ISYE 6402 Homework 8

Background

We have explored how various U.S. economic indicators are related to each other, which is a classic application for the VAR modeling. In this problem, we will study the inter-dependence and Granger causality between various economic indicators.

Instructions on reading the data

To read the data in R, save the file in your working directory (make sure you have changed the directory if different from the R working directory) and read the data using the R function `read.csv()`

```
# Read the monthly and quarterly data
#fname <- file.choose()
data <- read.csv("QDataR.csv",head = TRUE)
#fname2 <- file.choose()
data2 <- read.csv("MDataR.csv",head = TRUE)
date.quarter <- as.Date(data[,1],"%m/%d/%Y")
date.month <- as.Date(data2[,1],"%m/%d/%Y")
```

Here are the libraries you will need:

```
library(data.table)
library(vars)
library(xts)
library(mgcv)
library(stats)
library(tseries)
library(aod)
```

Question 1: Univariate Analysis

Question 1a

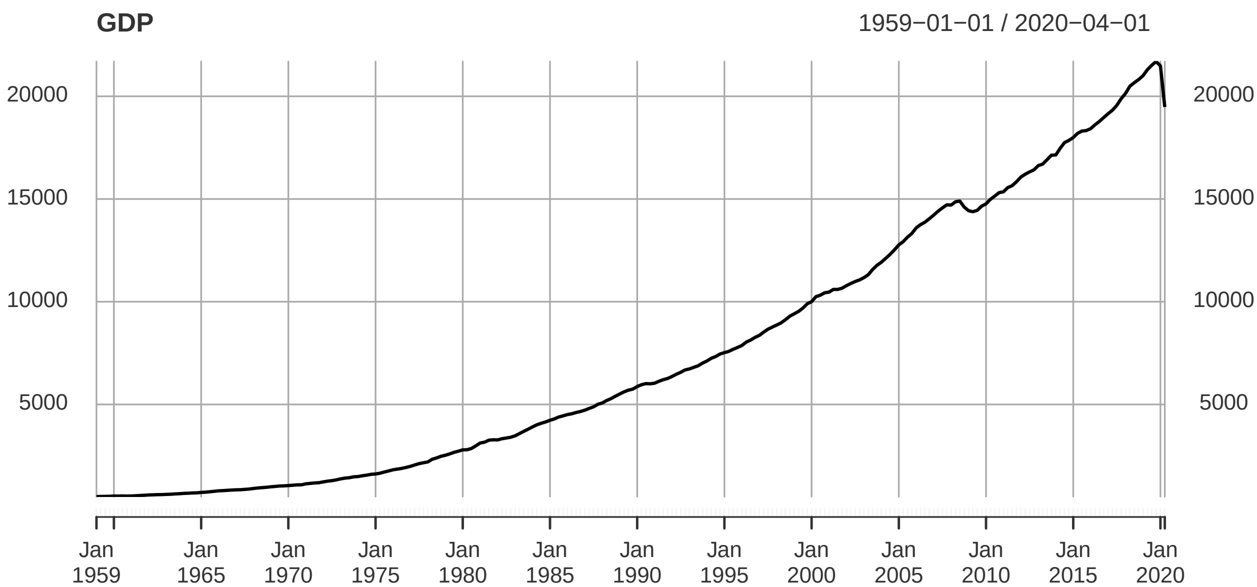
Plot the time series of all indicators for comparison and discuss whether you find any similarities in terms of trend or other features. Plot also the 1st order difference plots and the corresponding ACF plots. Interpret in terms of stationarity and volatility.

Keep in mind, 2 variables have monthly data, while three have quarterly data. You will need to standardise all into quarterly data time series in order to effectively answer all questions below.

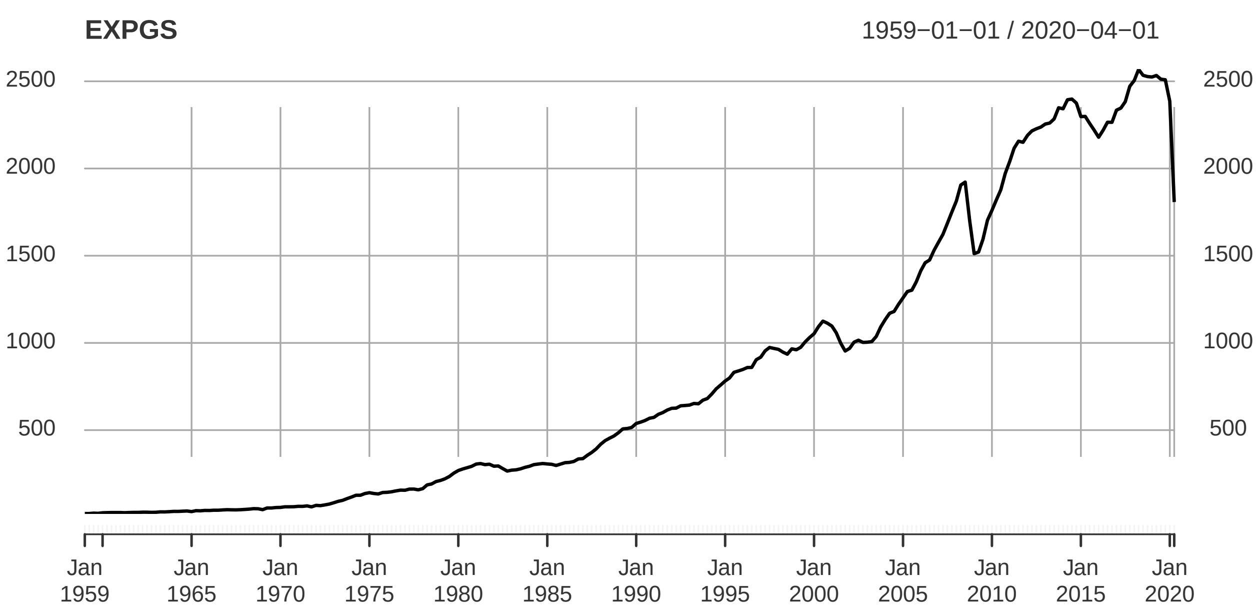
```
# transform into time series
gdp.ts <- xts(data$GDP, date.quarter)
expgs.ts <- xts(data$EXPGS, date.quarter)
imgpgsc.ts <- xts(data$IMPGSC1, date.quarter)
unrate.ts <- xts(data2$UNRATE, date.month)
pce.ts <- xts(data2$PCE, date.month)
```

```r
# merge into multivariate time series
ts.merge <- merge(gdp.ts, expgs.ts, join = 'inner')
ts.merge <- merge(ts.merge, imgpgsc.ts, join = 'inner')
ts.merge <- merge(ts.merge, unrate.ts, join = 'inner')
ts.merge <- merge(ts.merge, pce.ts, join = 'inner')
colnames(ts.merge) <- c("tsgdp", "tsexpgs", "tsimgpsc", "tsunrate", "tspce")
```

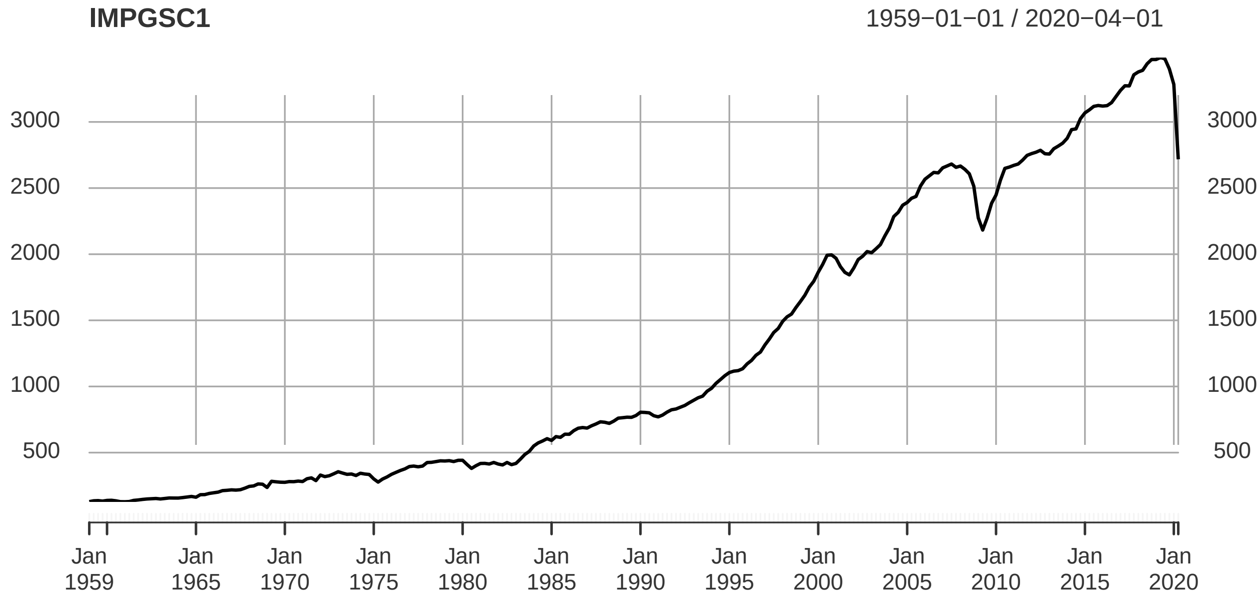
```r
#plots of original time series data.
plot(gdp.ts,main='GDP')
```



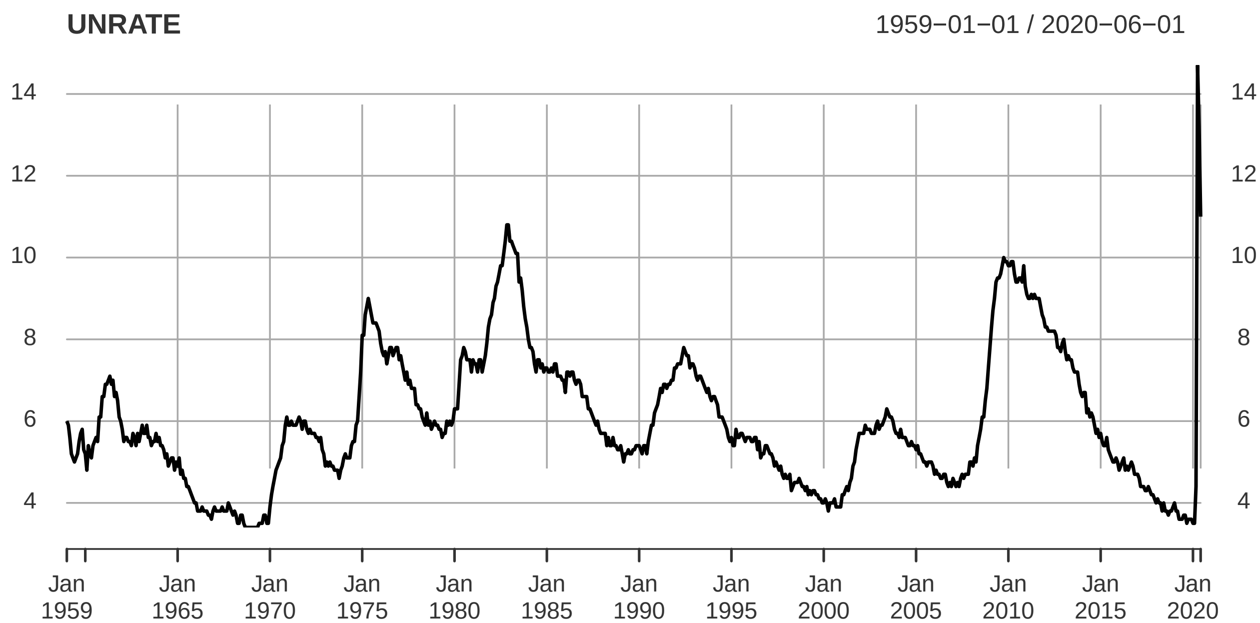

```r
plot(expgs.ts,main='EXPGS')
```



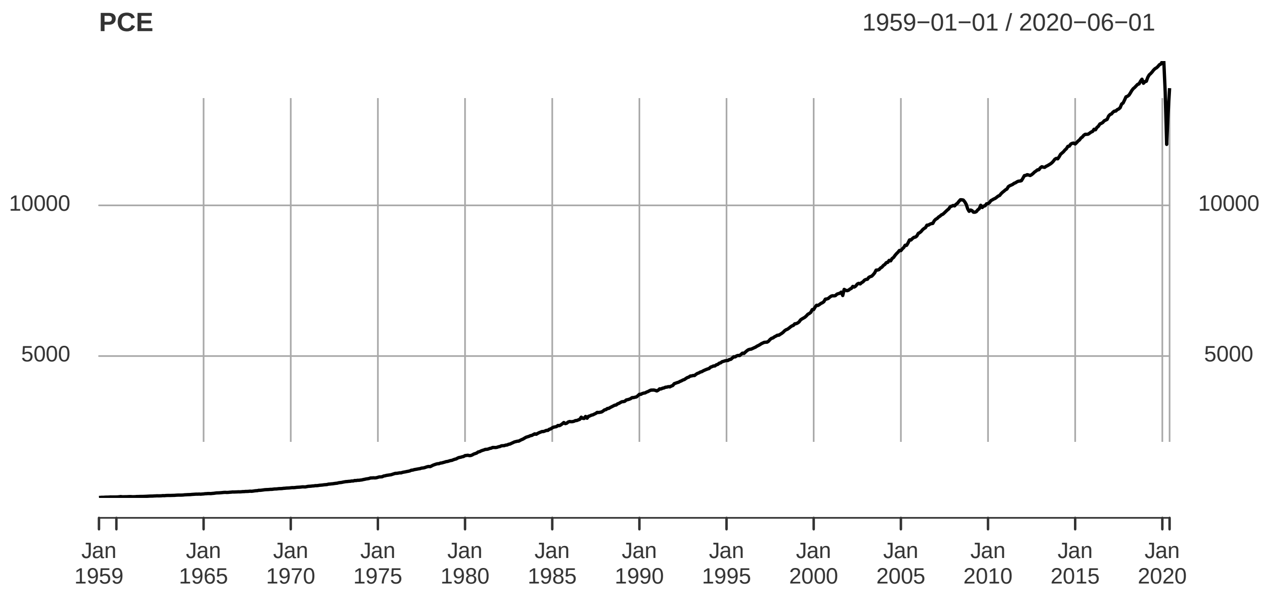

```r
plot(imgpgsc.ts,main='IMPGSC1')
```



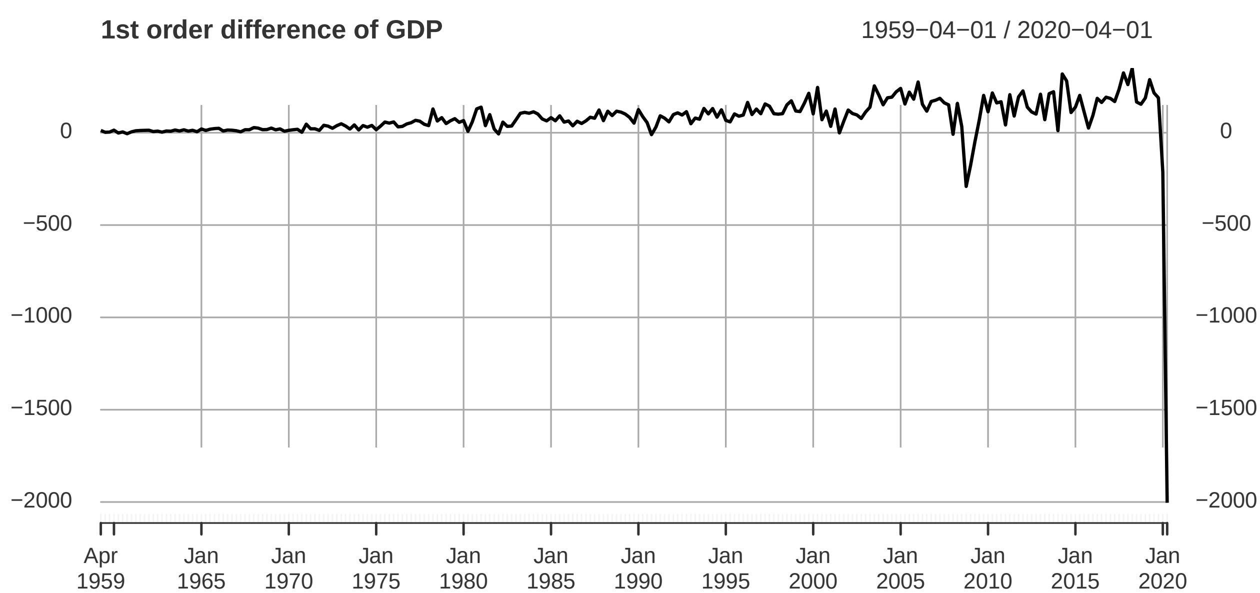

```
plot(unrate.ts,main='UNRATE')
```



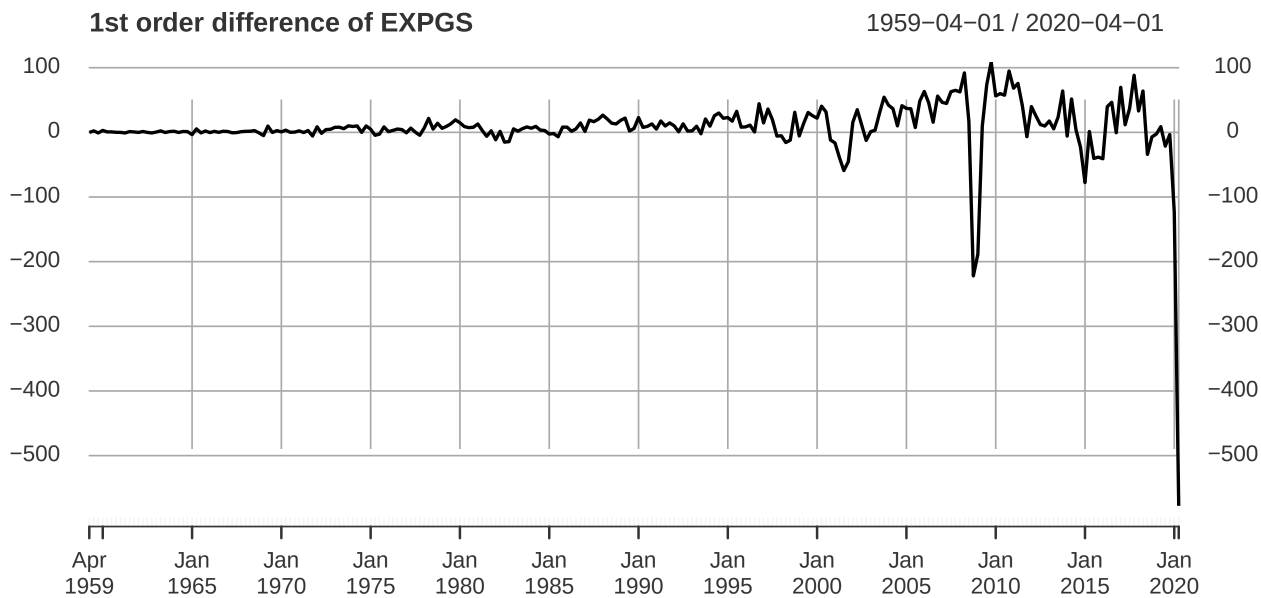

```
plot(pce.ts,main='PCE')
```



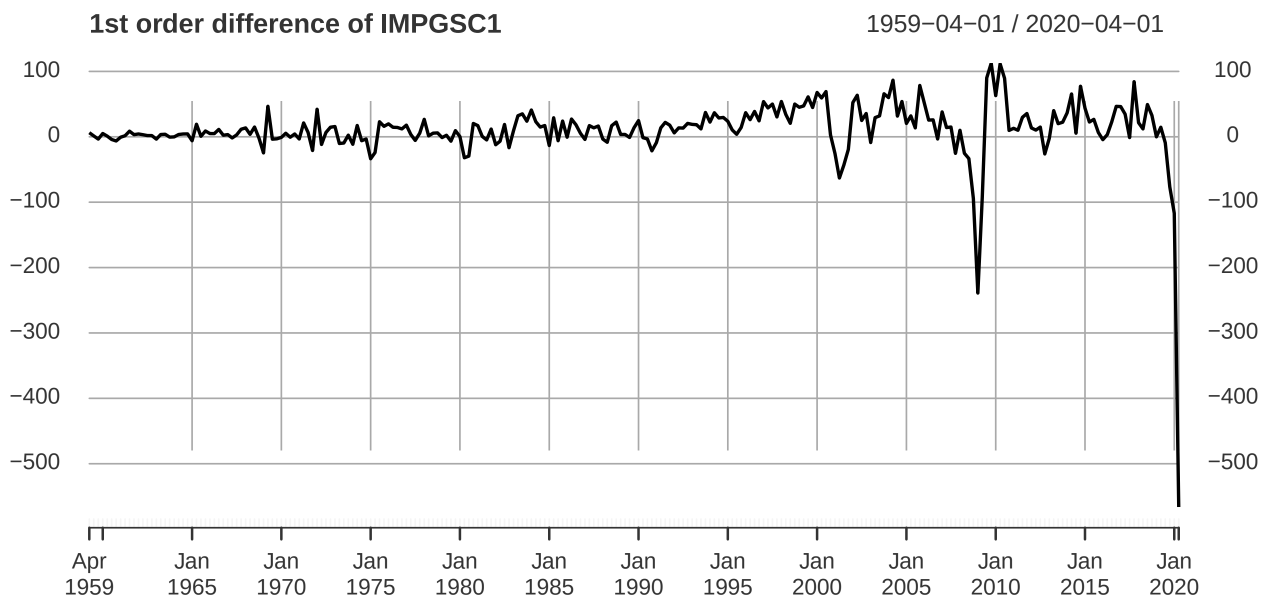

```
#combts<-ts.union(gdp.ts,expgs.ts,imgpgsc.ts, unrate.ts,pce.ts)
#plot(combts, xlab="time",main="",type="l")
#acf(combts, mar=c(3.5,3,1.9,0))
#pacf(combts, mar=c(3.5,3,1.9,0))

#Plot the first order difference of the data.
dgdp.ts <- diff(gdp.ts)[-1]
dexpgs.ts <- diff(expgs.ts)[-1]
dimgpgsc.ts<- diff(imgpgsc.ts)[-1]
dunrate.ts <-diff(unrate.ts)[-1]
dpce.ts <- diff(pce.ts)[-1]

plot(dgdp.ts,main='1st order difference of GDP')
```



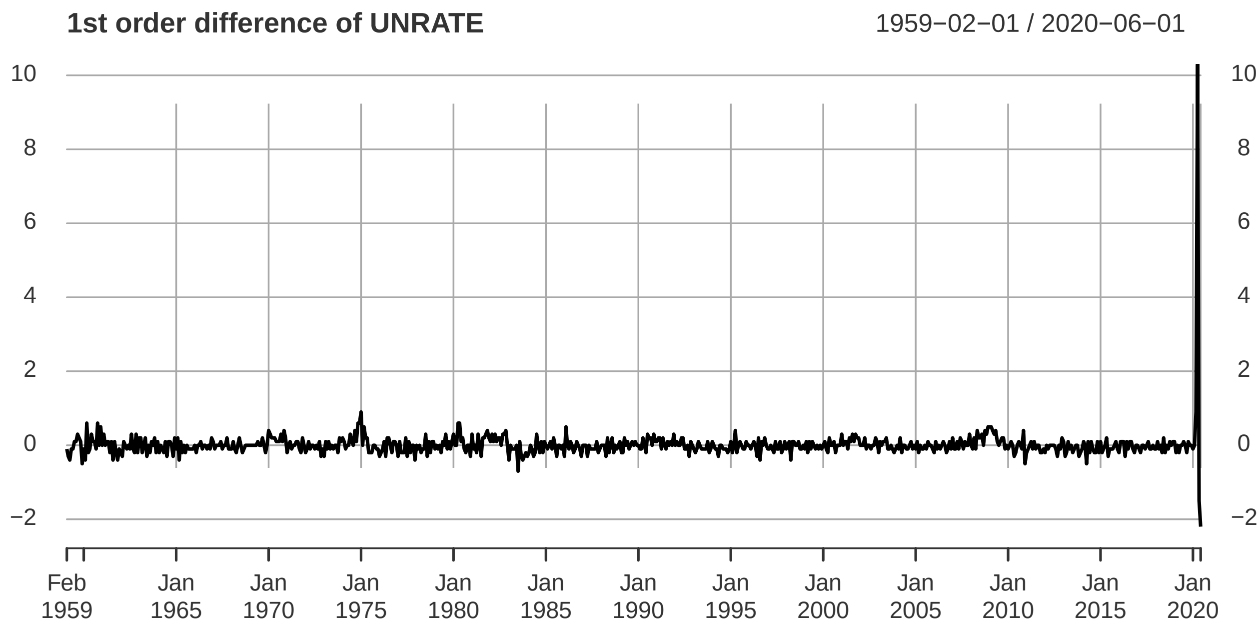

```
plot(dexpgs.ts,main='1st order difference of EXPGS')
```



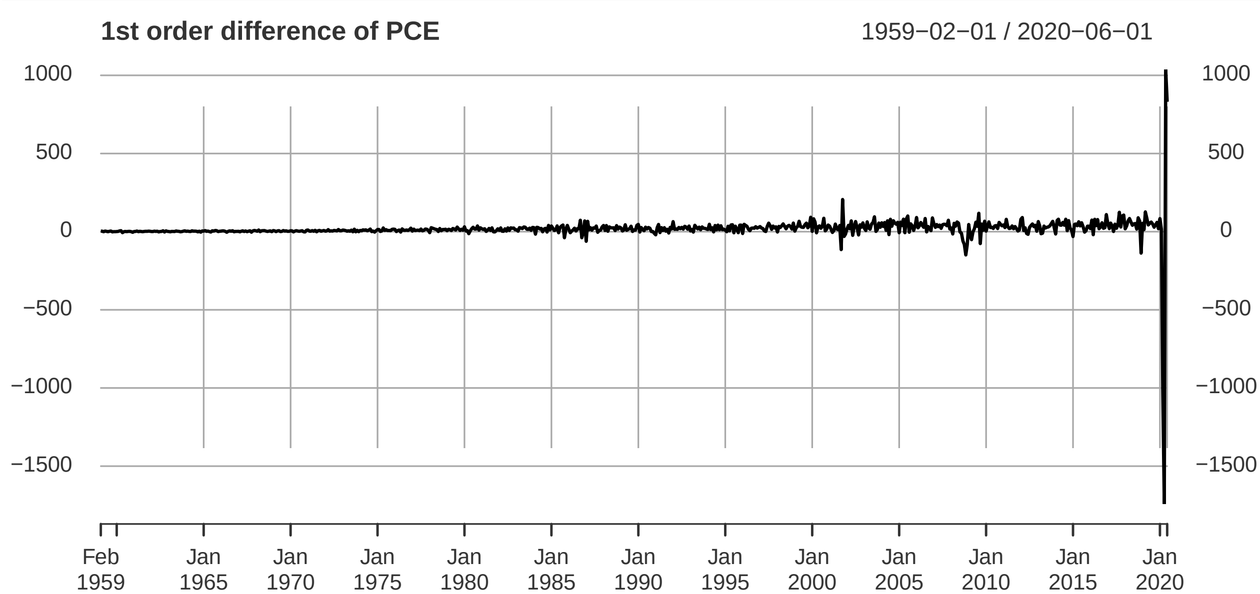

```
plot(dimgpgsc.ts,main='1st order difference of IMPGSC1')
```



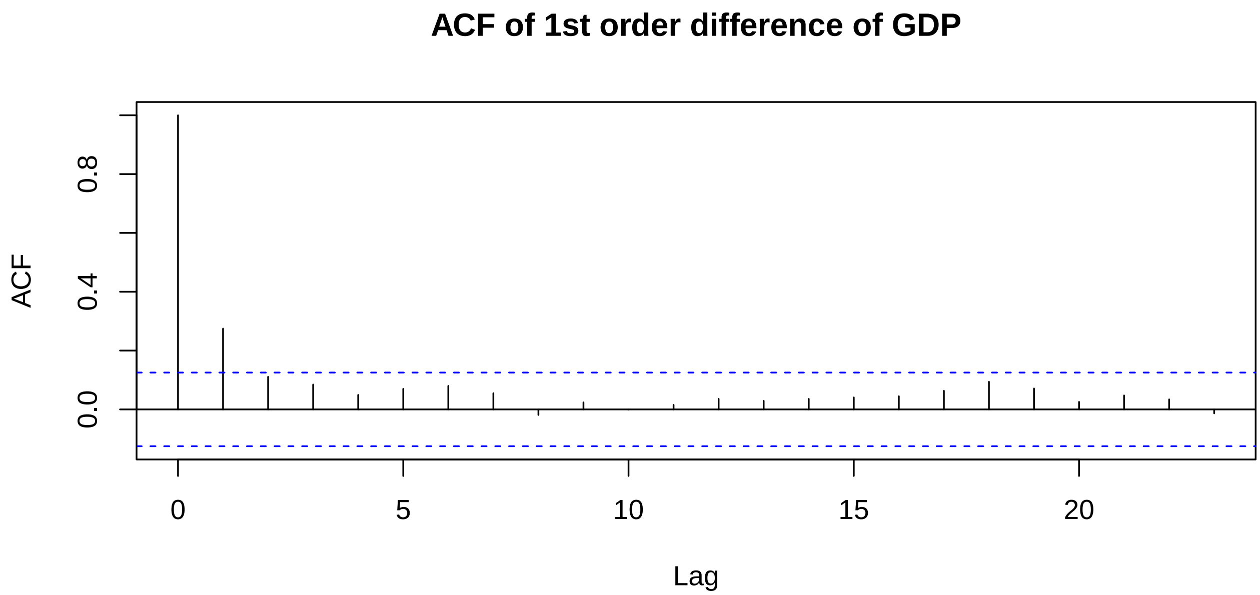

```
plot(dunrate.ts,main='1st order difference of UNRATE')
```



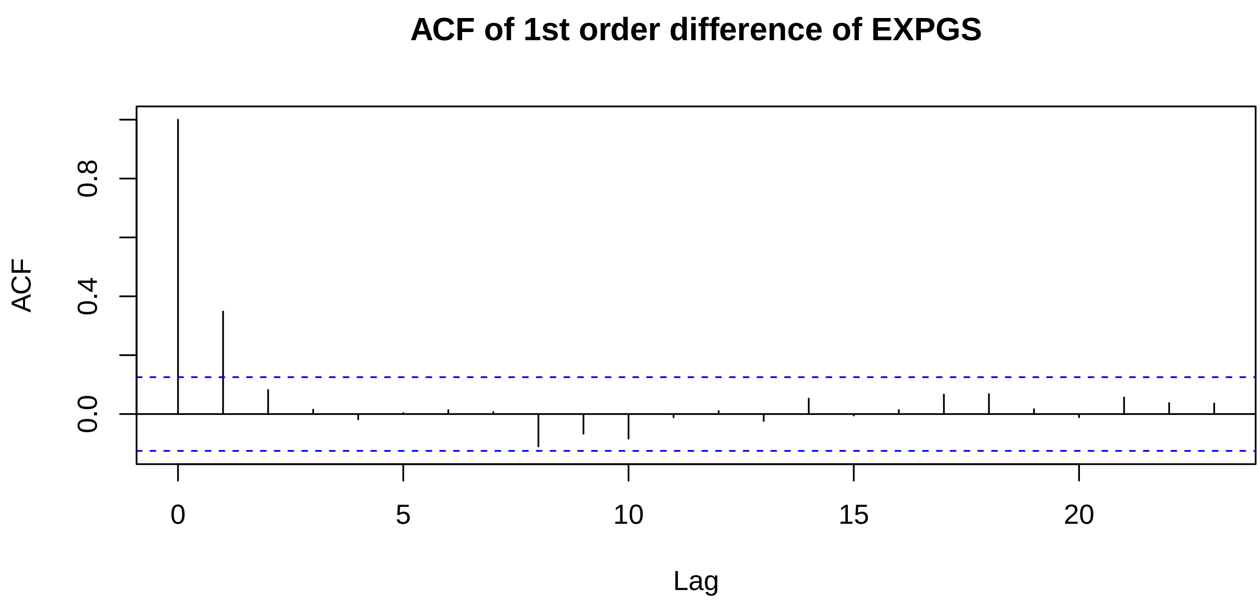

```
plot(dpce.ts,main='1st order difference of PCE')
```



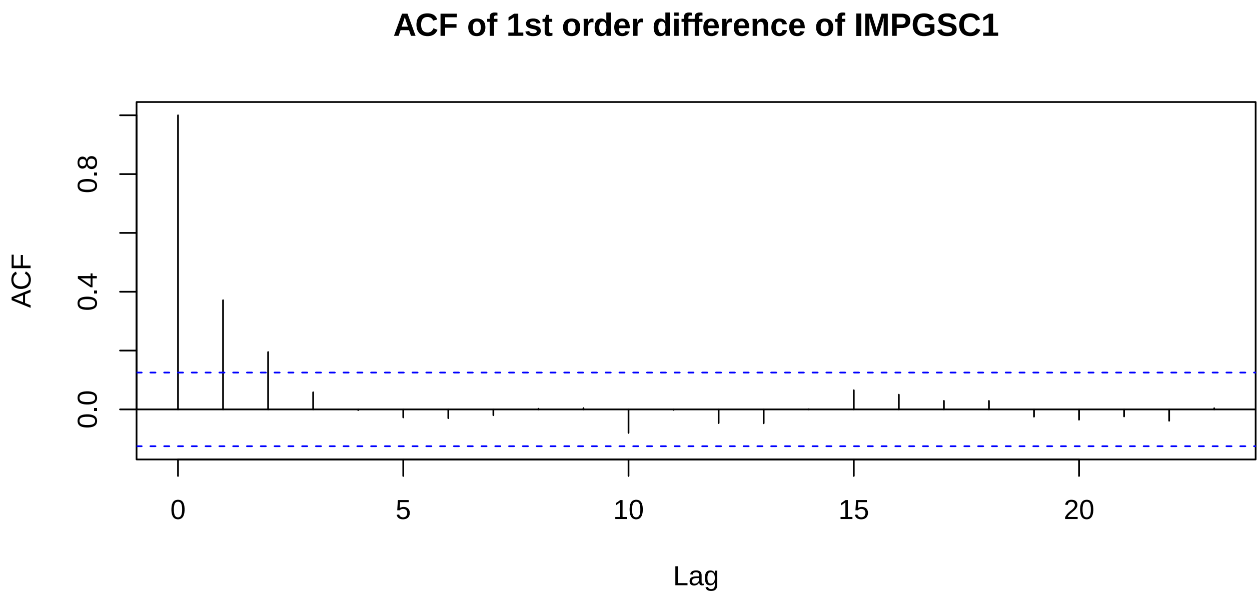

```
acf(dgdp.ts,main='ACF of 1st order difference of GDP')
```



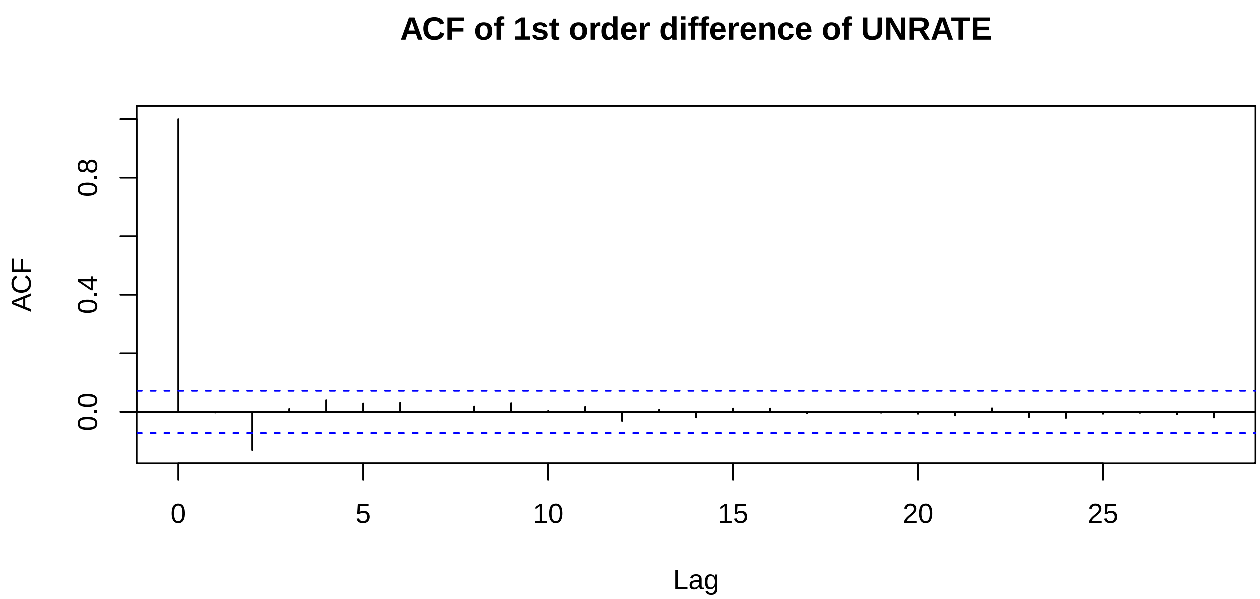

```r
acf(dexpgs.ts,main='ACF of 1st order difference of EXPGS')
```



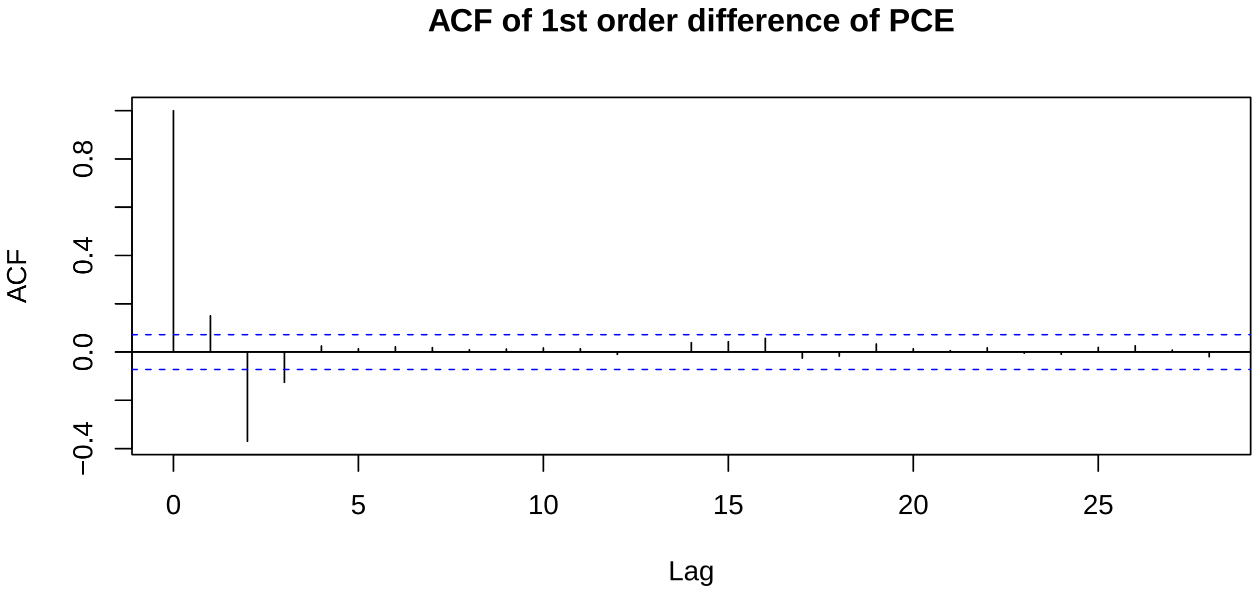

```r
acf(dimgpgsc.ts,main='ACF of 1st order difference of IMPGSC1')
```




```
acf(dunrate.ts,main='ACF of 1st order difference of UNRATE')
```




```
acf(dpce.ts,main='ACF of 1st order difference of PCE')
```



Response From the original time series plots, there is a linear increasing trend in Gross Domestic Product (GDP), Real imports of goods and services (IMPGSC1), Real exports of goods and services (EXPGS) and Personal Consumption Expenditure (PCE) but a steep fall during the pandemic hit. Unemployment (UNRATE) shows some trend and cyclical patterns and it reached the maximum when the pandemic was hit. First order differenced data removed the trend for Gross Domestic Product (GDP), Real imports of goods and services (IMPGSC1), Real exports of goods and services (EXPGS), Personal Consumption Expenditure (PCE) and Unemployment (UNRATE). The mean of all the First order difference are centered around zero mean but the variance seems to be non-constant for Gross Domestic Product (GDP), Real imports of goods and services (IMPGSC1), Real exports of goods and services (EXPGS) over the time period.Pandemic data shows significant change in variance levels. First order difference of Personal Consumption Expenditure (PCE) and Unemployment (UNRATE) shows constant variance as well except the pandemic season.

ACF plots of the first order difference shows Auto-correlation for all the the time series. So, the data doesn't show any stationarity.

Question 1b

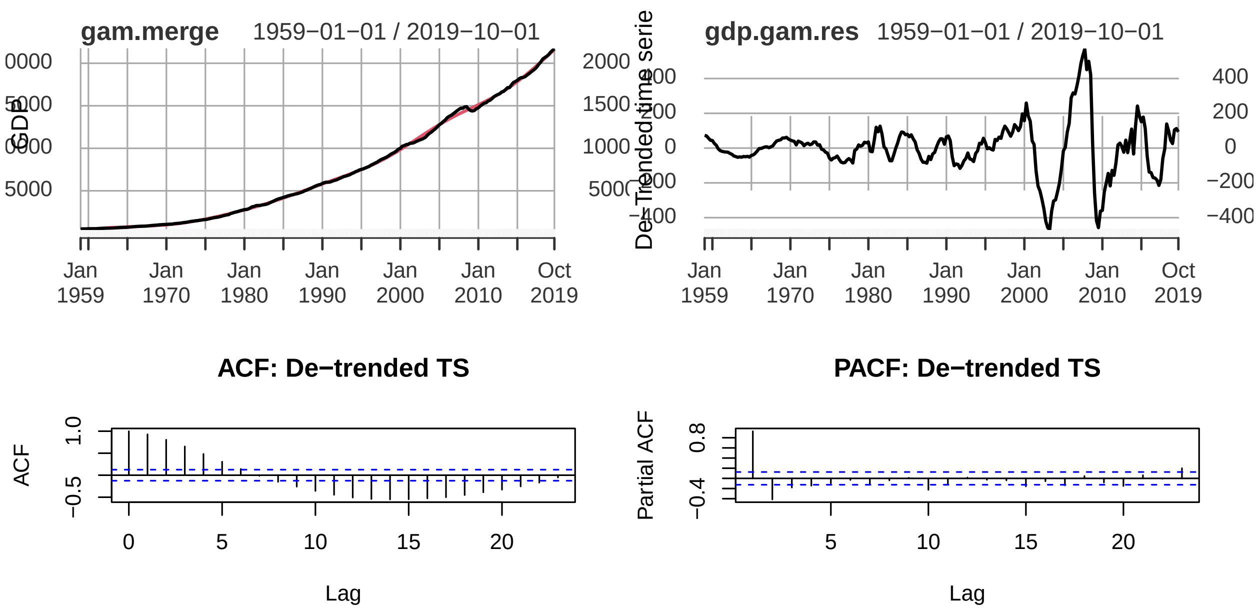
Using the original, undifferenced data, divide the GDP data into training data including the data for years 1959 to 2019 with the last two quarters being the testing data. Fit the trend using the splines regression to the GDP training time series. Then, apply ARMA to the residuals obtained from this splines fitting. Use max order of 6. Evaluate goodness of fit for the ARMA model. Forecast the first two quarters of 2020 (testing data) and compare to the observed values. Discuss why there are (or not!) significant differences between predicted vs observed. To do this, you should also evaluate the prediction intervals with a 95% confidence level.

```
n<-length(gdp.ts)
gdp.ts.tr<-gdp.ts[1:(n-2)]
gdp.ts.test<-gdp.ts[(n-1):n]
time.pts = c(1:(n-2))
time.pts = c(time.pts - min(time.pts))/max(time.pts)
gam.fit = gam(gdp.ts.tr~s(time.pts))
summary(gam.fit)
```

```
##
## Family: gaussian
## Link function: identity
##
```

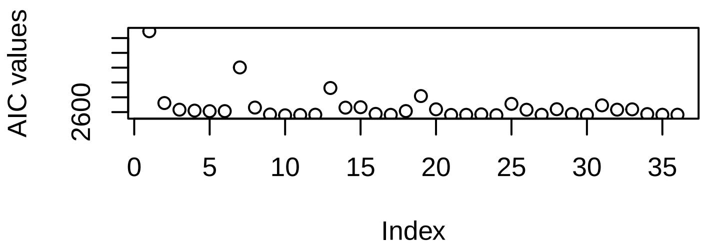
```
## Formula:
## gdp.ts.tr ~ s(time.pts)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 7346.328      9.892   742.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##              edf Ref.df     F p-value
## s(time.pts) 8.95  8.999 44575  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.999   Deviance explained = 99.9%
## GCV =  24893  Scale est. = 23878     n = 244
```

```r
gdp.fit.gam = fitted(gam.fit)
gdp.fit.gam= xts(gdp.fit.gam, date.quarter[1:length(gdp.fit.gam)])

gdp.gam.res<-gdp.ts.tr - gdp.fit.gam
gam.merge<-merge(gdp.ts.tr, gdp.fit.gam, join="inner")

par(mfrow=c(2,2))
plot(gam.merge,ylab="GDP", xlab="Time", type="l", auto.legend = TRUE)
plot(gdp.gam.res, ylab="De-Trended time series", xlab="Time")
acf(gdp.gam.res,main="ACF: De-trended TS")
pacf(gdp.gam.res,main="PACF: De-trended TS")
```

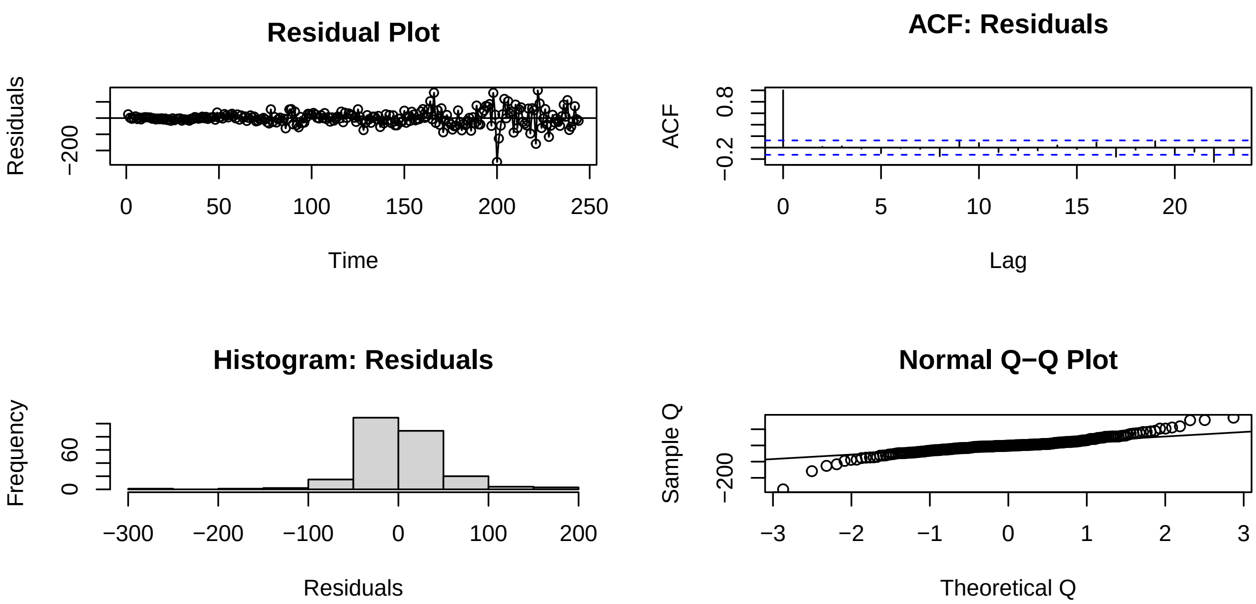



```r
#n = length(gdp.gam.res)
norder = 6
p = c(1:norder)-1; q = c(1:norder)-1
aic = matrix(0,norder,norder)
for(i in 1:norder){
```

```
   for(j in 1:norder){
    modij = arima(gdp.gam.res,order = c(p[i],0,q[j]), optim.control = list(maxit = 1000), method='ML')
    aic[i,j] = modij$aic-2*(p[i]+q[j]+1)+2*(p[i]+q[j]+1)*n/(n-p[i]-q[j]-2)
   }
 }

aicv = as.vector(aic)
plot(aicv,ylab="AIC values")
indexp = rep(c(1:norder),norder)
indexq = rep(c(1:norder),each=norder)
indexaic = which(aicv == min(aicv))
porder = indexp[indexaic]-1
qorder = indexq[indexaic]-1
final_model = arima(gdp.gam.res,order = c(porder,0,qorder), method='ML')
## GOF: residual analysis
par(mfrow=c(2,2))
```




```
plot(resid(final_model), ylab='Residuals',type='o',main="Residual Plot")
abline(h=0)
acf(resid(final_model),main="ACF: Residuals")
hist(resid(final_model),xlab='Residuals',main='Histogram: Residuals')
qqnorm(resid(final_model),ylab="Sample Q",xlab="Theoretical Q")
qqline(resid(final_model))
```




```
Box.test(final_model$resid, lag = (porder+qorder+1), type = "Box-Pierce", fitdf = (porder+qorder))
```

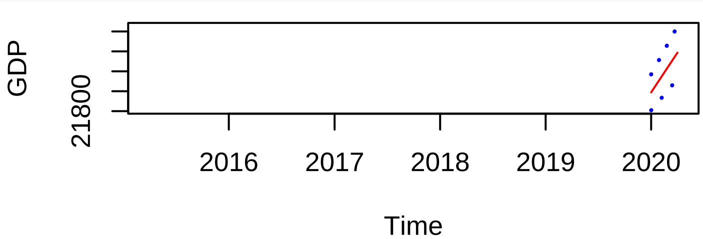
```
##
```

```
##  Box-Pierce test
##
## data:  final_model$resid
## X-squared = 2.5789, df = 1, p-value = 0.1083
```

```r
Box.test(final_model$resid, lag = (porder+qorder+1), type = "Ljung-Box", fitdf = (porder+qorder))
```

```
##
##  Box-Ljung test
##
## data:  final_model$resid
## X-squared = 2.6523, df = 1, p-value = 0.1034
```

```r
## Forecasting with ARIMA

##### Forecasting Trend+ARMA: 2 Quarters ahead ######
nfit = n-2
time.pts = c(1:n)
time.pts = c(time.pts - min(time.pts))/max(time.pts)
train.gdp= gdp.ts[1:nfit]
x = time.pts[1:nfit]
gam.fit.tr = gam(train.gdp~s(x))

## Step 1: Predict trend
newdata = data.frame(x=time.pts[(nfit+1):n])
gam.pred= predict(gam.fit.tr,newdata = newdata,interval=c("prediction"))

## Step 2: Predict ARMA
outpredresid = predict(final_model,n.ahead=2)
## Add up the predictions
final.pred.1 = outpredresid$pred+gam.pred


## 2 Quarters Ahead:
ubound = final.pred.1+1.96*outpredresid$se
lbound = final.pred.1-1.96*outpredresid$se
ymin = min(lbound)
ymax = max(ubound)
plot(date.quarter[(n-20):n],gdp.ts[(n-20):n],type="l", ylim=c(ymin,ymax), xlab="Time", ylab="GDP")
lines(date.quarter[(nfit+1):n],final.pred.1,col="red")
lines(date.quarter[(nfit+1):n],ubound,lty=3,lwd= 2, col="blue")
lines(date.quarter[(nfit+1):n],lbound,lty=3,lwd= 2, col="blue")
```



Response Using the original, undifferenced data, divide the GDP data into training data including the data for years 1959 to 2019 with the last two quarters being the testing data. Fit the trend using the splines regression to the GDP training time series. Then, apply ARMA to the residuals obtained from this splines fitting. Use max order of 6. Evaluate goodness of fit for the ARMA model. Forecast the first two quarters of 2020 (testing data) and compare to the observed values. Discuss why there are (or not!) significant differences between predicted vs observed. To do this, you should also evaluate the prediction intervals with a 95% confidence level.

In the first plot, we have the observed time series versus the fitted trend. We can see that the black line, which is the observed time series, is very similar to the trend line, which is the fitted line. The upper right plot represents the detrended time series. The detrended time series shows that the trend has been removed but there are some cylical patterns. The ACF and PACF plots show also that it is plausible that the detrended time series is non-stationary due to some cyclical patterns.

I applied the ARMA modeling to the detrended time series called here the residual process. First selected the AR and MA order using the AIC approach.Selected the order for AR and MA polynomials such that we have a minimum AIC value. The selected orders are 3 for AR polynomial and 1 for MA polynomial.

Goodness of ARMA residuals fit was evaluated using the residuals from final model. The residuals plot show some non-constant variance, while the acf plot show that higher order lags are above significant levels. The histogram and the probability normal plot show that the residuals have some deviation from the Gaussian distribution with left skew. It could be an outlier or dependent on other factors.

A test for uncorrelated residuals using the Box.test() reveals that the p-value for the test for uncorrelated data is at 0.1 for the lags at p_order+q_order+1 i.e. 5 and fitdf=4 . But if we increase the lags to 10 then the p-value goes down to 0.04, meaning that we have very weak evidence for not rejecting the null hypothesis of uncorrelated residuals.

The final plot provides the forecasts that includes trend + residuals. The forecasts show the upward trend but the observed values are close to the upper bound and have a slight chance of not within the 95% confidence bands. It could be due to the small evidence due to the corrleation at higherlags or other factors.

Question 1c

Perform a similar analysis as in (1b) but this time applying ARIMA to the GDP time series training dataset. Compare the forecast and discuss why these are different or similar from the testing data. Assume p,q belong to [0,5] and d belongs to [0,1]. Also evaluate the prediction intervals with a 95% confidence level, and compare the results of the forecast with the analysis in Question 1b.

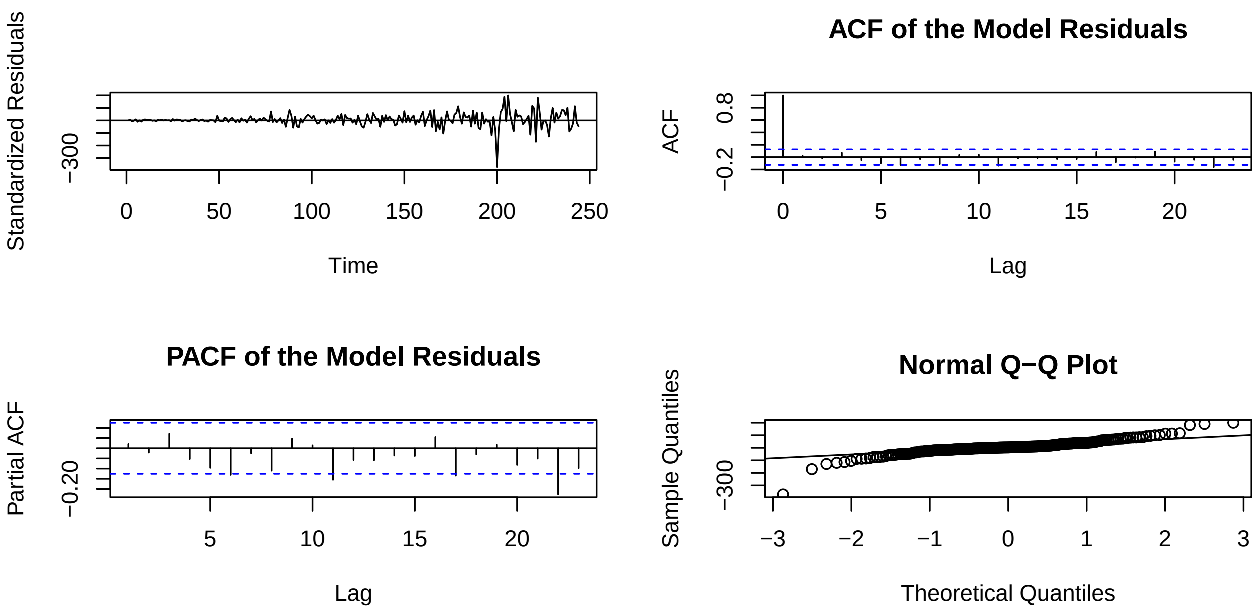
```r
final.aic = Inf
final.order = c(0,0,0)
orders = data.frame(Inf,Inf,Inf,Inf)
names(orders) <- c("p","d","q","AIC")
test_modelA<-function(p,d,q)
{
  mod = arima(gdp.ts.tr,order = c(p,d,q), optim.control = list(maxit = 1000), method='ML')
  current.aic = AIC(mod)
  df = data.frame(p,d,q,current.aic)
  names(df) <- c("p","d","q","AIC")
  #print(paste(p,d,q,current.aic,sep=" "))
  return(df)
}
for (p in 0:5) for (d in 0:1) for (q in 0:5) {
   #current.aic = AIC(arima(gdp.ts.tr, order=c(p, d, q), method="ML"))
      possibleError <- tryCatch(
        orders<-rbind(orders,test_modelA(p,d,q)),
        error=function(e) e
      )
      if(inherits(possibleError, "error")) next
}
orders <- orders[order(-orders$AIC),]
tail(orders,5)
```

```
##    p d q      AIC
## 51 4 1 1 2648.955
## 41 3 1 1 2648.247
## 32 2 1 2 2647.441
## 63 5 1 3 2646.822
## 65 5 1 5 2646.630
```

```
final.order<-c(5,1,5)
cat("Final order :", final.order)
```

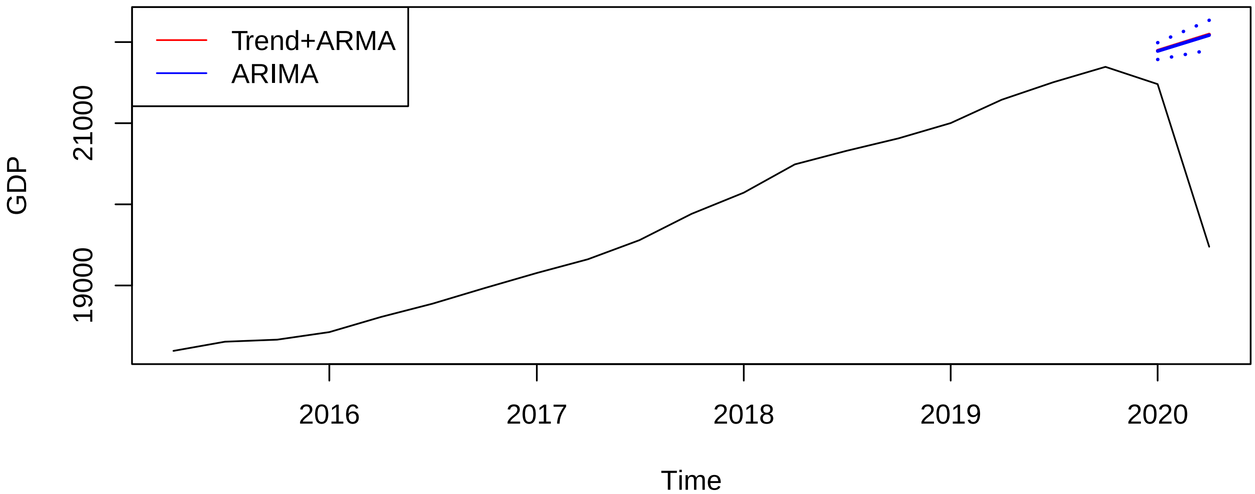
```
## Final order : 5 1 5
```

```
final_model.2 = arima(gdp.ts.tr, order = final.order, method = "ML")
resids.2 = resid(final_model.2)
## Residual Analysis
par (mfrow=c(2,2))
plot(resids.2, ylab='Standardized Residuals')
abline(h=0)
acf(resids.2,main= 'ACF of the Model Residuals')
pacf(resids.2,main='PACF of the Model Residuals')
qqnorm(resids.2)
qqline(resids.2)
```




```
## Forecasting with ARIMA 2 quarters Ahead:
outtotal = arima(gdp.ts[1:nfit], order = final.order,method = "ML")
final.pred.2 = predict(outtotal,n.ahead=2)$pred
final.se.2 = predict(outtotal,n.ahead=2)$se

############### Compare Forecasting ############################
par(mfrow=c(1,1))
ubound = final.pred.2+1.96*final.se.2
lbound = final.pred.2-1.96*final.se.2
ymin = min(c(gdp.ts[(n-20):n],lbound, final.pred.2))
ymax = max(c(gdp.ts[(n-20):n],ubound, final.pred.2))
plot(date.quarter[(n-20):n], gdp.ts[(n-20):n],type="l", ylim=c(ymin,ymax), xlab="Time", ylab="GDP")
lines(date.quarter[(nfit+1):n],final.pred.1,col="red",lwd=2)
```

```
lines(date.quarter[(nfit+1):n],final.pred.2,col="blue",lwd=2)
lines(date.quarter[(nfit+1):n],ubound,lty=3,lwd= 2, col="blue")
lines(date.quarter[(nfit+1):n],lbound,lty=3,lwd= 2, col="blue")
legend("topleft",legend=c("Trend+ARMA","ARIMA"),col=c("red","blue"),lty=1)
```



Response

The comparative plot is on the slide. The black line shows the observed time series for years, 2015 to 2020. The red line is the prediction of the first 2 quarters of 2020 using the first approach (Trend +ARMA) and the blue line is for the second approach (ARIMA) and dotted lines shows the 95% confidence intervals. The ACF plot of residuals shows higher order lags are above significant levels. Plot of residuuals shows that there is some non-constant variance.Both ARIMA and Trend+ARMA shows the upward trend and doesn't reflect the dip for the GDP occurred in 2020.

Question 2: Multivariate Analysis using VAR modeling

For this question, divide the quarterly data into training data (excluding the first two quarters of 2020) and testing data (including the two quarters). You will apply the modeling to the training data, and we will forecast the first two quarters of 2020.

Question 2a

Apply the VAR model to the multivariate time series including all five economic indicators observed quarterly. (Note that you will apply VAR to the training data.) Identify the VAR order using both AIC and BIC and compare. If the selected order using AIC is larger than the selected order than selected using BIC, apply the Wald test to evaluate whether a smaller order than the one selected with AIC would be a better choice, meaning the smaller order model would perform similarly than the larger order model. Interpret the order selection.

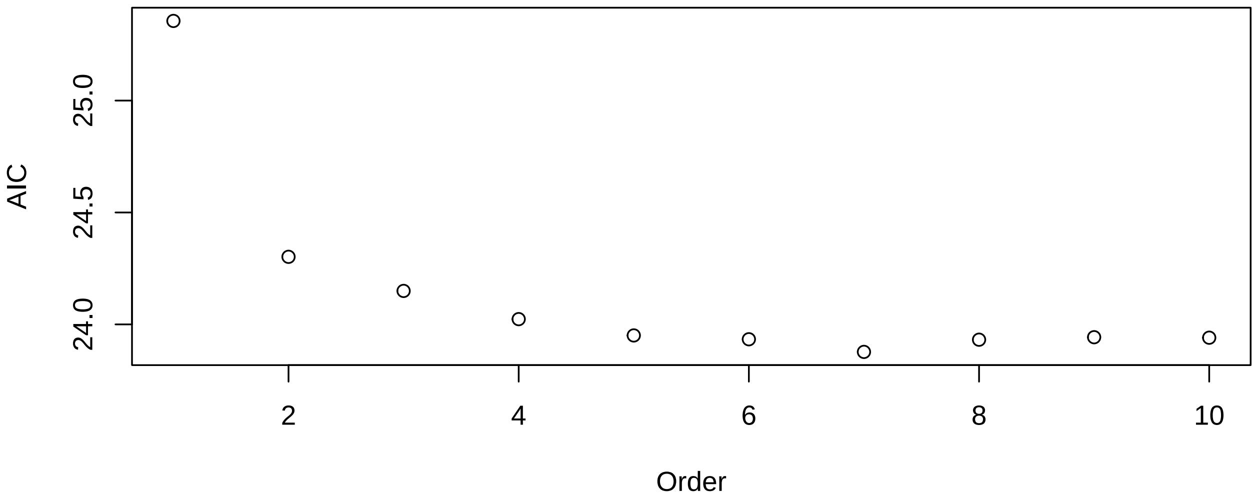
This can be done by following the below substeps:- 1)Combine the variables into a multivariate dataset 2)select/display var orders and isolate models using AIC and BIC orders 3)Isolate coefficients and covariances from the AIC model 4)applying the Wald test to the values obtained from the AIC model, but would not be present in the BIC model. You can run a single Wald test for each variable, with all the lagged coefficients for that particular variable, in all resulting in 5 wald tests.

```
n = nrow(ts.merge)
data.train=ts.merge[1:(n-2),]
data.test=ts.merge[(n-1):n,]
```

```r
##Model Selection
VARselect(data.train, lag.max = 10)$selection
```

```
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      7      3      2      7
```

```r
plot(VARselect(data.train, lag.max = 10)$criteria[1,], xlab="Order", ylab="AIC")
```




```r
mod_aic_1<-VAR(data.train, lag.max=10,ic="AIC", type="both" )
cat("Selected VAR order based on AIC criteria",mod_aic_1$p )
```

```
## Selected VAR order based on AIC criteria 7
```

```r
mod_bic_1<-VAR(data.train, lag.max=10,ic="SC", type="both" )
cat("Selected VAR order based on BIC criteria",mod_bic_1$p )
```

```
## Selected VAR order based on BIC criteria 2
```

```r
model_aic_var_1<-VAR(data.train, lag.max=10,p=mod_aic_1$p, type="both" )
summary(model_aic_var_1)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: tsgdp, tsexpgs, tsimgpsc, tsunrate, tspce
## Deterministic variables: both
## Sample size: 237
## Log Likelihood: -4315.783
## Roots of the characteristic polynomial:
## 1.017 0.9937 0.9624 0.9624 0.9014 0.9014 0.8882 0.8882 0.8743 0.8743 0.865 0.865 0.8646 0.8646 0.833
## Call:
## VAR(y = data.train, p = mod_aic_1$p, type = "both", lag.max = 10)
##
##
## Estimation results for equation tsgdp:
## ======================================
## tsgdp = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsimg
##
##                Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1       1.147451   0.103775  11.057   < 2e-16 ***
```

```
## tsexpgs.l1    0.131545   0.207324   0.634 0.526487
## tsimgpsc.l1   0.686166   0.177056   3.875 0.000144 ***
## tsunrate.l1 -12.413305  12.839641  -0.967 0.334814
## tspce.l1     -0.034671   0.162289  -0.214 0.831049
## tsgdp.l2      0.058873   0.145289   0.405 0.685754
## tsexpgs.l2   -0.499140   0.380047  -1.313 0.190564
## tsimgpsc.l2  -0.901228   0.273213  -3.299 0.001150 **
## tsunrate.l2  32.519067  20.209519   1.609 0.109173
## tspce.l2     -0.211756   0.178300  -1.188 0.236385
## tsgdp.l3      0.196061   0.144563   1.356 0.176554
## tsexpgs.l3   -0.001711   0.400252  -0.004 0.996593
## tsimgpsc.l3  -0.106544   0.286717  -0.372 0.710584
## tsunrate.l3 -16.758150  20.386551  -0.822 0.412044
## tspce.l3      0.123409   0.176552   0.699 0.485368
## tsgdp.l4     -0.202796   0.153780  -1.319 0.188763
## tsexpgs.l4    0.641074   0.392011   1.635 0.103549
## tsimgpsc.l4   0.307146   0.295226   1.040 0.299422
## tsunrate.l4  -8.764367  20.294529  -0.432 0.666309
## tspce.l4     -0.171654   0.185281  -0.926 0.355327
## tsgdp.l5     -0.349264   0.159067  -2.196 0.029264 *
## tsexpgs.l5   -0.445308   0.366890  -1.214 0.226279
## tsimgpsc.l5   0.189253   0.299534   0.632 0.528222
## tsunrate.l5   8.877132  20.181453   0.440 0.660509
## tspce.l5      0.435699   0.184826   2.357 0.019372 *
## tsgdp.l6      0.268583   0.153734   1.747 0.082161 .
## tsexpgs.l6   -0.504036   0.368210  -1.369 0.172572
## tsimgpsc.l6   0.223541   0.277433   0.806 0.421345
## tsunrate.l6   5.506239  20.037763   0.275 0.783759
## tspce.l6     -0.076232   0.175308  -0.435 0.664142
## tsgdp.l7     -0.232774   0.114888  -2.026 0.044084 *
## tsexpgs.l7    0.595730   0.235121   2.534 0.012052 *
## tsimgpsc.l7  -0.481808   0.181995  -2.647 0.008758 **
## tsunrate.l7  -9.882746  12.279893  -0.805 0.421896
## tspce.l7      0.135532   0.146876   0.923 0.357240
## const        10.279556  21.496142   0.478 0.633027
## trend         0.558164   0.464851   1.201 0.231273
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 50.2 on 200 degrees of freedom
## Multiple R-Squared: 0.9999,  Adjusted R-squared: 0.9999
## F-statistic: 1.02e+05 on 36 and 200 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsexpgs:
## ========================================
## tsexpgs = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsi
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1      0.03817    0.04699   0.812 0.417568
## tsexpgs.l1    1.62824    0.09388  17.343  < 2e-16 ***
## tsimgpsc.l1   0.01871    0.08018   0.233 0.815696
## tsunrate.l1  -3.31425    5.81417  -0.570 0.569297
```

```
## tspce.l1     -0.24852    0.07349  -3.382 0.000866 ***
## tsgdp.l2      0.04052    0.06579   0.616 0.538714
## tsexpgs.l2   -0.82013    0.17210  -4.766 3.62e-06 ***
## tsimgpsc.l2   0.04823    0.12372   0.390 0.697089
## tsunrate.l2   4.05849    9.15147   0.443 0.657898
## tspce.l2      0.03997    0.08074   0.495 0.621086
## tsgdp.l3      0.22580    0.06546   3.449 0.000685 ***
## tsexpgs.l3   -0.01749    0.18125  -0.097 0.923202
## tsimgpsc.l3  -0.34730    0.12983  -2.675 0.008092 **
## tsunrate.l3   9.29468    9.23164   1.007 0.315233
## tspce.l3     -0.05736    0.07995  -0.717 0.473905
## tsgdp.l4     -0.17667    0.06964  -2.537 0.011941 *
## tsexpgs.l4    0.21990    0.17751   1.239 0.216878
## tsimgpsc.l4   0.46678    0.13369   3.492 0.000591 ***
## tsunrate.l4 -17.81388    9.18997  -1.938 0.053981 .
## tspce.l4      0.06975    0.08390   0.831 0.406772
## tsgdp.l5     -0.17169    0.07203  -2.384 0.018079 *
## tsexpgs.l5    0.01217    0.16614   0.073 0.941677
## tsimgpsc.l5  -0.14377    0.13564  -1.060 0.290449
## tsunrate.l5  13.86006    9.13876   1.517 0.130941
## tspce.l5      0.10552    0.08369   1.261 0.208873
## tsgdp.l6      0.02787    0.06962   0.400 0.689282
## tsexpgs.l6   -0.24589    0.16674  -1.475 0.141861
## tsimgpsc.l6   0.21778    0.12563   1.734 0.084547 .
## tsunrate.l6  -3.88842    9.07370  -0.429 0.668721
## tspce.l6     -0.02832    0.07938  -0.357 0.721627
## tsgdp.l7      0.03163    0.05202   0.608 0.543886
## tsexpgs.l7    0.13531    0.10647   1.271 0.205262
## tsimgpsc.l7  -0.25164    0.08241  -3.053 0.002570 **
## tsunrate.l7  -0.96046    5.56070  -0.173 0.863044
## tspce.l7      0.11563    0.06651   1.739 0.083654 .
## const        -9.91388    9.73409  -1.018 0.309685
## trend        -0.04407    0.21050  -0.209 0.834370
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 22.73 on 200 degrees of freedom
## Multiple R-Squared: 0.9993,  Adjusted R-squared: 0.9992
## F-statistic:  8188 on 36 and 200 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsimgpsc:
## =========================================
## tsimgpsc = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + ts
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1     0.131180   0.045109   2.908 0.004047 **
## tsexpgs.l1   0.244785   0.090119   2.716 0.007181 **
## tsimgpsc.l1  1.222403   0.076962  15.883  < 2e-16 ***
## tsunrate.l1 -4.517390   5.581102  -0.809 0.419242
## tspce.l1    -0.028823   0.070543  -0.409 0.683278
## tsgdp.l2    -0.080657   0.063154  -1.277 0.203031
## tsexpgs.l2  -0.444431   0.165198  -2.690 0.007742 **
```

```
## tsimgpsc.l2 -0.273763   0.118760  -2.305 0.022182 *
## tsunrate.l2  8.569910   8.784622   0.976 0.330462
## tspce.l2     0.002491   0.077503   0.032 0.974389
## tsgdp.l3     0.038841   0.062838   0.618 0.537201
## tsexpgs.l3  -0.122731   0.173980  -0.705 0.481362
## tsimgpsc.l3  0.031052   0.124629   0.249 0.803500
## tsunrate.l3 -0.648143   8.861574  -0.073 0.941767
## tspce.l3    -0.008536   0.076743  -0.111 0.911543
## tsgdp.l4     0.020968   0.066845   0.314 0.754089
## tsexpgs.l4   0.152086   0.170398   0.893 0.373179
## tsimgpsc.l4  0.130611   0.128328   1.018 0.310009
## tsunrate.l4  0.221225   8.821574   0.025 0.980018
## tspce.l4    -0.119519   0.080537  -1.484 0.139378
## tsgdp.l5    -0.166108   0.069143  -2.402 0.017203 *
## tsexpgs.l5   0.345897   0.159479   2.169 0.031269 *
## tsimgpsc.l5 -0.202530   0.130200  -1.556 0.121403
## tsunrate.l5 -6.100456   8.772422  -0.695 0.487604
## tspce.l5     0.052516   0.080340   0.654 0.514076
## tsgdp.l6     0.013895   0.066825   0.208 0.835489
## tsexpgs.l6  -0.429933   0.160053  -2.686 0.007835 **
## tsimgpsc.l6  0.376627   0.120594   3.123 0.002055 **
## tsunrate.l6  4.582898   8.709963   0.526 0.599355
## tspce.l6    -0.004895   0.076203  -0.064 0.948842
## tsgdp.l7     0.028818   0.049939   0.577 0.564551
## tsexpgs.l7   0.231406   0.102202   2.264 0.024634 *
## tsimgpsc.l7 -0.293307   0.079109  -3.708 0.000271 ***
## tsunrate.l7 -2.041784   5.337792  -0.383 0.702486
## tspce.l7     0.130892   0.063844   2.050 0.041650 *
## const       -5.682576   9.343888  -0.608 0.543772
## trend        0.220346   0.202060   1.090 0.276807
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 21.82 on 200 degrees of freedom
## Multiple R-Squared: 0.9996,  Adjusted R-squared: 0.9996
## F-statistic: 1.582e+04 on 36 and 200 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsunrate:
## =========================================
## tsunrate = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + ts
##
##               Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1    -1.178e-03  5.752e-04  -2.047   0.0419 *
## tsexpgs.l1  -5.202e-04  1.149e-03  -0.453   0.6513
## tsimgpsc.l1 -2.449e-03  9.814e-04  -2.495   0.0134 *
## tsunrate.l1  1.222e+00  7.117e-02  17.177   <2e-16 ***
## tspce.l1     1.359e-03  8.995e-04   1.510   0.1325
## tsgdp.l2    -2.798e-05  8.053e-04  -0.035   0.9723
## tsexpgs.l2   1.625e-03  2.107e-03   0.771   0.4415
## tsimgpsc.l2  3.427e-03  1.514e-03   2.263   0.0247 *
## tsunrate.l2 -5.291e-02  1.120e-01  -0.472   0.6372
## tspce.l2    -8.500e-04  9.883e-04  -0.860   0.3908
```

```
## tsgdp.l3     1.100e-03  8.013e-04   1.373   0.1714
## tsexpgs.l3  -8.755e-04  2.219e-03  -0.395   0.6935
## tsimgpsc.l3 -1.762e-03  1.589e-03  -1.109   0.2689
## tsunrate.l3 -2.787e-01  1.130e-01  -2.466   0.0145 *
## tspce.l3    -1.644e-04  9.786e-04  -0.168   0.8667
## tsgdp.l4     3.281e-04  8.524e-04   0.385   0.7007
## tsexpgs.l4  -7.259e-05  2.173e-03  -0.033   0.9734
## tsimgpsc.l4  2.528e-04  1.636e-03   0.155   0.8774
## tsunrate.l4 -3.049e-02  1.125e-01  -0.271   0.7866
## tspce.l4    -7.842e-04  1.027e-03  -0.764   0.4460
## tsgdp.l5     3.856e-04  8.817e-04   0.437   0.6624
## tsexpgs.l5   3.527e-04  2.034e-03   0.173   0.8625
## tsimgpsc.l5  4.237e-04  1.660e-03   0.255   0.7989
## tsunrate.l5  1.571e-01  1.119e-01   1.404   0.1618
## tspce.l5    -6.004e-04  1.024e-03  -0.586   0.5585
## tsgdp.l6    -5.506e-04  8.521e-04  -0.646   0.5189
## tsexpgs.l6   1.123e-03  2.041e-03   0.550   0.5828
## tsimgpsc.l6 -1.853e-03  1.538e-03  -1.205   0.2296
## tsunrate.l6 -9.723e-02  1.111e-01  -0.875   0.3824
## tspce.l6     5.576e-04  9.717e-04   0.574   0.5667
## tsgdp.l7    -4.318e-05  6.368e-04  -0.068   0.9460
## tsexpgs.l7  -1.662e-03  1.303e-03  -1.275   0.2037
## tsimgpsc.l7  2.170e-03  1.009e-03   2.151   0.0327 *
## tsunrate.l7  2.869e-02  6.807e-02   0.422   0.6738
## tspce.l7     3.783e-04  8.141e-04   0.465   0.6426
## const        1.948e-01  1.191e-01   1.635   0.1036
## trend        3.335e-03  2.577e-03   1.294   0.1970
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2782 on 200 degrees of freedom
## Multiple R-Squared: 0.9747,  Adjusted R-squared: 0.9702
## F-statistic: 214.1 on 36 and 200 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tspce:
## ======================================
## tspce = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsimg
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1      0.41201    0.05812   7.088 2.28e-11 ***
## tsexpgs.l1    0.34015    0.11612   2.929 0.003793 **
## tsimgpsc.l1   0.04438    0.09917   0.448 0.654988
## tsunrate.l1  -5.36054    7.19146  -0.745 0.456903
## tspce.l1      0.36818    0.09090   4.050 7.31e-05 ***
## tsgdp.l2     -0.02911    0.08138  -0.358 0.720920
## tsexpgs.l2   -0.20106    0.21286  -0.945 0.346025
## tsimgpsc.l2  -0.21145    0.15303  -1.382 0.168577
## tsunrate.l2  23.24115   11.31932   2.053 0.041352 *
## tspce.l2      0.09100    0.09987   0.911 0.363264
## tsgdp.l3     -0.13210    0.08097  -1.631 0.104373
## tsexpgs.l3   -0.38984    0.22418  -1.739 0.083577 .
## tsimgpsc.l3   0.13601    0.16059   0.847 0.398027
```

```
## tsunrate.l3 -12.25424   11.41848  -1.073 0.284477    
## tspce.l3      0.33211    0.09889   3.358 0.000938 ***
## tsgdp.l4      0.11148    0.08613   1.294 0.197054    
## tsexpgs.l4   -0.02328    0.21956  -0.106 0.915659    
## tsimgpsc.l4   0.40105    0.16536   2.425 0.016179 *  
## tsunrate.l4   5.41912   11.36693   0.477 0.634065    
## tspce.l4     -0.17284    0.10378  -1.666 0.097367 .  
## tsgdp.l5     -0.32238    0.08909  -3.618 0.000375 ***
## tsexpgs.l5    0.15177    0.20549   0.739 0.461049    
## tsimgpsc.l5  -0.03621    0.16777  -0.216 0.829339    
## tsunrate.l5  -2.73498   11.30360  -0.242 0.809062    
## tspce.l5      0.06897    0.10352   0.666 0.506002    
## tsgdp.l6      0.06439    0.08611   0.748 0.455425    
## tsexpgs.l6   -0.20440    0.20623  -0.991 0.322839    
## tsimgpsc.l6   0.06826    0.15539   0.439 0.660924    
## tsunrate.l6  -3.29818   11.22312  -0.294 0.769159    
## tspce.l6      0.12759    0.09819   1.299 0.195306    
## tsgdp.l7      0.02381    0.06435   0.370 0.711718    
## tsexpgs.l7    0.26428    0.13169   2.007 0.046115 *  
## tsimgpsc.l7  -0.36379    0.10194  -3.569 0.000449 ***
## tsunrate.l7  -3.33658    6.87795  -0.485 0.628128    
## tspce.l7      0.01458    0.08226   0.177 0.859496    
## const       -15.47546   12.03996  -1.285 0.200159    
## trend        -0.41875    0.26036  -1.608 0.109335    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## 
## Residual standard error: 28.12 on 200 degrees of freedom
## Multiple R-Squared:     1,   Adjusted R-squared:     1 
## F-statistic: 1.516e+05 on 36 and 200 DF,  p-value: < 2.2e-16
## 
## 
## 
## Covariance matrix of residuals:
##             tsgdp  tsexpgs tsimgpsc tsunrate    tspce
## tsgdp    2519.742 745.0377  333.215 -1.95386 902.7557
## tsexpgs   745.038 516.6851  167.637 -0.52984 313.2394
## tsimgpsc  333.215 167.6374  476.091 -1.09751 113.6979
## tsunrate   -1.954  -0.5298   -1.098  0.07741  -0.3758
## tspce     902.756 313.2394  113.698 -0.37575 790.4689
## 
## Correlation matrix of residuals:
##            tsgdp  tsexpgs tsimgpsc tsunrate    tspce
## tsgdp     1.0000  0.65296   0.3042 -0.13990  0.63966
## tsexpgs   0.6530  1.00000   0.3380 -0.08378  0.49014
## tsimgpsc  0.3042  0.33800   1.0000 -0.18078  0.18534
## tsunrate -0.1399 -0.08378  -0.1808  1.00000 -0.04803
## tspce     0.6397  0.49014   0.1853 -0.04803  1.00000
```

```
# Granger Causality Tests
#tsgdp, tsexpgs, tsimgpsc, tsunrate, tspce

## Estimated coefficients and their variance for gdp regression Equation
```

```r
coef.tsgdp = coefficients(model_aic_var_1)$tsgdp[-c((7*5+1),(7*5+2)) ,1]
var.tsgdp = vcov(model_aic_var_1)[c(2:(7*5+1)),c(2:(7*5+1))]
## Estimated coefficients and their variance for tsexpgs regression Equaltion
coef.tsexpgs = coefficients(model_aic_var_1)$tsexpgs[--c((7*5+1),(7*5+2)) ,1]
tsexpgs_int_index=which(rownames(vcov(model_aic_var_1)) == "tsexpgs:(Intercept)")
tsexpgs_l7_index=which(rownames(vcov(model_aic_var_1)) == "tsexpgs:tspce.l7")
tsexpgs.index = c((tsexpgs_int_index+1):(tsexpgs_l7_index))
coef.tsexpgs = coefficients(model_aic_var_1)$tsexpgs[-c((7*5+1),(7*5+2)) ,1]
var.tsexpgs = vcov(model_aic_var_1)[tsexpgs.index,tsexpgs.index]

## Estimated coefficients and their variance for tsimgpsc regression Equaltion
coef.tsimgpsc = coefficients(model_aic_var_1)$tsimgpsc[--c((7*5+1),(7*5+2)) ,1]
tsimgpsc_int_index=which(rownames(vcov(model_aic_var_1)) == "tsimgpsc:(Intercept)")
tsimgpsc_l7_index=which(rownames(vcov(model_aic_var_1)) == "tsimgpsc:tspce.l7")
tsimgpsc.index = c((tsimgpsc_int_index+1):(tsimgpsc_l7_index))
coef.tsimgpsc = coefficients(model_aic_var_1)$tsimgpsc[-c((7*5+1),(7*5+2)) ,1]
var.tsimgpsc = vcov(model_aic_var_1)[tsimgpsc.index,tsimgpsc.index]

## Estimated coefficients and their variance for tsunrate regression Equaltion
coef.tsunrate = coefficients(model_aic_var_1)$tsunrate[--c((7*5+1),(7*5+2)) ,1]
tsunrate_int_index=which(rownames(vcov(model_aic_var_1)) == "tsunrate:(Intercept)")
tsunrate_l7_index=which(rownames(vcov(model_aic_var_1)) == "tsunrate:tspce.l7")
tsunrate.index = c((tsunrate_int_index+1):(tsunrate_l7_index))
coef.tsunrate = coefficients(model_aic_var_1)$tsunrate[-c((7*5+1),(7*5+2)) ,1]
var.tsunrate = vcov(model_aic_var_1)[tsunrate.index,tsunrate.index]

## Estimated coefficients and their variance for tspce regression Equaltion
coef.tspce = coefficients(model_aic_var_1)$tspce[--c((7*5+1),(7*5+2)) ,1]
tspce_int_index=which(rownames(vcov(model_aic_var_1)) == "tspce:(Intercept)")
tspce_l7_index=which(rownames(vcov(model_aic_var_1)) == "tspce:tspce.l7")
tspce.index = c((tspce_int_index+1):(tspce_l7_index))
coef.tspce = coefficients(model_aic_var_1)$tspce[-c((7*5+1),(7*5+2)) ,1]
var.tspce = vcov(model_aic_var_1)[tspce.index,tspce.index]



library(aod)
# Is there a lead-lag relationship for gdp-other economic factors
#tsgdp = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsimgps
tsgdpnames=c("tsgdp:tsgdp.l1", "tsgdp:tsgdp.l2","tsgdp:tsgdp.l3", "tsgdp:tsgdp.l4", "tsgdp:tsgdp.l5", "
tsnongdpvars<-which(!colnames(var.tsgdp)%in%tsgdpnames)
wald.test(b=coef.tsgdp, var.tsgdp,Terms = tsnongdpvars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 69.7, df = 28, P(> X2) = 2e-05
```

```r
#tsexpgs = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsimg

# Is there a lead-lag relationship for expgs-other economic factors
tsexpgsnames=c("tsexpgs:tsexpgs.l1", "tsexpgs:tsexpgs.l2","tsexpgs:tsexpgs.l3", "tsexpgs:tsexpgs.l4", "
tsnonexpgsvars<-which(!colnames(var.tsexpgs)%in%tsexpgsnames)
```

```r
wald.test(b=coef.tsexpgs, var.tsexpgs,Terms = tsnonexpgsvars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 96.8, df = 28, P(> X2) = 1.6e-09
```

```r
# Is there a lead-lag relationship for imgpsc-other economic factors
tsimgpscnames=c("tsimgpsc:tsimgpsc.l1", "tsimgpsc:tsimgpsc.l2","tsimgpsc:tsimgpsc.l3", "tsimgpsc:tsimgps
tsnonimgpscvars<-which(!colnames(var.tsimgpsc)%in%tsimgpscnames)
wald.test(b=coef.tsimgpsc, var.tsimgpsc,Terms = tsnonimgpscvars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 146.4, df = 28, P(> X2) = 0.0
```

```r
# Is there a lead-lag relationship for unrate-other economic factors
tsunratenames=c("tsunrate:tsunrate.l1", "tsunrate:tsunrate.l2","tsunrate:tsunrate.l3", "tsunrate:tsunra
tsnonunratevars<-which(!colnames(var.tsunrate)%in%tsunratenames)
wald.test(b=coef.tsunrate, var.tsunrate,Terms = tsnonunratevars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 48.1, df = 28, P(> X2) = 0.01
```

```r
# Is there a lead-lag relationship for pce-other economic factors
tspcenames=c("tspce:tspce.l1", "tspce:tspce.l2","tspce:tspce.l3", "tspce:tspce.l4", "tspce:tspce.l5", "
tsnonpcevars<-which(!colnames(var.tspce)%in%tspcenames)
wald.test(b=coef.tspce, var.tspce,Terms = tsnonpcevars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 275.6, df = 28, P(> X2) = 0.0
```

Response AIC(n) HQ(n) SC(n) FPE(n) 7 3 2 7

Selected VAR order(p) based on AIC criteria: 7 Selected VAR order(p) based on BIC criteria: 2

All the 5 wald tests are showing some sort of lead relation on the co-effieicnts from the other economic factors. Unemployment rate may have a weak relation though.

Question 2b

Based on the analysis in 2a, select the VAR order using BIC and fit that model. Print out the model summary and comment on the statistical significance of the coefficients. Apply a model selection analysis using stepwise regression to select the models for each individual time series. What do you conclude from this model selection? Apply the restrict() command in R to restrict the model of order. How do the restricted models compare?

Follow the below steps to implement this: 1)Analyze the coefficients of the unrestricted model. 2)Then treat each of the series separately and do a stepwise regression by apply lm() and step() (in the backward direction with 3 steps) functions to each of the component time series to examine the coefficients 3)The stepwise regressions will return three separate models. Analyze if the same coefficients are significant in the overall VAR versus each of the stepwise models. Discuss. 4)Then build a model using restrict and see which predictors were significant in the restricted VAR model.

```
#BIC var model
model_bic_var_1<-VAR(data.train, lag.max=10,p=mod_bic_1$p, type="both" )

summary(model_bic_var_1)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: tsgdp, tsexpgs, tsimgpsc, tsunrate, tspce
## Deterministic variables: both
## Sample size: 237
## Log Likelihood: -4315.783
## Roots of the characteristic polynomial:
## 1.017 0.9937 0.9624 0.9624 0.9014 0.9014 0.8882 0.8882 0.8743 0.8743 0.865 0.865 0.8646 0.8646 0.8335
## Call:
## VAR(y = data.train, p = mod_bic_1$p, type = "both", lag.max = 10)
##
##
## Estimation results for equation tsgdp:
## ======================================
## tsgdp = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsimgp
##
##                Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1       1.147451   0.103775  11.057  < 2e-16 ***
## tsexpgs.l1     0.131545   0.207324   0.634 0.526487
## tsimgpsc.l1    0.686166   0.177056   3.875 0.000144 ***
## tsunrate.l1 -12.413305  12.839641  -0.967 0.334814
## tspce.l1      -0.034671   0.162289  -0.214 0.831049
## tsgdp.l2       0.058873   0.145289   0.405 0.685754
## tsexpgs.l2    -0.499140   0.380047  -1.313 0.190564
## tsimgpsc.l2   -0.901228   0.273213  -3.299 0.001150 **
## tsunrate.l2   32.519067  20.209519   1.609 0.109173
## tspce.l2      -0.211756   0.178300  -1.188 0.236385
## tsgdp.l3       0.196061   0.144563   1.356 0.176554
## tsexpgs.l3    -0.001711   0.400252  -0.004 0.996593
## tsimgpsc.l3   -0.106544   0.286717  -0.372 0.710584
## tsunrate.l3 -16.758150  20.386551  -0.822 0.412044
## tspce.l3       0.123409   0.176552   0.699 0.485368
## tsgdp.l4      -0.202796   0.153780  -1.319 0.188763
## tsexpgs.l4     0.641074   0.392011   1.635 0.103549
## tsimgpsc.l4    0.307146   0.295226   1.040 0.299422
## tsunrate.l4   -8.764367  20.294529  -0.432 0.666309
## tspce.l4      -0.171654   0.185281  -0.926 0.355327
## tsgdp.l5      -0.349264   0.159067  -2.196 0.029264 *
## tsexpgs.l5    -0.445308   0.366890  -1.214 0.226279
## tsimgpsc.l5    0.189253   0.299534   0.632 0.528222
## tsunrate.l5    8.877132  20.181453   0.440 0.660509
## tspce.l5       0.435699   0.184826   2.357 0.019372 *
## tsgdp.l6       0.268583   0.153734   1.747 0.082161 .
## tsexpgs.l6    -0.504036   0.368210  -1.369 0.172572
```

```
## tsimgpsc.l6   0.223541   0.277433   0.806 0.421345
## tsunrate.l6   5.506239  20.037763   0.275 0.783759
## tspce.l6     -0.076232   0.175308  -0.435 0.664142
## tsgdp.l7     -0.232774   0.114888  -2.026 0.044084 *
## tsexpgs.l7    0.595730   0.235121   2.534 0.012052 *
## tsimgpsc.l7  -0.481808   0.181995  -2.647 0.008758 **
## tsunrate.l7  -9.882746  12.279893  -0.805 0.421896
## tspce.l7      0.135532   0.146876   0.923 0.357240
## const        10.279556  21.496142   0.478 0.633027
## trend         0.558164   0.464851   1.201 0.231273
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 50.2 on 200 degrees of freedom
## Multiple R-Squared: 0.9999,  Adjusted R-squared: 0.9999
## F-statistic: 1.02e+05 on 36 and 200 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsexpgs:
## ========================================
## tsexpgs = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsi
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1      0.03817    0.04699   0.812 0.417568
## tsexpgs.l1    1.62824    0.09388  17.343  < 2e-16 ***
## tsimgpsc.l1   0.01871    0.08018   0.233 0.815696
## tsunrate.l1  -3.31425    5.81417  -0.570 0.569297
## tspce.l1     -0.24852    0.07349  -3.382 0.000866 ***
## tsgdp.l2      0.04052    0.06579   0.616 0.538714
## tsexpgs.l2   -0.82013    0.17210  -4.766 3.62e-06 ***
## tsimgpsc.l2   0.04823    0.12372   0.390 0.697089
## tsunrate.l2   4.05849    9.15147   0.443 0.657898
## tspce.l2      0.03997    0.08074   0.495 0.621086
## tsgdp.l3      0.22580    0.06546   3.449 0.000685 ***
## tsexpgs.l3   -0.01749    0.18125  -0.097 0.923202
## tsimgpsc.l3  -0.34730    0.12983  -2.675 0.008092 **
## tsunrate.l3   9.29468    9.23164   1.007 0.315233
## tspce.l3     -0.05736    0.07995  -0.717 0.473905
## tsgdp.l4     -0.17667    0.06964  -2.537 0.011941 *
## tsexpgs.l4    0.21990    0.17751   1.239 0.216878
## tsimgpsc.l4   0.46678    0.13369   3.492 0.000591 ***
## tsunrate.l4 -17.81388    9.18997  -1.938 0.053981 .
## tspce.l4      0.06975    0.08390   0.831 0.406772
## tsgdp.l5     -0.17169    0.07203  -2.384 0.018079 *
## tsexpgs.l5    0.01217    0.16614   0.073 0.941677
## tsimgpsc.l5  -0.14377    0.13564  -1.060 0.290449
## tsunrate.l5  13.86006    9.13876   1.517 0.130941
## tspce.l5      0.10552    0.08369   1.261 0.208873
## tsgdp.l6      0.02787    0.06962   0.400 0.689282
## tsexpgs.l6   -0.24589    0.16674  -1.475 0.141861
## tsimgpsc.l6   0.21778    0.12563   1.734 0.084547 .
## tsunrate.l6  -3.88842    9.07370  -0.429 0.668721
## tspce.l6     -0.02832    0.07938  -0.357 0.721627
```

```
## tsgdp.l7      0.03163    0.05202   0.608 0.543886
## tsexpgs.l7    0.13531    0.10647   1.271 0.205262
## tsimgpsc.l7  -0.25164    0.08241  -3.053 0.002570 **
## tsunrate.l7  -0.96046    5.56070  -0.173 0.863044
## tspce.l7      0.11563    0.06651   1.739 0.083654 .
## const        -9.91388    9.73409  -1.018 0.309685
## trend        -0.04407    0.21050  -0.209 0.834370
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 22.73 on 200 degrees of freedom
## Multiple R-Squared: 0.9993,  Adjusted R-squared: 0.9992
## F-statistic:  8188 on 36 and 200 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsimgpsc:
## =========================================
## tsimgpsc = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + ts
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1     0.131180   0.045109   2.908 0.004047 **
## tsexpgs.l1   0.244785   0.090119   2.716 0.007181 **
## tsimgpsc.l1  1.222403   0.076962  15.883  < 2e-16 ***
## tsunrate.l1 -4.517390   5.581102  -0.809 0.419242
## tspce.l1    -0.028823   0.070543  -0.409 0.683278
## tsgdp.l2    -0.080657   0.063154  -1.277 0.203031
## tsexpgs.l2  -0.444431   0.165198  -2.690 0.007742 **
## tsimgpsc.l2 -0.273763   0.118760  -2.305 0.022182 *
## tsunrate.l2  8.569910   8.784622   0.976 0.330462
## tspce.l2     0.002491   0.077503   0.032 0.974389
## tsgdp.l3     0.038841   0.062838   0.618 0.537201
## tsexpgs.l3  -0.122731   0.173980  -0.705 0.481362
## tsimgpsc.l3  0.031052   0.124629   0.249 0.803500
## tsunrate.l3 -0.648143   8.861574  -0.073 0.941767
## tspce.l3    -0.008536   0.076743  -0.111 0.911543
## tsgdp.l4     0.020968   0.066845   0.314 0.754089
## tsexpgs.l4   0.152086   0.170398   0.893 0.373179
## tsimgpsc.l4  0.130611   0.128328   1.018 0.310009
## tsunrate.l4  0.221225   8.821574   0.025 0.980018
## tspce.l4    -0.119519   0.080537  -1.484 0.139378
## tsgdp.l5    -0.166108   0.069143  -2.402 0.017203 *
## tsexpgs.l5   0.345897   0.159479   2.169 0.031269 *
## tsimgpsc.l5 -0.202530   0.130200  -1.556 0.121403
## tsunrate.l5 -6.100456   8.772422  -0.695 0.487604
## tspce.l5     0.052516   0.080340   0.654 0.514076
## tsgdp.l6     0.013895   0.066825   0.208 0.835489
## tsexpgs.l6  -0.429933   0.160053  -2.686 0.007835 **
## tsimgpsc.l6  0.376627   0.120594   3.123 0.002055 **
## tsunrate.l6  4.582898   8.709963   0.526 0.599355
## tspce.l6    -0.004895   0.076203  -0.064 0.948842
## tsgdp.l7     0.028818   0.049939   0.577 0.564551
## tsexpgs.l7   0.231406   0.102202   2.264 0.024634 *
## tsimgpsc.l7 -0.293307   0.079109  -3.708 0.000271 ***
```

```
## tsunrate.l7 -2.041784   5.337792  -0.383 0.702486
## tspce.l7     0.130892   0.063844   2.050 0.041650 *
## const       -5.682576   9.343888  -0.608 0.543772
## trend        0.220346   0.202060   1.090 0.276807
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 21.82 on 200 degrees of freedom
## Multiple R-Squared: 0.9996,  Adjusted R-squared: 0.9996
## F-statistic: 1.582e+04 on 36 and 200 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsunrate:
## =========================================
## tsunrate = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsi
##
##               Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1    -1.178e-03  5.752e-04  -2.047   0.0419 *
## tsexpgs.l1  -5.202e-04  1.149e-03  -0.453   0.6513
## tsimgpsc.l1 -2.449e-03  9.814e-04  -2.495   0.0134 *
## tsunrate.l1  1.222e+00  7.117e-02  17.177   <2e-16 ***
## tspce.l1     1.359e-03  8.995e-04   1.510   0.1325
## tsgdp.l2    -2.798e-05  8.053e-04  -0.035   0.9723
## tsexpgs.l2   1.625e-03  2.107e-03   0.771   0.4415
## tsimgpsc.l2  3.427e-03  1.514e-03   2.263   0.0247 *
## tsunrate.l2 -5.291e-02  1.120e-01  -0.472   0.6372
## tspce.l2    -8.500e-04  9.883e-04  -0.860   0.3908
## tsgdp.l3     1.100e-03  8.013e-04   1.373   0.1714
## tsexpgs.l3  -8.755e-04  2.219e-03  -0.395   0.6935
## tsimgpsc.l3 -1.762e-03  1.589e-03  -1.109   0.2689
## tsunrate.l3 -2.787e-01  1.130e-01  -2.466   0.0145 *
## tspce.l3    -1.644e-04  9.786e-04  -0.168   0.8667
## tsgdp.l4     3.281e-04  8.524e-04   0.385   0.7007
## tsexpgs.l4  -7.259e-05  2.173e-03  -0.033   0.9734
## tsimgpsc.l4  2.528e-04  1.636e-03   0.155   0.8774
## tsunrate.l4 -3.049e-02  1.125e-01  -0.271   0.7866
## tspce.l4    -7.842e-04  1.027e-03  -0.764   0.4460
## tsgdp.l5     3.856e-04  8.817e-04   0.437   0.6624
## tsexpgs.l5   3.527e-04  2.034e-03   0.173   0.8625
## tsimgpsc.l5  4.237e-04  1.660e-03   0.255   0.7989
## tsunrate.l5  1.571e-01  1.119e-01   1.404   0.1618
## tspce.l5    -6.004e-04  1.024e-03  -0.586   0.5585
## tsgdp.l6    -5.506e-04  8.521e-04  -0.646   0.5189
## tsexpgs.l6   1.123e-03  2.041e-03   0.550   0.5828
## tsimgpsc.l6 -1.853e-03  1.538e-03  -1.205   0.2296
## tsunrate.l6 -9.723e-02  1.111e-01  -0.875   0.3824
## tspce.l6     5.576e-04  9.717e-04   0.574   0.5667
## tsgdp.l7    -4.318e-05  6.368e-04  -0.068   0.9460
## tsexpgs.l7  -1.662e-03  1.303e-03  -1.275   0.2037
## tsimgpsc.l7  2.170e-03  1.009e-03   2.151   0.0327 *
## tsunrate.l7  2.869e-02  6.807e-02   0.422   0.6738
## tspce.l7     3.783e-04  8.141e-04   0.465   0.6426
## const        1.948e-01  1.191e-01   1.635   0.1036
```

```
## trend        3.335e-03  2.577e-03   1.294   0.1970
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2782 on 200 degrees of freedom
## Multiple R-Squared: 0.9747,  Adjusted R-squared: 0.9702
## F-statistic: 214.1 on 36 and 200 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tspce:
## ======================================
## tspce = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsimg
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1      0.41201    0.05812   7.088 2.28e-11 ***
## tsexpgs.l1    0.34015    0.11612   2.929 0.003793 **
## tsimgpsc.l1   0.04438    0.09917   0.448 0.654988
## tsunrate.l1  -5.36054    7.19146  -0.745 0.456903
## tspce.l1      0.36818    0.09090   4.050 7.31e-05 ***
## tsgdp.l2     -0.02911    0.08138  -0.358 0.720920
## tsexpgs.l2   -0.20106    0.21286  -0.945 0.346025
## tsimgpsc.l2  -0.21145    0.15303  -1.382 0.168577
## tsunrate.l2  23.24115   11.31932   2.053 0.041352 *
## tspce.l2      0.09100    0.09987   0.911 0.363264
## tsgdp.l3     -0.13210    0.08097  -1.631 0.104373
## tsexpgs.l3   -0.38984    0.22418  -1.739 0.083577 .
## tsimgpsc.l3   0.13601    0.16059   0.847 0.398027
## tsunrate.l3 -12.25424   11.41848  -1.073 0.284477
## tspce.l3      0.33211    0.09889   3.358 0.000938 ***
## tsgdp.l4      0.11148    0.08613   1.294 0.197054
## tsexpgs.l4   -0.02328    0.21956  -0.106 0.915659
## tsimgpsc.l4   0.40105    0.16536   2.425 0.016179 *
## tsunrate.l4   5.41912   11.36693   0.477 0.634065
## tspce.l4     -0.17284    0.10378  -1.666 0.097367 .
## tsgdp.l5     -0.32238    0.08909  -3.618 0.000375 ***
## tsexpgs.l5    0.15177    0.20549   0.739 0.461049
## tsimgpsc.l5  -0.03621    0.16777  -0.216 0.829339
## tsunrate.l5  -2.73498   11.30360  -0.242 0.809062
## tspce.l5      0.06897    0.10352   0.666 0.506002
## tsgdp.l6      0.06439    0.08611   0.748 0.455425
## tsexpgs.l6   -0.20440    0.20623  -0.991 0.322839
## tsimgpsc.l6   0.06826    0.15539   0.439 0.660924
## tsunrate.l6  -3.29818   11.22312  -0.294 0.769159
## tspce.l6      0.12759    0.09819   1.299 0.195306
## tsgdp.l7      0.02381    0.06435   0.370 0.711718
## tsexpgs.l7    0.26428    0.13169   2.007 0.046115 *
## tsimgpsc.l7  -0.36379    0.10194  -3.569 0.000449 ***
## tsunrate.l7  -3.33658    6.87795  -0.485 0.628128
## tspce.l7      0.01458    0.08226   0.177 0.859496
## const       -15.47546   12.03996  -1.285 0.200159
## trend        -0.41875    0.26036  -1.608 0.109335
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
##
## Residual standard error: 28.12 on 200 degrees of freedom
## Multiple R-Squared:     1,   Adjusted R-squared:     1
## F-statistic: 1.516e+05 on 36 and 200 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##             tsgdp  tsexpgs tsimgpsc tsunrate    tspce
## tsgdp    2519.742 745.0377  333.215 -1.95386 902.7557
## tsexpgs   745.038 516.6851  167.637 -0.52984 313.2394
## tsimgpsc  333.215 167.6374  476.091 -1.09751 113.6979
## tsunrate   -1.954  -0.5298   -1.098  0.07741  -0.3758
## tspce     902.756 313.2394  113.698 -0.37575 790.4689
##
## Correlation matrix of residuals:
##            tsgdp  tsexpgs tsimgpsc tsunrate    tspce
## tsgdp     1.0000  0.65296   0.3042 -0.13990  0.63966
## tsexpgs   0.6530  1.00000   0.3380 -0.08378  0.49014
## tsimgpsc  0.3042  0.33800   1.0000 -0.18078  0.18534
## tsunrate -0.1399 -0.08378  -0.1808  1.00000 -0.04803
## tspce     0.6397  0.49014   0.1853 -0.04803  1.00000
```

```r
### Estimated coefficients and their variance for gdp regression Equation
#coef.tsgdp.b = coefficients(model_bic_var_1)$tsgdp[-c((7*5+1),(7*5+2)) ,1]
#var.tsgdp.b = vcov(model_bic_var_1)[c(2:(7*5+1)),c(2:(7*5+1))]
### Estimated coefficients and their variance for tsexpgs regression Equaltion
#coef.tsexpgs.b = coefficients(model_bic_var_1)$tsexpgs[--c((7*5+1),(7*5+2)) ,1]
#tsexpgs_int_index.b=which(rownames(vcov(model_bic_var_1)) == "tsexpgs:(Intercept)")
#tsexpgs_l7_index.b=which(rownames(vcov(model_bic_var_1)) == "tsexpgs:tspce.l7")
#tsexpgs.index.b = c((tsexpgs_int_index.b+1):(tsexpgs_l7_index.b))
#var.tsexpgs.b = vcov(model_bic_var_1)[tsexpgs.index.b,tsexpgs.index.b]
#
### Estimated coefficients and their variance for tsimgpsc regression Equaltion
#coef.tsimgpsc.b = coefficients(model_bic_var_1)$tsimgpsc[--c((7*5+1),(7*5+2)) ,1]
#tsimgpsc_int_index.b=which(rownames(vcov(model_bic_var_1)) == "tsimgpsc:(Intercept)")
#tsimgpsc_l7_index.b=which(rownames(vcov(model_bic_var_1)) == "tsimgpsc:tspce.l7")
#tsimgpsc.index.b = c((tsimgpsc_int_index.b+1):(tsimgpsc_l7_index.b))
#var.tsimgpsc.b = vcov(model_bic_var_1)[tsimgpsc.index.b,tsimgpsc.index.b]
#
### Estimated coefficients and their variance for tsunrate regression Equaltion
#coef.tsunrate.b = coefficients(model_bic_var_1)$tsunrate[--c((7*5+1),(7*5+2)) ,1]
#tsunrate_int_index.b=which(rownames(vcov(model_bic_var_1)) == "tsunrate:(Intercept)")
#tsunrate_l7_index.b=which(rownames(vcov(model_bic_var_1)) == "tsunrate:tspce.l7")
#tsunrate.index.b = c((tsunrate_int_index.b+1):(tsunrate_l7_index.b))
#var.tsunrate.b = vcov(model_bic_var_1)[tsunrate.index.b,tsunrate.index.b]
#
### Estimated coefficients and their variance for tspce regression Equaltion
#coef.tspce.b = coefficients(model_bic_var_1)$tspce[--c((7*5+1),(7*5+2)) ,1]
#tspce_int_index.b=which(rownames(vcov(model_bic_var_1)) == "tspce:(Intercept)")
#tspce_l7_index.b=which(rownames(vcov(model_bic_var_1)) == "tspce:tspce.l7")
#tspce.index.b = c((tspce_int_index.b+1):(tspce_l7_index.b))
#var.tspce.b = vcov(model_bic_var_1)[tspce.index.b,tspce.index.b]
```

```r
gdp.lm<-lm(tsgdp~tsexpgs+tsimgpsc+tsunrate+tspce, data=data.train)
step(gdp.lm, direction="backward", steps=3)
```

```
## Start:  AIC=2367.43
## tsgdp ~ tsexpgs + tsimgpsc + tsunrate + tspce
##
##            Df Sum of Sq      RSS    AIC
## <none>                   3831565 2367.4
## - tsunrate  1    123239  3954804 2373.2
## - tsexpgs   1    369782  4201347 2387.9
## - tsimgpsc  1    672275  4503841 2404.9
## - tspce     1  79566756 83398321 3117.0

##
## Call:
## lm(formula = tsgdp ~ tsexpgs + tsimgpsc + tsunrate + tspce, data = data.train)
##
## Coefficients:
## (Intercept)      tsexpgs     tsimgpsc     tsunrate        tspce
##    321.4670      -0.3394      -0.4887     -15.7494       1.6496
```

```r
tsexpgs.lm<-lm(tsexpgs~tsgdp+tsimgpsc+tsunrate+tspce, data=data.train)
step(tsexpgs.lm, direction="backward", steps=3)
```

```
## Start:  AIC=2301.74
## tsexpgs ~ tsgdp + tsimgpsc + tsunrate + tspce
##
##            Df Sum of Sq     RSS    AIC
## - tsunrate  1      5722 2932853 2300.2
## <none>                  2927131 2301.7
## - tsimgpsc  1     68782 2995913 2305.4
## - tsgdp     1    282496 3209626 2322.2
## - tspce     1    595944 3523075 2344.9
##
## Step:  AIC=2300.21
## tsexpgs ~ tsgdp + tsimgpsc + tspce
##
##            Df Sum of Sq     RSS    AIC
## <none>                  2932853 2300.2
## - tsimgpsc  1    112451 3045304 2307.4
## - tsgdp     1    310991 3243844 2322.8
## - tspce     1    679199 3612052 2349.0

##
## Call:
## lm(formula = tsexpgs ~ tsgdp + tsimgpsc + tspce, data = data.train)
##
## Coefficients:
## (Intercept)        tsgdp     tsimgpsc        tspce
##    -27.9554      -0.2666      -0.1899       0.6234
```

```r
tsimgpsc.lm<-lm(tsimgpsc~tsgdp+tsexpgs+tsunrate+tspce, data=data.train)
step(tsimgpsc.lm, direction="backward", steps=3)
```

```
## Start:  AIC=2252.79
## tsimgpsc ~ tsgdp + tsexpgs + tsunrate + tspce
```

```
##
##             Df Sum of Sq     RSS    AIC
## <none>                   2395049 2252.8
## - tsexpgs   1     56279 2451329 2256.4
## - tsgdp     1    420229 2815278 2290.2
## - tsunrate  1    609127 3004176 2306.1
## - tspce     1    988162 3383212 2335.1
```

```
##
## Call:
## lm(formula = tsimgpsc ~ tsgdp + tsexpgs + tsunrate + tspce, data = data.train)
##
## Coefficients:
## (Intercept)        tsgdp      tsexpgs     tsunrate        tspce
##    284.9774      -0.3055      -0.1371     -31.7623       0.7216
```

```r
tsunrate.lm<-lm(tsunrate~tsgdp+tsimgpsc+tsexpgs+tspce, data=data.train)
step(tsunrate.lm, direction="backward", steps=3)
```

```
## Start:  AIC=175.79
## tsunrate ~ tsgdp + tsimgpsc + tsexpgs + tspce
##
##             Df Sum of Sq    RSS    AIC
## - tsexpgs   1     0.941 482.30 174.26
## <none>                  481.36 175.79
## - tsgdp     1    15.483 496.84 181.51
## - tspce     1    27.142 508.50 187.17
## - tsimgpsc  1   122.424 603.79 229.08
##
## Step:  AIC=174.26
## tsunrate ~ tsgdp + tsimgpsc + tspce
##
##             Df Sum of Sq    RSS    AIC
## <none>                   482.30 174.26
## - tsgdp     1    19.838 502.14 182.10
## - tspce     1    39.044 521.35 191.26
## - tsimgpsc  1   131.437 613.74 231.07
```

```
##
## Call:
## lm(formula = tsunrate ~ tsgdp + tsimgpsc + tspce, data = data.train)
##
## Coefficients:
## (Intercept)        tsgdp     tsimgpsc        tspce
##    6.743187    -0.002130    -0.006491     0.004726
```

```r
tspce.lm<-lm(tspce~tsgdp+tsimgpsc+tsunrate+tsexpgs, data=data.train)
step(tspce.lm, direction="backward", steps=3)
```

```
## Start:  AIC=2111.7
## tspce ~ tsgdp + tsimgpsc + tsunrate + tsexpgs
##
##             Df Sum of Sq     RSS    AIC
## <none>                    1343389 2111.7
## - tsunrate  1     75749  1419138 2123.1
## - tsexpgs   1    273505  1616893 2154.9
```

```
## - tsimgpsc  1    554262  1897651 2194.0
## - tsgdp     1  27896973 29240361 2861.3
```

```
##
## Call:
## lm(formula = tspce ~ tsgdp + tsimgpsc + tsunrate + tsexpgs, data = data.train)
##
## Coefficients:
## (Intercept)        tsgdp     tsimgpsc     tsunrate      tsexpgs
##   -198.9294       0.5784       0.4048      12.2051       0.2786
```

```
model_bic_var_restric_1<-restrict(model_bic_var_1)
summary(model_bic_var_restric_1)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: tsgdp, tsexpgs, tsimgpsc, tsunrate, tspce
## Deterministic variables: both
## Sample size: 237
## Log Likelihood: -4386.402
## Roots of the characteristic polynomial:
## 1.007 1.007 0.9847 0.9549 0.9549 0.8832 0.8832 0.8811 0.8811 0.8508 0.8508 0.8497 0.8497 0.8394 0.83
## Call:
## VAR(y = data.train, p = mod_bic_1$p, type = "both", lag.max = 10)
##
##
## Estimation results for equation tsgdp:
## ======================================
## tsgdp = tsgdp.l1 + tsimgpsc.l1 + tsexpgs.l2 + tsimgpsc.l2 + tsunrate.l2 + tsgdp.l5 + tsimgpsc.l5 + ts
##
##             Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1     1.15139    0.03307  34.818  < 2e-16 ***
## tsimgpsc.l1  0.59064    0.12917   4.572 7.92e-06 ***
## tsexpgs.l2  -0.22020    0.05089  -4.327 2.27e-05 ***
## tsimgpsc.l2 -0.69677    0.14922  -4.669 5.17e-06 ***
## tsunrate.l2  3.20539    1.10938   2.889  0.00423 **
## tsgdp.l5    -0.21786    0.04938  -4.412 1.58e-05 ***
## tsimgpsc.l5  0.26175    0.09573   2.734  0.00674 **
## tspce.l5     0.14197    0.04516   3.143  0.00189 **
## tsexpgs.l7   0.11626    0.04937   2.355  0.01939 *
## tsimgpsc.l7 -0.23552    0.07520  -3.132  0.00196 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 49.78 on 227 degrees of freedom
## Multiple R-Squared:     1,   Adjusted R-squared:     1
## F-statistic: 9.182e+05 on 10 and 227 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsexpgs:
## ========================================
## tsexpgs = tsexpgs.l1 + tspce.l1 + tsexpgs.l2 + tsgdp.l3 + tsimgpsc.l3 + tsgdp.l4 + tsexpgs.l4 + tsim
##
```

```
##             Estimate Std. Error t value Pr(>|t|)
## tsexpgs.l1   1.61867    0.05771  28.048  < 2e-16 ***
## tspce.l1    -0.13843    0.02815  -4.917 1.69e-06 ***
## tsexpgs.l2  -0.80272    0.07832 -10.249  < 2e-16 ***
## tsgdp.l3     0.25556    0.04074   6.273 1.78e-09 ***
## tsimgpsc.l3 -0.22286    0.06724  -3.314  0.00107 **
## tsgdp.l4    -0.14290    0.05180  -2.759  0.00628 **
## tsexpgs.l4   0.09338    0.04107   2.274  0.02392 *
## tsimgpsc.l4  0.34052    0.07505   4.537 9.26e-06 ***
## tsgdp.l5    -0.11904    0.03747  -3.177  0.00170 **
## tsimgpsc.l7 -0.12906    0.03107  -4.154 4.63e-05 ***
## tspce.l7     0.17173    0.02174   7.900 1.21e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 22.49 on 226 degrees of freedom
## Multiple R-Squared: 0.9996,  Adjusted R-squared: 0.9996
## F-statistic: 5.619e+04 on 11 and 226 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsimgpsc:
## =========================================
## tsimgpsc = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsexpgs.l2 + tsimgpsc.l2 + tsexpgs.l4 + tspce.l4 +
##
##             Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1     0.08482    0.01608   5.277 3.11e-07 ***
## tsexpgs.l1   0.29975    0.06154   4.871 2.10e-06 ***
## tsimgpsc.l1  1.22897    0.06886  17.848  < 2e-16 ***
## tsexpgs.l2  -0.61828    0.07478  -8.268 1.22e-14 ***
## tsimgpsc.l2 -0.26005    0.07794  -3.336 0.000994 ***
## tsexpgs.l4   0.20420    0.08769   2.329 0.020776 *
## tspce.l4    -0.16723    0.03868  -4.323 2.32e-05 ***
## tsgdp.l5    -0.08146    0.02094  -3.890 0.000132 ***
## tsexpgs.l5   0.25704    0.11775   2.183 0.030091 *
## tsexpgs.l6  -0.38182    0.11503  -3.319 0.001054 **
## tsimgpsc.l6  0.25268    0.07206   3.507 0.000548 ***
## tsexpgs.l7   0.19824    0.07198   2.754 0.006372 **
## tsimgpsc.l7 -0.23865    0.06242  -3.823 0.000171 ***
## tspce.l7     0.17622    0.03281   5.372 1.96e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 21.4 on 223 degrees of freedom
## Multiple R-Squared: 0.9998,  Adjusted R-squared: 0.9998
## F-statistic: 1.022e+05 on 14 and 223 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsunrate:
## =========================================
## tsunrate = tsgdp.l1 + tsimgpsc.l1 + tsunrate.l1 + tsexpgs.l2 + tsimgpsc.l2 + tspce.l2 + tsgdp.l3 + ts
##
##               Estimate Std. Error t value Pr(>|t|)
```

```
## tsgdp.l1    -0.0006930  0.0002653  -2.612 0.009613 **
## tsimgpsc.l1 -0.0017151  0.0008119  -2.112 0.035759 *
## tsunrate.l1  1.2196753  0.0407523  29.929  < 2e-16 ***
## tsexpgs.l2   0.0007759  0.0002782   2.789 0.005743 **
## tsimgpsc.l2  0.0029617  0.0012310   2.406 0.016942 *
## tspce.l2    -0.0010007  0.0002712  -3.691 0.000281 ***
## tsgdp.l3     0.0013222  0.0003199   4.133 5.05e-05 ***
## tsimgpsc.l3 -0.0018931  0.0008334  -2.272 0.024058 *
## tsunrate.l3 -0.3186415  0.0656753  -4.852 2.29e-06 ***
## tsunrate.l5  0.0804019  0.0396962   2.025 0.044004 *
## tsexpgs.l7  -0.0009161  0.0002637  -3.474 0.000616 ***
## tsimgpsc.l7  0.0011677  0.0003335   3.502 0.000557 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2699 on 225 degrees of freedom
## Multiple R-Squared: 0.9982,  Adjusted R-squared: 0.9981
## F-statistic: 1.036e+04 on 12 and 225 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tspce:
## ======================================
## tspce = tsgdp.l1 + tsexpgs.l1 + tspce.l1 + tsexpgs.l2 + tsunrate.l2 + tsgdp.l3 + tsexpgs.l3 + tspce.l
##
##             Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1     0.37770    0.03791   9.964  < 2e-16 ***
## tsexpgs.l1   0.38099    0.08390   4.541 9.17e-06 ***
## tspce.l1     0.40088    0.06414   6.251 2.06e-09 ***
## tsexpgs.l2  -0.28072    0.13560  -2.070 0.039583 *
## tsunrate.l2  7.55167    2.01430   3.749 0.000226 ***
## tsgdp.l3    -0.12204    0.05024  -2.429 0.015925 *
## tsexpgs.l3  -0.34369    0.11000  -3.124 0.002019 **
## tspce.l3     0.36349    0.07143   5.089 7.65e-07 ***
## tsimgpsc.l4  0.34559    0.04377   7.895 1.30e-13 ***
## tsgdp.l5    -0.19160    0.03477  -5.511 9.84e-08 ***
## tspce.l6     0.15241    0.04641   3.284 0.001189 **
## tsexpgs.l7   0.20363    0.03843   5.299 2.79e-07 ***
## tsimgpsc.l7 -0.32748    0.04317  -7.587 8.83e-13 ***
## tsunrate.l7 -8.97058    2.08647  -4.299 2.56e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 27.88 on 223 degrees of freedom
## Multiple R-Squared:     1,   Adjusted R-squared:     1
## F-statistic: 9.367e+05 on 14 and 223 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##             tsgdp tsexpgs tsimgpsc tsunrate    tspce
## tsgdp    2812.167 778.690  358.756 -2.10320 924.5492
## tsexpgs   778.690 571.478  187.746 -0.72796 320.8291
```

```
## tsimgpsc  358.756 187.746  510.581 -1.20434 118.2643
## tsunrate   -2.103  -0.728   -1.204  0.08196  -0.5292
## tspce     924.549 320.829  118.264 -0.52925 866.7236
##
## Correlation matrix of residuals:
##            tsgdp tsexpgs tsimgpsc tsunrate    tspce
## tsgdp     1.0000  0.6142   0.2994 -0.13854  0.59220
## tsexpgs   0.6142  1.0000   0.3476 -0.10637  0.45586
## tsimgpsc  0.2994  0.3476   1.0000 -0.18617  0.17778
## tsunrate -0.1385 -0.1064  -0.1862  1.00000 -0.06279
## tspce     0.5922  0.4559   0.1778 -0.06279  1.00000
```

Response Coefficient analysis for unrestricted BIC model selected indicates the following: * Lead relationship of Real imports of goods and services (IMPGSC1) are statistically significant on GDP along with the factors corresponding to lags of GDP. * Lead relationship of Personal Consumption Expenditure (PCE), GDP and IMPGSC1 are statistically significant on Real exports of goods and services (EXPGS) along with the factors corresponding to lags of EXPGS. * Lead relationship of Real exports of goods and services (EXPGS) statistically significant on Real imports of goods and services (IMPGSC1) along with the factors corresponding to lags of IMPGSC1 * Unemployment rate doesn't seem to have a lead relation on other factors, apart from the lags of the same. * Lead relationships of GDP and Real exports of goods and services (EXPGS) are statistically significant on Personal Consumption Expenditure (PCE) along with the factors corresponding to lags of PCE.

Stepwise regression and VAR models are differing interns coefficients significance for some models.

Restricted model is picking only the coefficients that are significant compared to that of the unrestricted model. Restricted model is easy to explain.

Question 2c

Evaluate the goodness of fit for the restricted BIC model using the multivariate ARCH test, the Jarque-Bera test and the Portmanteau test. State which assumptions are satisfied, and which are violated. (Note: While we evaluate the residuals for the normality assumption, we do not necessarily assume normality of the data. We use the normality assumption if we use the t-test to evaluate statistical significance.)

```r
model.bic.restrict.resid<-residuals(model_bic_var_restric_1)
normality.test(model_bic_var_restric_1)
```

```
## $JB
##
##  JB-Test (multivariate)
##
## data:  Residuals of VAR object model_bic_var_restric_1
## Chi-squared = 1682.1, df = 10, p-value < 2.2e-16
##
##
## $Skewness
##
##  Skewness only (multivariate)
##
## data:  Residuals of VAR object model_bic_var_restric_1
## Chi-squared = 97.315, df = 5, p-value < 2.2e-16
##
##
## $Kurtosis
##
```

```
##  Kurtosis only (multivariate)
##
## data:  Residuals of VAR object model_bic_var_restric_1
## Chi-squared = 1584.8, df = 5, p-value < 2.2e-16
```

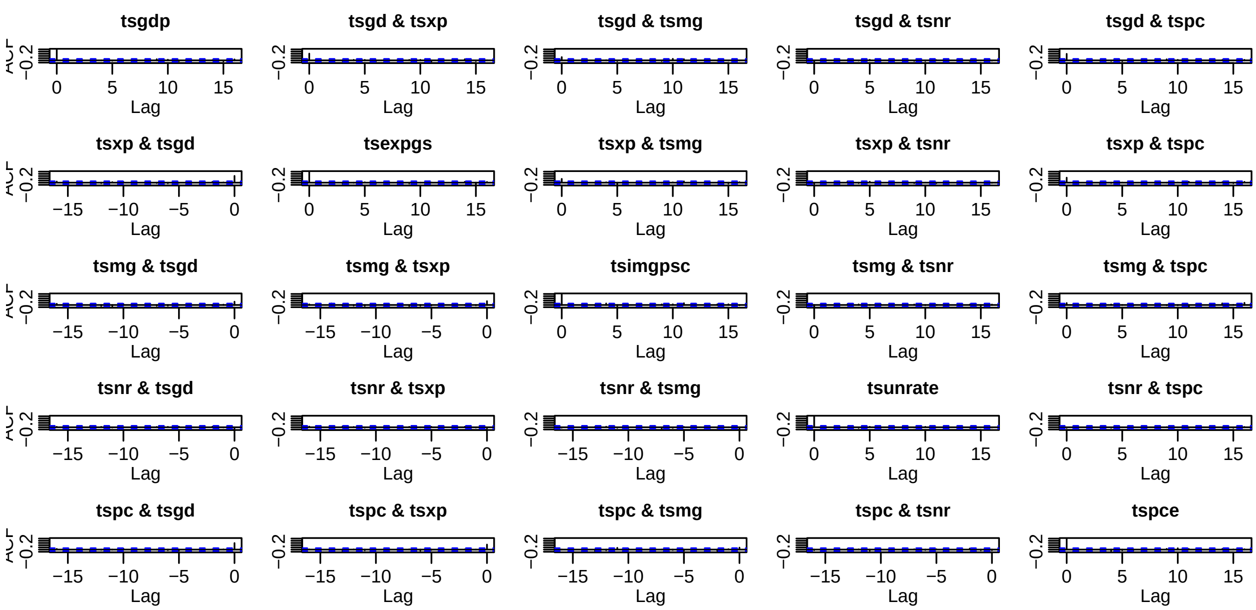
```
arch.test(model_bic_var_restric_1)
```

```
##
##  ARCH (multivariate)
##
## data:  Residuals of VAR object model_bic_var_restric_1
## Chi-squared = 1776.9, df = 1125, p-value < 2.2e-16
```

```
serial.test(model_bic_var_restric_1)
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object model_bic_var_restric_1
## Chi-squared = 371.92, df = 225, p-value = 2.498e-09
```

```
acf(model.bic.restrict.resid)
```



Response The normality test provides inference on multiple aspects of normality including skeweness and kurtosis, where the former is a measure of departure from symmetry and the latter is a measure of a departure from normal tails. It was provided using normality.test with null hypotheisis as the data (the residuals in this case) are normally distributed. arch.test(), which is a hypothesis testing procedure where the null hypothesis is that the residuals have constant variance.

Residual correlation is tested using a hypothesis testing procedure for independence called serial.test() in R. The null hypothesis of this test is that the residuals are uncorrelated

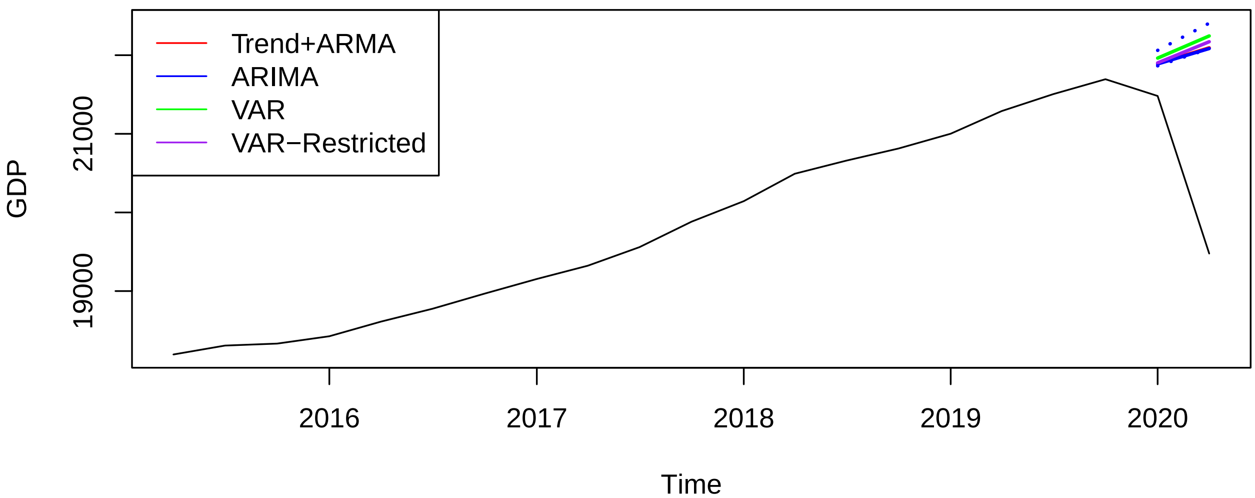
P-values are small for all three tests indicating that the residuals are correlated, they are not normally distributed and have a non-constant variance.

ACF plot alos reveals the same on serial correlation.

Question 2d

Using the VAR model with the order selected using BIC, forecast the first two quarters of 2020 using the unrestricted and restricted VAR. Include 95% confidence intervals. Compare the predictions to the observed data. (You don't need to plot them (but can if you'd like). Using mean absolute percentage error and the precision measure, compare the predictions for GDP derived from the univariate analysis (Question 1) and this multivariate analysis. Discuss on the differences or similarities.

```
nfit=n-2
pred.model.bic.var1=predict(model_bic_var_1,n.ahead=2)
gdp.fcst.var1 = xts(pred.model.bic.var1[[1]]$tsgdp[,1], date.quarter[(n-1):n])
gdp.fcst.lbound.var1 = xts(pred.model.bic.var1[[1]]$tsgdp[,2], date.quarter[(n-1):n])
gdp.fcst.ubound.var1 = xts(pred.model.bic.var1[[1]]$tsgdp[,3], date.quarter[(n-1):n])

pred.model.bic.var2=predict(model_bic_var_restric_1,n.ahead=2)
gdp.fcst.var2 = xts(pred.model.bic.var2[[1]]$tsgdp[,1], date.quarter[(n-1):n])
gdp.fcst.lbound.var2 = xts(pred.model.bic.var2[[1]]$tsgdp[,2], date.quarter[(n-1):n])
gdp.fcst.ubound.var2 = xts(pred.model.bic.var2[[1]]$tsgdp[,3], date.quarter[(n-1):n])

ymin = min(c(gdp.ts[(n-20):n],final.pred.2,gdp.fcst.lbound.var1))
ymax = max(c(gdp.ts[(n-20):n],final.pred.2,gdp.fcst.ubound.var1))
plot(date.quarter[(n-20):n], gdp.ts[(n-20):n],type="l", ylim=c(ymin,ymax), xlab="Time", ylab="GDP")
lines(date.quarter[(nfit+1):n],final.pred.1,col="red",lwd=2)
lines(date.quarter[(nfit+1):n],final.pred.2,col="blue",lwd=2)
lines(date.quarter[(nfit+1):n],gdp.fcst.var1,col="green",lwd=2)
lines(date.quarter[(nfit+1):n],gdp.fcst.var2,col="purple",lwd=2)
lines(date.quarter[(nfit+1):n],gdp.fcst.ubound.var1,lty=3,lwd= 2, col="blue")
lines(date.quarter[(nfit+1):n],gdp.fcst.lbound.var1,lty=3,lwd= 2, col="blue")
legend("topleft",legend=c("Trend+ARMA","ARIMA","VAR", "VAR-Restricted"),col=c("red","blue","green", "pu
```




```
print("UnRestricted VAR Summary --------")
```

```
## [1] "UnRestricted VAR Summary --------"
```

```
print("MAPE:")
```

```
## [1] "MAPE:"
```

```
print(mean(abs(gdp.fcst.var1 - gdp.ts.test) / gdp.ts.test))
```

```
## [1] 0.08224683
```

```r
print("PM:")
```

```
## [1] "PM:"
```

```r
print(sum((gdp.fcst.var1 - gdp.ts.test)^2) / sum((gdp.fcst.var1 - mean(gdp.fcst.var1))^2))
```

```
## [1] 198.4992
```

```r
print("Does the observed data fall outside the prediction intervals?")
```

```
## [1] "Does the observed data fall outside the prediction intervals?"
```

```r
print(sum(gdp.ts.test < gdp.fcst.lbound.var1) & sum(gdp.ts.test > gdp.fcst.ubound.var1))
```

```
## [1] FALSE
```

```r
print("Restricted VAR Summary --------")
```

```
## [1] "Restricted VAR Summary --------"
```

```r
print("MAPE:")
```

```
## [1] "MAPE:"
```

```r
print(mean(abs(gdp.fcst.var2 - gdp.ts.test) / gdp.ts.test))
```

```
## [1] 0.07894489
```

```r
print("PM:")
```

```
## [1] "PM:"
```

```r
print(sum((gdp.fcst.var2 - gdp.ts.test)^2) / sum((gdp.fcst.var2 - mean(gdp.fcst.var1))^2))
```

```
## [1] 164.3513
```

```r
print("Does the observed data fall outside the prediction intervals?")
```

```
## [1] "Does the observed data fall outside the prediction intervals?"
```

```r
print(sum(gdp.ts.test < gdp.fcst.lbound.var2) & sum(gdp.ts.test > gdp.fcst.ubound.var1))
```

```
## [1] FALSE
```

```r
print("Univariate Summary --------")
```

```
## [1] "Univariate Summary --------"
```

```r
print("MAPE:")
```

```
## [1] "MAPE:"
```

```r
print(mean(abs(final.pred.2 - gdp.ts.test) / gdp.ts.test))
```

```
## [1] 0.07641706
```

```r
print("PM:")
```

```
## [1] "PM:"
```

```r
print(sum((final.pred.2 - gdp.ts.test)^2) / sum((final.pred.2 - mean(gdp.fcst.var1))^2))
```

```
## [1] 149.36
```

```
print("Does the observed data fall outside the prediction intervals?")
```

```
## [1] "Does the observed data fall outside the prediction intervals?"
```

```
print(sum(gdp.ts.test < lbound) & sum(gdp.ts.test > ubound))
```

```
## [1] FALSE
```

Response All of the predictions including the VAR model doesn't show the decreasing trend in the last 2 quarters. This is due to the correlations and shock that came in as Covid which wwas to be accounted for. All the models are showing high PM values and indicating high variability in the data.

Question 2e

Perform a Granger Causality analysis using Wald test to evaluate whether any of the economic indicators lead GDP. Would any of the indicators help in predicting or explaining GDP for next quarters? Provide your interpretation based on the Granger causality as well as for forecasting comparison in (2d).For this, use the unrestricted bic model from Question 2a.

```
### Estimated coefficients and their variance for gdp regression Equation
coef.tsgdp.b = coefficients(model_bic_var_1)$tsgdp[-c((7*5+1),(7*5+2)) ,1]
var.tsgdp.b = vcov(model_bic_var_1)[c(2:(7*5+1)),c(2:(7*5+1))]
library(aod)
# Is there a lead-lag relationship for gdp-other economic factors
#tsgdp = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsimgps
tsgdpnames=c("tsgdp:tsgdp.l1", "tsgdp:tsgdp.l2","tsgdp:tsgdp.l3", "tsgdp:tsgdp.l4", "tsgdp:tsgdp.l5", "

tsnongdpvars<-which(!colnames(var.tsgdp.b)%in%tsgdpnames)
wald.test(b=coef.tsgdp.b, var.tsgdp.b,Terms = tsnongdpvars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 69.7, df = 28, P(> X2) = 2e-05
```

```
# Is there a lead-lag relationship for gdp-expgs economic factors
tsgdpexpgs<-c("tsgdp:tsexpgs.l1", "tsgdp:tsexpgs.l2","tsgdp:tsexpgs.l3", "tsgdp:tsexpgs.l4", "tsgdp:tse
tsgdpexpgsvars<-which(colnames(var.tsgdp.b)%in%tsgdpexpgs)
wald.test(b=coef.tsgdp.b, var.tsgdp.b,Terms = tsgdpexpgsvars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 24.6, df = 7, P(> X2) = 0.00091
```

```
# Is there a lead-lag relationship for gdp-imgpsc economic factors
tsgdpimgpsc<-c("tsgdp:tsimgpsc.l1", "tsgdp:tsimgpsc.l2","tsgdp:tsimgpsc.l3", "tsgdp:tsimgpsc.l4", "tsgdp
tsgdpimgpscvars<-which(colnames(var.tsgdp.b)%in%tsgdpimgpsc)
wald.test(b=coef.tsgdp.b, var.tsgdp.b,Terms = tsgdpimgpscvars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 28.6, df = 7, P(> X2) = 0.00017
```

```
# Is there a lead-lag relationship for gdp-unrate economic factors
tsgdpunrate<-c("tsgdp:tsunrate.l1", "tsgdp:tsunrate.l2","tsgdp:tsunrate.l3", "tsgdp:tsunrate.l4", "tsgd
tsgdpunratevars<-which(colnames(var.tsgdp.b)%in%tsgdpunrate)
wald.test(b=coef.tsgdp.b, var.tsgdp.b,Terms = tsgdpunratevars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 4.0, df = 7, P(> X2) = 0.78
```

```
# Is there a lead-lag relationship for gdp-pce economic factors
tsgdppce<-c("tsgdp:tspce.l1", "tsgdp:tspce.l2","tsgdp:tspce.l3", "tsgdp:tspce.l4", "tsgdp:tspce.l5", "ts
tsgdppcevars<-which(colnames(var.tsgdp.b)%in%tsgdppce)
wald.test(b=coef.tsgdp.b, var.tsgdp.b,Terms = tsgdppcevars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 15.4, df = 7, P(> X2) = 0.031
```

Response Wald test for gdp-unrate lead lag relation shows no-evidence as the P value is 0.78 which is above significant levels. For the rest of the economic factors the wald tests shows that there is a lead relationship with gdp. In the VAR forecasting the same behavior is depicted. However, the var coeffieicnts indicates that there is only some significant lead relations with EXPGS and PCE compared to that of IMGPSC. # Question 3

For this question, consider the training data to include the time values up to December 2017 and the testing data to include the first two quarters of 2018.

Question 3a

Apply the VAR modeling approach with the order selected using the BIC approach giving the unrestricted VAR model. Apply a model selection analysis using stepwise regression to select the models for each individual time series. Based on the selected models, form the restricted VAR model, much like what was presented in the Moose R example code. Compare these two models in terms of coefficients and their statistical significance with the models derived in Question 2.

Follow the below steps to implement this: 1)Analyze the coefficients of the unrestricted model. 2)Then treat each of the series separately and do a stepwise regression by apply lm() and step()(in the backward direction with 3 steps) functions to each of the component time series to examine the coefficients 3)The stepwise regressions will return three separate models. Analyze if the same coefficients are significant in the overall VAR versus each of the stepwise models. Discuss. 4)Then build a model using restrict and see which predictors were significant in the restricted VAR model.

```
data.train.3<-data.train[lubridate::year(index(data.train))<2018]
ts_gdp.3.test<-data.train[237:238][,"tsgdp"]
ts_gdp.3.tr=data.train.3[,"tsgdp"]
ts_expgs.3.tr=data.train.3[,"tsexpgs"]
ts_imgpsc.3.tr=data.train.3[,"tsimgpsc"]
ts_unrate.3.tr=data.train.3[,"tsunrate"]
ts_pce.3.tr=data.train.3[,"tspce"]
ts_data.train.3=cbind(ts_gdp.3.tr, ts_expgs.3.tr, ts_imgpsc.3.tr, ts_unrate.3.tr, ts_pce.3.tr )

VARselect(ts_data.train.3, lag.max = 10)$selection
```

```
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      5      4      2      5
```

```
model_bic_var_3<-VAR(ts_data.train.3, lag.max=10,p=2 )
summary(model_bic_var_3)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: tsgdp, tsexpgs, tsimgpsc, tsunrate, tspce
## Deterministic variables: const
## Sample size: 231
## Log Likelihood: -4217.141
## Roots of the characteristic polynomial:
## 1.006 0.9878 0.9483 0.9483 0.8774 0.857 0.857 0.769 0.769 0.7585 0.7585 0.702 0.702 0.6988 0.6988 0.6
## Call:
## VAR(y = ts_data.train.3, p = 2, lag.max = 10)
##
##
## Estimation results for equation tsgdp:
## ======================================
## tsgdp = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsimgp
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1      1.12307    0.10552  10.643  < 2e-16 ***
## tsexpgs.l1    0.07464    0.20672   0.361 0.718409
## tsimgpsc.l1   0.79530    0.17835   4.459 1.35e-05 ***
## tsunrate.l1 -11.61913   12.28642  -0.946 0.345421
## tspce.l1      0.13984    0.15028   0.931 0.353199
## tsgdp.l2     -0.03315    0.14183  -0.234 0.815450
## tsexpgs.l2   -0.57448    0.37693  -1.524 0.129025
## tsimgpsc.l2  -1.03563    0.27812  -3.724 0.000254 ***
## tsunrate.l2  27.56100   19.19134   1.436 0.152493
## tspce.l2     -0.13479    0.17680  -0.762 0.446723
## tsgdp.l3      0.16155    0.14257   1.133 0.258490
## tsexpgs.l3    0.27418    0.37648   0.728 0.467290
## tsimgpsc.l3   0.06093    0.29425   0.207 0.836163
## tsunrate.l3 -12.73845   19.05462  -0.669 0.504552
## tspce.l3      0.03539    0.17121   0.207 0.836428
## tsgdp.l4     -0.06589    0.14146  -0.466 0.641875
## tsexpgs.l4    0.33032    0.36258   0.911 0.363342
## tsimgpsc.l4   0.21059    0.27169   0.775 0.439165
## tsunrate.l4  -9.42963   19.06605  -0.495 0.621428
## tspce.l4     -0.31833    0.16919  -1.882 0.061316 .
## tsgdp.l5     -0.21193    0.11348  -1.868 0.063246 .
## tsexpgs.l5   -0.20301    0.21582  -0.941 0.347992
## tsimgpsc.l5  -0.06982    0.16848  -0.414 0.679031
## tsunrate.l5   9.19991   11.74020   0.784 0.434165
## tspce.l5      0.34976    0.14394   2.430 0.015960 *
## const        -7.62487   21.93836  -0.348 0.728528
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 49.3 on 205 degrees of freedom
```

```
## Multiple R-Squared: 0.9999,  Adjusted R-squared: 0.9999
## F-statistic: 1.291e+05 on 25 and 205 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsexpgs:
## ========================================
## tsexpgs = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsi
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1      0.05636    0.04868   1.158 0.248320
## tsexpgs.l1    1.63126    0.09537  17.105  < 2e-16 ***
## tsimgpsc.l1   0.03439    0.08228   0.418 0.676440
## tsunrate.l1  -2.86830    5.66843  -0.506 0.613390
## tspce.l1     -0.17433    0.06933  -2.514 0.012689 *
## tsgdp.l2     -0.05463    0.06543  -0.835 0.404716
## tsexpgs.l2   -0.84188    0.17390  -4.841 2.54e-06 ***
## tsimgpsc.l2   0.01856    0.12831   0.145 0.885113
## tsunrate.l2   2.73436    8.85406   0.309 0.757768
## tspce.l2      0.06277    0.08157   0.769 0.442492
## tsgdp.l3      0.23143    0.06578   3.518 0.000535 ***
## tsexpgs.l3   -0.02224    0.17369  -0.128 0.898228
## tsimgpsc.l3  -0.17787    0.13576  -1.310 0.191590
## tsunrate.l3   9.41123    8.79099   1.071 0.285629
## tspce.l3     -0.04702    0.07899  -0.595 0.552303
## tsgdp.l4     -0.19152    0.06527  -2.935 0.003721 **
## tsexpgs.l4    0.32369    0.16728   1.935 0.054357 .
## tsimgpsc.l4   0.22215    0.12535   1.772 0.077830 .
## tsunrate.l4 -14.63344    8.79626  -1.664 0.097721 .
## tspce.l4      0.03166    0.07806   0.406 0.685481
## tsgdp.l5     -0.06710    0.05235  -1.282 0.201438
## tsexpgs.l5   -0.16849    0.09957  -1.692 0.092135 .
## tsimgpsc.l5  -0.11740    0.07773  -1.510 0.132506
## tsunrate.l5   5.66185    5.41643   1.045 0.297110
## tspce.l5      0.18590    0.06641   2.799 0.005607 **
## const        -0.89079   10.12142  -0.088 0.929955
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 22.75 on 205 degrees of freedom
## Multiple R-Squared: 0.9992,  Adjusted R-squared: 0.9991
## F-statistic:  9999 on 25 and 205 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsimgpsc:
## =========================================
## tsimgpsc = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + ts
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1     0.206166   0.046278   4.455 1.38e-05 ***
## tsexpgs.l1   0.140709   0.090656   1.552   0.1222
## tsimgpsc.l1  1.184274   0.078216  15.141  < 2e-16 ***
## tsunrate.l1 -3.303556   5.388273  -0.613   0.5405
## tspce.l1     0.035122   0.065905   0.533   0.5947
```

```
## tsgdp.l2    -0.144175   0.062201  -2.318   0.0214 *
## tsexpgs.l2  -0.314893   0.165304  -1.905   0.0582 .
## tsimgpsc.l2 -0.226821   0.121969  -1.860   0.0644 .
## tsunrate.l2  6.669388   8.416461   0.792   0.4290
## tspce.l2    -0.078538   0.077538  -1.013   0.3123
## tsgdp.l3    -0.039837   0.062525  -0.637   0.5247
## tsexpgs.l3  -0.104002   0.165109  -0.630   0.5295
## tsimgpsc.l3  0.125583   0.129047   0.973   0.3316
## tsunrate.l3 -2.282003   8.356501  -0.273   0.7851
## tspce.l3    -0.006275   0.075083  -0.084   0.9335
## tsgdp.l4    -0.012558   0.062040  -0.202   0.8398
## tsexpgs.l4   0.182524   0.159009   1.148   0.2524
## tsimgpsc.l4 -0.015960   0.119151  -0.134   0.8936
## tsunrate.l4  5.050835   8.361514   0.604   0.5465
## tspce.l4    -0.016987   0.074198  -0.229   0.8191
## tsgdp.l5    -0.041153   0.049767  -0.827   0.4093
## tsexpgs.l5   0.066148   0.094650   0.699   0.4854
## tsimgpsc.l5 -0.119411   0.073889  -1.616   0.1076
## tsunrate.l5 -7.337699   5.148725  -1.425   0.1556
## tspce.l5     0.131404   0.063124   2.082   0.0386 *
## const       10.385777   9.621181   1.079   0.2816
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 21.62 on 205 degrees of freedom
## Multiple R-Squared: 0.9996,  Adjusted R-squared: 0.9995
## F-statistic: 2.007e+04 on 25 and 205 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsunrate:
## =========================================
## tsunrate = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + ts
##
##               Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1    -1.177e-03  6.017e-04  -1.956   0.0518 .
## tsexpgs.l1  -5.870e-05  1.179e-03  -0.050   0.9603
## tsimgpsc.l1 -1.947e-03  1.017e-03  -1.914   0.0570 .
## tsunrate.l1  1.252e+00  7.005e-02  17.878   <2e-16 ***
## tspce.l1     4.285e-04  8.568e-04   0.500   0.6175
## tsgdp.l2     2.733e-04  8.087e-04   0.338   0.7358
## tsexpgs.l2   1.548e-03  2.149e-03   0.720   0.4722
## tsimgpsc.l2  2.833e-03  1.586e-03   1.787   0.0754 .
## tsunrate.l2 -8.987e-02  1.094e-01  -0.821   0.4124
## tspce.l2    -8.460e-04  1.008e-03  -0.839   0.4023
## tsgdp.l3     1.528e-03  8.129e-04   1.880   0.0616 .
## tsexpgs.l3  -1.434e-03  2.147e-03  -0.668   0.5049
## tsimgpsc.l3 -1.772e-03  1.678e-03  -1.056   0.2921
## tsunrate.l3 -2.083e-01  1.086e-01  -1.917   0.0566 .
## tspce.l3    -8.967e-05  9.762e-04  -0.092   0.9269
## tsgdp.l4    -2.375e-04  8.066e-04  -0.294   0.7687
## tsexpgs.l4   1.202e-03  2.067e-03   0.581   0.5616
## tsimgpsc.l4  1.313e-04  1.549e-03   0.085   0.9326
## tsunrate.l4 -5.102e-02  1.087e-01  -0.469   0.6393
```

```
## tspce.l4    -3.065e-04  9.647e-04  -0.318   0.7510
## tsgdp.l5    -1.805e-05  6.470e-04  -0.028   0.9778
## tsexpgs.l5  -1.382e-03  1.231e-03  -1.123   0.2627
## tsimgpsc.l5  9.604e-04  9.606e-04   1.000   0.3186
## tsunrate.l5  6.564e-02  6.694e-02   0.981   0.3279
## tspce.l5     2.659e-04  8.207e-04   0.324   0.7463
## const        1.555e-01  1.251e-01   1.244   0.2151
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2811 on 205 degrees of freedom
## Multiple R-Squared: 0.9718,  Adjusted R-squared: 0.9684
## F-statistic: 282.5 on 25 and 205 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tspce:
## ======================================
## tspce = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsunrate.l1 + tspce.l1 + tsgdp.l2 + tsexpgs.l2 + tsimg
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1      0.30385    0.06110   4.973 1.39e-06 ***
## tsexpgs.l1    0.41977    0.11969   3.507 0.000557 ***
## tsimgpsc.l1   0.11213    0.10327   1.086 0.278833
## tsunrate.l1  -9.89141    7.11404  -1.390 0.165913
## tspce.l1      0.56296    0.08701   6.470 7.08e-10 ***
## tsgdp.l2     -0.12811    0.08212  -1.560 0.120311
## tsexpgs.l2   -0.43579    0.21825  -1.997 0.047173 *
## tsimgpsc.l2  -0.09303    0.16103  -0.578 0.564093
## tsunrate.l2  24.13753   11.11210   2.172 0.030989 *
## tspce.l2      0.13422    0.10237   1.311 0.191303
## tsgdp.l3     -0.13008    0.08255  -1.576 0.116607
## tsexpgs.l3   -0.17483    0.21799  -0.802 0.423470
## tsimgpsc.l3   0.09780    0.17038   0.574 0.566584
## tsunrate.l3  -8.63543   11.03294  -0.783 0.434709
## tspce.l3      0.36451    0.09913   3.677 0.000301 ***
## tsgdp.l4      0.21521    0.08191   2.627 0.009256 **
## tsexpgs.l4   -0.20609    0.20994  -0.982 0.327426
## tsimgpsc.l4   0.21969    0.15731   1.397 0.164069
## tsunrate.l4   3.02902   11.03956   0.274 0.784070
## tspce.l4     -0.29373    0.09796  -2.998 0.003050 **
## tsgdp.l5     -0.19023    0.06571  -2.895 0.004201 **
## tsexpgs.l5    0.36158    0.12496   2.893 0.004222 **
## tsimgpsc.l5  -0.29110    0.09755  -2.984 0.003191 **
## tsunrate.l5  -7.06362    6.79777  -1.039 0.299978
## tspce.l5      0.12949    0.08334   1.554 0.121779
## const       -20.12538   12.70267  -1.584 0.114657
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 28.55 on 205 degrees of freedom
## Multiple R-Squared:     1,   Adjusted R-squared: 0.9999
## F-statistic: 1.796e+05 on 25 and 205 DF,  p-value: < 2.2e-16
```

```
##
##
##
## Covariance matrix of residuals:
##             tsgdp  tsexpgs tsimgpsc tsunrate    tspce
## tsgdp    2430.694 749.9592   435.48 -2.10304 901.3154
## tsexpgs   749.959 517.3742   207.29 -0.64744 354.0319
## tsimgpsc  435.479 207.2851   467.50 -1.08029 194.9391
## tsunrate   -2.103  -0.6474    -1.08  0.07902  -0.6771
## tspce     901.315 354.0319   194.94 -0.67709 814.9145
##
## Correlation matrix of residuals:
##            tsgdp tsexpgs tsimgpsc tsunrate    tspce
## tsgdp     1.0000  0.6688   0.4085 -0.15174  0.64041
## tsexpgs   0.6688  1.0000   0.4215 -0.10126  0.54524
## tsimgpsc  0.4085  0.4215   1.0000 -0.17774  0.31583
## tsunrate -0.1517 -0.1013  -0.1777  1.00000 -0.08438
## tspce     0.6404  0.5452   0.3158 -0.08438  1.00000
```

```r
lm.all <- lm(ts_gdp.3.tr~ts_expgs.3.tr + ts_imgpsc.3.tr + ts_unrate.3.tr + ts_pce.3.tr)
summary(lm.all)
```

```
##
## Call:
## lm(formula = ts_gdp.3.tr ~ ts_expgs.3.tr + ts_imgpsc.3.tr + ts_unrate.3.tr +
##     ts_pce.3.tr)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -405.25 -106.84   23.76   85.14  288.37
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)    351.71749   40.76992   8.627 1.03e-15 ***
## ts_expgs.3.tr   -0.33952    0.07123  -4.767 3.31e-06 ***
## ts_imgpsc.3.tr  -0.55848    0.08117  -6.880 5.57e-11 ***
## ts_unrate.3.tr -21.40904    6.17940  -3.465 0.000633 ***
## ts_pce.3.tr      1.66902    0.02490  67.037  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 126.8 on 231 degrees of freedom
## Multiple R-squared:  0.9995, Adjusted R-squared:  0.9995
## F-statistic: 1.251e+05 on 4 and 231 DF,  p-value: < 2.2e-16
```

```r
## Stepwise Regression
full = lm.all
minimum = lm(ts_gdp.3.tr~ts_imgpsc.3.tr)
step.model <-step(full, scope = list(lower=minimum, upper = full), direction = "backward")
```

```
## Start:  AIC=2290.65
## ts_gdp.3.tr ~ ts_expgs.3.tr + ts_imgpsc.3.tr + ts_unrate.3.tr +
##     ts_pce.3.tr
##
##                  Df Sum of Sq      RSS    AIC
```

```
## <none>                          3713992 2290.7
## - ts_unrate.3.tr  1    192988   3906980 2300.6
## - ts_expgs.3.tr   1    365317   4079309 2310.8
## - ts_pce.3.tr     1  72252327 75966319 3000.9
```

```r
summary(step.model)
```

```
##
## Call:
## lm(formula = ts_gdp.3.tr ~ ts_expgs.3.tr + ts_imgpsc.3.tr + ts_unrate.3.tr +
##     ts_pce.3.tr)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -405.25 -106.84   23.76   85.14  288.37
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)    351.71749   40.76992   8.627 1.03e-15 ***
## ts_expgs.3.tr   -0.33952    0.07123  -4.767 3.31e-06 ***
## ts_imgpsc.3.tr  -0.55848    0.08117  -6.880 5.57e-11 ***
## ts_unrate.3.tr -21.40904    6.17940  -3.465 0.000633 ***
## ts_pce.3.tr      1.66902    0.02490  67.037  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 126.8 on 231 degrees of freedom
## Multiple R-squared:  0.9995, Adjusted R-squared:  0.9995
## F-statistic: 1.251e+05 on 4 and 231 DF,  p-value: < 2.2e-16
```

```r
adf.test(step.model$resid)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  step.model$resid
## Dickey-Fuller = -3.3087, Lag order = 6, p-value = 0.07054
## alternative hypothesis: stationary
```

```r
#Restricted VAR model

model_bic_var_3<-VAR(ts_data.train.3, lag.max=10,p=2 )
model_bic_var_3_restrict<-restrict(model_bic_var_3)
summary(model_bic_var_3_restrict)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: tsgdp, tsexpgs, tsimgpsc, tsunrate, tspce
## Deterministic variables: const
## Sample size: 231
## Log Likelihood: -4273.284
## Roots of the characteristic polynomial:
## 1.007 0.9777 0.9612 0.9612 0.8763 0.8596 0.8596 0.7873 0.7873 0.7786 0.7786 0.7081 0.7081 0.6805 0.68
## Call:
## VAR(y = ts_data.train.3, p = 2, lag.max = 10)
```

```
##
##
## Estimation results for equation tsgdp:
## ======================================
## tsgdp = tsgdp.l1 + tsimgpsc.l1 + tsexpgs.l2 + tsimgpsc.l2 + tsunrate.l2 + tsexpgs.l4 + tsgdp.l5 + tsp
##
##             Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1     1.15573    0.02980  38.789  < 2e-16 ***
## tsimgpsc.l1  0.68638    0.12610   5.443 1.38e-07 ***
## tsexpgs.l2  -0.32364    0.07712  -4.196 3.92e-05 ***
## tsimgpsc.l2 -0.74097    0.11969  -6.191 2.85e-09 ***
## tsunrate.l2  2.43627    1.04294   2.336  0.02038 *
## tsexpgs.l4   0.22996    0.07783   2.955  0.00347 **
## tsgdp.l5    -0.19959    0.04960  -4.024 7.84e-05 ***
## tspce.l5     0.10035    0.04343   2.311  0.02176 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 48.98 on 223 degrees of freedom
## Multiple R-Squared:     1,   Adjusted R-squared:     1
## F-statistic: 1.002e+06 on 8 and 223 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsexpgs:
## ========================================
## tsexpgs = tsexpgs.l1 + tspce.l1 + tsexpgs.l2 + tsgdp.l3 + tsgdp.l4 + tsexpgs.l4 + tsexpgs.l5 + tspce
##
##            Estimate Std. Error t value Pr(>|t|)
## tsexpgs.l1  1.72627    0.06164  28.007  < 2e-16 ***
## tspce.l1   -0.12331    0.02841  -4.341 2.15e-05 ***
## tsexpgs.l2 -0.96515    0.08544 -11.296  < 2e-16 ***
## tsgdp.l3    0.25437    0.04140   6.144 3.65e-09 ***
## tsgdp.l4   -0.26672    0.04246  -6.282 1.73e-09 ***
## tsexpgs.l4  0.35723    0.08145   4.386 1.78e-05 ***
## tsexpgs.l5 -0.18612    0.06050  -3.076  0.00236 **
## tspce.l5    0.15647    0.02385   6.562 3.67e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 22.68 on 223 degrees of freedom
## Multiple R-Squared: 0.9996,  Adjusted R-squared: 0.9995
## F-statistic: 6.355e+04 on 8 and 223 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsimgpsc:
## =========================================
## tsimgpsc = tsgdp.l1 + tsexpgs.l1 + tsimgpsc.l1 + tsgdp.l2 + tsexpgs.l2 + tsgdp.l3 + tsexpgs.l5 + tsi
##
##             Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1     0.19867    0.03629   5.475 1.18e-07 ***
## tsexpgs.l1   0.22489    0.06865   3.276 0.001223 **
## tsimgpsc.l1  1.06741    0.02565  41.608  < 2e-16 ***
```

```
## tsgdp.l2    -0.13684    0.04962  -2.758 0.006307 **
## tsexpgs.l2  -0.44954    0.07665  -5.865 1.62e-08 ***
## tsgdp.l3    -0.08633    0.03631  -2.378 0.018269 *
## tsexpgs.l5   0.19049    0.02780   6.853 7.06e-11 ***
## tsimgpsc.l5 -0.11256    0.02563  -4.391 1.75e-05 ***
## tspce.l5     0.05364    0.01361   3.940 0.000109 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 21.43 on 222 degrees of freedom
## Multiple R-Squared: 0.9998,  Adjusted R-squared: 0.9998
## F-statistic: 1.356e+05 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tsunrate:
## ============================================
## tsunrate = tsgdp.l1 + tsimgpsc.l1 + tsunrate.l1 + tsexpgs.l2 + tsimgpsc.l2 + tsgdp.l3 + tsunrate.l3 +
##
##               Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1    -0.0007571  0.0002819  -2.685 0.007798 **
## tsimgpsc.l1 -0.0019484  0.0007992  -2.438 0.015557 *
## tsunrate.l1  1.1992492  0.0355423  33.741  < 2e-16 ***
## tsexpgs.l2   0.0006757  0.0003121   2.165 0.031458 *
## tsimgpsc.l2  0.0018360  0.0008046   2.282 0.023441 *
## tsgdp.l3     0.0008106  0.0002910   2.785 0.005808 **
## tsunrate.l3 -0.2456528  0.0366934  -6.695 1.74e-10 ***
## tsexpgs.l5  -0.0008599  0.0003097  -2.776 0.005968 **
## const        0.3069313  0.0900744   3.408 0.000778 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2761 on 222 degrees of freedom
## Multiple R-Squared: 0.9981,  Adjusted R-squared: 0.998
## F-statistic: 1.312e+04 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation tspce:
## =========================================
## tspce = tsgdp.l1 + tsexpgs.l1 + tspce.l1 + tsexpgs.l2 + tsunrate.l2 + tsgdp.l3 + tspce.l3 + tsgdp.l4
##
##              Estimate Std. Error t value Pr(>|t|)
## tsgdp.l1     0.27884    0.04242   6.573 3.67e-10 ***
## tsexpgs.l1   0.44739    0.09232   4.846 2.41e-06 ***
## tspce.l1     0.65621    0.07249   9.053  < 2e-16 ***
## tsexpgs.l2  -0.56186    0.12317  -4.562 8.52e-06 ***
## tsunrate.l2  7.34145    2.89326   2.537 0.011876 *
## tsgdp.l3    -0.21083    0.06936  -3.040 0.002661 **
## tspce.l3     0.40929    0.08899   4.599 7.23e-06 ***
## tsgdp.l4     0.19481    0.07917   2.461 0.014658 *
## tsexpgs.l4  -0.31965    0.15624  -2.046 0.041984 *
## tsimgpsc.l4  0.36631    0.09195   3.984 9.29e-05 ***
## tspce.l4    -0.29218    0.09464  -3.087 0.002286 **
```

```
## tsgdp.l5    -0.22331    0.05907  -3.780 0.000203 ***
## tsexpgs.l5   0.39701    0.11695   3.395 0.000818 ***
## tsimgpsc.l5 -0.36098    0.09116  -3.960 0.000102 ***
## tsunrate.l5 -8.18484    2.93488  -2.789 0.005765 **
## tspce.l5     0.17905    0.07761   2.307 0.022011 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 28.63 on 215 degrees of freedom
## Multiple R-Squared:     1,   Adjusted R-squared:     1
## F-statistic: 6.556e+05 on 16 and 215 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##            tsgdp  tsexpgs tsimgpsc tsunrate    tspce
## tsgdp    2610.04 751.4730  457.001 -2.03992 908.7575
## tsexpgs   751.47 559.5201  207.619 -0.67771 360.6179
## tsimgpsc  457.00 207.6186  496.554 -1.18701 195.1553
## tsunrate   -2.04  -0.6777   -1.187  0.08256  -0.6988
## tspce     908.76 360.6179  195.155 -0.69881 859.3788
##
## Correlation matrix of residuals:
##            tsgdp  tsexpgs tsimgpsc tsunrate    tspce
## tsgdp     1.0000  0.62184   0.4014 -0.13897  0.60678
## tsexpgs   0.6218  1.00000   0.3939 -0.09972  0.52005
## tsimgpsc  0.4014  0.39389   1.0000 -0.18539  0.29875
## tsunrate -0.1390 -0.09972  -0.1854  1.00000 -0.08296
## tspce     0.6068  0.52005   0.2987 -0.08296  1.00000
```

Response Coefficient analysis for unrestricted BIC model selected indicates the following: * Lead relationship of Real imports of goods and services (IMPGSC1) are statistically significant on GDP along with the factors corresponding to lags of GDP.

Coefficient analysis for Stepwise LR model selected indicates the following: * All the economic factor coefficients are significant in linear regression model of GDP.

Coefficient analysis for restricted BIC model selected indicates the following: * Lead relation of IMPGSC1, EXPGS are stastically significant on GDP along with the factors corresponding to lags of GDP. Unemployment rate and PCE also shows some level of lead relationship on GDP but as as high as other two factors.

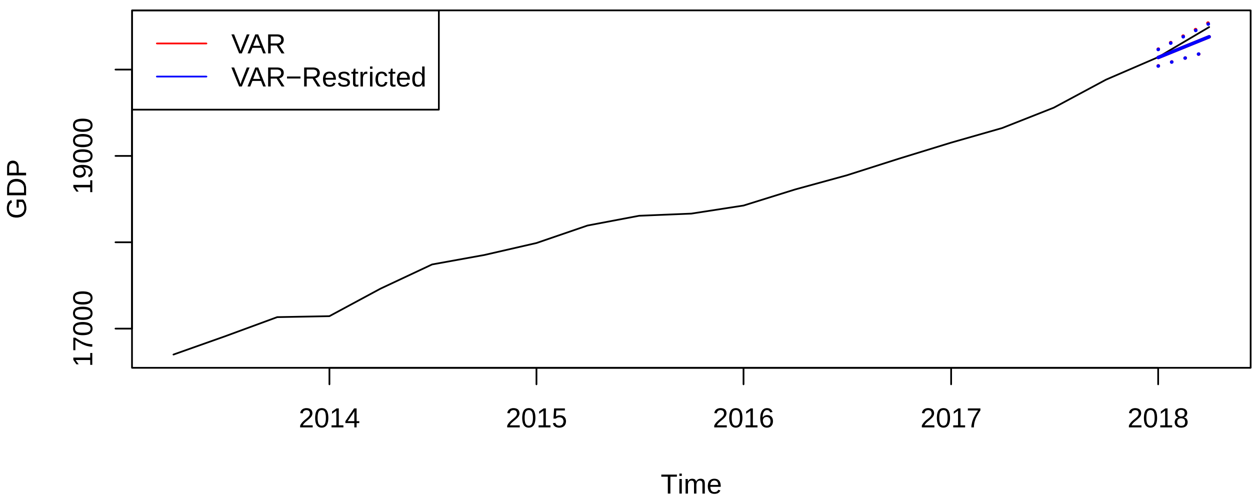
Restricted model is picking only the coefficients that are significant compared to that of the unrestricted model. Restricted model is easy to explain.

Question 3b

Forecast the first two quarters of 2018 using the unrestricted and restricted VAR models derived in (3a). Include 95% confidence intervals. Compare the predictions to the observed data using mean absolute percentage error and the precision measure for GDP. Compare the predictions to those derived in (2d). Comment on the accuracy of the predictions.

```r
n=nrow(data.train.3)+2
nfit=n-2
pred.model.bic.var3=predict(model_bic_var_3,n.ahead=2)
gdp.fcst.var3 = xts(pred.model.bic.var3[[1]]$tsgdp[,1], date.quarter[(n-1):n])
```

```r
gdp.fcst.lbound.var3 = xts(pred.model.bic.var3[[1]]$tsgdp[,2], date.quarter[(n-1):n])
gdp.fcst.ubound.var3 = xts(pred.model.bic.var3[[1]]$tsgdp[,3], date.quarter[(n-1):n])

pred.model.bic.var4=predict(model_bic_var_3_restrict,n.ahead=2)
gdp.fcst.var4 = xts(pred.model.bic.var4[[1]]$tsgdp[,1], date.quarter[(n-1):n])
gdp.fcst.lbound.var4 = xts(pred.model.bic.var4[[1]]$tsgdp[,2], date.quarter[(n-1):n])
gdp.fcst.ubound.var4 = xts(pred.model.bic.var4[[1]]$tsgdp[,3], date.quarter[(n-1):n])

ymin = min(c(gdp.ts[(n-20):n],gdp.fcst.lbound.var3))
ymax = max(c(gdp.ts[(n-20):n],gdp.fcst.ubound.var4))
plot(date.quarter[(n-20):n], gdp.ts[(n-20):n],type="l", ylim=c(ymin,ymax), xlab="Time", ylab="GDP")
lines(date.quarter[(nfit+1):n],gdp.fcst.var3,col="red",lwd=2)
lines(date.quarter[(nfit+1):n],gdp.fcst.var3,col="blue",lwd=2)
lines(date.quarter[(nfit+1):n],gdp.fcst.ubound.var3,lty=3,lwd= 2, col="red")
lines(date.quarter[(nfit+1):n],gdp.fcst.lbound.var3,lty=3,lwd= 2, col="red")
lines(date.quarter[(nfit+1):n],gdp.fcst.ubound.var4,lty=3,lwd= 2, col="blue")
lines(date.quarter[(nfit+1):n],gdp.fcst.lbound.var4,lty=3,lwd= 2, col="blue")
legend("topleft",legend=c("VAR", "VAR-Restricted"),col=c("red","blue"),lty=1)
```




```r
print("UnRestricted VAR Summary --------")
```

```
## [1] "UnRestricted VAR Summary --------"
```

```r
print("MAPE:")
```

```
## [1] "MAPE:"
```

```r
print(mean(abs(gdp.fcst.var3 - ts_gdp.3.test) / ts_gdp.3.test))
```

```
## [1] 0.002849173
```

```r
print("PM:")
```

```
## [1] "PM:"
```

```r
print(sum((gdp.fcst.var3 - ts_gdp.3.test)^2) / sum((gdp.fcst.var3 - mean(gdp.fcst.var3))^2))
```

```
## [1] 0.4407157
```

```r
print("Does the observed data fall outside the prediction intervals?")
```

```
## [1] "Does the observed data fall outside the prediction intervals?"
```

```r
print(sum(ts_gdp.3.test < gdp.fcst.lbound.var3) & sum(ts_gdp.3.test > gdp.fcst.ubound.var3))
```

```
## [1] FALSE
```

```r
print("Restricted VAR Summary --------")
```

```
## [1] "Restricted VAR Summary --------"
```

```r
print("MAPE:")
```

```
## [1] "MAPE:"
```

```r
print(mean(abs(gdp.fcst.var4 - ts_gdp.3.test) / ts_gdp.3.test))
```

```
## [1] 0.003066979
```

```r
print("PM:")
```

```
## [1] "PM:"
```

```r
print(sum((gdp.fcst.var4 - ts_gdp.3.test)^2) / sum((gdp.fcst.var4 - mean(gdp.fcst.var4))^2))
```

```
## [1] 0.4925487
```

```r
print("Does the observed data fall outside the prediction intervals?")
```

```
## [1] "Does the observed data fall outside the prediction intervals?"
```

```r
print(sum(ts_gdp.3.test < gdp.fcst.lbound.var4) & sum(ts_gdp.3.test > gdp.fcst.ubound.var4))
```

```
## [1] FALSE
```

Response Both un-restricted and restricted VAR produced the similar results and the predictions & observations are within the lower and upper bands. MAPE and PM are providing better results for 2018 Q1 and Q2. This is totally different from that of the predictions for 2020 Q1 and Q2.
Question 3c

Perform a Granger Causality analysis using Wald test to evaluate whether any of the economic indicators lead GDP. Would any of the indicators help in predicting or explaining GDP for next quarters? Provide your interpretation based on the Granger causality as well as for forecasting comparison in (3b). Compare this analysis with the findings in (2e). For this question, use the unrestricted VAR model from Question 3a.

```r
### Estimated coefficients and their variance for gdp regression Equation
coef.tsgdp.c = coefficients(model_bic_var_3)$tsgdp[-c((5*5+1),(5*5+2)) ,1]
var.tsgdp.c = vcov(model_bic_var_3)[c(2:(5*5+1)),c(2:(5*5+1))]
library(aod)
# Is there a lead-lag relationship for gdp-other economic factors
tsgdpnames=c("tsgdp:tsgdp.l1", "tsgdp:tsgdp.l2","tsgdp:tsgdp.l3", "tsgdp:tsgdp.l4", "tsgdp:tsgdp.l5" )

tsnongdpvars<-which(!colnames(var.tsgdp.c)%in%tsgdpnames)
wald.test(b=coef.tsgdp.c, var.tsgdp.c,Terms = tsnongdpvars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 65.6, df = 20, P(> X2) = 9.2e-07
```

```r
# Is there a lead-lag relationship for gdp-expgs economic factors
tsgdpexpgs<-c("tsgdp:tsexpgs.l1", "tsgdp:tsexpgs.l2","tsgdp:tsexpgs.l3", "tsgdp:tsexpgs.l4", "tsgdp:tse
```

```r
tsgdpexpgsvars<-which(colnames(var.tsgdp.c)%in%tsgdpexpgs)
wald.test(b=coef.tsgdp.c, var.tsgdp.c,Terms = tsgdpexpgsvars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 16.1, df = 5, P(> X2) = 0.0065
```

```r
# Is there a lead-lag relationship for gdp-imgpsc economic factors
tsgdpimgpsc<-c("tsgdp:tsimgpsc.l1", "tsgdp:tsimgpsc.l2","tsgdp:tsimgpsc.l3", "tsgdp:tsimgpsc.l4", "tsgd
tsgdpimgpscvars<-which(colnames(var.tsgdp.c)%in%tsgdpimgpsc)
wald.test(b=coef.tsgdp.c, var.tsgdp.c,Terms = tsgdpimgpscvars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 23.3, df = 5, P(> X2) = 0.00029
```

```r
# Is there a lead-lag relationship for gdp-unrate economic factors
tsgdpunrate<-c("tsgdp:tsunrate.l1", "tsgdp:tsunrate.l2","tsgdp:tsunrate.l3", "tsgdp:tsunrate.l4", "tsgd
tsgdpunratevars<-which(colnames(var.tsgdp.c)%in%tsgdpunrate)
wald.test(b=coef.tsgdp.c, var.tsgdp.c,Terms = tsgdpunratevars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 3.4, df = 5, P(> X2) = 0.65
```

```r
# Is there a lead-lag relationship for gdp-pce economic factors
tsgdppce<-c("tsgdp:tspce.l1", "tsgdp:tspce.l2","tsgdp:tspce.l3", "tsgdp:tspce.l4", "tsgdp:tspce.l5" )
tsgdppcevars<-which(colnames(var.tsgdp.c)%in%tsgdppce)
wald.test(b=coef.tsgdp.c, var.tsgdp.c,Terms = tsgdppcevars)
```

```
## Wald test:
## ----------
##
## Chi-squared test:
## X2 = 7.2, df = 5, P(> X2) = 0.2
```

Response In this analysis it shows that GDP has a lead-lag relation with EXPGS and IMGPSC based on the wald test results that have a p-value less than 0.05. For the Unemployment rate and PCE the p-values are higher and they don't show any Granger causality on GDP.

Question 4: Reflection

From what you encountered above and your conceptual understanding of VAR modelling, reflect on the relative strengths and weaknesses of the modelling approach. Particularly, you will need to put this analysis into the perspective of the results you found and any relevant economic events you might be potentially able to link them to.

Response VAR modeling is extremely useful in understanding the endogeneous as well as exogeneous factors and explain the relation. In this exercise, we developed multiple models and each of the VAR model either restricted or unrestricted was able to provide the significance levels of the coefficients of the corresponding economic factors on the GDP. The forecasts seems to work well unless there is a big shock to the environment.

The Covid situation is a better example towards the weakness of the VAR models. They are not able to capture the volatility well. This is where VEC (Vector Error Correction) models will help.