.test)^2)
[1] 7.448684e-06
> #Mean Absolute Prediction Error (MAE)
> mean(abs(fore.series.1 - data.test))
[1] 0.002316292
> mean(abs(fore.series.2 - data.test))
[1] 0.002371034
> #Mean Absolute Percentage Error (MAPE)
> mean(abs(fore.series.1 - data.test)/abs(data.test+0.000001))
[1] 0.9445393
> mean(abs(fore.series.2 - data.test)/abs(data.test+0.000001))
[1] 0.9948057
> #Precision Measure (PM)
> sum((fore.series.1 - data.test)^2)/sum((data.test-mean(data.test))^2)
[1] 1.029351
> sum((fore.series.2 - data.test)^2)/sum((data.test-mean(data.test))^2)
[1] 1.042908
```

Both models perform similarly across all measures with Model 1 performing slightly better;

Prediction Accuracy (USD/BRL)

```
> #Mean Squared Prediction Error (MSPE)
> mean((fore.series.1 - data.test)^2)
[1] 0.005115842
> mean((fore.series.2 - data.test)^2)
[1] 0.005165258
> #Mean Absolute Prediction Error (MAE)
> mean(abs(fore.series.1 - data.test))
[1] 0.05764268
> mean(abs(fore.series.2 - data.test))
[1] 0.05852945
> #Mean Absolute Percentage Error (MAPE)
> mean(abs(fore.series.1 - data.test)/abs(data.test+0.000001))
[1] 1.458612
> mean(abs(fore.series.2 - data.test)/abs(data.test+0.000001))
[1] 1.418509
> #Precision Measure (PM)
> sum((fore.series.1 - data.test)^2)/sum((data.test-mean(data.test))^2)
[1] 1.011352
> sum((fore.series.2 - data.test)^2)/sum((data.test-mean(data.test))^2)
[1] 1.021121
```

Both models perform similarly across all measures with Model 1 performing slightly better

Prediction Accuracy (USD/CYN): 1993-2020

```
> mean((fore.series.1 - data.test)^2)
[1] 0.000260578
> mean((fore.series.2 - data.test)^2)
[1] 0.0002593786
> #Mean Absolute Prediction Error (MAE)
> mean(abs(fore.series.1 - data.test))
[1] 0.01286297
> mean(abs(fore.series.2 - data.test))
[1] 0.012644
> #Mean Absolute Percentage Error (MAPE)
> mean(abs(fore.series.1 - data.test)/abs(data.test+0.000001))
[1] 1.396633
> mean(abs(fore.series.2 - data.test)/abs(data.test+0.000001))
[1] 0.994802
> #Precision Measure (PM)
> sum((fore.series.1 - data.test)^2)/sum((data.test-mean(data.test))^2)
[1] 1.052421
> sum((fore.series.2 - data.test)^2)/sum((data.test-mean(data.test))^2)
[1] 1.047577
```

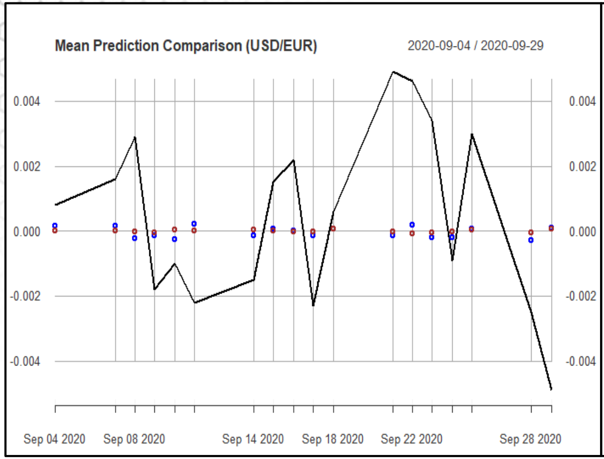
Both models perform similarly across all measures with Model 2 performing slightly better

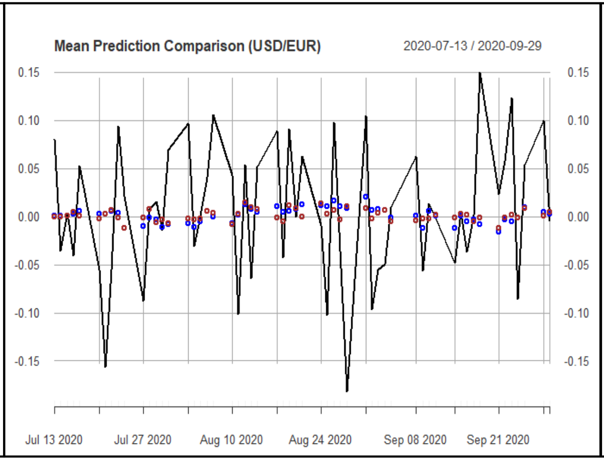
Prediction Accuracy (USD/CYN): 2000-2020

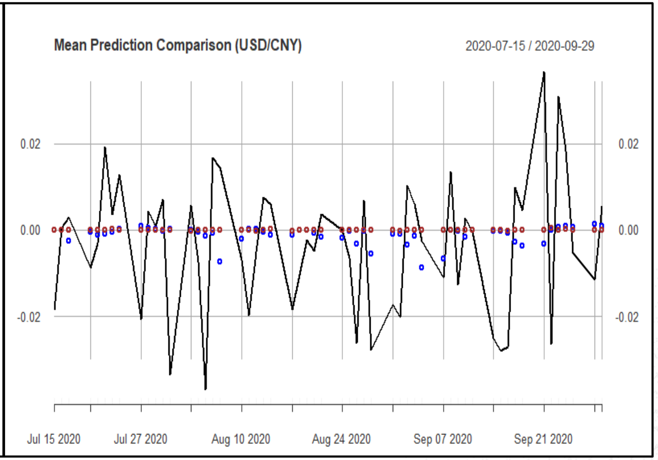
```
#Mean Squared Prediction Error (MSPE)
> mean((fore.series.1 - data.test)^2)
[1] 0.0002584315
> mean((fore.series.2 - data.test)^2)
[1] 0.0002596784
> #Mean Absolute Prediction Error (MAE)
> mean(abs(fore.series.1 - data.test))
[1] 0.01246465
> mean(abs(fore.series.2 - data.test))
[1] 0.01246016
> #Mean Absolute Percentage Error (MAPE)
> mean(abs(fore.series.1 - data.test)/(data.test+0.000001))
[1] 0.1328716
> mean(abs(fore.series.2 - data.test)/(data.test+0.000001))
[1] -0.002343166
> #Precision Measure (PM)
> sum((fore.series.1 - data.test)^2)/sum((data.test-mean(data.test))^2)
[1] 1.043752
> sum((fore.series.2 - data.test)^2)/sum((data.test-mean(data.test))^2)
[1] 1.048787
```

The accuracy measures are similar regardless of the time period considered

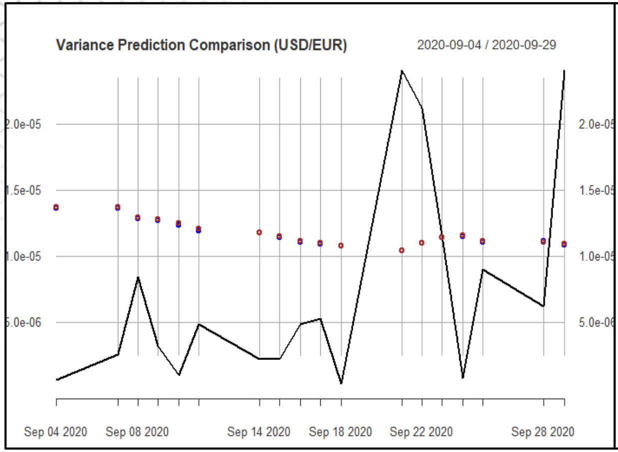
Mean Prediction Comparison

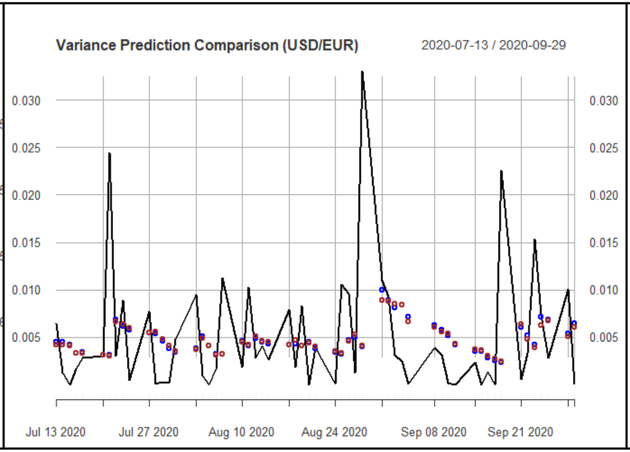


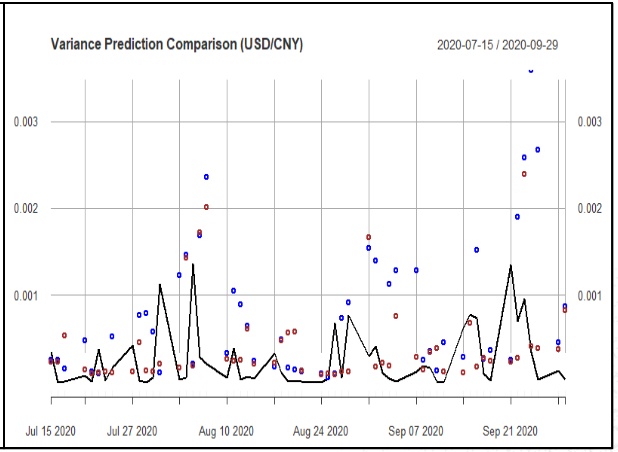




Variance Prediction Comparison







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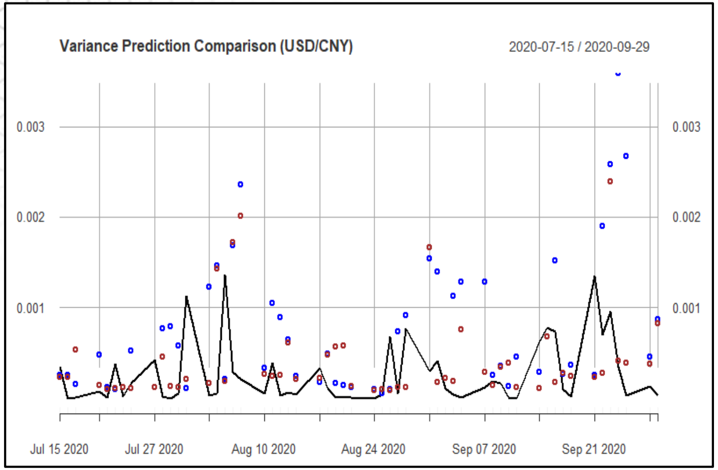
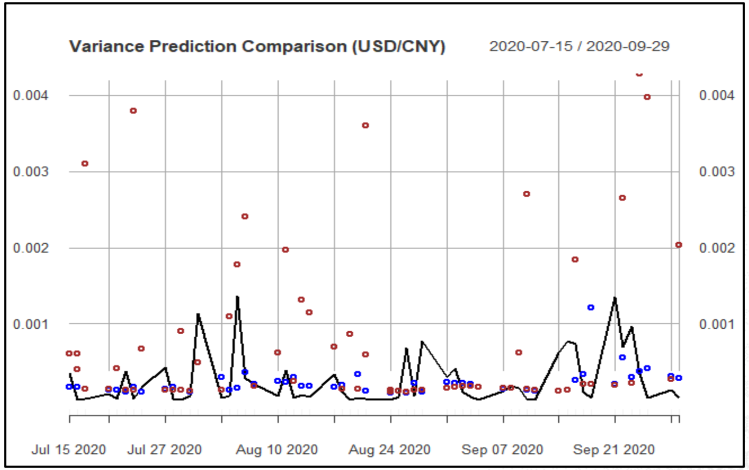
Variance Prediction Comparison: USD/CYN

Full Time Series



Reduced Time Series



Model 1 shown in blue & Model 2 shown in brown:

- Model 1 using full time series captures more of the time-varying volatility
- Model 2 using reduced time series captures more of the time-varying volatility

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Other Models: R Implementation

#GARCH

```
spec.1 = ugarchspec(variance.model=list(garchOrder=c(1,1)), mean.model=
      list(armaOrder=c(2,1), include.mean=T), distribution.model="std")
```

#GJR-GARCH

```
spec.2 = ugarchspec(variance.model=list(model = "gjrGARCH",garchOrder=c(1,1)),
      mean.model=list(armaOrder=c(2,1), include.mean=T), distribution.model="std")
```

#EGARCH

```
spec.3 = ugarchspec(variance.model=list(model ="eGARCH",garchOrder=c(1,1)),
      mean.model=list(armaOrder=c(2,1),include.mean=T), distribution.model="std")
```

#APARCH

```
spec.4 = ugarchspec(variance.model=list(model = "apARCH",garchOrder=c(1,1)),
      mean.model=list(armaOrder=c(2,1), include.mean=T), distribution.model="std")
```

#IGARCH

```
spec.5 = ugarchspec(variance.model=list(model = "iGARCH",garchOrder=c(1,1)),
      mean.model=list(armaOrder=c(2,1), include.mean=T), distribution.model="std")
```

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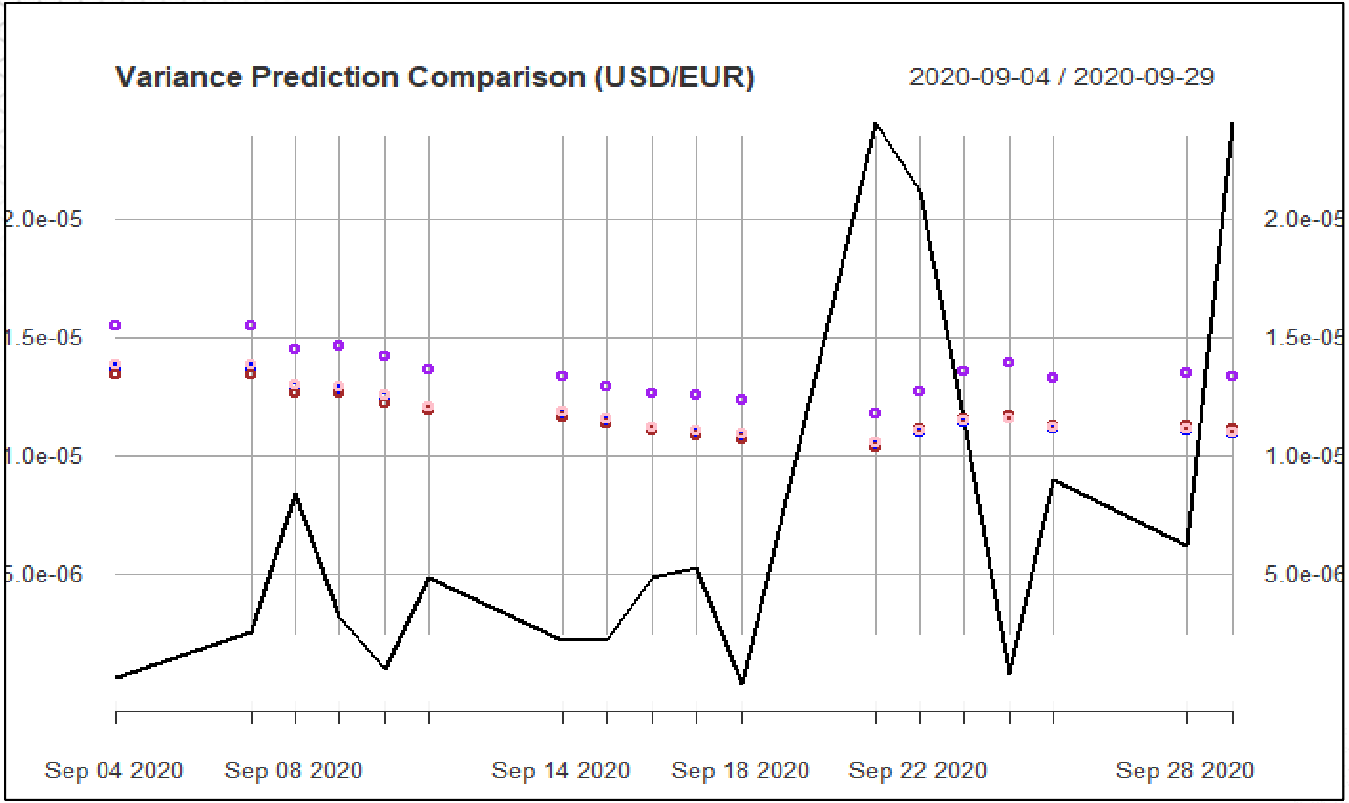
Variance Prediction Comparison



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Take Home Conclusions

- Exchange currencies behave differently primarily with respect to the volatility in the change of the exchange rate.

- Volatility in the USD-Euro is much lower than that for USD-BLR and USD-CYN.

- Predicted volatility captures the temporal variations better for the exchange rate changes with large volatility.

- Different time periods of the same time series can result in different predictions although not necessarily of better or worse accuracy as illustrated with the USD-CYN exchange rate.

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Summary