Time Series Analysis

Modeling Heteroskedasticity

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Other Heteroskedasticity Models

About This Lesson

Other Heteroskedasticity Models: EGARCH

Standardized residuals from GARCH models still display some presence of fat tails. This led to the following models:

- R_t is modelled as having a non-Gaussian distribution (e.g. t-distribution)
- EGARCH (exponential GARCH)
  1. It models $\log(\sigma_t^2)$ instead of σ_t^2.
  2. It allows the variance to have asymmetric behavior to reflect any asymmetries in the data. This model uses $|Z_{t-i}|$ along with Z_{t-i} to capture the asymmetries in the returns data in the heteroskedasticity model.

Reference: Daniel B. Nelson (1991). “Conditional heteroskedasticity in asset returns: A new approach.” Econometrica, 59, pages 347-370

Other Heteroskedasticity Models: APARCH

- In some time series, large negative observations appear to increase volatility more than do positive observations of the same magnitude. This is called the *leverage effect*.
- GARCH models cannot model the leverage effect because they model σ_t^2 as a function of past values of Z_t^2-- whether positive or negative is not taken into account
- The solution is to replace the square function with a flexible class of nonnegative functions that include asymmetric functions.
- APARCH(p; q) model for the conditional standard deviation

$$\sigma_t^\delta = \gamma_0 + \sum_{i=1}^{p} \alpha_i (|Z_{t-i}| - \gamma_i Z_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta$$

- For δ=2 and $\gamma_1 = \cdots = \gamma_p = 0$ it becomes GARCH

Other Heteroskedasticity Models

- IGARCH (Integrated GARCH). These are models where the variance is nonstationary. It has the form of the GARCH model, where the GARCH parameters add to one. For example, an IGARCH(1,1) model will have the form:
  $$\sigma_t^2 = \gamma_0 + \gamma_1 Z_{t-1}^2 + (1-\gamma_1)\sigma_{t-1}^2$$
- Threshold GARCH model of Zakoian (1994) or GJR model of Glosten, Jagannathan, and Runkle (1993).
- Conditional heteroscedastic ARMA (CHARMA) model of Tsay (1987).
- Random coefficient autoregressive (RCA) model of Nicholls and Quinn (1982).
- Stochastic volatility (SV) models of Melino and Turnbull (1990), Harvey, Ruiz and Shephard (1994), and Jacquier, Polson and Rossi (1994).

Summary