z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Distributions:

$Gamma: \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$
$E[X] = \alpha\beta$; $Var(X) = \alpha\beta^2$
$M_x(t) = \frac{1}{(1-\beta t)^\alpha}$

$Exponential: \frac{1}{\beta}e^{-x/\beta}$ $(Gamma\ when\ \alpha = 1)$
$E[X] = \beta$; $Var(X) = \beta^2$ with x>0 and b>0
(use when distribution resembles poisson)
Poisson distribution:

$$f(x) = \frac{e^{-\lambda w}(\lambda w)^x}{x!} \qquad \beta = \frac{1}{\lambda}$$

$Chi-square: \frac{1}{\Gamma(k/2)2^{k/2}}x^{k/2-1}e^{-x/2}$
$(Gamma\ with\ \beta = 2,\ \alpha = k/2)$ $E[X] = k$
$Var(X) = 2k$
K = degrees of freedom

$Normal: \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$
E[X] = μ ; V[X] = σ^2
To convert to standard normal use $z = \frac{x-\mu}{\sigma}$
Standard Normal: σ=1, μ=0

Using Normal Distribution to approximate Binomial
Is appropriate if both np and n(1-p) >10. $\mu = np \quad \sigma = \sqrt{np(1-p)}$
use P(X< x $\pm .5$) to include or exclude a point; x> you shift up 0.5 and x< you shift down 0.5
Appropriate if either: $p \le 0.5\ AND\ np > 5$ **OR** $p > 0.5\ AND\ n(1-p) > 5$

ChebyShev's Inequality:
$k = number\ of\ standard\ deviations\ away\ from\ \mu$
$P(|x-\mu| \le k\sigma) \ge 1 - 1/k^2$

$Cauchy: \frac{1}{\pi}\frac{a}{a^2+(x-b)^2}$
$Mx(t): DNE$
$Mean: DNE$
$VarX: DNE$

Normal Probability Rule: Let X be normally distributed with parameters μ and σ. Then
P[$-\sigma < X-\mu < \sigma$]= .68
P[$-2\sigma < X-\mu < 2\sigma$]= .95
P[$-3\sigma < X-\mu < 3\sigma$]= .997

Uniform: $\frac{1}{b-a}\ a < x < b$
$Mx(t) = \frac{e^{tb}-e^{ta}}{t(b-a)}\ for\ t \neq 0$
$Mx(t) = 1\ for\ t = 0$
$E[X] = \frac{a+b}{2}\quad VarX = (b-a)^2/12$

$Weibull: \alpha\beta x^{\beta-1}e^{-\alpha x^\beta}$
$E[X] = \alpha^{-1/\beta}\Gamma(1+1/\beta)$
$VarX = \alpha^{-2/\beta}\Gamma(1+2/\beta) - \mu^2$

General for continuous functions: Let X be a continuous random variable with density f.
Ch. 4

Required to be a continuous density:
$$f(x) > 0 \qquad \int_{-\infty}^{\infty} f(x)dx = 1$$

General form for expected value:
$$E[H(x)] = \int_{-\infty}^{\infty} H(x)f(x)dx$$

$$Mx(t) = E[e^{tx}] = \int e^{tx}f(x)dx$$
Use it by taking derivative with respect to t

Gamma function:
$$\Gamma(a) = \int_0^\infty z^{a-1}e^{-z}dz$$

$$P[a \le X \le b] = \int_a^b f(x)dx$$

$$E[X] = \int_{-\infty}^{\infty} x\,f(x)dx = \mu$$

If you know the cumulative density you can find $f(x)$:
$f(x) = F'(x)$

$\Gamma(1) = 1$
$for\ \alpha > 1,\ \Gamma(a) = (a-1)!$

Cumulative density:
$$F(x) = P[X \le x] = \int_{-\infty}^{x} f(t)dt$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)dx$$

Transformation formula:
Let X be a continuous random variable with density f_x Let $Y = g(x)$, where g is strictly monotonic and differentiable. The density for Y is denoted by f_y and is given by
$$f_y(y) = f_x(g^{-1}(y))\left|\frac{dg^{-1}(y)}{dy}\right|$$

Ch. 5: To be a density: $fxy(x,y)$ >= 0 AND $\sum_x\sum_y fxy(x,y) = 1$

Ch.5 Joint distribution $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{xy}(x,y)dydx = 1$ $= P[X = x \text{ and } Y = y]$	$E(x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xf_{xy}(x,y)dydx$ $E(y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} yf_{xy}(x,y)dydx$	$P[a \le X \le b \text{ and } c \le Y \le d]$ $= \int_a^b\int_c^d f_{xy}(x,y)dydx$	$E(x^2) =$ $= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f_{xy}(x,y)dydx$
X and Y are independent iff $f_x(x)\, f_y(y) = f_{xy}(x,y)$ And the intervals match	$E(xy) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf_{xy}(x,y)dydx$	$f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)dy$ $\quad f_y(y) = \int_{-\infty}^{\infty} f_{xy}(x,y)dx$	
If X and Y independent then $E(XY) = E(X)E(Y)$	$E[H(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(x,y) f_{xy}(x,y)dydx$	$Cov(x,y) = E(xy) - E(x)E(y)$	
$Correlation = \rho_{XY} = \frac{Cov(X,Y)}{\sqrt{(VarX)(VarY)}} \quad -1 \le \rho \le 1$			

Discrete counting and probability

General Addition Rule: $P(A_1 \cup A_2) = P(A_1) + P(A_1) - P(A_1 \cap A_2)$	Conditional Probability: $P[A_1 \vert A_2] = P(A_1 \cap A_2) / P(A_2)$	If events are independent: $P(A_1 \cap A_2) = P(A_1)P(A_2)$	A_1 and A_2 are independent if and only if: $P(A_1 \vert A_2) = P(A_1)$; $P(A_1)$=/=0
Convergence of geometric series: Let ∑(k=1 to n) ar^{k-1} be a geometric series. Converges to a/(1-r) if |r| < 1	Expected value: E[H[X]] = ∑H(x)f(x)	Rules for expectation: E[c] = c (constant) ; E[cX] = cE[X] E[X+Y] = E[X] + E[Y]	Variance: Var X = σ^2 = E[(X - μ)²] = E[X²] - (E[X])²
Standard deviation: σ = sqrt(Var X) = sqrt(σ²)	Rules for variance: Var c = 0 ; Var cX = c²Var X Var(X+Y) = Var X + Var Y (if independent)	Geometric distribution: $f(x) = (1-p)^{x-1}p$ 0 < p < 1 ; x = 1, 2, 3, ...	Moment generating function: $m_x(t) = E[e^{tX}] = \sum e^{tX} f(x)$ $E[X^k]$ = the kth derivative of $m_x(t)$ eval at t=0
Geometric moment generating function: $m_x(t) = (pe^t)/(1 - qe^t)$	Known values for geometric distribution: E[X] = 1/p Var X = q/(p²)	P[x≥ 5] = 1- P[x < 5]	

Example problems:

1.Find k such that $f(x) = kx^{1/2}$ for 1 ≤ x ≤ 4 is a probability density function.

$k\int_1^4 x^{1/2}dx = 1$
$k[(2/3)x^{3/2}|1 \text{ to } 4] = 1$
$(2/3)k[4^{3/2} - 1^{3/2}] = 1$
$(2/3)k[8 - 1] = 1$
$(2/3)k[7] = 1$
$(14/3)k = 1$
$k = 3/14$

2.Let X be a continuous random variable with f(x) = $(3/4)x^{1/2}$ for $1 \le x \le 4$. Find $F(x)$.
$Definition: F(x) = \int_{-\infty}^{x} f(x)dx$; but our problem is bounded from below by 1.
$F(x) = \int_1^x (3/14)x^{1/2}dx$
$= (3/14)[(2/3)x^{3/2}|1 \text{ to } x]$
$= (3*2/14*3)[x^{3/2} - 1]$
$= (6/42)[x^{3/2} - 1]$
$= (1/7)[x^{3/2} - 1]$

3.Let X be a continuous random variable with f(x) = $(3/4)x^{1/2}$ for $1 \le x \le 4$. What is $E[X]$?
$Definition: E[X] = \int_{-\infty}^{\infty} xf(x)dx$; but our problem is bounded from 1 to 4
$E[X] = \int_1^4 x*(3/14)x^{1/2}dx$
$= (3/14)\int_1^4 x^{3/2}dx$
$= (3/14)[(2/5)x^{5/2} \mid 1 \text{ to } 4]$
$= (3/35)[x^{5/2} \mid 1 \text{ to } 4]$
$= (3/35)[32 - 1]$
$= (3/35)[31] = 93/35 = 2.66$

4.Let X be a continuous random variable with f(x) = $(3/4)x^{1/2}$ for $1 \le x \le 4$. What is $E[1/X]$?
$Definition: E[H(x)] = \int_{-\infty}^{\infty} H(x)f(x)dx$
our $H(x) = 1/x$

$E[1/x] = \int_1^4 (1/x)(3/14)(x^{1/2})dx$
$= (3/14)\int_1^4 x^{-1}x^{1/2}dx$
$= (3/14)\int_1^4 x^{1/2-1}dx$
$= (3/14)\int_1^4 x^{-1/2}dx$
$= (3/14)[2x^{1/2} \mid 1 \text{ to } 4]$
$= (3/7)[2 - 1] = (3/7)$

6.Assume that test scores are normally distributed with mean 70 and variance 90. What is the probability that a randomly selected student scores higher than 82 points?
$\mu = 70,\ \sigma^2 = 90,\ \sigma = 3\sqrt{10} = 9.487$

First we standardize so we can use z-scores.
Use $P(X \le z)$ where $z = \frac{x-\mu}{\sigma}$

$P[X > 82]$
$= P[(X-70)/9.487 > (82-70)/9.487]$
$= P[z > 12/9.847] = P[z > 1.265]$
But our chart is that of a cumulative distribution, so we only know what's below z
$= 1 - P[z \le 1.26]$
Look this up on the chart
$= 1 - 0.8962$
$= 0.1038$

Remember the 68-95-99.7 rule.

1 standard deviation above our mean of 70 is roughly 79.5, the rest not accounted for is the final ~20% on both sides. We only care about the top scoring side, so roughly the top ~10%

7.Assume that the lengths of widgets are normally distributed with a mean of 14 inches and a standard deviation of 4 inches. The shortest 15% of widgets are shorter than how many inches?
$P[X < x_0] = 0.15$
Standardize so we can use z-scores
$P[(X-14)/4 < (x_0 - 14)/4] = 0.15$
$P[z < (x_0 - 14)/4] = 0.15$

$(x_0 - 14)/4$ is the point on the standard normal curve with 15% of the area under the curve to its left.

Find a value for 0.15 on the table: the closest is $z = -1.04$

$(x_0 - 14)/4 = -1.04$
$x_0 - 14 = -1.04(4)$
$x_0 = -4.16 + 14$
$x_0 = -4.16 + 14 = 9.84$
Remember the 68-95-99.7 rule. This makes sense as 1 standard deviation away from the mean of 14 is 10 and 18. This 10 to 18 range contains 68% of the widgets.
The remainder of 32% is on either side, but we only care about the the lower 16%. We want the point at which the lower 15% is contained under the curve, so our answer should be a little less than 10 and we got 9.84

8.Assume that the lengths of widgets are normally distributed with a mean of 14 inches and a standard deviation of 4 inches. The middle 70% of widgets are

9.Let X be an exponential random variable with parameter beta = 4. a) What is the mean of X? b) What is the median of X? Describe how you found both of the quantities above.
Exponential: Gamma distribution with $\alpha = 1$ is $\frac{1}{\beta}e^{-x/\beta}$ $with\ x > 0$; $ours = \frac{1}{4}e^{-x/4}$
a)The moment generating function for Gamma dist: $Mx(t) = (1 - \beta t)^{-\alpha}$; $Mx(t) = (1 - 4t)^{-1}$
Find first derivative and set t=1

between ____ and _____ inches.

From the previous answer, we know that the lower 15% is bounded x=9.84. As the normal curve is symmetrical we can extrapolate the value for the upper 15%.

The difference between the mean and the point that bounds the lower 15% is 14-9.84=4.16

Lower bound = 14-4.16 = 9.84
Upper bound = 14+4.16 = 18.16

$E[X] = -1(1-4t)^{-2}(-4)\ |t=0$
$E[X] = 4(1-0)^{-2} = 4$ (also worth noting that in exponential form $E[X] = \alpha\beta = \beta$ by definition)

b) the mean is point at which half the area lies to the left and half lies to the right. We need to integrate our function from 0 to k and set it equal to ½. Solve for k.

$\frac{1}{4}\int_0^k e^{-x/4}dx = 1/2$
$\frac{1}{4}[4-4e^{-k/4}] = 1/2$
$1-e^{-k/4} = 1/2$
$e^{-k/4} = 1/2$
$-k/4 = ln(1/2) \quad ln(1/2) = -0.693$
$k = (-4)(-0.693) = 2.77$

10. I expect 4 students, on average, to visit me during an office hour. What is the probability that I have to wait more than 10 minutes for the first student to arrive?

This is an exponential density. 1 every 15 minutes. $\lambda = 1/15$ *and* $\beta = 1/\lambda = 15$

We can find out the probability of waiting up to 10 minutes:
$P[X \le 10] = \int_0^{10} \frac{1}{15}e^{-x/15}dx$
$= \frac{1}{15}[e^{-x/15}(-15)\ |\ 0\ to\ 10]$
$= -[e^{-x/15}\ |\ 0\ to\ 10]$
$= -[e^{-2/3}-1]$
$= [1-0.5134]$
$= 0.487$

And subtract this from 1
$p[X>10] = 1-P[X \le 10]$
$= 1-0.487$
$= 0.513$

Let X be a random variable with pdf f(x) = 1/3 for 1<x<4. Compute the probability that x is within 2 standard deviations of the mean. Compare that to the comparable results from the normal probability rule and Chebyshev's inequality. Explain why there are discrepancies between the 3 values.

$\mu = E[X] = \int_1^4 x\, f(x)dx = \int_1^4 x\ (1/3)\ dx$
$= (1/3)[x^2/2\ |\ 1\ to\ 4]$
$= (1/6)[16\ -\ 1]$
$= 15/6 =$ **2.5**
$E[x^2] = \int_1^4 x^2 f(x)dx = \int_1^4 x^2\ (1/3)\ dx$
$= (1/3)[x^3/3\ |\ 1\ to\ 4]$
$= (1/9)[64-1]$
$= 63/9 =$ **7**

$VarX = \sigma^2 = 7-2.5^2 = 7-6.25 = 0.75$
$standard\ deviation = \sqrt{\sigma^2} = \sigma = \sqrt{0.75} = 0.866$
$\mu = 2.5\ ,\ \sigma = 0.866$

Within 2 standard deviations of the mean, $2\sigma = 1.732$.
2 standard devs lower = $\mu - 2\sigma = 2.5-1.732 = 0.768$
and 2 standard devs above = $\mu + 2\sigma = 2.5+1.732 = 4.232$.

Both of these values are outside of our 1<x<4 range so 100% of our area is within 2 standard deviations.

The discrepancy between this and normal distribution is because our shape is a rectangle and cannot be modeled by a normal distribution.

Chebyshev's $P(|x-\mu| < k\sigma) \ge 1-\frac{1}{k^2}$
$P(|x-2.5| < 1.732) \ge 1-1/4 = 0.75$

Not a discrepancy here, at least 75% of our distribution is contained within 2 standard deviations. This is still true.

60% of all WSU students taking upper division math classes think that Cora is FABULOUS! If 42 randomly selected students WSU students taking upper division math classes are surveyed, what is the probability that more than 20 of them say that Cora is fabulous? Use the normal approximation to the binomial.

n=42, p=.6, (1-p)=.4 Find P(X>20) We will use P(x>20.5) to exclude this point.

$\mu = np = 42(.6) = 25.2$
$\sigma = \sqrt{np(1-p)} = \sqrt{42(.6)(.4)} = 3.17$

$P(X>20.5) = 1-P(X \le 20.5)$
$= 1-P(Z \le \frac{20.5-25.2}{3.17})$
$= 1-P(Z \le -1.48) = 1-.0694 = .9306$

Let X be a random variable with $f_X(x) = -2x + 2$ for 0 < x < 1. Let Y = 1/X. Find $f_Y(y)$.
$f_Y(y) = f_X(g^{-1}(y))\left|\frac{dg^{-1}(y)}{dy}\right|$
Y=g(x)=1/x
Thus $g^{-1}(y) = 1/y$
$and \left|\frac{dg^{-1}(y)}{dy}\right| = \left|\frac{-1}{y^2}\right| = \left|\frac{1}{y^2}\right|$
$= \frac{1}{y^2}$ *Since all values will be positive*

Then
$f_Y(y) = (\frac{-2}{y}+2)\frac{1}{y^2} = \frac{-2}{y^3}+\frac{2}{y^2}$ 1<y<∞

With our given equation of $Y = 1/x$

$\lim_{x\to 0^+} 1/x = y \to Infinity$

or inversely as $y \to \infty,\ x \to 0$
So $y = \infty$ *when* $x = 0$ *and* $y = 1$ *when* $x = 1$

Let X be a random variable with pdf f(x) = 1/x for 1 < x < *e*.

Describe how to simulate data distributed like X.

1.Find $F(X)$

$F(X) = \int_1^x 1/x\ dx = ln(x)\ |1\ to\ x$
$= ln(x) - ln(1) = ln(x)$

2.Find $F^{-1}(X)$

F(X)=ln(x) then $F^{-1}(x) = e^x$

3.Generate Uniform random numbers between 0 and 1

4.Plug them into F'(X) to generate desired distribution

Homework chapter 5

1. Let X denote the air temp (in Celsius) and let Y denote the time in minutes that it takes for the diesel engine on a car to start. Assume that the joint density for (X,Y) is given by:

2. Let X denote the air temp (in Celsius) and let Y denote the time in minutes that it takes for the diesel engine on a car to start. Assume that the joint density for (X,Y) is given by:
Let fXY(x,y) = (1/6640)(4x + 2y + 1) for 0 ≤ x ≤ 40, 0 ≤ y ≤ 2.

3. Let X denote the air temp (in Celsius) and let Y denote the time in minutes that it takes for the diesel engine on a car to start. Assume that the joint density for (X,Y) is given by:

Let fXY(x,y) = (1/6640)(4x + 2y + 1) for 0 ≤ x ≤

Let fXY(x,y) = (1/6640)(4x + 2y + 1) for 0 ≤ x ≤ 40, 0 ≤ y ≤ 2.

Find the probability that on a randomly selected day the air temp will exceed 20 degrees Celsius and it will take at least 1 minute for the car to start.

Finding:

$(1/6640) \int_{20}^{40} \int_{1}^{2} 4x + 2y + 1 \, dydx$
$= (1/6640) \int_{20}^{40} [4xy + y^2 + y |_1^2]dx$
$= (1/6640) \int_{20}^{40} [8x + 6 - (4x + 2)]dx$
$= (1/6640) \int_{20}^{40} 4x + 4 \, dx$
$= (1/6640)[2x^2 + 4x |_{20}^{40}]$
$= (1/6640)[2(40)^2 + 4(40) - (2(20)^2 + 4(20))]$
= (1/6640)[3200 + 160 - (800 + 80)]
= (1/6640)[3360 - 880]
= (1/6640)[2480]
= 0.3735

Find the marginal density of X.

Find the probability that on a randomly selected day the air temp will exceed 20 degrees Celsius.

$f_x(x) = \int_0^2 f_{xy}(x,y) \, dy$
$= (1/6640) \int_0^2 4x + 2y + 1 \, dy$
$= (1/6640) (4xy + y^2 + y |_0^2)$
= (1/6640) (4x(2) + 4 + 2)
= (1/6640)(8x + 6) for 0≤x≤40

P[X > 20]
$= \int_{20}^{40} f_x(x) \, dx$
$= (1/6640) \int_{20}^{40} (8x + 6) \, dx$
$= (1/6640) [4x^2 + 6x |_{20}^{40}]$
$= (1/6640) [4(40)^2 + 6(40) - (4(20)^2 + 6(20))]$
= (1/6640) [4(1600) + 240 - (4(400) + 120)]
= (1/6640) [6400 + 240 - (1600 + 120)]
= (1/6640) [6640 - 1720]
= (1/6640) [4920] = 123/166 = **0.741**

40, 0 ≤ y ≤ 2.
-Find the marginal density of Y.
-Find the probability that on a randomly selected day it will take at least one minute for the car to start.

$f_Y(y) = \int_0^{40} fxy(x,y) \, dx$
$= (1/6640) \int_0^{40} 4x + 2y + 1 \, dx$
$= (1/6640) (2x2 + 2xy + x|_0^{40})$
= (1/6640) (2(1600) + 2(40)y + 40)
= (1/6640) (3200 + 80y + 40)
=(1/6640)(80y+3240)= (1/166)(2y + 81) for 0≤y≤2

P[Y > 1]
$= \int_1^2 f_Y(y) \, dy$
$= (1/6640) \int_1^2 3240 + 80y \, dy$
$= (1/6640)[3240y + 40y2 |_1^2]$
= (1/6640)[3240(2) + 40(2)2 - (3240(1) + 40(1)2)]
= (1/6640)[6480 + 160 - (3240 + 40)]
= (1/6640)[6640 - 3280]
= (1/6640)[3360] = 42/83 = **0.506**

4. Let X denote the air temp (in Celsius) and let Y denote the time in minutes that it takes for the diesel engine on a car to start. Assume that the joint density for (X,Y) is given by:

Let fXY(x,y) = (1/6640)(4x + 2y + 1) for 0 ≤ x ≤ 40, 0 ≤ y ≤ 2.

Are X and Y independent? Explain on a mathematical basis.

X and Y are independent iff
fx(x)fY(y) = fxy(x,y)
and the intervals match

$f_X(x)f_Y(y)$ = (1/6640)(8x + 6)(1/6640)(3240 + 80y)

= (1/44089600)[(8x + 6)(3240 + 80y)]

= (1/44089600)[8x(3240) + 8x(80y) + 6(3240) + 6(80y)]

= (1/4408960)[64xy + 2592x + 48y +1944]

= (1/551120)[8xy + 324x + 6y +243]

$=/= f_{XY}(x,y)$

Not independent

5. Let X denote the air temp (in Celsius) and let Y denote the time in minutes that it takes for the diesel engine on a car to start. Assume that the joint density for (X,Y) is given by:

Let fXY(x,y) = (1/6640)(4x + 2y + 1) for 0 ≤ x ≤ 40, 0 ≤ y ≤ 2.

Find Cov(X,Y). Explain your process.
From a physical standpoint, should Cov(X,Y) be positive or negative? Explain.

Cov(x,y) = E(xy) - E(x)E(y) : Need to find E(xy), E(x) and E(y)

$E(xy) = (1/6640) \int_0^{40} \int_0^2 xy(4x + 2y + 1) \, dydx$
$= (1/6640) \int_0^{40} \int_0^2 4x2y + 2xy2 + xy \, dydx$
$= (1/6640) \int_0^{40} [2x2y2 + 2xy3/3 + xy2/2 |_0^2] \, dx$
$= (1/6640) \int_0^{40} [2x2(2)2 + 2x(2)3/3 + x(2)2/2 \ _0^2] \, dx$
$= (1/6640) \int_0^{40} 8x2 + 16x/3 + 2x \, dx$
= (1/6640) [8x3/3 + 16x2/6 + x2 |040]
= (1/6640) [8x3/3 + 22x2/6 |040]
= (1/6640) [8(40)3/3 + 22(40)2/6]
= (1/6640) [529600/3] = **26.586**

$E(x) = \int_0^{40} xf(x) \, dx$
$= (1/6640) \int_0^{40} x(8x + 6) \, dx$
$= (1/6640) \int_0^{40} 8x2 + 6x \, dx$
$= (1/6640)[8x3/3 + 3x2 |_0^{40}]$
= (1/6640)[8(40)3/3 + 3(40)2]
= (1/6640)[8(64000)/3 + 3(1600)]
= **26.426**

$E(y) = \int_0^2 yfY(y) \, dy$
$= (1/6640) \int_0^2 y(3240 + 80y) \, dy$
$= (1/6640) \int_0^2 3240y + 80y2 \, dy$
$= (1/6640)[1620y2 + 80y3/3 |_0^2]$
= (1/6640)[1620(4) + 80(8)/3]
= **1.008**

Cov(x,y) = E(xy) - E(x)E(y) = 26.586 - 26.426(1.008) = **-0.051408**

6. Let X denote the air temp (in Celsius) and let Y denote the time in minutes that it takes for the diesel engine on a car to start. Assume that the joint density for (X,Y) is given by:

Let fXY(x,y) = (1/6640)(4x + 2y + 1) for 0 ≤ x ≤ 40, 0 ≤ y ≤ 2.

Find ρXY. Explain how your arrived at your answer. (You can show your work or describe in words.)

Cov(x,y) = -0.051408
E(xy) = 26.586
E(x)= 26.426
E(y) = 1.008
Correlation = ρXY = Cov(X,Y) / sqrt[(VarX)(VarY)]

Need to find VarX and VarY, which means we need to find $E(x^2)$ and $E(y^2)$
.

$E(x^2) = \int_0^{40} x^2 f(x) \, dx$
$= (1/6640) \int_0^{40} x2(8x + 6) \, dx$
$= (1/6640) \int_0^{40} 8x3 + 6x2 \, dx$
$= (1/6640) [2x4 + 2x3 |_0^{40}]$
= (1/6640) [2(40)4 + 2(40)3]
= (1/6640) [2(2560000) + 2(64000)]
= **790.361**
VarX = 790.361 - 698.333 = **92.028**

$E(y^2) = \int_0^2 y^2 f_Y(y) \, dy$
$= (1/6640) \int_0^2 y2(3240 + 80y) \, dy$
$= (1/6640) \int_0^2 3240y2 + 80y3 \, dy$
$= (1/6640)[1080y3 + 20y4 |_0^2]$
= (1/6640)[1080(2)3 + 20(2)4]
= (1/6640)[1080(8) + 20(16)]
= **1.349**
VarY = 1.349 - 1.016 = **0.333**

ρXY = Cov(X,Y) / sqrt[(VarX)(VarY)]
= -0.051408 / sqrt[(92.028)(0.333)]
= **-0.00928643**