# Time Series Analysis

## ARMA Models

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## Basic Concepts: ARMA Simulation

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# About This Lesson

# White Noise Process

```
## simulate normal/exponential WN
 w1 = rnorm(1000,0,1)
w2 = rexp(1000,1)
## Re-scale to mean 0 and std dev 1
w1 = (w1-mean(w1))/sqrt(var(w1))
w2 = (w2-mean(w2))/sqrt(var(w2))
## Plot ts and their acf's
w1 = ts(w1,start=1,deltat=1)
w2 = ts(w2,start=1,deltat=1)
par(mfrow=c(2,2))
ts.plot(w1,main='Normal')
ts.plot(w2,main='Exponential')
acf(w1,main=''); acf(w2,main='')
```

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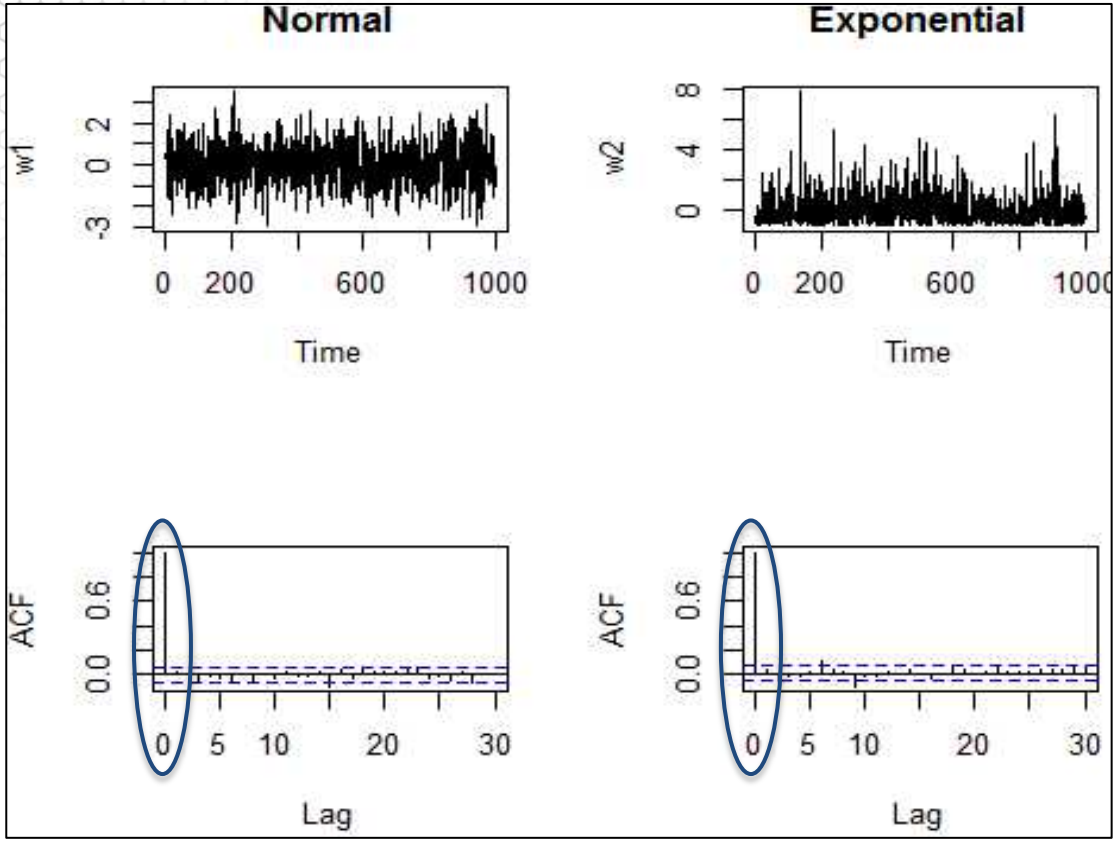
# White Noise

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# Moving Average Processes

```
## Simulate White Noise
w1 = rnorm(502)
w2 = rexp(502)-1
## Set coefficients
a = c(1,-.5,.2)
a1 = c(1,.5,.2)
## Simulate MA(2) with Normal/Exp WN
ma2.11 = filter(w1,filter=a,side=1)
ma2.11 = ma2.11[3:502]
ma2.12 = filter(w1,filter=a1,side=1)
ma2.12 = ma2.12[3:502]
ma2.21 = filter(w2,filter=a,side=1)
ma2.21 = ma2.21[3:502]
ma2.22 = filter(w2,filter=a1,side=1)
ma2.22 = ma2.22[3:502]
```

$$X_t = Z_t - 0.5\,Z_{t-1} + 0.2\,Z_{t-2}$$
$$X_t = Z_t + 0.5\,Z_{t-1} + 0.2\,Z_{t-2}$$

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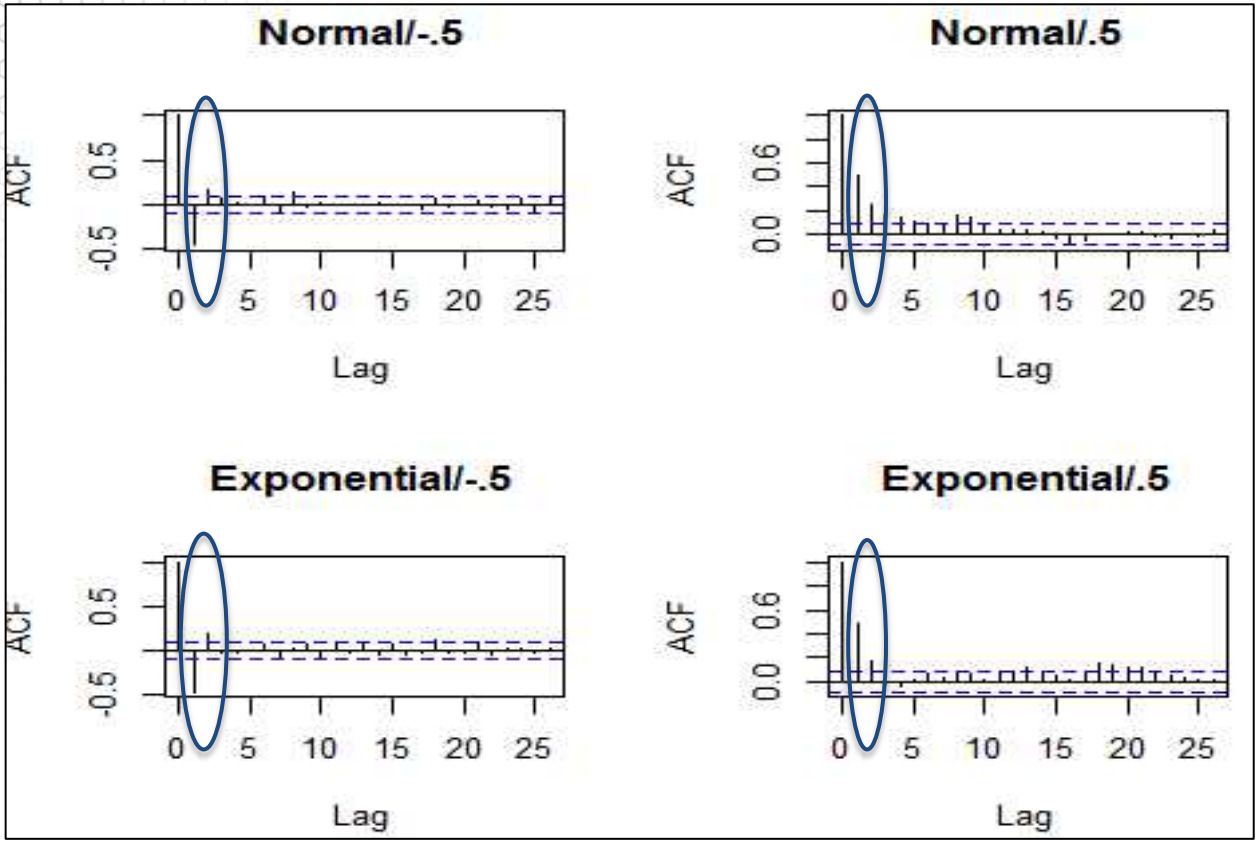
# Moving Average

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# Moving Average: Non-Stationary Noise

```
## Simulate White Noise
w1 = rnorm(502)
## Set coefficients
a4 = c(1,.2,.8,1.2)
## Simulate MA(3) with non-stationary noise
ma2.4 = filter(w1*(2*(1:502)+0.5),filter=a4,side=1)
ma2.4 = ma2.4[4:502]
```

$X_t = Z_t + 0.2\ Z_{t-1} + 0.8\ Z_{t-2} + 1.2\ Z_{t-3}$ with $Z_t = \mathrm{WN}(0,1)*2t+0.5$

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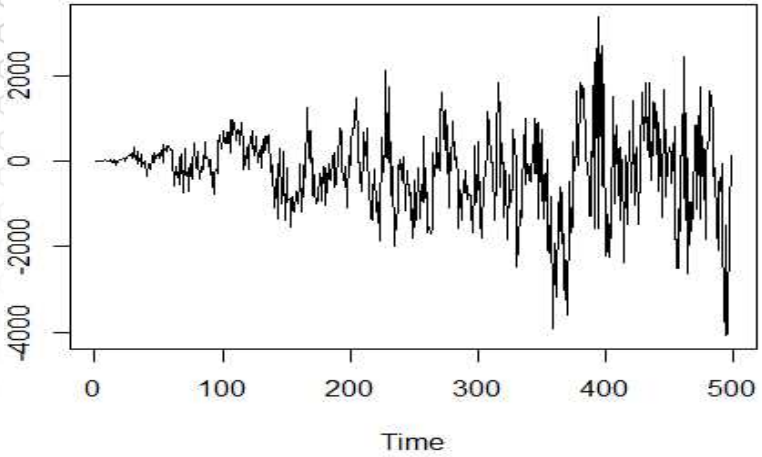
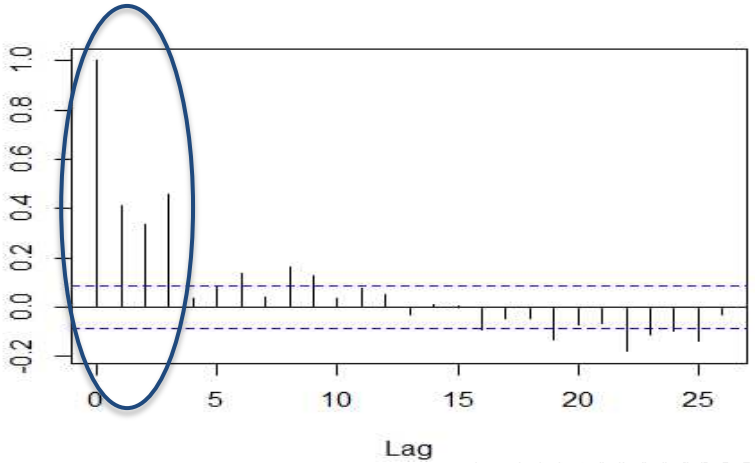
# Moving Average: Non-Stationary Noise

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# Autoregressive Process

```
## Nonstationary AR(2)
w2 = rnorm(1500)
a2 = c(0.8,0.2)
ar2 = filter(w2,filter=a2,method='recursive')
ar2 = ar2[1251:1500]
## Stationary AR(1) process
a1 = 0.5
ar1 = filter(w2,filter=a1,method='recursive')
ar1 = ar1[1251:1500]
```

$$X_t = 0.8\, X_{t-1} + 0.2\, X_{t-2} + Z_t$$

$$X_t = 0.5\, X_{t-1} + Z_t$$

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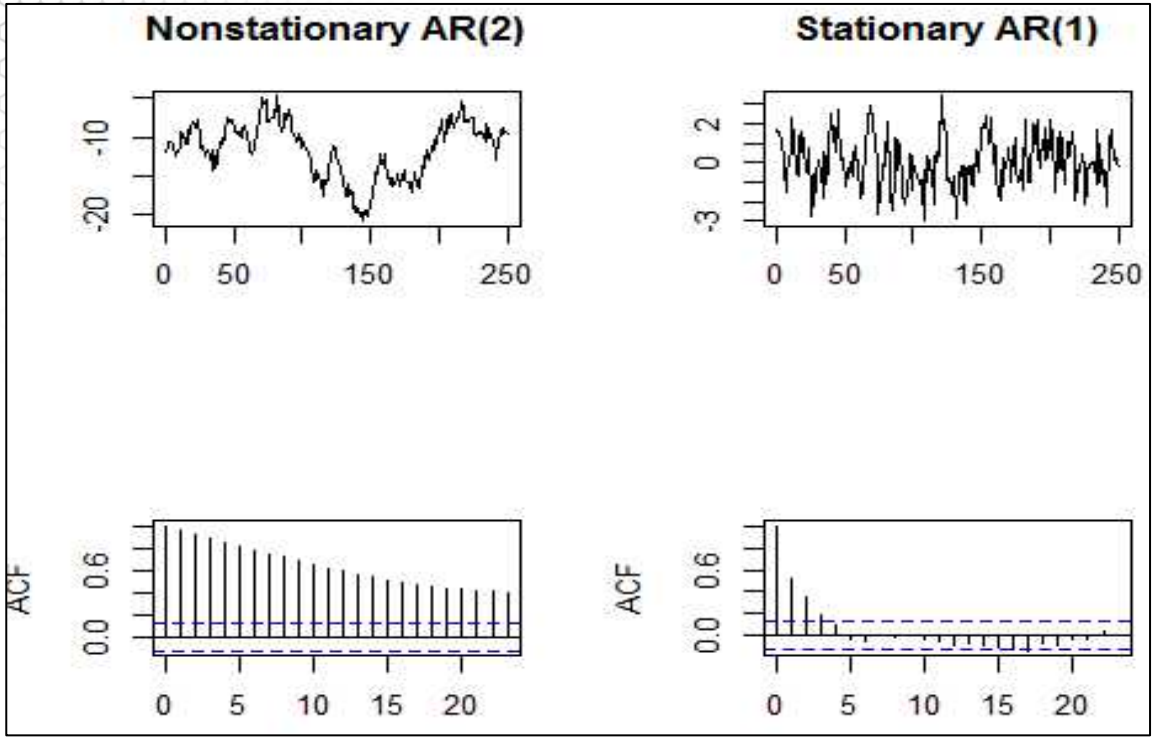
# Autoregressive Process

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# AR & MA Processes: Take Home Points

- ARMA processes -- stationarity vs non-stationarity
  - Defined for stationary processes
  - Simulated MA & AR as non-stationary processes
  - Non-stationary in MA processes not detected using the ACF plot
  - Non-stationary in AR processes detected using the ACF plot

# Summary