



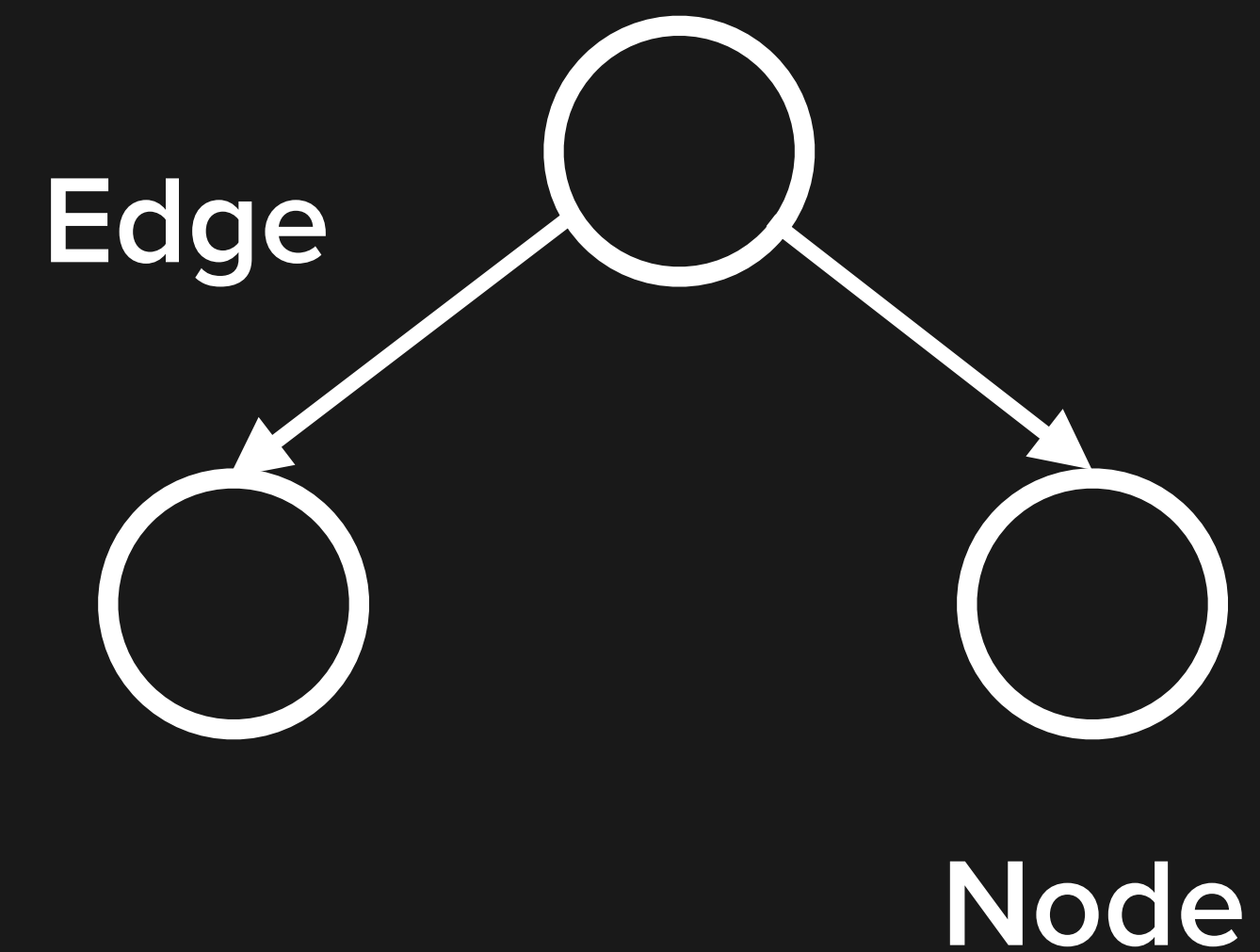
MAKE
SCHOOL

TREES

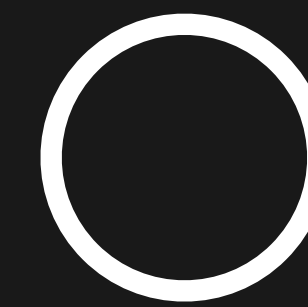
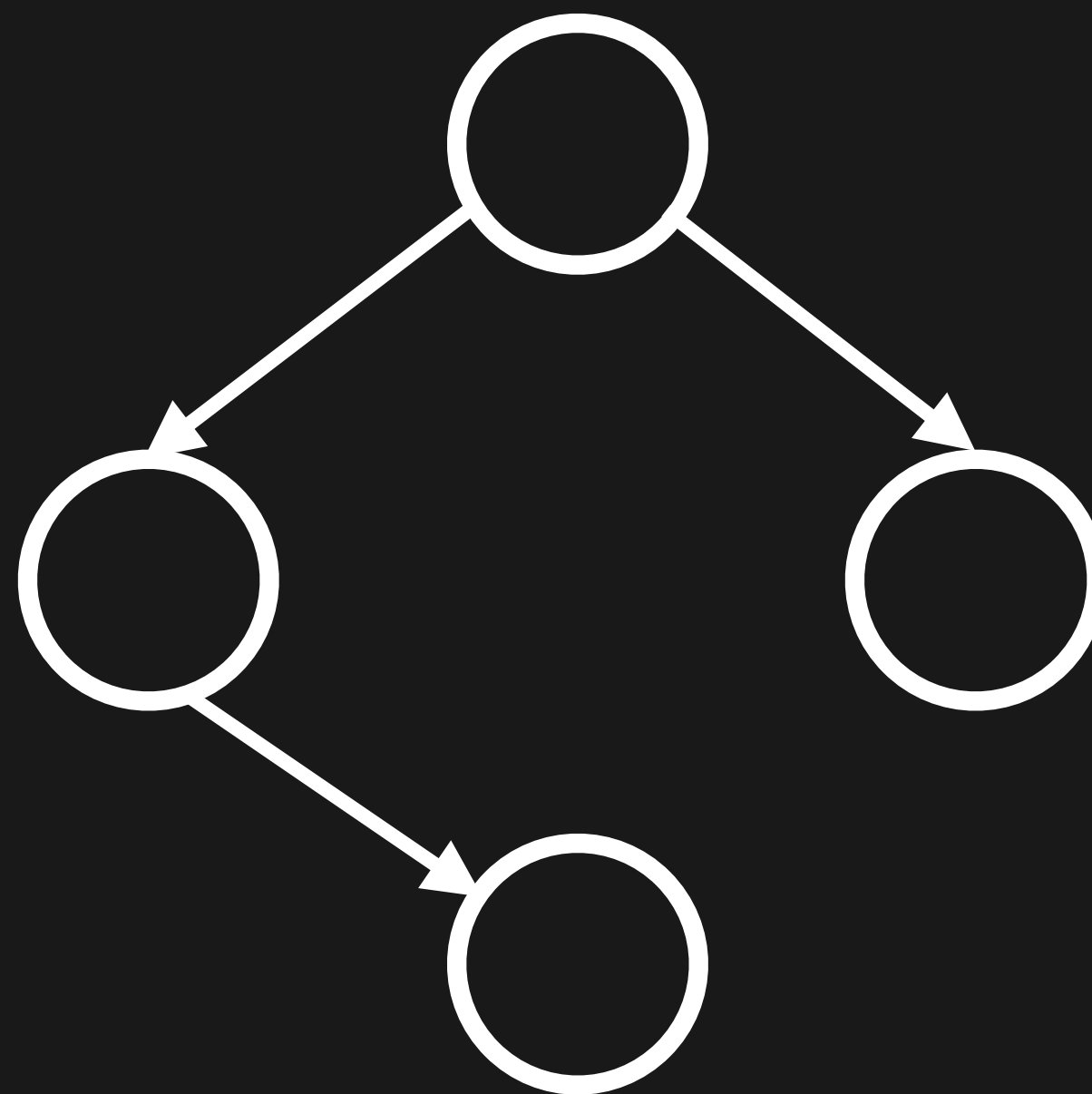
So log-arithmetic. (Get it!?)

TREE

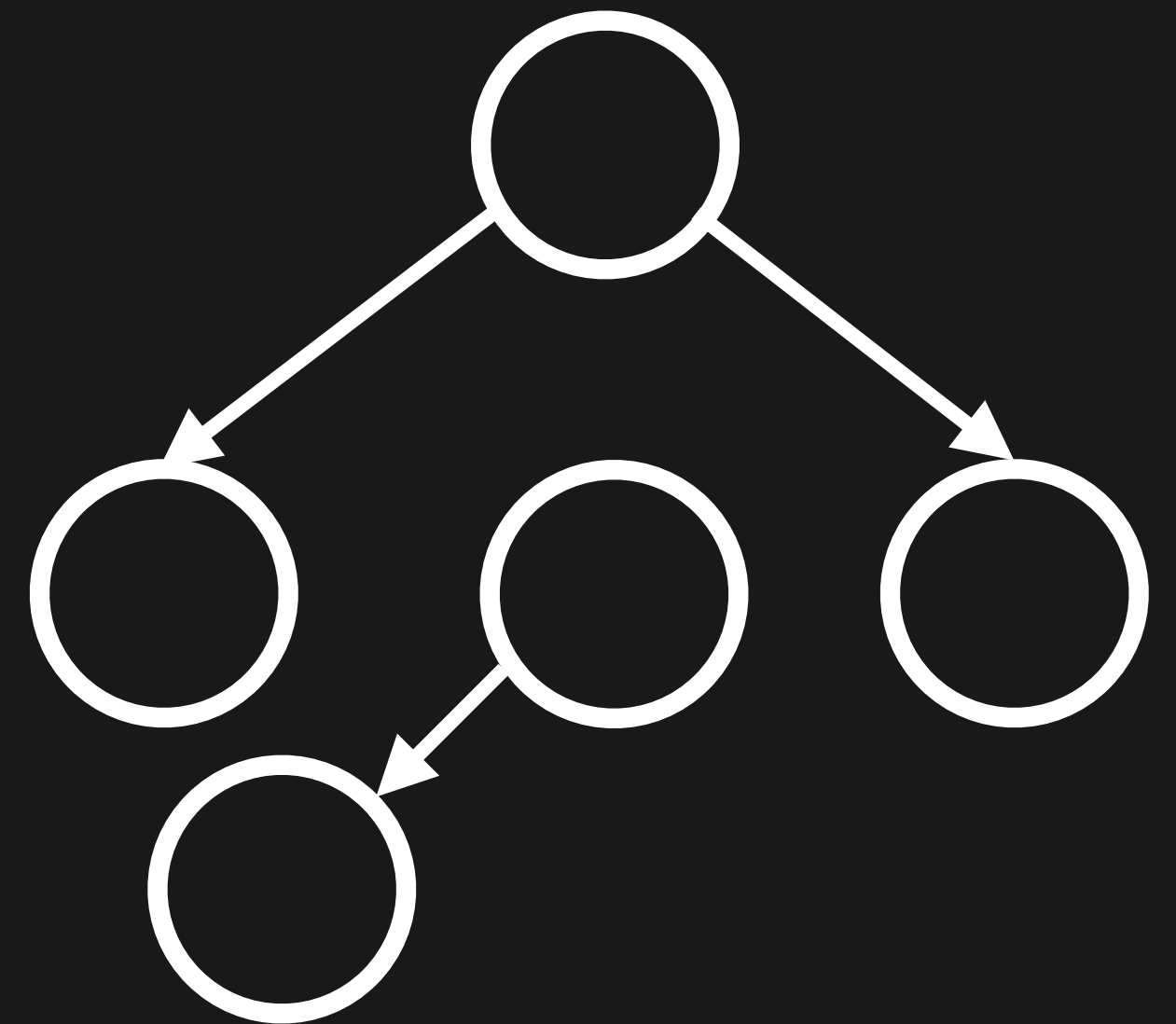
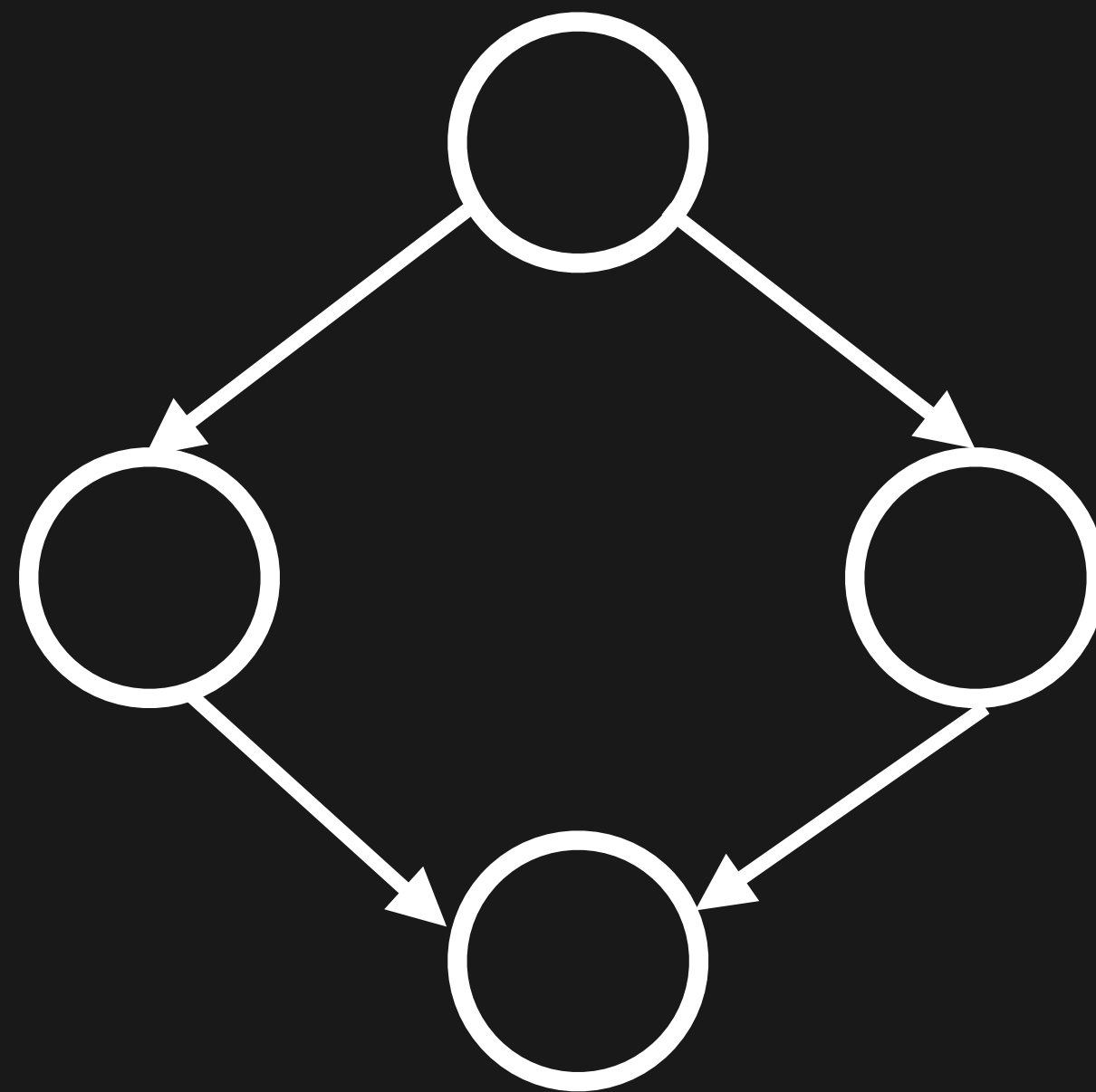
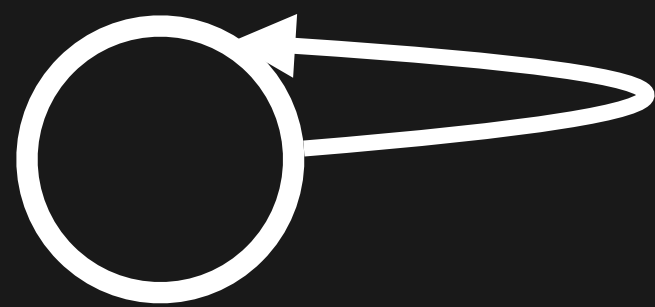
Nodes and edges
(references to child
nodes) without any cycle



TREES



NOT TREES



root

topmost node

parent

converse of child

descendant

node reachable from parent
to child

ancestor

node reachable from child
to parent

leaf / external node

node with no children

internal node

node with at least one child

height (tree)

number of edges on longest downward path from root to leaf

height (node)

number of edges on longest downward path from node to leaf

level

1 + number of edges between the node and the root

depth

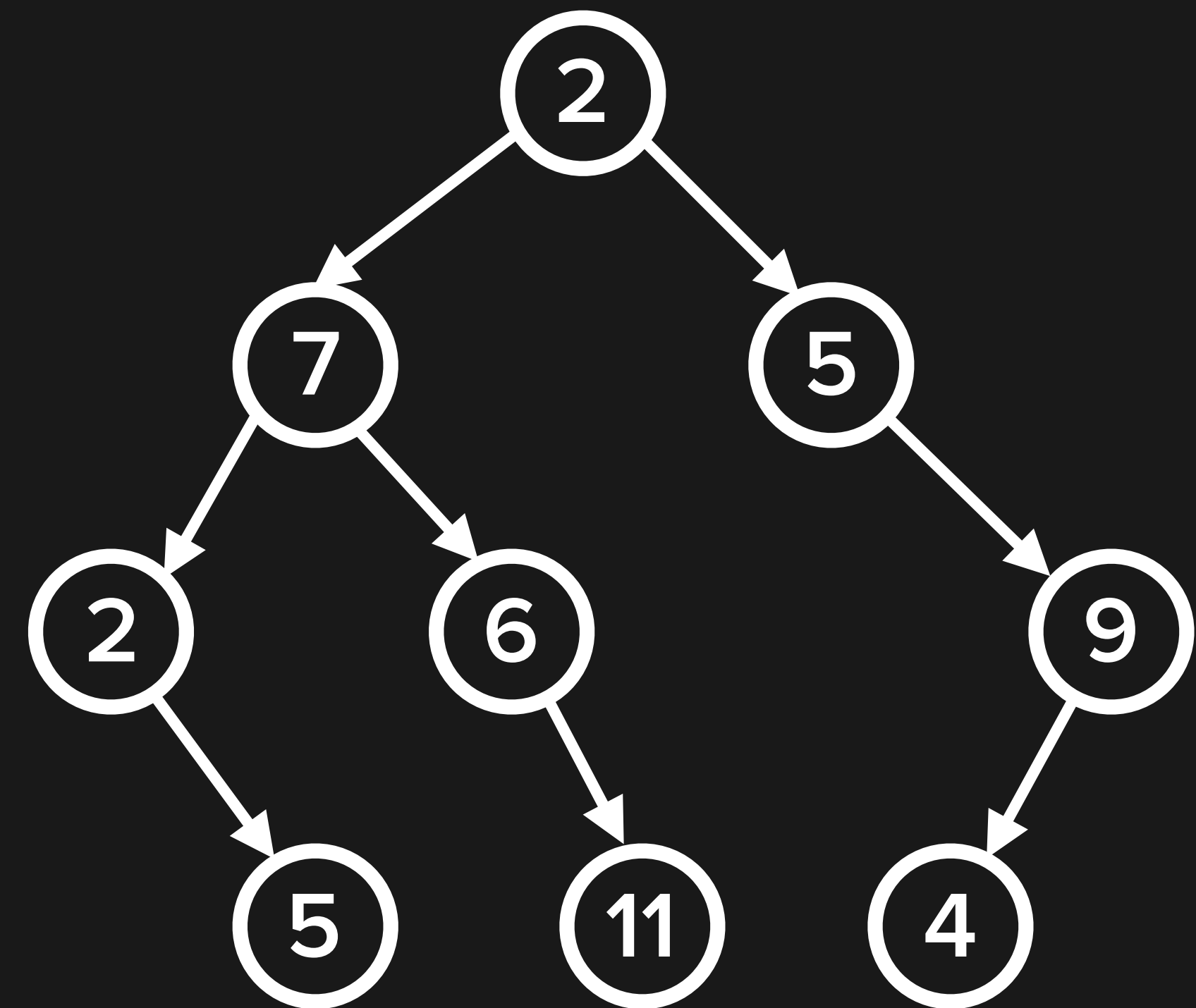
number of edges between the node and the root

size

number of nodes in the tree

BINARY TREE

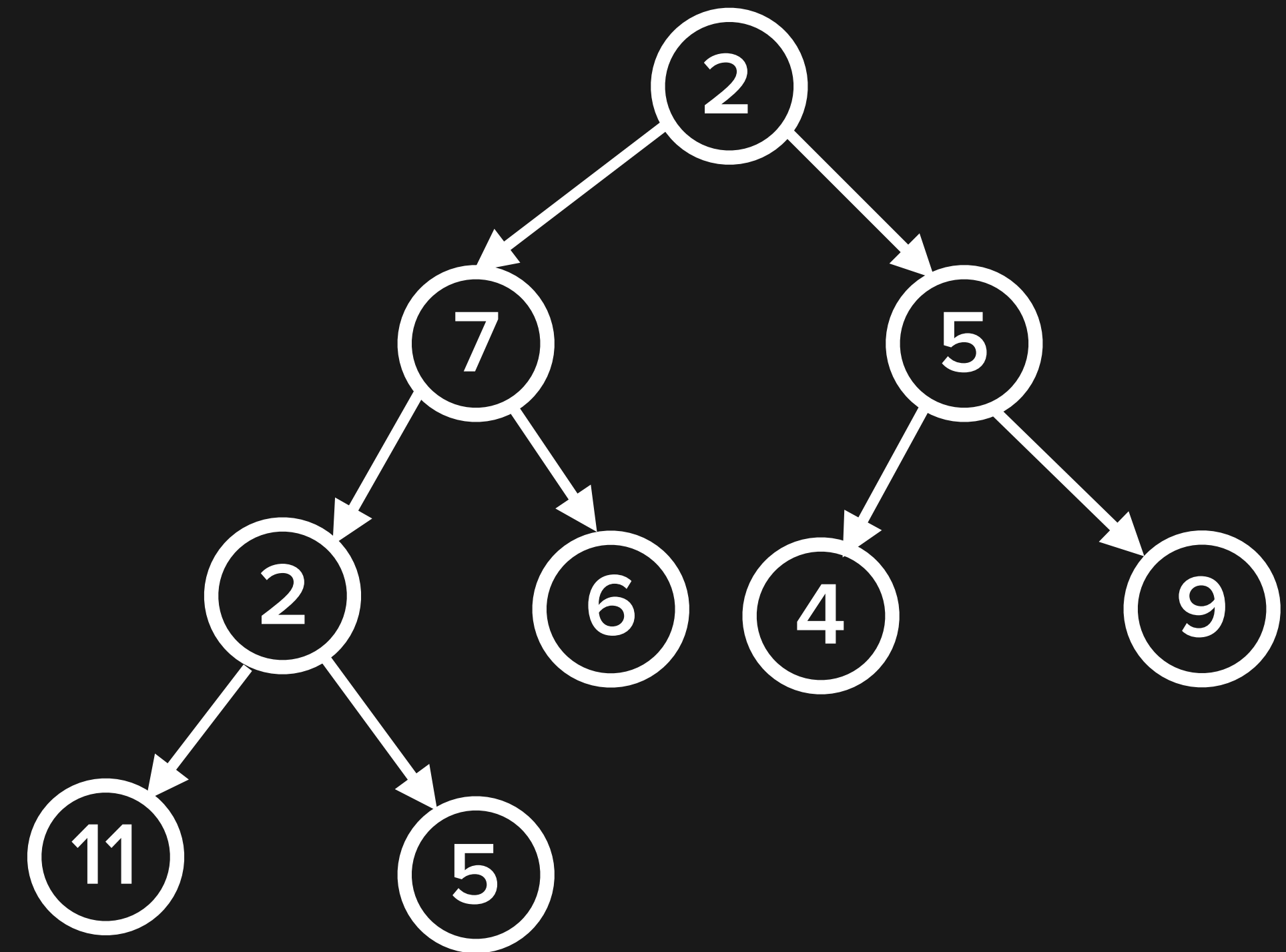
Tree in which each node has at most two children



Size 9 Height 3

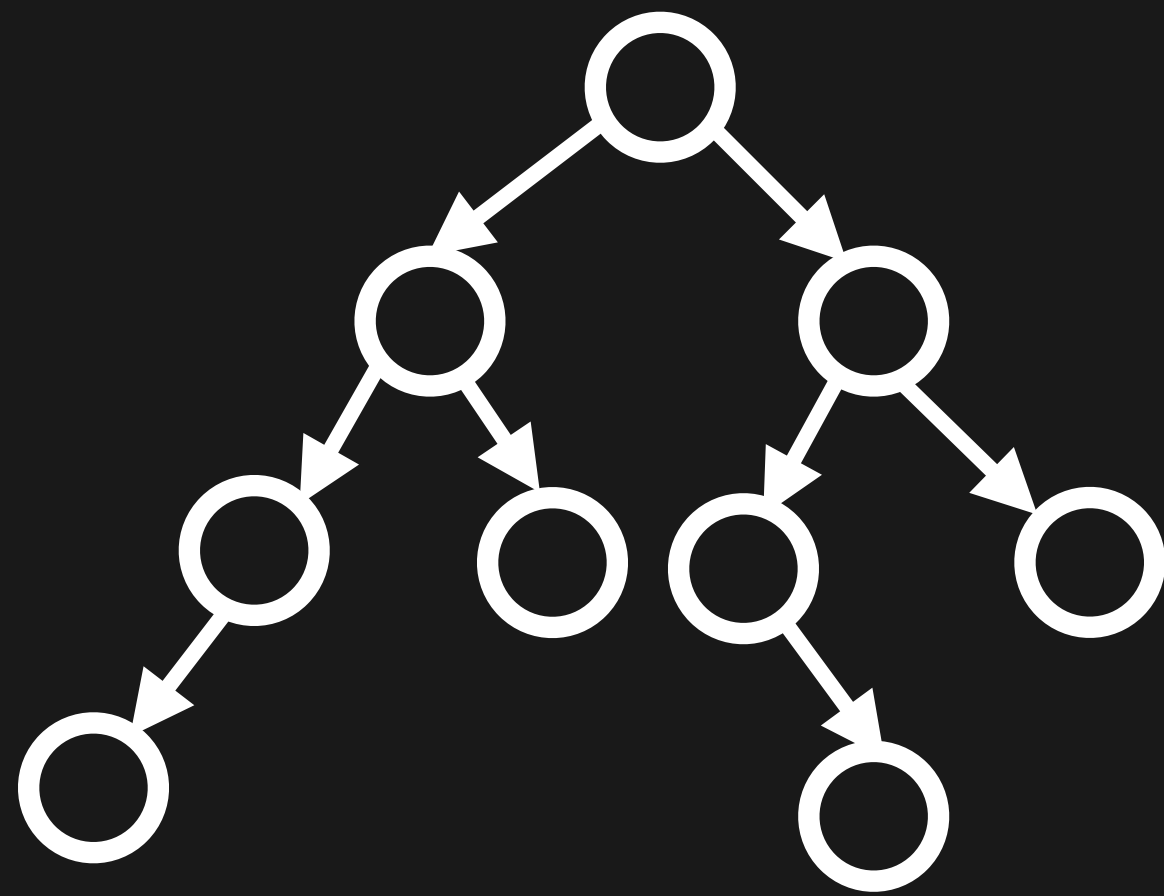
COMPLETE TREE

Every level except possibly last is completely filled and nodes are as far left as possible

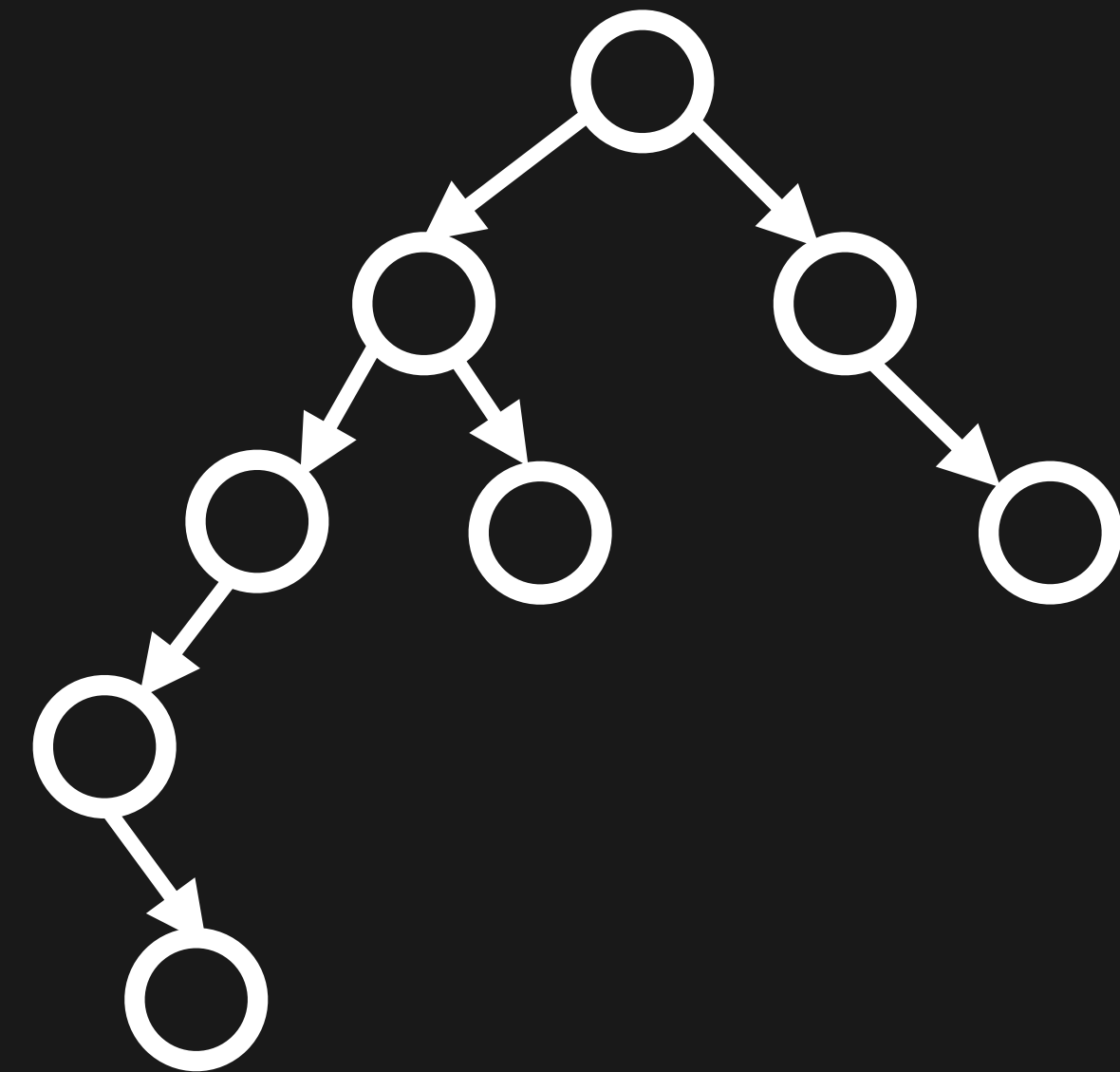


BALANCED TREE

All leaves are at minimum
possible depth



Balanced



Unbalanced

BINARY SEARCH TREE

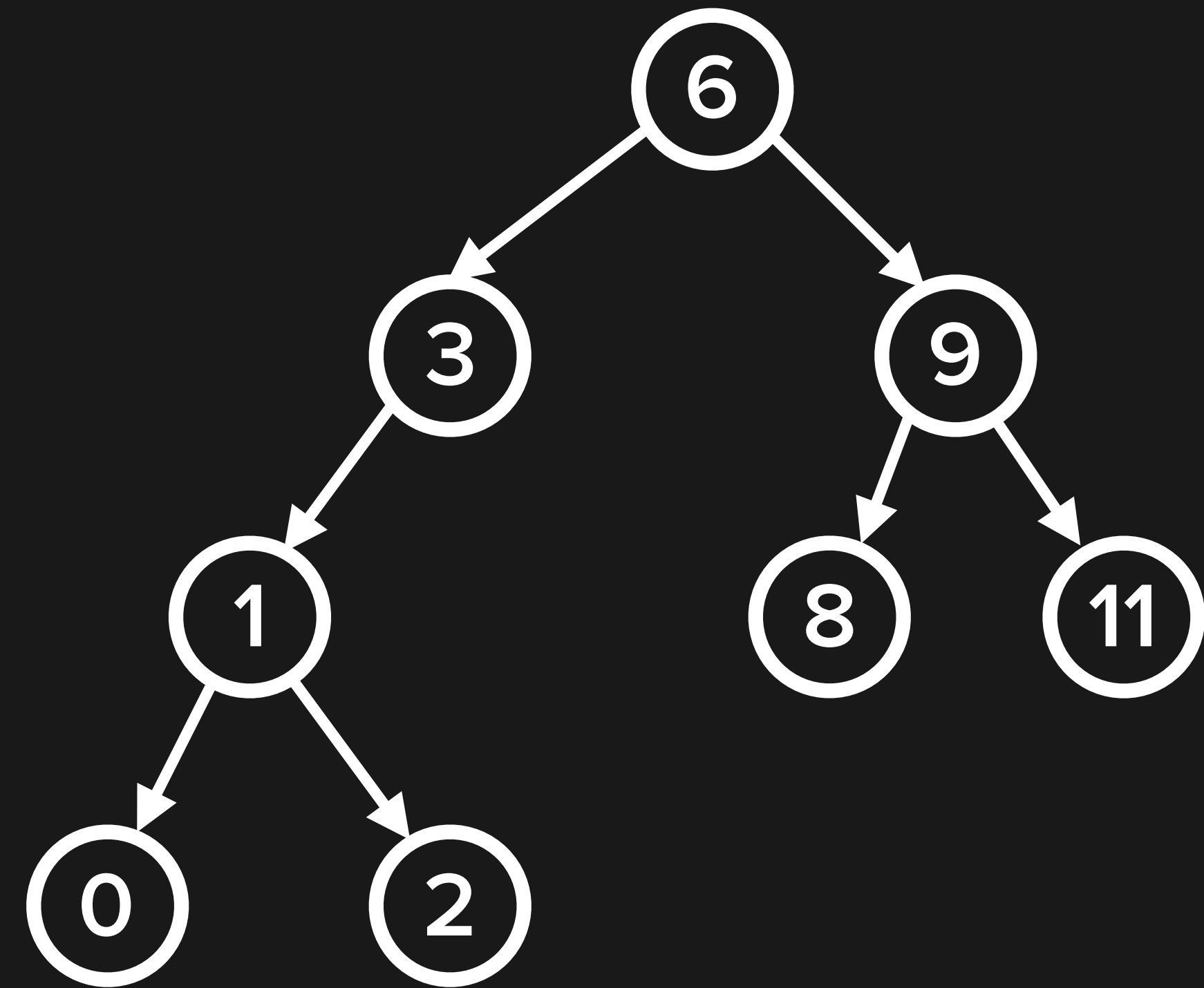
Always sorted

For each node

Left children are smaller

Right children are larger

No duplicate keys



WHY USE A BST?

Fast search, insertion, deletion - especially when balanced

Sort as you go instead of all at once

Fairly simple implementation for good performance

WHY USE A BST?

Only allocates memory as it's needed

Doesn't have to reallocate memory to grow
(like a hash table)

A NOTE ON LOG N

When discussing complexity of computational algorithms, $\log n$ means $\log_2 n$

BINARY LOGARITHM

$$\log_2 n = x \leftrightarrow 2^x = n$$

the power by which 2 must be raised by to obtain n

$$\log_2 16 = 4 \quad \log_2 32 = 5 \quad \log_2 143 = 7.15987$$

SEARCH

```
# call initially with node == root node
def find_recursive(key, node):
    if node is None or node.key == key:
        return node
    elif key < node.key:
        return find_recursive(key, node.left)
    else:
        return find_recursive(key, node.right)
```


INSERTION

Same as search except once you find a node without a child on the next side you're traversing, add it there.

DELETION

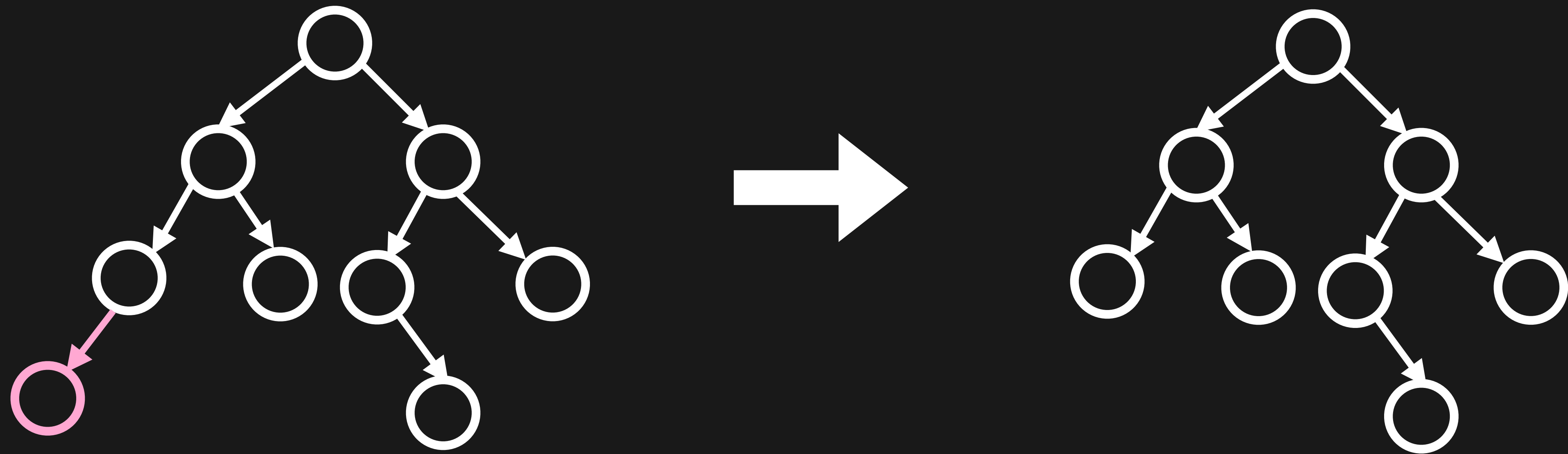
Three cases

No children

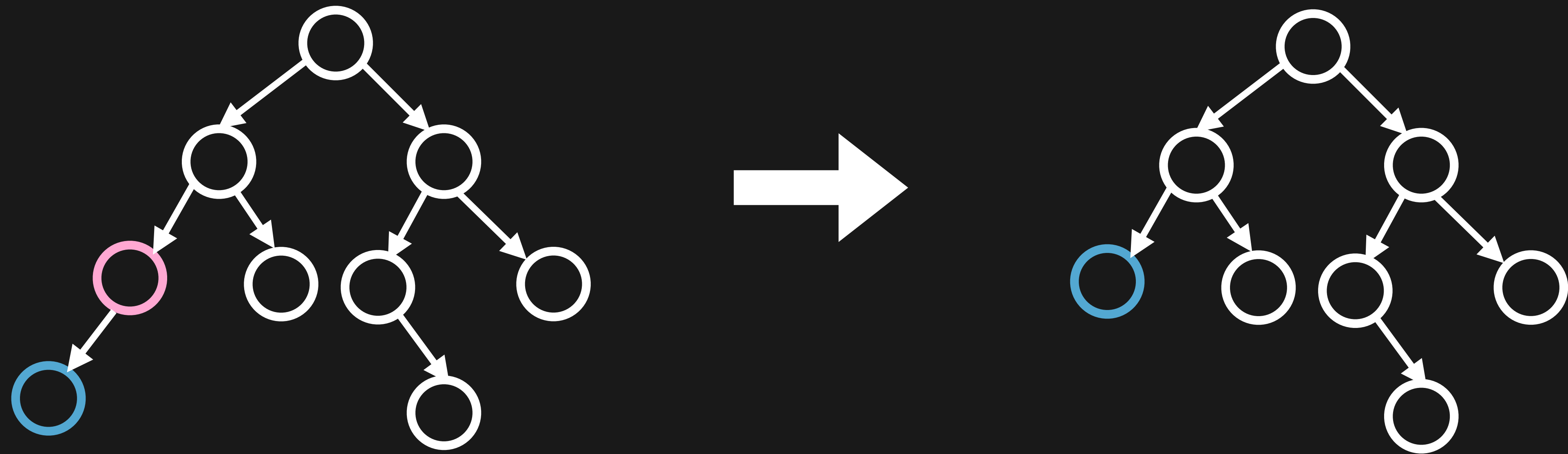
One child

Two children

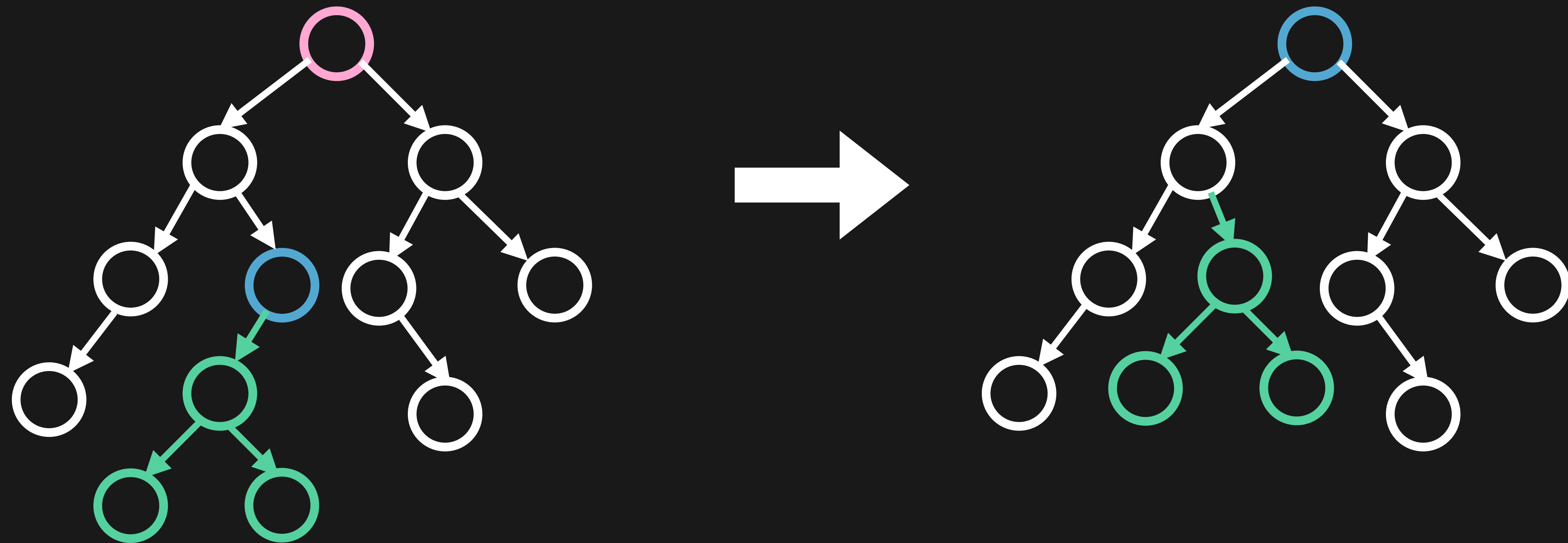
NO CHILDREN



ONE CHILD



TWO CHILDREN



COMPLEXITY

	Average Case	Worst Case
Space	$O(n)$	$O(n)$
Search	$O(\log n)$	$O(n)$
Insert	$O(\log n)$	$O(n)$
Delete	$O(\log n)$	$O(n)$

WHY IS LOG N GOOD?

Imagine a BST with 2^{32} items

2^{32} is 4,294,967,296

When searching, we only have to touch a maximum of 32 nodes to find the node we're looking for



MAKE
SCHOOL