# Table of Common Distributions

## Discrete Distributions

***Bernoulli***$(p)$

*pmf* $P(X = x|p) = p^x(1-p)^{1-x}; \quad x = 0, 1; \quad 0 \le p \le 1$

*mean and variance* $\mathrm{E}X = p, \quad \mathrm{Var}\, X = p(1-p)$

*mgf* $M_X(t) = (1-p) + pe^t$

***Binomial***$(n, p)$

*pmf* $P(X = x|n, p) = \binom{n}{x} p^x(1-p)^{n-x}; \quad x = 0, 1, 2, \ldots, n; \quad 0 \le p \le 1$

*mean and variance* $\mathrm{E}X = np, \quad \mathrm{Var}\, X = np(1-p)$

*mgf* $M_X(t) = [pe^t + (1-p)]^n$

*notes* Related to Binomial Theorem (Theorem 3.2.2). The *multinomial* d bution (Definition 4.6.2) is a multivariate version of the binomial d bution.

***Discrete uniform***

*pmf* $P(X = x|N) = \frac{1}{N}; \quad x = 1, 2, \ldots, N; \quad N = 1, 2, \ldots$

*mean and variance* $\mathrm{E}X = \frac{N+1}{2}, \quad \mathrm{Var}\, X = \frac{(N+1)(N-1)}{12}$

*mgf* $M_X(t) = \frac{1}{N} \sum_{i=1}^{N} e^{it}$

***Geometric***$(p)$

*pmf* $P(X = x|p) = p(1-p)^{x-1}; \quad x = 1, 2, \ldots; \quad 0 \le p \le 1$

*mean and variance* $\mathrm{E}X = \frac{1}{p}, \quad \mathrm{Var}\, X = \frac{1-p}{p^2}$

*mgf* $M_X(t) = \frac{pe^t}{1-(1-p)e^t}, \quad t < -\log(1-p)$

*notes* $Y = X - 1$ is negative binomial$(1, p)$. The distribution is *memoryless*: $P(X > s|X > t) = P(X > s - t)$.

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***Hypergeometric***

*pmf* $P(X = x|N, M, K) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}; \quad x = 0, 1, 2, \ldots, K;$
$M - (N - K) \leq x \leq M; \quad N, M, K \geq 0$

*mean and variance* $EX = \frac{KM}{N}, \quad \text{Var}\, X = \frac{KM}{N}\frac{(N-M)(N-K)}{N(N-1)}$

*notes* If $K \ll M$ and $N$, the range $x = 0, 1, 2, \ldots, K$ will be appropriate.

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***Negative binomial***$(r, p)$

*pmf* $P(X = x|r, p) = \binom{r+x-1}{x}p^r(1-p)^x; \quad x = 0, 1, \ldots; \quad 0 \leq p \leq 1$

*mean and variance* $EX = \frac{r(1-p)}{p}, \quad \text{Var}\, X = \frac{r(1-p)}{p^2}$

*mgf* $M_X(t) = \left(\frac{p}{1-(1-p)e^t}\right)^r, \quad t < -\log(1-p)$

*notes* An alternate form of the pmf is given by $P(Y = y|r, p) = \binom{y-1}{r-1}p^r(1-p)^{y-r}$, $y = r, r+1, \ldots$. The random variable $Y = X + r$. The negative binomial can be derived as a gamma mixture of Poissons. (See Exercise 4.34.)

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***Poisson***$(\lambda)$

*pmf* $P(X = x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; \quad x = 0, 1, \ldots; \quad 0 \leq \lambda < \infty$

*mean and variance* $EX = \lambda, \quad \text{Var}\, X = \lambda$

*mgf* $M_X(t) = e^{\lambda(e^t-1)}$

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## Continuous Distributions

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***Beta***$(\alpha, \beta)$

*pdf* $f(x|\alpha, \beta) = \frac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \leq x \leq 1, \quad \alpha > 0, \quad \beta > 0$

*mean and variance* $EX = \frac{\alpha}{\alpha+\beta}, \quad \text{Var}\, X = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

*mgf* $M_X(t) = 1 + \sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1}\frac{\alpha+r}{\alpha+\beta+r}\right)\frac{t^k}{k!}$

*notes* The constant in the beta pdf can be defined in terms of gamma functior $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$. Equation (3.2.18) gives a general expression for tl moments.

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***Cauchy***$(\theta, \sigma)$

*pdf* $f(x|\theta, \sigma) = \frac{1}{\pi\sigma}\frac{1}{1+\left(\frac{x-\theta}{\sigma}\right)^2}, \quad -\infty < x < \infty; \quad -\infty < \theta < \infty, \quad \sigma > 0$

*mean and variance* do not exist

*mgf* does not exist

*notes* Special case of Student's $t$, when degrees of freedom = 1. Also, if $X$ a Y are independent $n(0, 1)$, $X/Y$ is Cauchy.

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***Chi squared***$(p)$

*pdf* $f(x|p) = \frac{1}{\Gamma(p/2)2^{p/2}}x^{(p/2)-1}e^{-x/2}; \quad 0 \leq x < \infty; \quad p = 1, 2, \ldots$

*mean and variance* $EX = p, \quad \text{Var}\, X = 2p$

*mgf* $M_X(t) = \left(\frac{1}{1-2t}\right)^{p/2}, \quad t < \frac{1}{2}$

*notes* Special case of the gamma distribution.

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***Double exponential***$(\mu, \sigma)$

*pdf* $f(x|\mu, \sigma) = \frac{1}{2\sigma}e^{-|x-\mu|/\sigma}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$

*mean and variance* $EX = \mu, \quad \text{Var}\, X = 2\sigma^2$

*mgf* $M_X(t) = \frac{e^{\mu t}}{1-(\sigma t)^2}, \quad |t| < \frac{1}{\sigma}$

*notes* Also known as the *Laplace* distribution.

***Exponential***$(\beta)$

*pdf* $f(x|\beta) = \frac{1}{\beta}e^{-x/\beta}, \quad 0 \le x < \infty, \quad \beta > 0$

*mean and variance* $\mathrm{E}X = \beta, \quad \mathrm{Var}\,X = \beta^2$

*mgf* $M_X(t) = \frac{1}{1-\beta t}, \quad t < \frac{1}{\beta}$

*notes* Special case of the gamma distribution. Has the *memoryless* property. Has many special cases: $Y = X^{1/\gamma}$ is *Weibull*, $Y = \sqrt{2X/\beta}$ is *Rayleigh*, $Y = \alpha - \gamma \log(X/\beta)$ is *Gumbel*.

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***F***

*pdf* $f(x|\nu_1,\nu_2) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} \frac{x^{(\nu_1-2)/2}}{\left(1+\left(\frac{\nu_1}{\nu_2}\right)x\right)^{(\nu_1+\nu_2)/2}};$
$0 \le x < \infty; \quad \nu_1,\nu_2 = 1,\ldots$

*mean and variance* $\mathrm{E}X = \frac{\nu_2}{\nu_2-2}, \quad \nu_2 > 2,$

$\mathrm{Var}\,X = 2\left(\frac{\nu_2}{\nu_2-2}\right)^2 \frac{(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}, \quad \nu_2 > 4$

*moments (mgf does not exist)* $\mathrm{E}X^n = \frac{\Gamma\left(\frac{\nu_1+2n}{2}\right)\Gamma\left(\frac{\nu_2-2n}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\left(\frac{\nu_2}{\nu_1}\right)^n, \quad n < \frac{\nu_2}{2}$

*notes* Related to chi squared ($F_{\nu_1,\nu_2} = \left(\frac{\chi^2_{\nu_1}}{\nu_1}\right) \Big/ \left(\frac{\chi^2_{\nu_2}}{\nu_2}\right)$, where the $\chi^2$s are independent) and $t$ ($F_{1,\nu} = t^2_\nu$).

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***Gamma***$(\alpha,\beta)$

*pdf* $f(x|\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}, \quad 0 \le x < \infty, \quad \alpha,\beta > 0$

*mean and variance* $\mathrm{E}X = \alpha\beta, \quad \mathrm{Var}\,X = \alpha\beta^2$

*mgf* $M_X(t) = \left(\frac{1}{1-\beta t}\right)^\alpha, \quad t < \frac{1}{\beta}$

*notes* Some special cases are exponential ($\alpha = 1$) and chi squared ($\alpha = p/2$, $\beta = 2$). If $\alpha = \frac{3}{2}$, $Y = \sqrt{X/\beta}$ is *Maxwell*. $Y = 1/X$ has the *inverted gamma distribution*. Can also be related to the Poisson (Example 3.2.1).

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***Logistic***$(\mu,\beta)$

*pdf* $f(x|\mu,\beta) = \frac{1}{\beta}\frac{e^{-(x-\mu)/\beta}}{[1+e^{-(x-\mu)/\beta}]^2}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \beta > 0$

*mean and variance* $\mathrm{E}X = \mu, \quad \mathrm{Var}\,X = \frac{\pi^2\beta^2}{3}$



*mgf* $M_X(t) = e^{\mu t}\Gamma(1-\beta t)\Gamma(1+\beta t), \quad |t| < \frac{1}{\beta}$

*notes* The cdf is given by $F(x|\mu,\beta) = \frac{1}{1+e^{-(x-\mu)/\beta}}$.

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***Lognormal***$(\mu,\sigma^2)$

*pdf* $f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}\frac{e^{-(\log x-\mu)^2/(2\sigma^2)}}{x}, \quad 0 \le x < \infty, \quad -\infty < \mu < \infty,$
$\sigma > 0$

*mean and variance* $\mathrm{E}X = e^{\mu+(\sigma^2/2)}, \quad \mathrm{Var}\,X = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$

*moments (mgf does not exist)* $\mathrm{E}X^n = e^{n\mu+n^2\sigma^2/2}$

*notes* Example 2.3.5 gives another distribution with the same moments.

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***Normal***$(\mu,\sigma^2)$

*pdf* $f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/(2\sigma^2)}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty,$
$\sigma > 0$

*mean and variance* $\mathrm{E}X = \mu, \quad \mathrm{Var}\,X = \sigma^2$

*mgf* $M_X(t) = e^{\mu t+\sigma^2t^2/2}$

*notes* Sometimes called the *Gaussian* distribution.

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***Pareto***$(\alpha,\beta)$

*pdf* $f(x|\alpha,\beta) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad a < x < \infty, \quad \alpha > 0, \quad \beta > 0$

*mean and variance* $\mathrm{E}X = \frac{\beta\alpha}{\beta-1}, \quad \beta > 1, \quad \mathrm{Var}\,X = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}, \quad \beta > 2$

*mgf* does not exist

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***t***

*pdf* $f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\left(\frac{x^2}{\nu}\right)\right)^{(\nu+1)/2}}, \quad -\infty < x < \infty, \quad \nu = 1,\ldots$

*mean and variance* $\mathrm{E}X = 0, \quad \nu > 1, \quad \mathrm{Var}\,X = \frac{\nu}{\nu-2}, \quad \nu > 2$

*moments (mgf does not exist)* $\mathrm{E}X^n = \frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{\nu-n}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\nu^{n/2}$ if $n < \nu$ and even,
$\mathrm{E}X^n = 0$ if $n < \nu$ and odd.

*notes* Related to $F$ ($F_{1,\nu} = t^2_\nu$).

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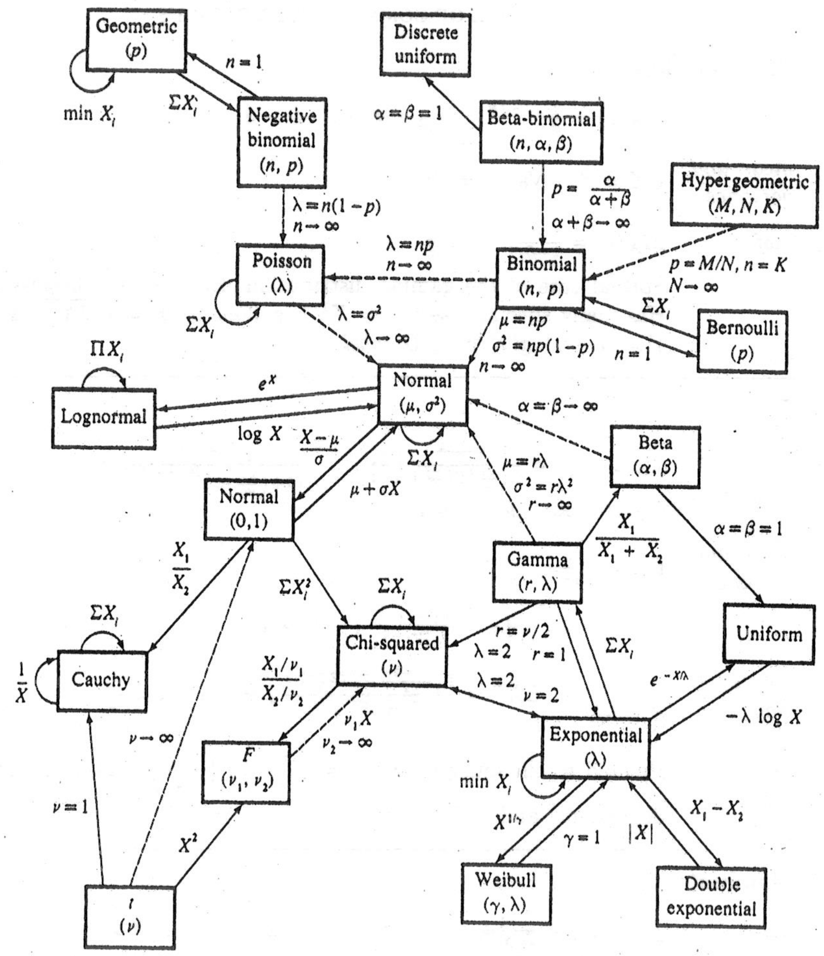
*Uniform*$(a,b)$

| | |
|---|---|
| *pdf* | $f(x\|a,b)=\frac{1}{b-a},\quad a\le x\le b$ |
| *mean and variance* | $\mathrm{E}X=\frac{b+a}{2},\quad \mathrm{Var}\,X=\frac{(b-a)^2}{12}$ |
| *mgf* | $M_X(t)=\frac{e^{bt}-e^{at}}{(b-a)t}$ |
| *notes* | If $a=0$ and $b=1$, this is a special case of the beta ($\alpha=\beta=1$). |

***Weibull***$(\gamma,\beta)$

| | |
|---|---|
| *pdf* | $f(x\|\gamma,\beta)=\frac{\gamma}{\beta}x^{\gamma-1}e^{-x^{\gamma}/\beta},\quad 0\le x<\infty,\quad \gamma>0,\quad \beta>0$ |
| *mean and variance* | $\mathrm{E}X=\beta^{1/\gamma}\Gamma\left(1+\frac{1}{\gamma}\right),\quad \mathrm{Var}\,X=\beta^{2/\gamma}\left[\Gamma\left(1+\frac{2}{\gamma}\right)-\Gamma^2\left(1+\frac{1}{\gamma}\right)\right]$ |
| *moments* | $\mathrm{E}X^n=\beta^{n/\gamma}\Gamma\left(1+\frac{n}{\gamma}\right)$ |
| *notes* | The mgf exists only for $\gamma\ge 1$. Its form is not very useful. A special case is exponential ($\gamma=1$). |






**Relationships among common distributions.** Solid lines represent transformations and special cases, dashed lines represent limits. Adapted from Leemis (1986).