Time Series Analysis

Modeling Heteroskedasticity

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ARCH Model: Data Example

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About This Lesson

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PDC Energy, Inc (PDCE)

Summary:

- Crude oil and natural gas producer with headquartered in Denver, Colorado
- PDC's portfolio is comprised of the Wattenberg Field in Colorado, the Delaware Basin in West Texas and the Utica Shale in Ohio

Time Series Data:

- Daily stock price for more than 12 years of data starting with January 2007
- Largely dependent on the crude oil price

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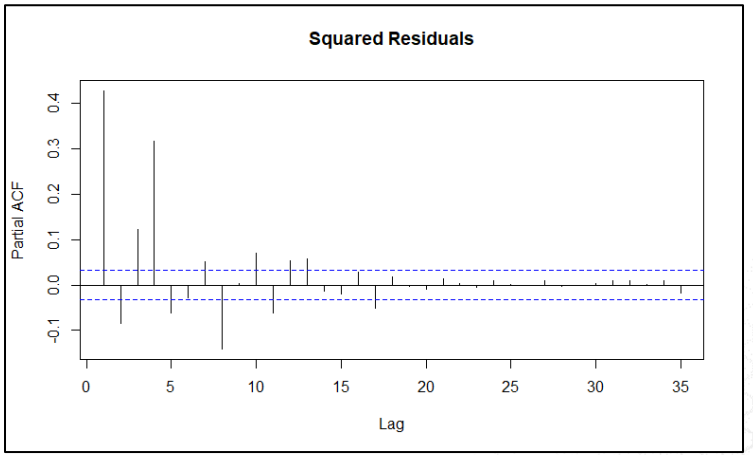
ARCH Fit

```
## garch from tseries library
library(tseries)
## What order?
pacf(resids^2, ,main="Squared Residuals")
garch.fit = garch(resids, order = c(0, 8)
,trace=F)
summary(garch.fit)
## Evaluate goodness of fit
resids.fgarch = residuals(garch.fit)[-c(1:8)]
acf(resids.fgarch,main="ACF of ARCH Residuals")
acf(resids.fgarch^2,main="ACF of Squared ARCH Residuals")
```



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ARCH Fit: Summary

```
Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 0.0003297   0.0000212   15.547  < 2e-16 ***
a1 0.2156195   0.0135533   15.909  < 2e-16 ***
a2 0.1452874   0.0148443    9.787  < 2e-16 ***
a3 0.0666798   0.0147351    4.525 6.03e-06 ***
a4 0.1226002   0.0147280    8.324  < 2e-16 ***
a5 0.0486741   0.0123270    3.949 7.86e-05 ***
a6 0.1433676   0.0110352   12.992  < 2e-16 ***
a7 0.0433621   0.0132301    3.278  0.00105 **
a8 0.0723855   0.0148555    4.873 1.10e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 1041.7, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.17626, df = 1, p-value = 0.6746
```

All coefficients are statistically significant

Reject the null of uncorrelated residuals but do not reject for the squared residuals

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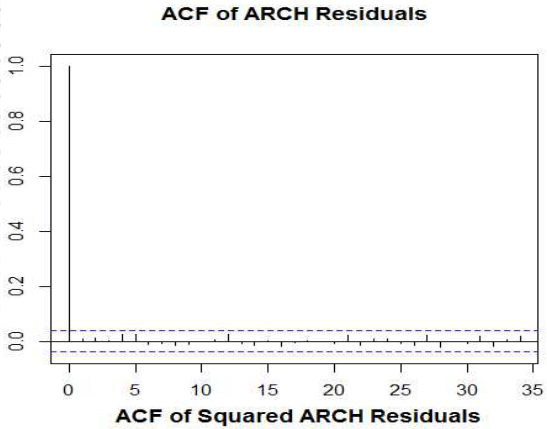
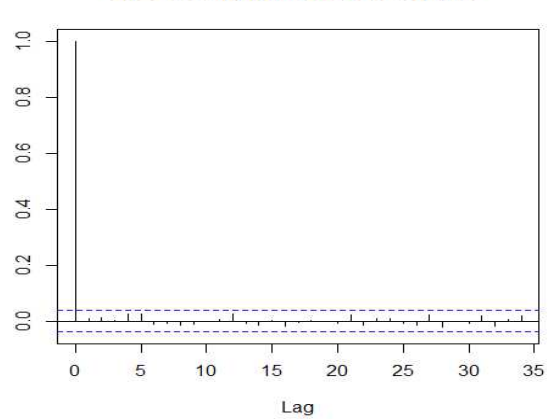
ARCH Fit: Residual Analysis



ACF of White Noise



ACF of White Noise

ARCH Fit: Different Implementations

```
## garchFit from the fGarch library
library(fGarch)
## Fit ARCH on the ARMA residuals
archFit.resid = garchFit(~ garch(8,0), data = resids, trace = FALSE)
summary(archFit.resid)
## Fit ARMA-ARCH
archFit.ts = garchFit(~ arma(4,4)+ garch(8,0), data=pdcert[-1], trace = FALSE)
summary(archFit.ts)
```

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ARCH Fit: Comparison

tseries – garch()

	Estimate	Std. Error	t value	Pr(>|t|)
a0	0.0003297	0.0000212	15.547	< 2e-16
a1	0.2156195	0.0135533	15.909	< 2e-16
a2	0.1452874	0.0148443	9.787	< 2e-16
a3	0.0666798	0.0147351	4.525	6.03e-06
a4	0.1226002	0.0147280	8.324	< 2e-16
a5	0.0486741	0.0123270	3.949	7.86e-05
a6	0.1433676	0.0110352	12.992	< 2e-16
a7	0.0433621	0.0132301	3.278	0.00105
a8	0.0723855	0.0148555	4.873	1.10e-06

fGarch – garchFit()

	Estimate	Std. Error	t value	Pr(>|t|)
mu	2.637e-04	4.938e-04	0.534	0.593342
omega	3.313e-04	2.741e-05	12.086	< 2e-16
alpha1	2.145e-01	2.816e-02	7.615	2.62e-14
alpha2	1.441e-01	2.595e-02	5.551	2.84e-08
alpha3	6.775e-02	2.191e-02	3.092	0.001987
alpha4	1.210e-01	2.173e-02	5.568	2.58e-08
alpha5	4.789e-02	2.038e-02	2.350	0.018750
alpha6	1.422e-01	2.185e-02	6.506	7.73e-11
alpha7	4.292e-02	1.841e-02	2.331	0.019763
alpha8	7.240e-02	2.103e-02	3.443	0.000575

$$\sigma_t{}^2 = 0.0003 + 0.215\, Z_{t-1}^2 + 0.145\, Z_{t-2}^2 + 0.066\, Z_{t-3}^2 + 0.122\, Z_{t-4}^2 + 0.048\, Z_{t-5}^2 + 0.143\, Z_{t-6}^2 + 0.043\, Z_{t-7}^2 + 0.072\, Z_{t-8}^2$$

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ARMA-ARCH Fit: Summary

	Estimate	Std. Error	t value	Pr(>|t|)
mu	3.527e-04	3.542e-04	0.996	0.319284
ar1	7.485e-01	1.834e-01	4.082	4.46e-05
ar2	3.890e-01	1.508e-01	2.579	0.009908
ar3	-8.756e-01	1.373e-01	-6.379	1.78e-10
ar4	3.798e-02	1.666e-01	0.228	0.819645
ma1	-7.120e-01	1.864e-01	-3.821	0.000133
ma2	-4.060e-01	1.540e-01	-2.636	0.008401
ma3	8.307e-01	1.416e-01	5.865	4.50e-09
ma4	6.772e-03	1.681e-01	0.040	0.967871
omega	3.312e-04	2.705e-05	12.243	< 2e-16
alpha1	2.148e-01	2.833e-02	7.583	3.40e-14
alpha2	1.373e-01	2.517e-02	5.456	4.88e-08
alpha3	6.277e-02	2.217e-02	2.831	0.004634
alpha4	1.243e-01	2.207e-02	5.635	1.75e-08
alpha5	4.022e-02	1.969e-02	2.043	0.041055
alpha6	1.422e-01	2.245e-02	6.336	2.35e-10
alpha7	4.419e-02	1.857e-02	2.379	0.017362
alpha8	8.344e-02	2.214e-02	3.769	0.000164

$$Y_t = 0.0004 + 0.748\,Y_{t-1} + 0.389\,Y_{t-2} - 0.875\,Y_{t-3} + 0.037\,Y_{t-4} + Z_t - 0.712\,Z_{t-1} - 0.405\,Z_{t-2} + 0.830\,Z_{t-3} + 0.006\,Z_{t-4}$$

The estimated ARMA coefficients from the ARMA-ARCH fit are different from the estimated coefficients from ARMA fit alone

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ARMA-ARCH Fit: Summary

	Estimate	Std. Error	t value	Pr(>|t|)
mu	3.527e-04	3.542e-04	0.996	0.319284
ar1	7.485e-01	1.834e-01	4.082	4.46e-05
ar2	3.890e-01	1.508e-01	2.579	0.009908
ar3	-8.756e-01	1.373e-01	-6.379	1.78e-10
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ma4	6.772e-03	1.681e-01	0.040	0.967871
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alpha3	6.277e-02	2.217e-02	2.831	0.004634
alpha4	1.243e-01	2.207e-02	5.635	1.75e-08
alpha5	4.022e-02	1.969e-02	2.043	0.041055
alpha6	1.422e-01	2.245e-02	6.336	2.35e-10
alpha7	4.419e-02	1.857e-02	2.379	0.017362
alpha8	8.344e-02	2.214e-02	3.769	0.000164

$$Z_t = \sigma_t R_t$$

$$\sigma_t{}^2 = 0.0003 + 0.214\, Z_{t-1}^2 + 0.137\, Z_{t-2}^2 + 0.062\, Z_{t-3}^2 + 0.124\, Z_{t-4}^2 + 0.040\, Z_{t-5}^2 + 0.142\, Z_{t-6}^2 + 0.044\, Z_{t-7}^2 + 0.083\, Z_{t-8}^2$$

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Summary

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