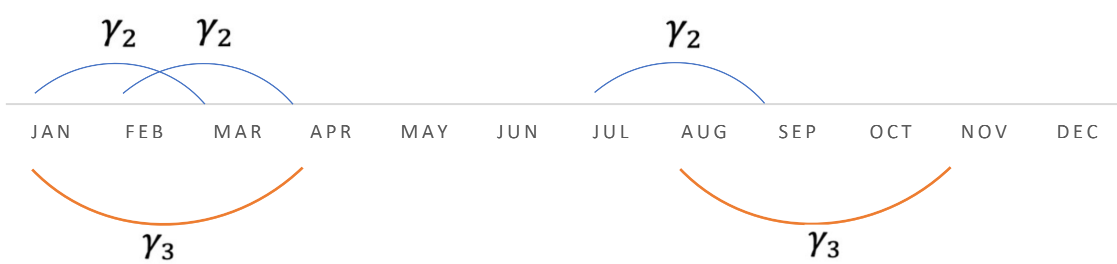
Module Demo.4

Stationary Processes. Notes on Weakly Stationary Condition.

The process is weakly stationary if the first two moments are time invariant.
$E[X_t] = \mu$ and $Cov[X_t, X_{t-h}] = \gamma_h$

In the case of the covariance, it will depend on the lag distance not on time period. For instance, if we are looking for the covariance between observations lagged two periods, i.e. $Cov[X_t, X_{t-2}]$, it will be the same across all the time line, $Cov[X_t, X_{t-2}] = \gamma_2$.
Consider monthly data, this will imply that if we get the covariance of the data corresponding to the months March (X_t) and January (X_{t-2}), it will be γ_2. Now, the covariance between April (X_t) and February (X_{t-2}) will also be γ_2. The same will apply for the covariance between September (X_t) and July (X_{t-2}), $Cov[\text{September}, \text{July}] = \gamma_2$.
Thus, we can see that, for this data set, the covariance for a lag of two will always be γ_2, independent of the time point we are analyzing it.
Now, if we look for the covariance for points lagged 3 periods $Cov[X_t, X_{t-3}]$, for the process to be stationary, it will also have to be constant over time. Let say we analyze the covariance between April (X_t) and January (X_{t-3}), and the result is γ_3. Now, if we look for the covariance between November (X_t) and August (X_{t-3}), for the process to be weakly stationary the result will also be γ_3.



Note that the covariance results varied when the lag distance changed:

- When $h = 2$, $Cov[X_t, X_{t-2}] = \gamma_2$, regardless of the months.
- When $h = 3$, $Cov[X_t, X_{t-3}] = \gamma_3$, regardless of the months.

Thus, the value of the covariance does not depend on time, but on lag distance.