Time Series Analysis

ARMA Models

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ACF and PACF: AR & MA Simulation

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About This Lesson

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Moving Average: Stationary Process

```
## Simulate White Noise
w1 = rnorm(502)
w2 = rexp(502)-1
## Set coefficients
a = c(1,-.5,.2)
a1 = c(1,.5,.2)
## Simulate MA(2) with Normal/Exp WN
ma2.11 = filter(w1,filter=a,side=1)
ma2.11 = ma2.11[3:502]
ma2.12 = filter(w1,filter=a1,side=1)
ma2.12 = ma2.12[3:502]
ma2.21 = filter(w2,filter=a,side=1)
ma2.21 = ma2.21[3:502]
ma2.22 = filter(w2,filter=a1,side=1)
ma2.22 = ma2.22[3:502]
```

$$X_t = Z_t - 0.5\,Z_{t-1} + 0.2\,Z_{t-2}$$
$$X_t = Z_t + 0.5\,Z_{t-1} + 0.2\,Z_{t-2}$$

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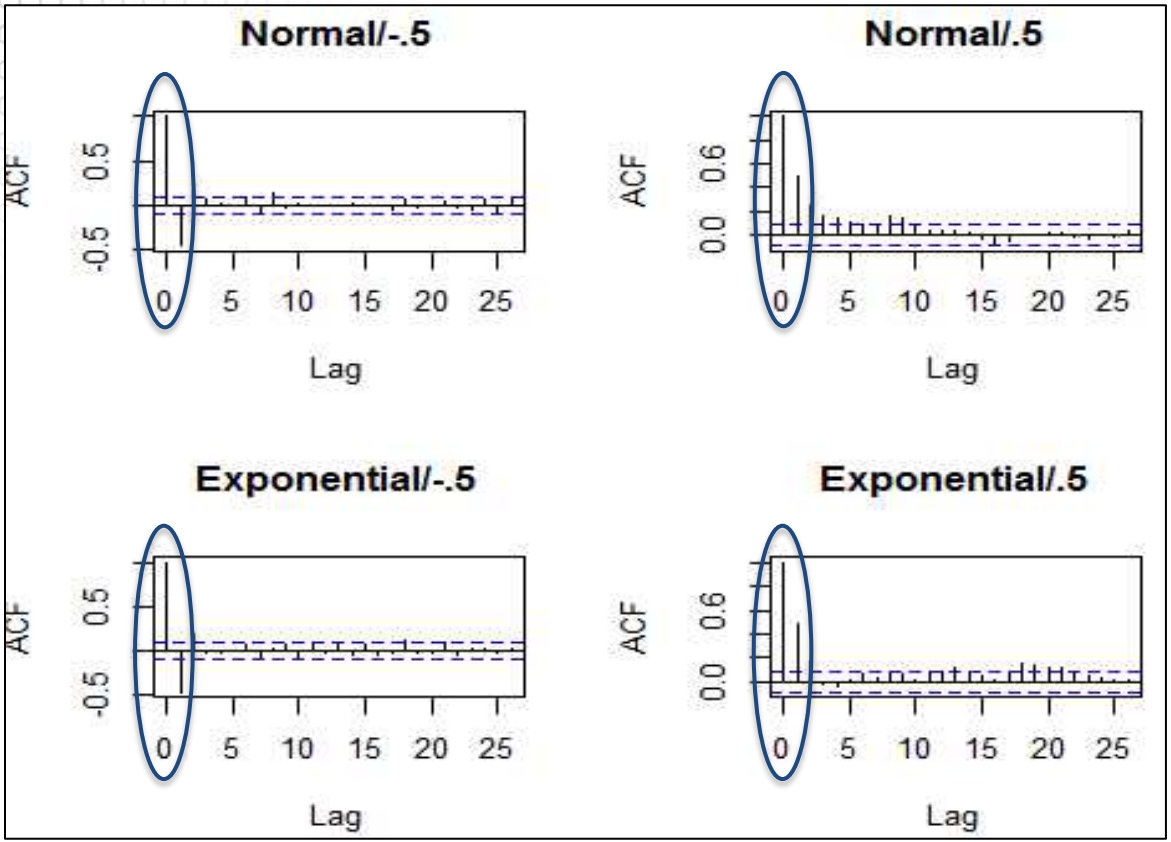
Moving Average: Stationary Process



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Moving Average: Non-Stationary Noise

```
## Simulate White Noise
w1 = rnorm(502)
## Set coefficients
a4 = c(1,.2,.8,1.2)
## Simulate MA(3) with non-stationary noise
ma2.4 = filter(w1*(2*(1:502)+0.5),filter=a4,side=1)
ma2.4 = ma2.4[4:502]
```

$X_t = Z_t + 0.2\ Z_{t-1} + 0.8\ Z_{t-2} + 1.2\ Z_{t-3}$ with
$Z_t = \mathrm{WN}(0,1) * 2t + 0.5$

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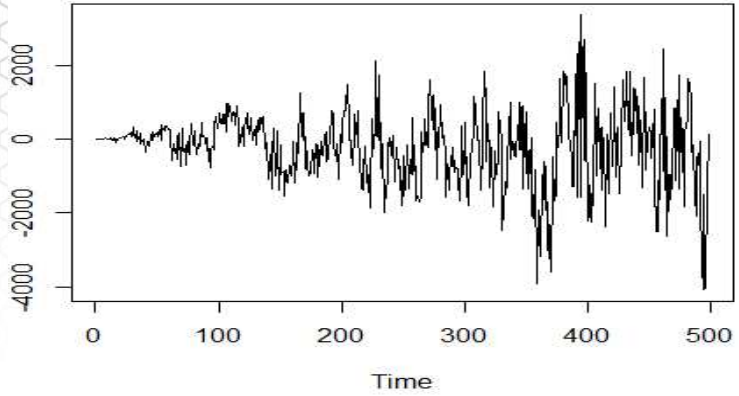
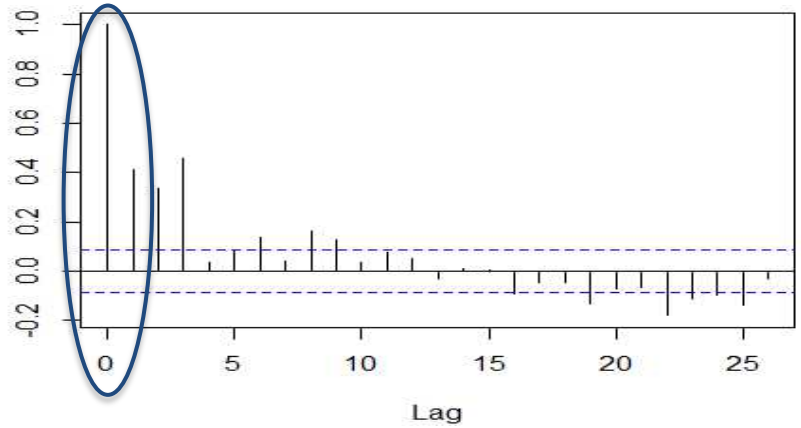
Moving Average: Non-Stationary Noise





$X_t = Z_t + 0.2\ Z_{t-1} + 0.8\ Z_{t-2} + 1.2\ Z_{t-3}$ with
$Z_t = \mathrm{WN}(0,1)*2t+0.5$

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Autoregressive Process

```
w2 = rnorm(1500)
## Nonstationary AR(2)
a2 = c(0.8,0.2)
ar2.n = filter(w2,filter=a2,method='recursive')
ar2.n = ar2.n[1251:1500]
## Stationary AR(2) process
a2 = c(1.8,-0.9)
ar2.s = filter(w2,filter=a2,method='recursive')
ar2.s = ar2.s[1251:1500]
## Compare PACF
par(mfrow=c(2,2))
ts.plot(ar2.n,main='Nonstationary AR(2)',ylab="",xlab="")
ts.plot(ar2.s,main='Stationary AR(2)',ylab="",xlab="")
pacf(ar2.n,main = '')
pacf(ar2.s,main = '')
```

$$X_t = 0.8\,X_{t-1} + 0.2\,X_{t-2} + Z_t$$

$$X_t = 1.8\,X_{t-1} - 0.9\,X_{t-2} + Z_t$$

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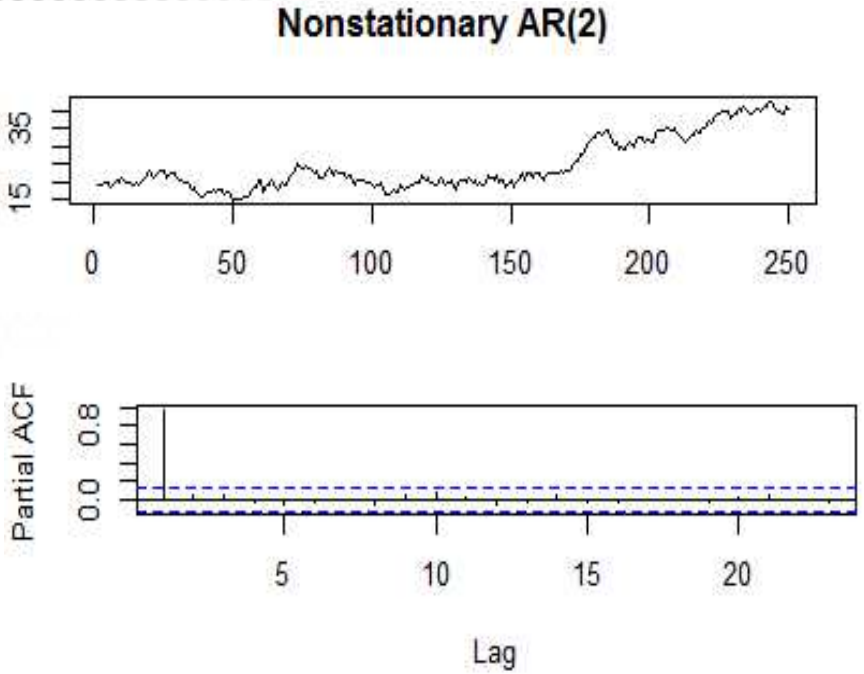
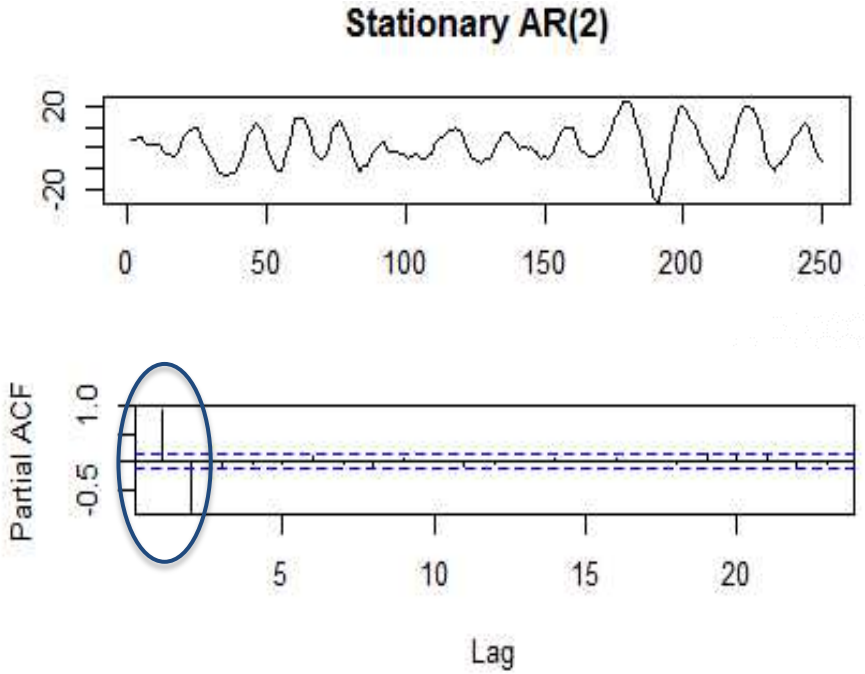
Autoregressive Process





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ARMA Process

Stationary ARMA(2,2)

arma22 = arima.sim(n = 500, list(ar = c(0.88, -0.49), ma = c(-0.23, 0.25)), sd = sqrt(0.18))

$$X_t = 0.88\,X_{t-1} - 0.49\,X_{t-2} - 0.23\,Z_{t-1} + 0.25\,Z_{t-2} + Z_t,\ Z_t \sim WN(0, 0.18)$$

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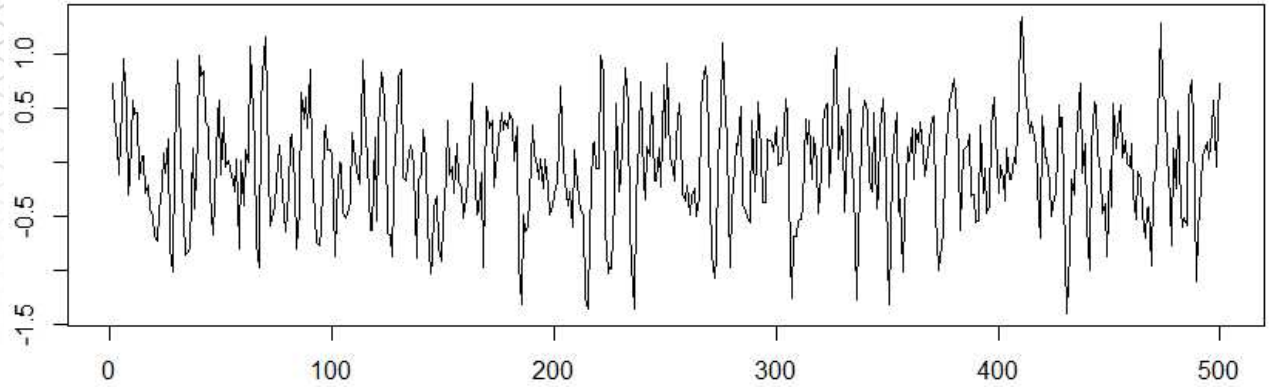
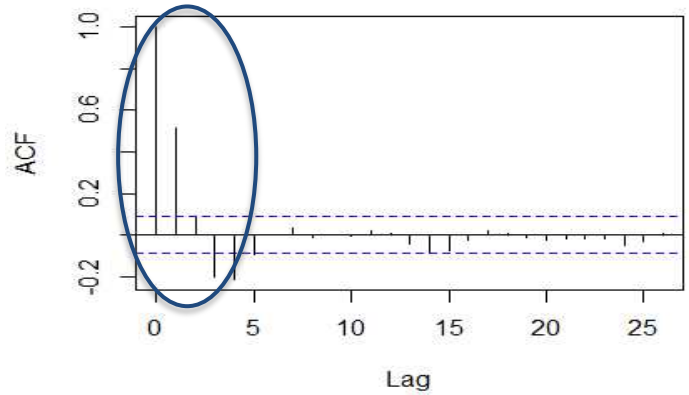
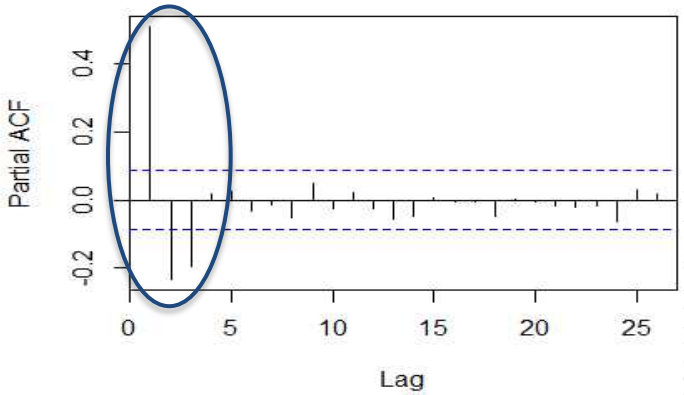
ARMA Process







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Summary

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