Computing through Simplicity: **Towards a Theory of Dynamics**

Emanuele Natale

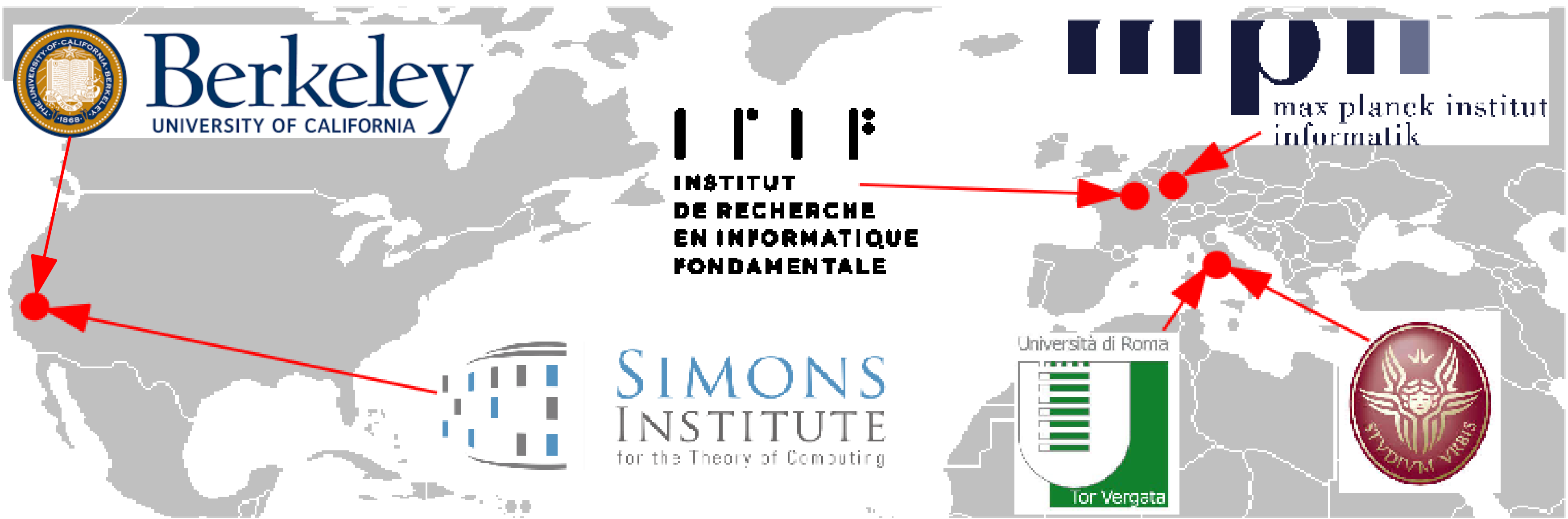
max planck institut informatik

Simons Institute for the Theory of Computing

Berkeley University of California

Academic Path

- 2017 - PhD in CS, Sapienza University
- 2014/15 (3 months)- Visiting IRIF
- 2016 & 2018-now (1 year) - **Fellow** of Simons Institute for the Theory of Computing (Brain Program)
- 2017-now - PostDoc, Max-Planck Institute for Informatics



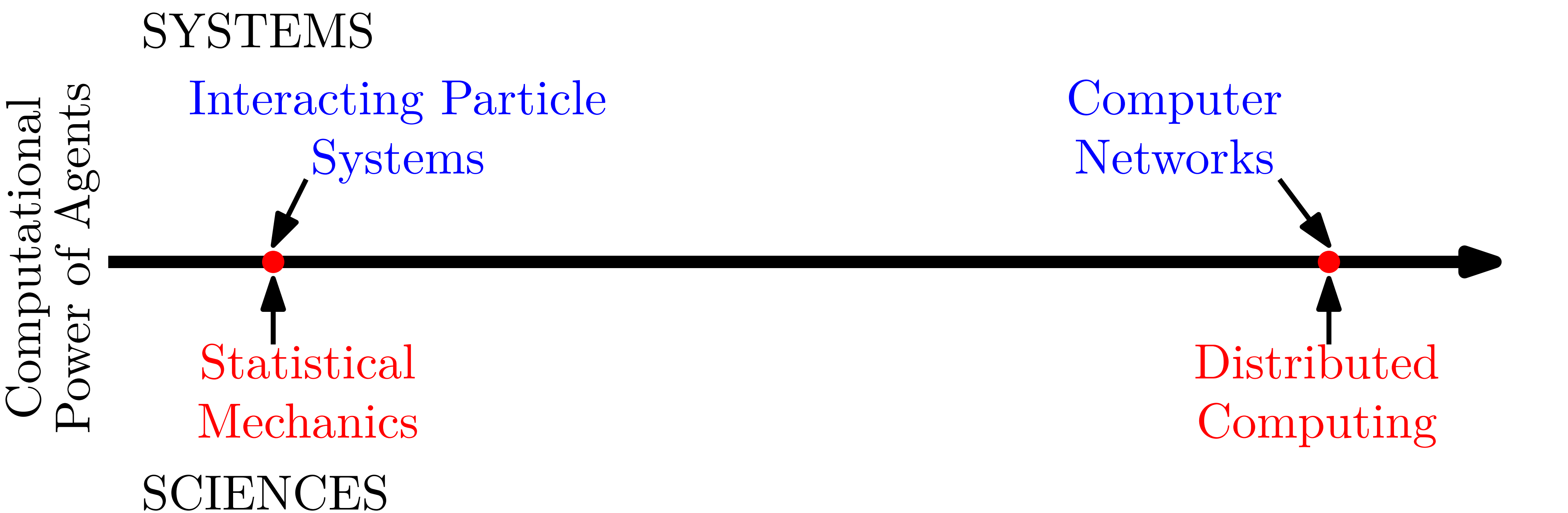
Awards

- ESA'16 **Best Student Paper**
- EATCS **Best Italian PhD Thesis** (Springer monograph in preparation)
- Sapienza 2016 **Outstanding PhD**
- Sapienza 2015 **Best CS Paper**

Invited speaker

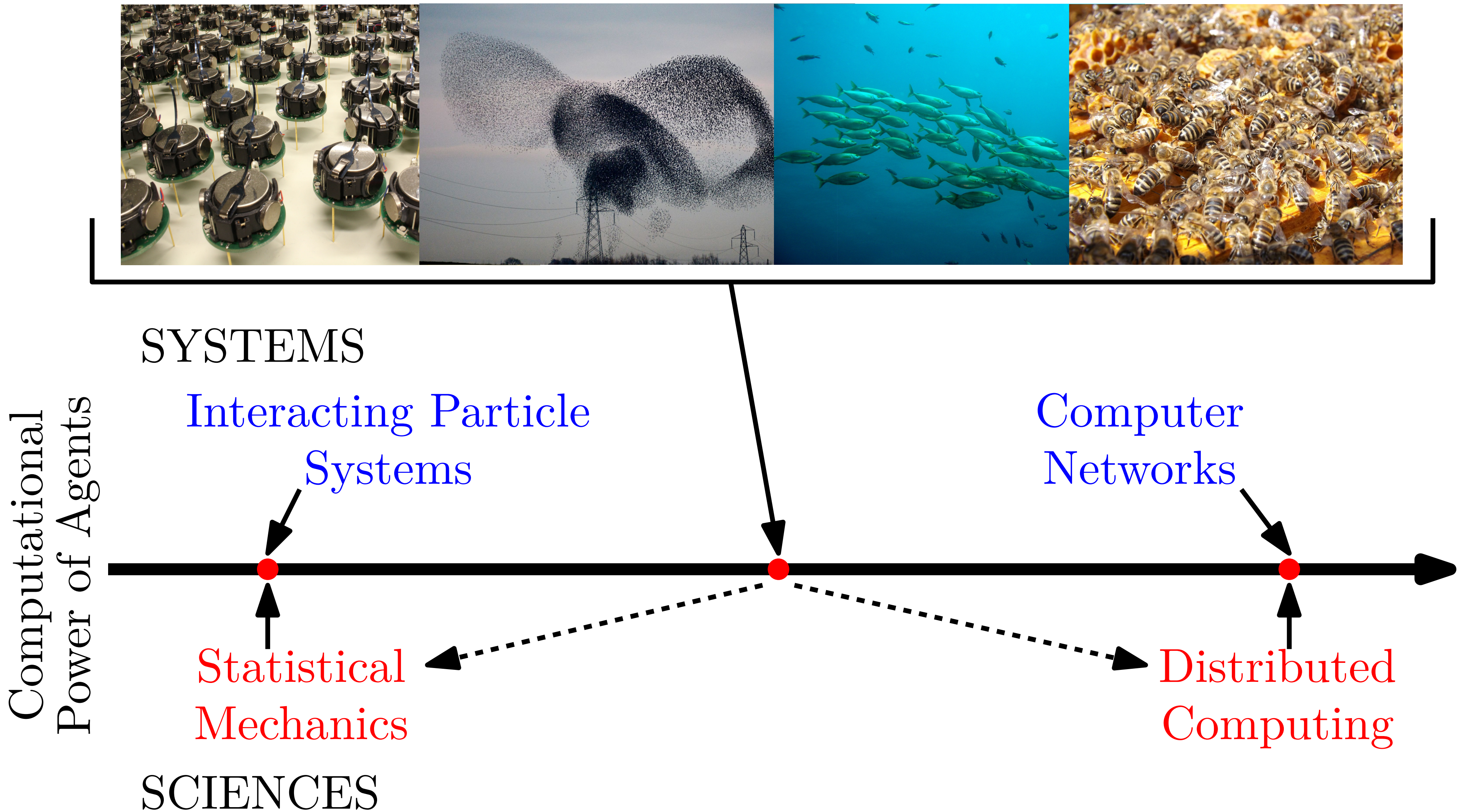
- ICTCS 2017
- Workshop on Random Processes in Discrete Structures, University of Warwick 2016

What can *Locally-Simple* Systems *Compute*?



What can *Locally-Simple* Systems *Compute*?

A **computational lens** on how
global behavior emerges from
simple local interactions among individuals



The Vision: A Theory of Dynamics

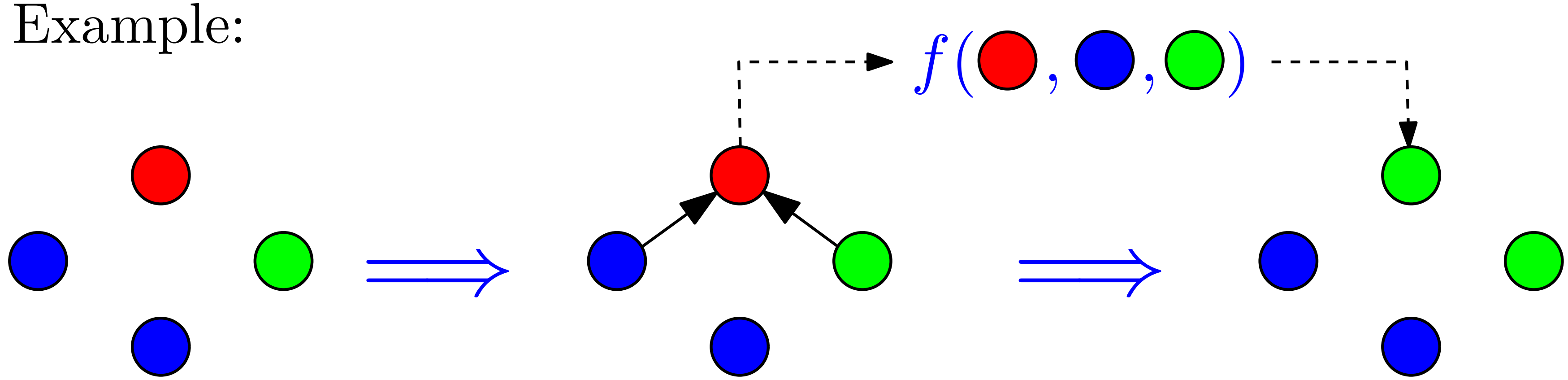
Dynamics [PhD Thesis]

Small set S and function f, agents have state $s \in S$ and

$$s^{new} = f(s^{old}, s^{\text{neighbor } u_1}, s^{\text{neighbor } u_2}, \ldots)$$

$\{u_1, u_2, \ldots\}$ typically random.

Example:



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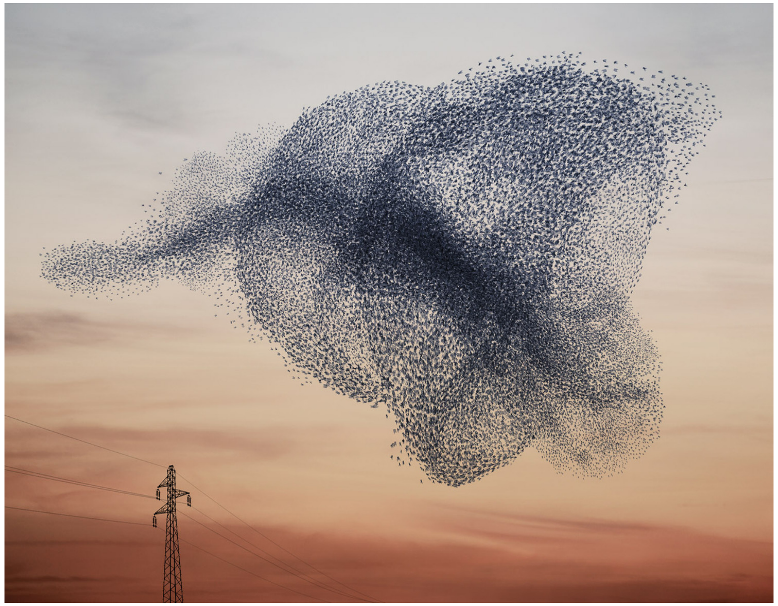
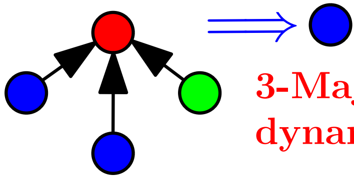
Wide applications in

- Collective behavior
- Neural networks (*Simons Institute, UC Berkeley*)
- Evolutionary dynamics (ecology)
- Many technological settings (dynamic networks)



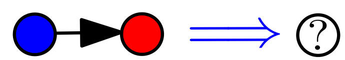
Towards the Project: Previous Work on Dynamics

Solve **majority consensus** **SPAA'14**



3-Majority dynamics

Solving majority consensus in $\mathcal{O}$(**monochromatic distance**) **SODA'15**

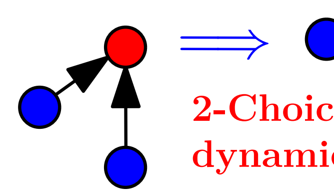


Undecided-State dynamics

General **noisy rumor-spreading** **PODC'16**

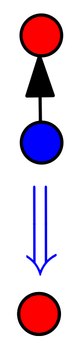
General **stabilizing consensus** **SODA'16**

Use dynamics for **clock synchronization** **SODA'17**



2-Choice dynamics

Comparative techniques: use results on one dynamics to understand other dynamics **PODC'17**



Voter dynamics

Relation between social-network structure in graph and evolution of dynamics **AAMAS'18**

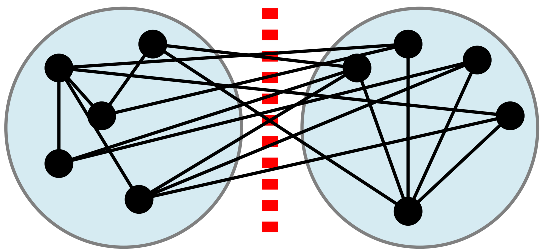
A Representative Result

Dynamics are shown
to solve difficult problems
in *distributed* settings

**Can we use dynamics
to solve difficult problems
even in *classical* settings?**

Start of Technical Part: Clustering

Minimum Bisection Problem
Find bipartition that minimizes cut
...NP-Complete!



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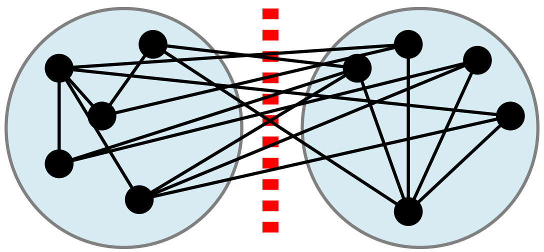

Minimum Bisection Problem
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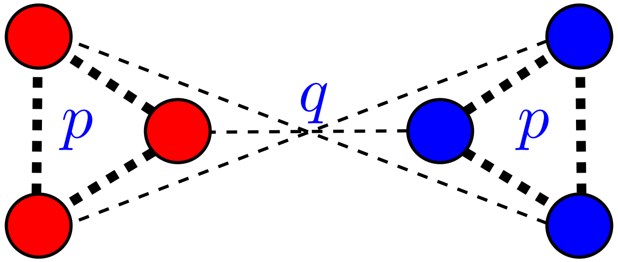
Stochastic Block Model



- "Communities" V_1, V_2, with $|V_1| = |V_2|$
- include each edge with probability
  - p if edge is inside V_1, V_2
  - q if edge is between V_1 and V_2

Start of Technical Part: Clustering

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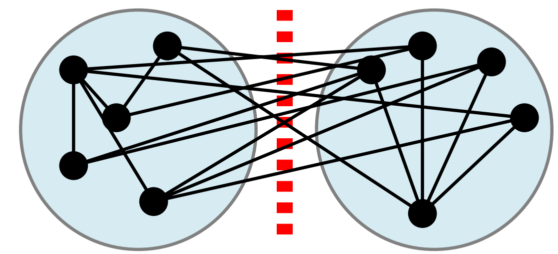

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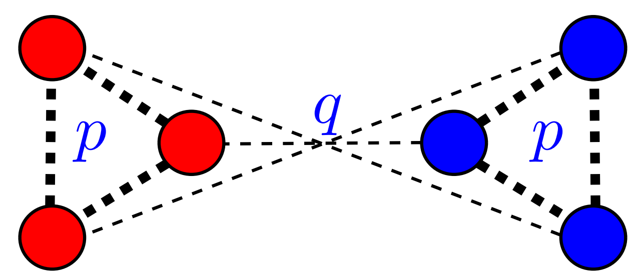
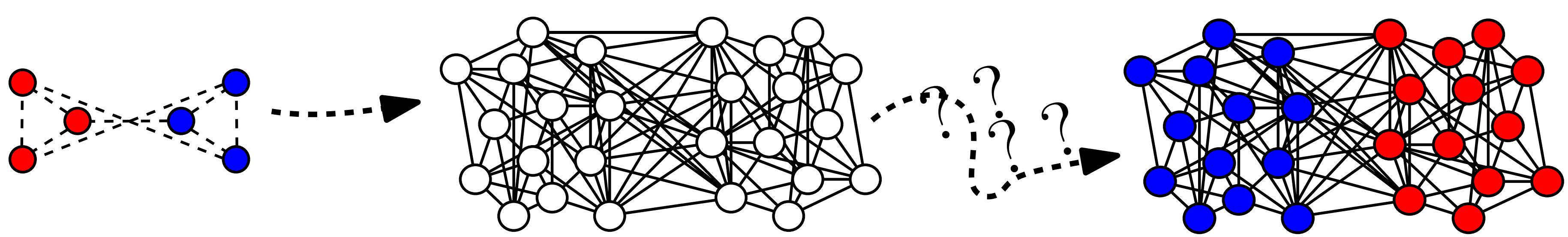


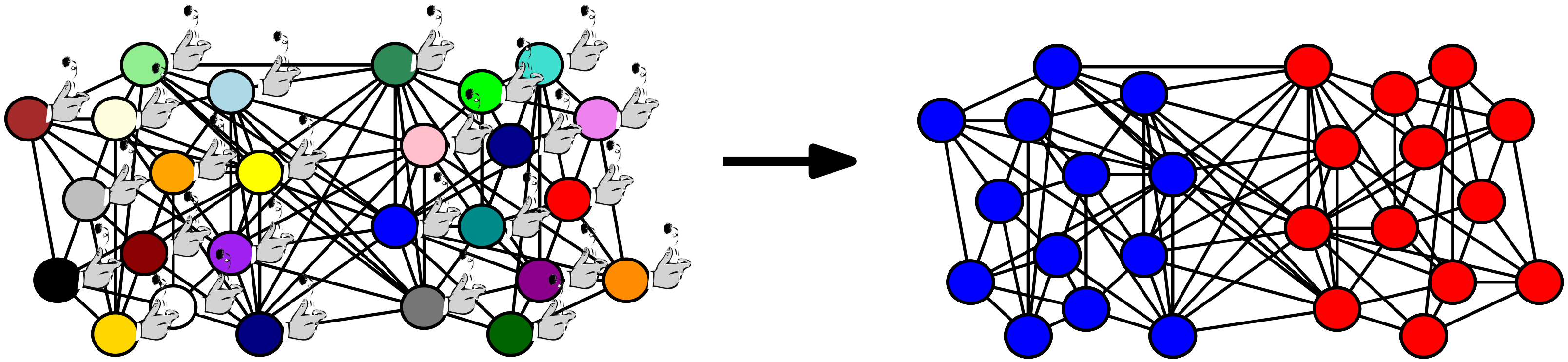
Clustering problem
Given graph generated by SBM, find original clusters



Known: clustering possible **if and only if** p and q in a precise regime

Clustering via **Averaging** [SODA'17]

Label **P**ropagation **A**lgorithms. Widely used heuristics in Data Mining: Each node initially gets random label, then updates label with simple rule of neighbors' states



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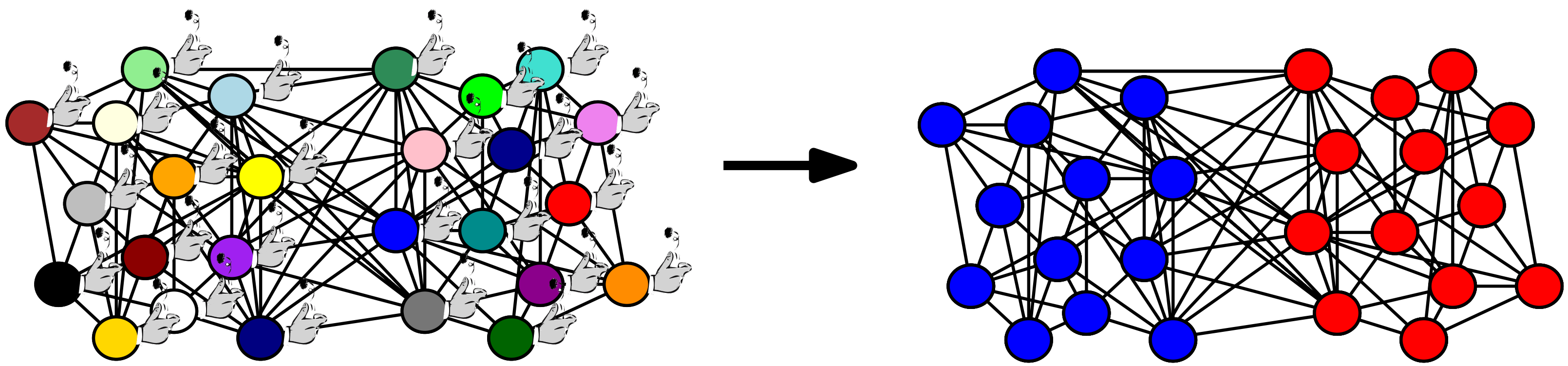


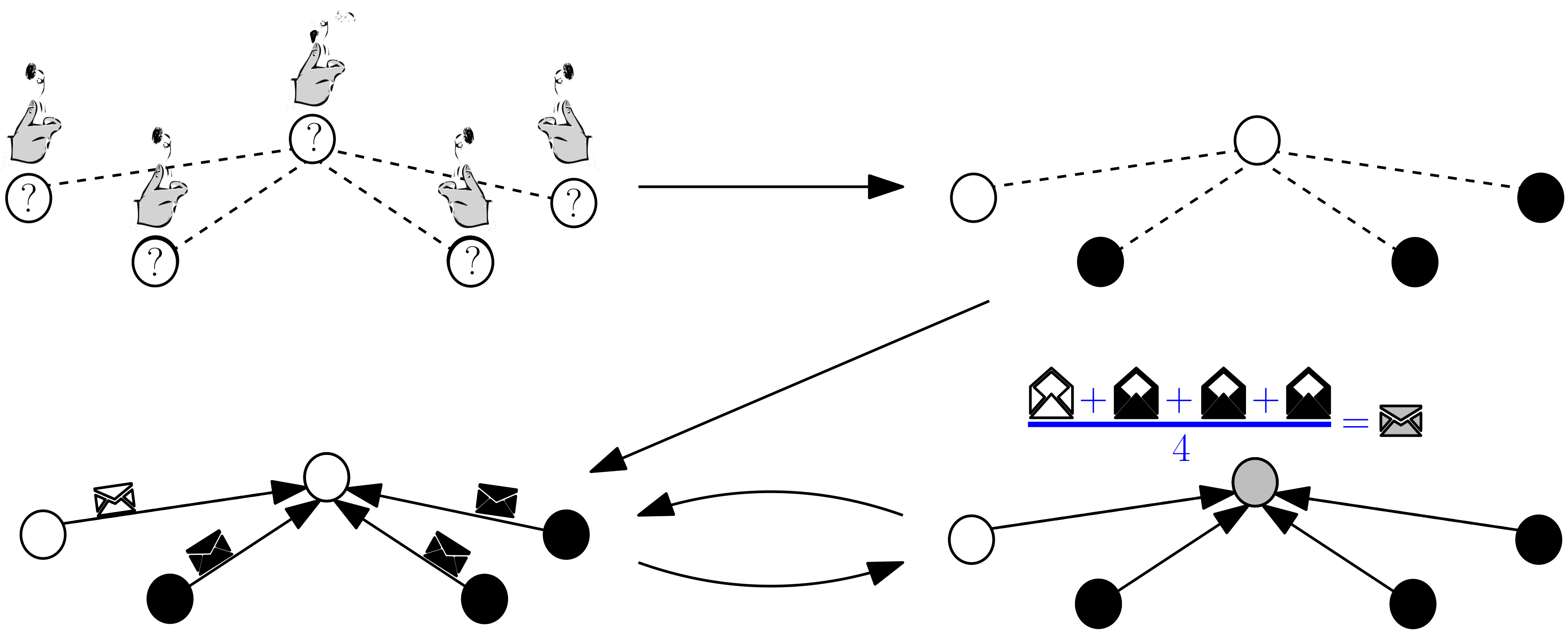
Theorem. There exists a dynamics that, in Stochastic Block Model with $p - q > \sqrt{const(p+q)/n} + \mathcal{O}(\log n/n)$, finds clusters with high probability after $\mathcal{O}(\frac{\log n}{\log \lambda_2/\lambda_3})$ steps, with $\mathcal{O}(\frac{p+q}{p-q})$ errors

- Improve state of the art in efficient distributed clustering
- Simulations show good performance in practice

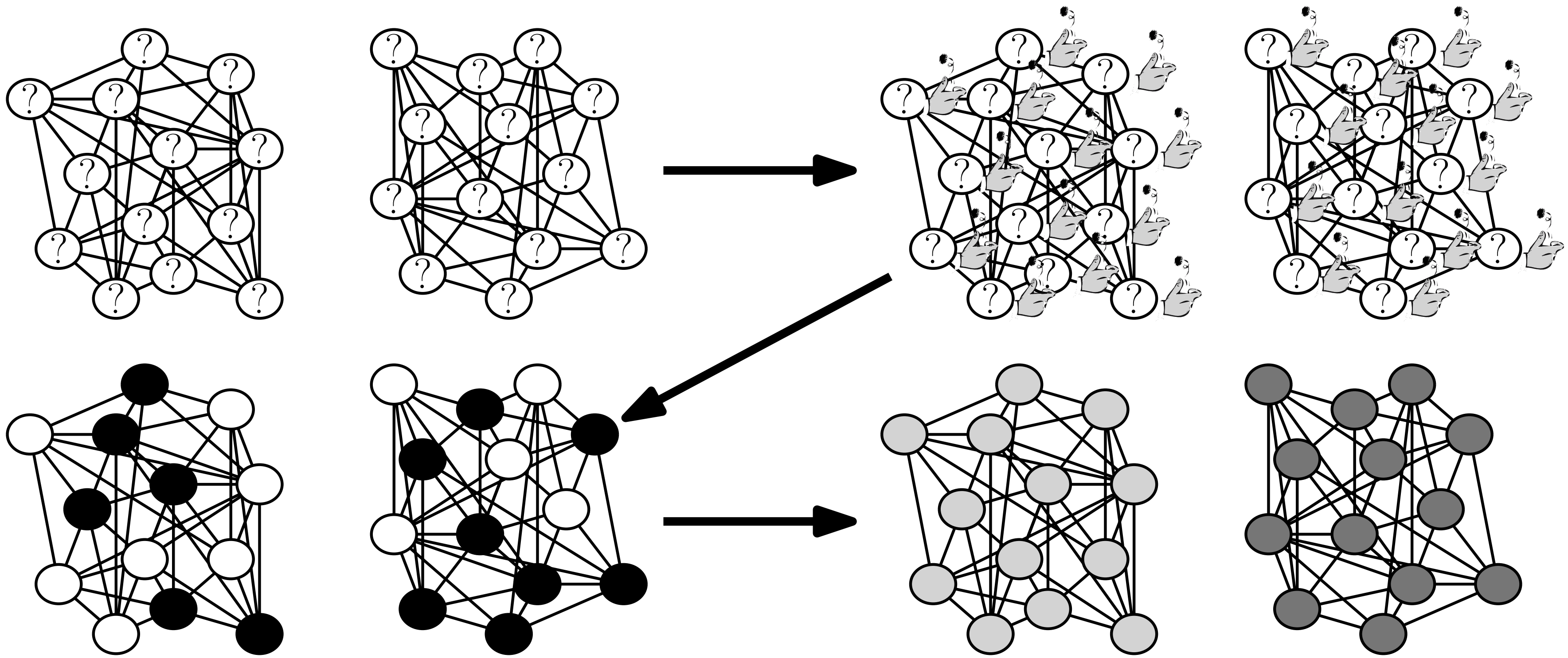
Averaging Dynamics [SODA'17]

All nodes at the same time:

- At $t = 0$, randomly pick value $x^{(t)} \in \{\mathbf{+1}, \mathbf{-1}\}$
- Then, at each round
  set value $x^{(t)}$ to average of neighbors



Why it Works: Intuition



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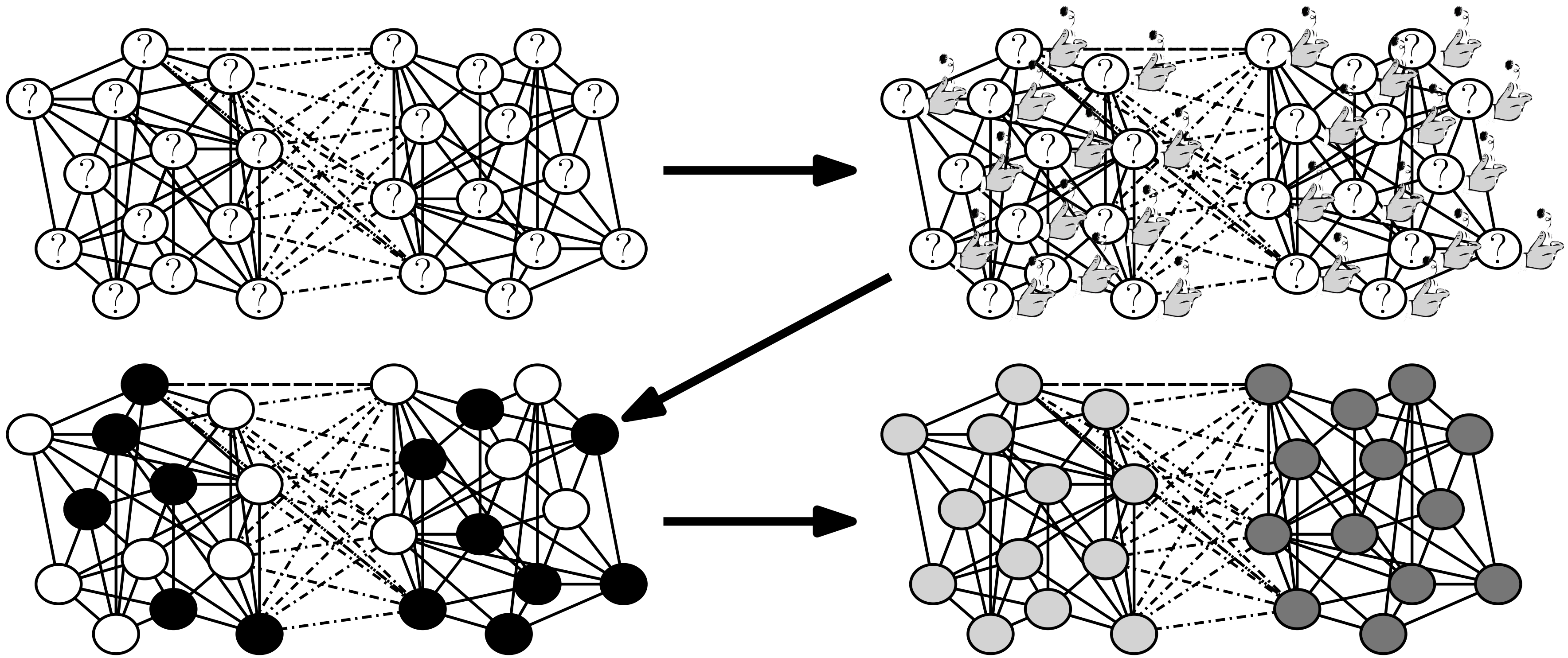

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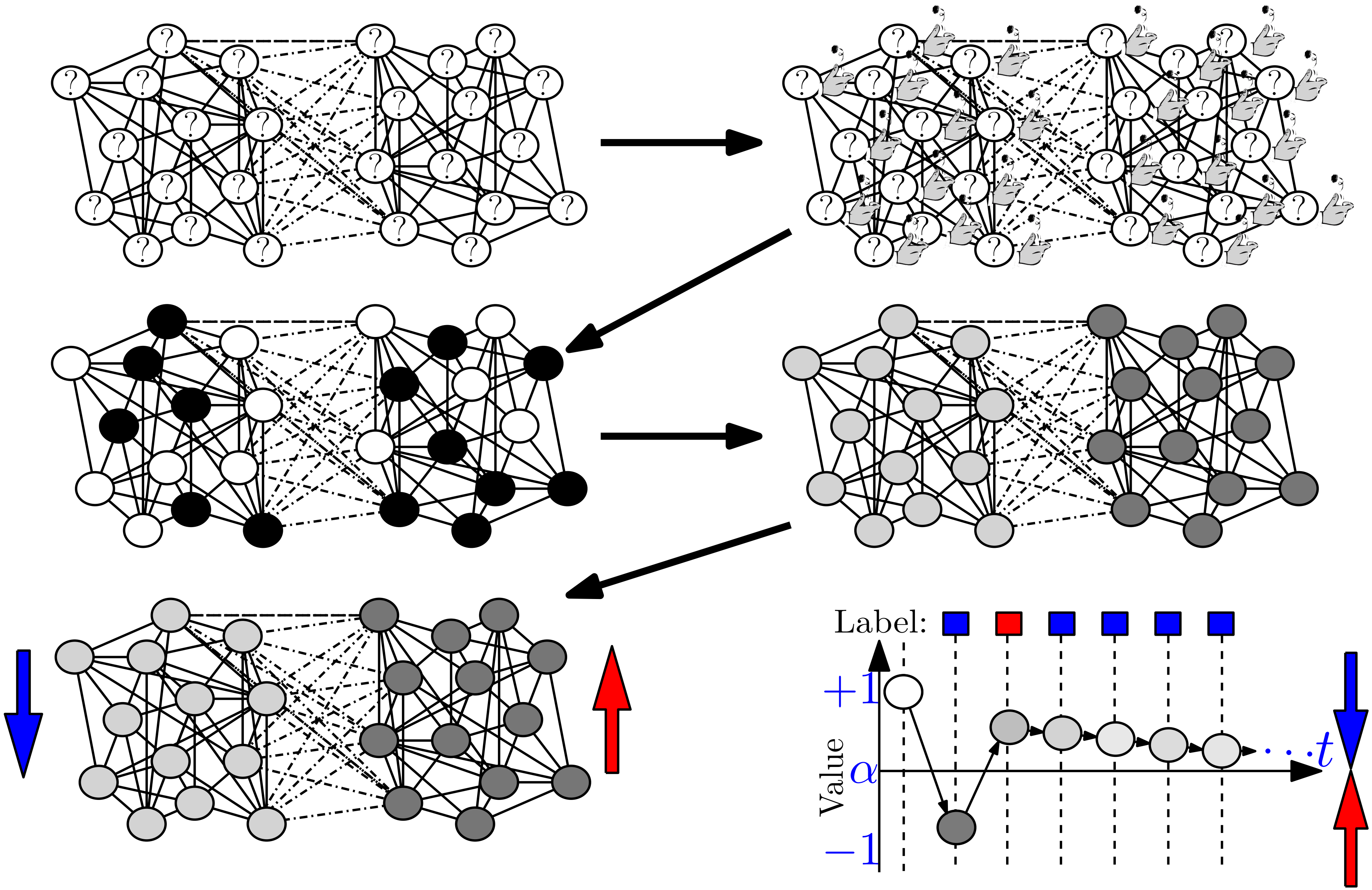
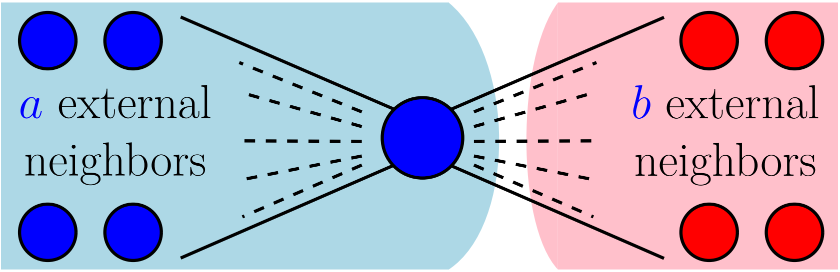


- Set label to **blue** if $x^{(t)} < x^{(t-1)}$, **red** otherwise

End of Technical Part: Idea of Proof [SODA'17]

G instance of Regular
Stochastic Block Model



Theorem. In Regular Stochastic Block Model with $a - b > \sqrt{2(a+b)}$, Averaging Dynamics finds clusters after $\frac{\log n}{\log \lambda_2 / \lambda_3}$ steps with high probability

Averaging is a **linear** dynamics:

$$\boxed{\mathbf{x}^{(t)} = P \cdot \mathbf{x}^{(t-1)} = P^t \cdot \mathbf{x}^{(0)}}$$

$$\mathbf{x}^{(t)} = \begin{pmatrix} \circ \\ \bullet \\ \circ \\ \bullet \\ \bullet \end{pmatrix}$$

P transition matrix of random walk on G

P symmetric $\Longrightarrow$ orthonormal eigenvectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$,
real eigenvalues $\lambda_1, \ldots, \lambda_n$.

End of Technical Part: Idea of Proof [SODA'17]

2nd eigenvalue $\gg \lambda_3(1+\delta)$ w.h.p.

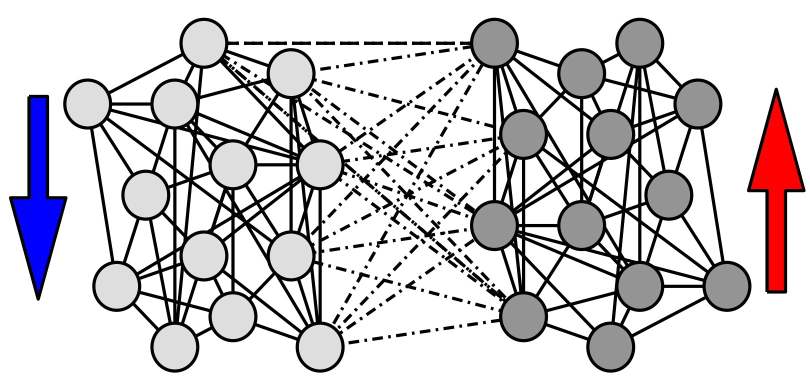
negligible after $t \gg \frac{\log n}{\log \lambda_2/\lambda_3}$

$$\mathbf{x}^{(t)} = \frac{1}{n}\mathbf{1}^\intercal \mathbf{x}^{(0)}\mathbf{1} + \left(\frac{a-b}{a+b}\right)^t \frac{1}{n}\chi^\intercal \mathbf{x}^{(0)}\chi + \mathbf{e}^{(t)}$$

1st eigenvector: all ones

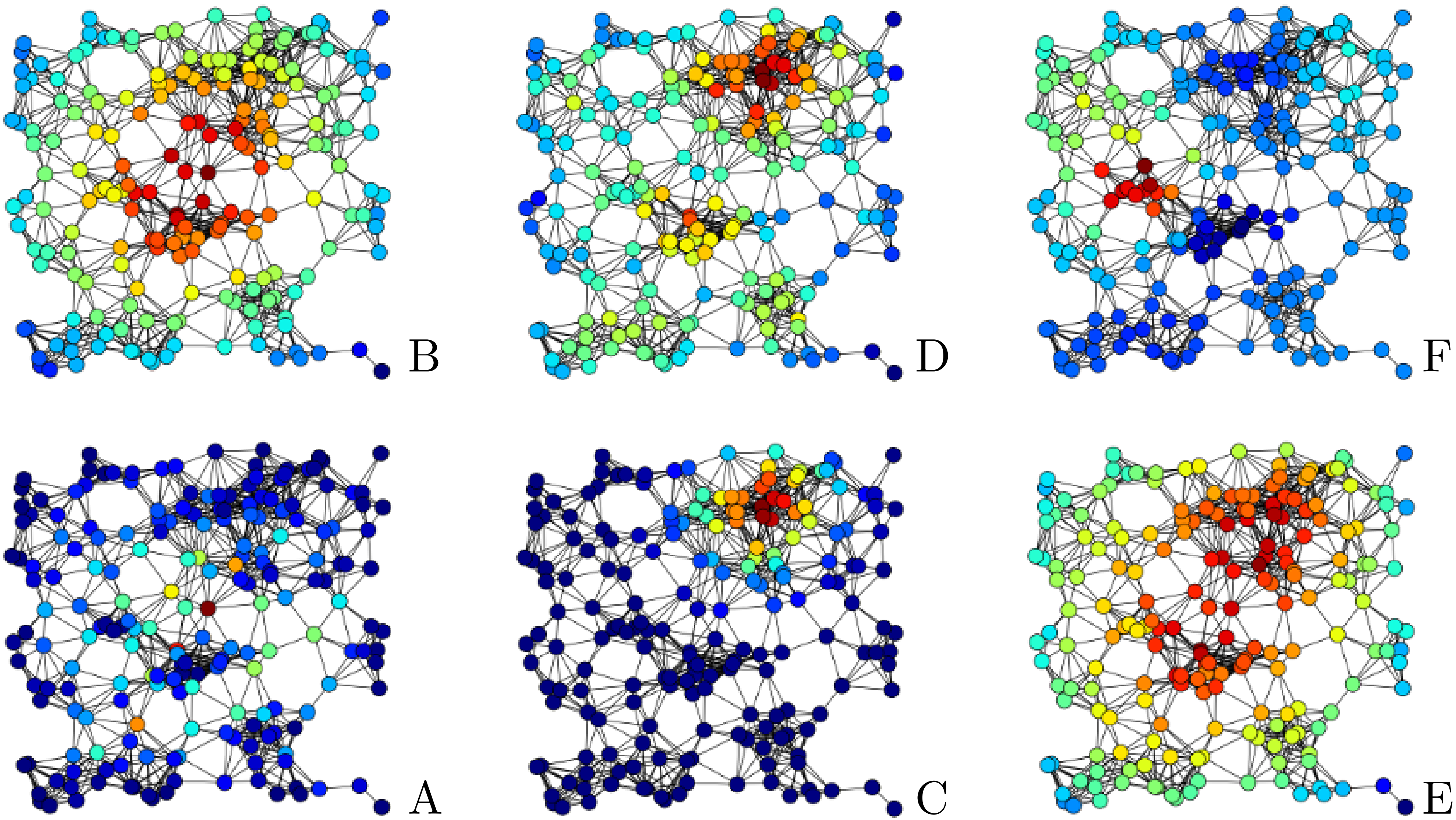
$(1, \ldots, 1, -1, \ldots, -1)$

$$\boxed{\mathrm{sign}(\mathbf{x}^{(t)}(u) - \mathbf{x}^{(t-1)}(u)) \propto \mathrm{sign}(\chi(u))}$$



QED

Theory-Driven Algorithm Engineering: Centrality



Examples of A) Betweenness centrality, B) Closeness centrality, C) Eigenvector centrality, D) Degree centrality, E) Harmonic Centrality and F) Katz centrality of the same graph*.

*Source: Wikipedia.

Betweenness Centrality

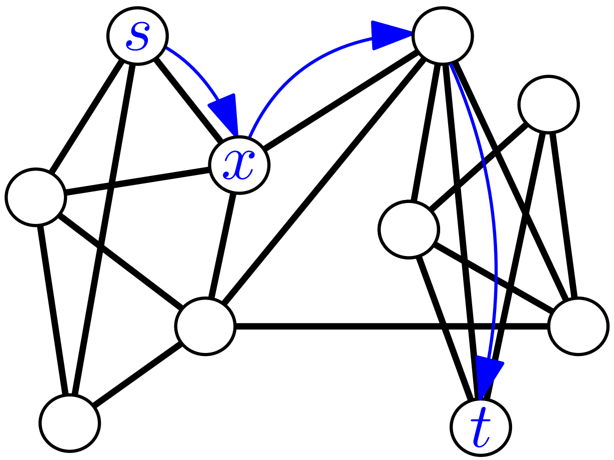
Probability $\Pr(X)$ of being in a shortest path

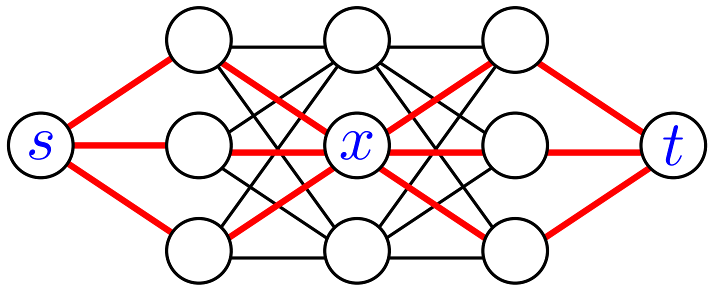
Betweenness centrality:

$$\sigma(x) = \frac{1}{n(n-1)} \sum_{s \neq x \neq t} \frac{\sigma_{st}(x)}{\sigma_{st}}$$

$\sigma_{st} :=$ # shortest paths from s to t

$\sigma_{st}(x) :=$ # shortest paths from s to t through x





Brandes '01: betweenness of all nodes in $\mathcal{O}(mn)$

Borassi et al. '15: Brandes is optimal assuming Strong Exponential Time Hypothesis

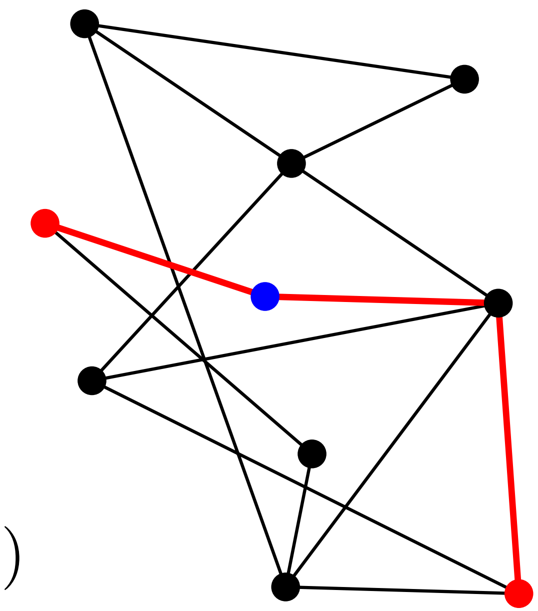
Betweenness Centrality, Before

Eppstein and Wang [SODA '01]: samples $S \subset V$ and compute measure w.r.t. $S \implies$ approx. of *closeness centrality* w.h.p. in $\mathcal{O}(CB) \cdot \mathcal{O}(SSSP)$

#samples   shortest paths

Betweenness centrality:

Idea: samples $s, t \in V$ and give 1 point to x if x is in the st-shortest path



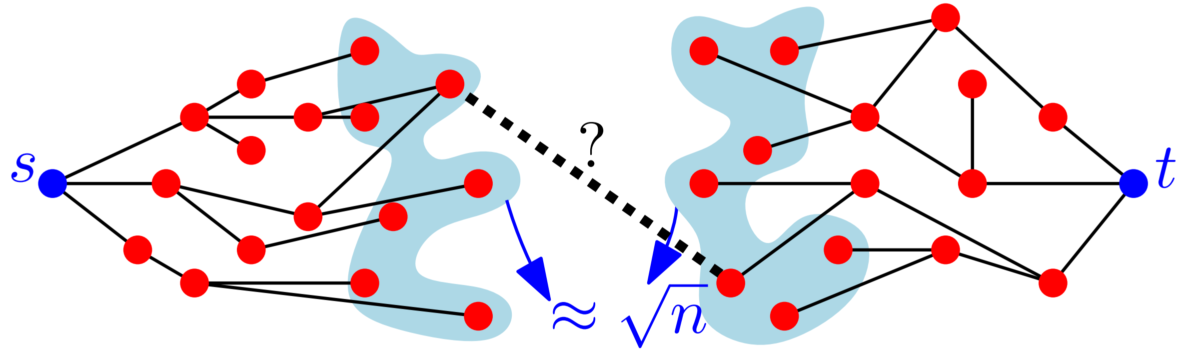
......, Riondato and Upfal [KDD '16]:
ABRA*, ϵ-approx. in time
$\mathcal{O}(g(\mathrm{RA})) \cdot \mathcal{O}(st\text{-}SP)$ (also adaptive sampling)

#samples   shortest paths

* Approximating Betweenness via Rademacher Averages

Ingredient 1/2: Bidirectional Balanced BFS

Complex Networks
$\approx$
"good" Random
Graph Models

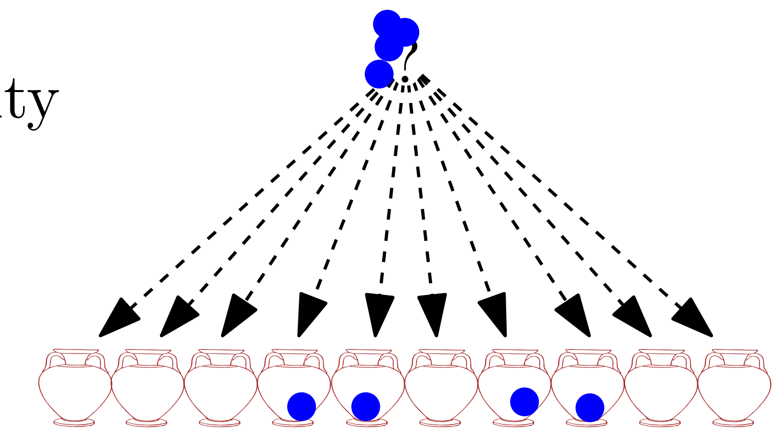


The Birthday (pseudo)Paradox

m balls u.a.r. in n bins: Probability p of ≥ 2 balls in one bin?

$$1 - p \leq \left(1 - \frac{m}{2n}\right)^{\frac{m}{2}} \approx e^{-\frac{c^2}{4}}$$

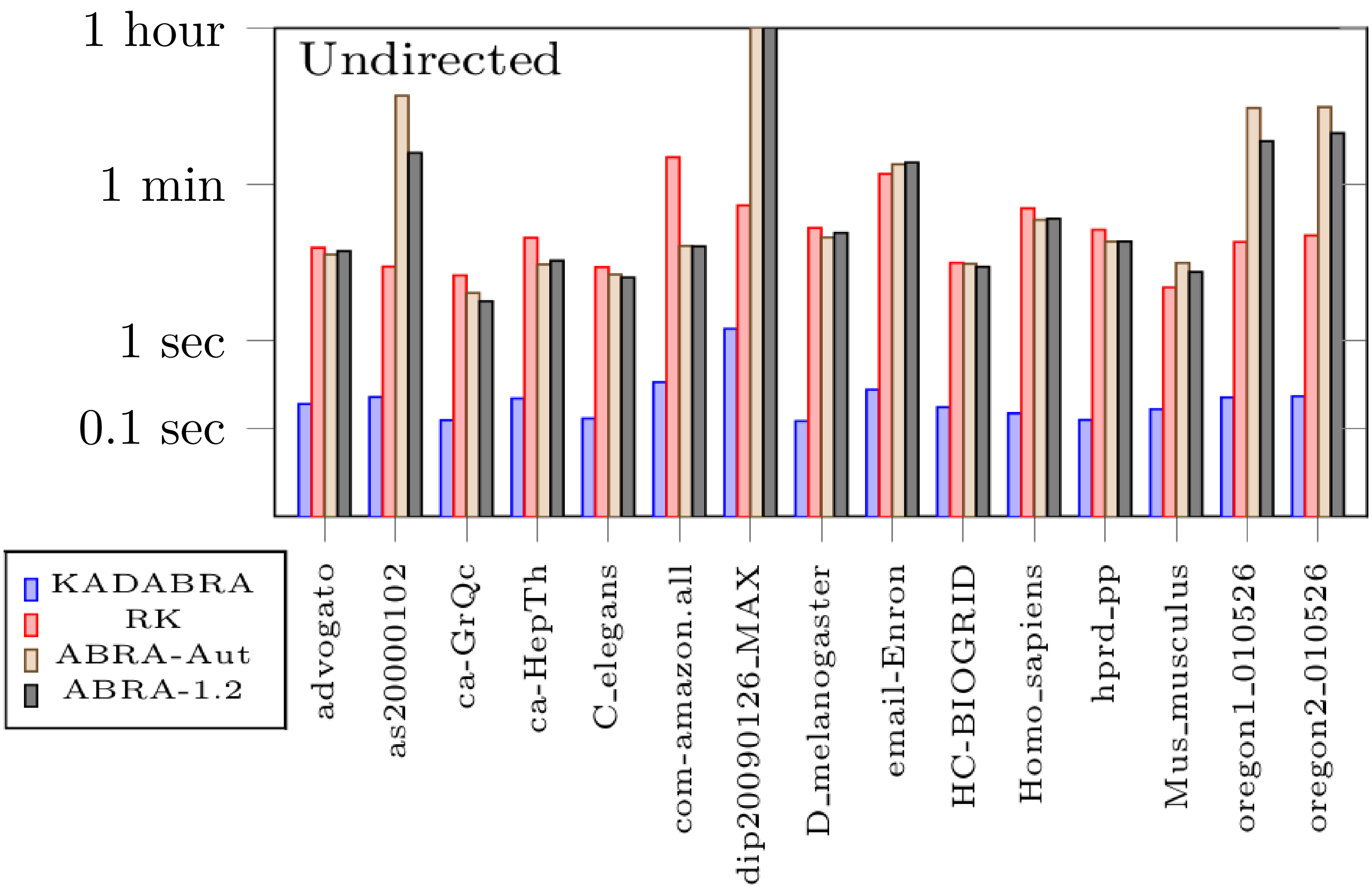
$$m = c\sqrt{n}$$



Theorem. Let G be a random graph (Configuration, Norros-Reittu, Chung-Lu, Generalized Random Graph). For each $\epsilon > 0$, and pair of nodes s and t, w.h.p. BBBFS computes st-shortest path in $\mathcal{O}(n^{\frac{1}{2}+\epsilon})$ if degree distribution λ has finite 2nd moment, $\mathcal{O}(n^{\frac{4-\beta}{2}+\epsilon})$ if λ is power law with $2 < \beta < 3$.

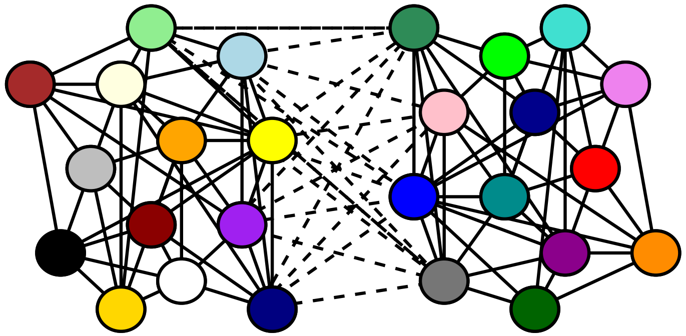
Betweenness Centrality, After

Borassi & N. [ESA'16]: Kadabra is an ADaptive Algorithm for Betweenness via Random Approximation

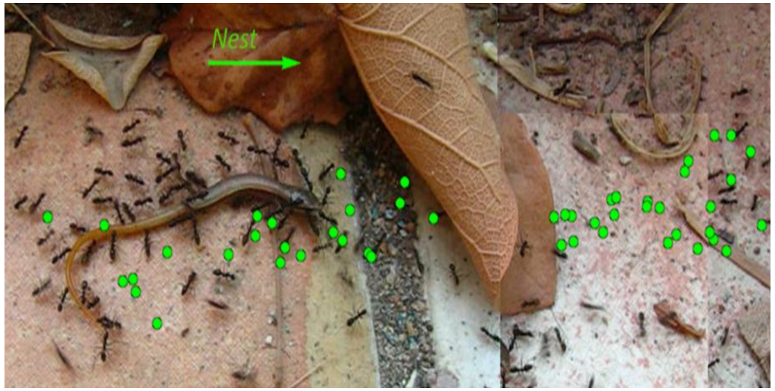


Research on Dynamics: Future with GANG

- Computing in **Dynamic Networks**
  - **L**abel **P**ropagation **A**lgorithms made rigorous (structural graph properties, stochastic processes)



- Biological experiment *from* TCS predictions
  - algorithmic analysis of ants, bacteria, molecules.



(DREAMY Project, TAPDANCE)

Research on Dynamics: Far Future

- The **Theory of Dynamics**
  - General comparative techniques
  - Computation through dynamic processes

  (population protocols, dynamic information in complex systems)

- From theoretical neuroscience to machine learning
  - Simons Institute, Brain Program
  - New approaches to neural networks

  (comparative compuational neuroscience, Denéve, O'Regan, Van Rullen)

Merci pour votre attention

Conference (16)

⋆16 Crucianiᶜ, E.; N., E.; Nusser, A.; Scornavacca†, G. **AAMAS'18**
⋆15 Becchetti L., Bonifaci V., N. E. **AAMAS'18** † = supervised interns
⋆14 Boczkowski, L.; Korman, A.; N. E. **ITCS'18**
⋆13 Becchetti, L.; Clementi, A.; N., E.; Pasquale, F.; Trevisan, L. **SODA'17**
⋆12 Boczkowski, L.; Korman, A.; N., E. **SODA'17**
⋆11 Berenbrink, P.; Clementi, A.; Elsässer, R.; Kling, P.; Mallmann-Trenn, F.; N., E. **PODC'17**.
⋆10 Becchetti, L.; Clementi, A.; N., E.; Pasquale, F.; Trevisan, L.. **SODA'16**
9 Gualà, L.; Leucci, S.; N. E.; Tauraso, R. **FUN'16**
8 Borassi, M.; N. E. **ESA'16 (Best Student Paper Award)**
⋆7 Kaaser, D.; Mallmann-Trenn, F.; N., E. **MFCS'16**
⋆6 Fraigniaud, P.; N., E. **PODC'16**
⋆5 Becchetti, L.; Clementi, A.; N., E.; Pasquale, F.; Silvestri, R. **SODA'15 (Best 2015 PhD Paper in CS at Sapienza Award)**
4 Becchetti, L.; Clementi, A.; N. E.; Pasquale, F.; Posta, G. **SPAA'15**
3 Gualà, L.; Leucci, S.; N. E. Bejeweled, **CIG'14**
⋆2 Becchetti, L.; Clementi, A.; N., E.; Pasquale, F.; Silvestri, R.; Trevisan, L. **SPAA'14**
⋆1 Clementi, A.; Di Ianni, M.; Gambosi, G.; N. E.; Silvestri, R. **SIROCCO'13**

Red text= Important/A^* conference
⋆ item = Related to Dynamics
Blue box = New

authors in alphabetical order

Journal (4)

⋆5 Fraigniaud P., N. E. **Distributed Computing**
⋆4 Boczkowski L., Korman A., N. E. **Distributed Computing 2018**
3 Becchetti, L.; Clementi, A.; N. E.; Pasquale, F.; Posta, G. **Distributed Computing 2017**
⋆2 Becchetti, L.; Clementi, A.; N. E.; Pasquale, F.; Silvestri, R.; Trevisan, L. **Distributed Computing 2017**
1 Clementi, A.; Di Ianni, M.; Gambosi, G.; N. E.; Silvestri, R. **Theor. Comp. Scie. 2015 (Special Issue)**

Submitted (2 conference, 3 journal)

⋆4 Boczkowski L., Feinerman O., Korman A., N. E. Submitted revision **PLOS Computational Biology**
⋆3 Becchetti, L.; Clementi, A.; Manurangsi, P.; N. E.; Pasquale, F.; Raghavendra, P.; Trevisan, L. **Sub. SPAA**
⋆2 Clementi, A.; Gualà, L.; Pasquale, F.; Scornavacca, G.; N. E.; Ghaffari, M. **Submitted to SPAA**
⋆1 Borassi, M.; N. E.. **Submitted to J. of Experimental Algorithms (Special Issue)**