Tightening the Two-Sided Bounds of Nevo and Rosen when Instruments are Weak:

An Application of the Property of Transitivity in Correlations

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DGP

DGP

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Consider the following data generating process:

$$y = x\beta + u$$

OLS

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If x is correlated with u, of course OLS will be biased:

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\begin{aligned}
plim[\beta_{OLS}] &= plim[(x'x)^{-1}(x'y)] \\
&= plim[(x'x)^{-1}(x'(x\beta + u))]
\end{aligned}
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$$\begin{aligned} plim[\beta_{OLS}] &= plim[(x'x)^{-1}(x'y)] \\ &= plim[(x'x)^{-1}(x'(x\beta + u))] \\ &= (x'x)^{-1}(x'x)\beta + plim[(x'x)^{-1}(x'u)] \end{aligned}$$

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&= \beta + \frac{\sigma_{xu}}{\sigma_x^2}
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And the asymptotic bias will be

$$\beta_{OLS} - \beta = \frac{\sigma_{xu}}{\sigma_x^2}$$

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We typically would assume that $\rho_{zu} = 0$ and move on,

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One solution is to find some z s.t. $\rho_{xz} \neq 0$ and use IV

$$\beta_{IV} = (z'x)^{-1}(z'y)$$

We typically would assume that $\rho_{zu} = 0$ and move on,

but here we will think of IV causing problems as well when this fails:

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$$plim[\beta_{IV}] \quad = \quad plim[(z'x)^{-1}(z'y)]$$

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And the asymptotic bias will be

$$\beta_{IV} - \beta = \frac{\sigma_{zu}}{\sigma_{xz}}$$

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4 Cases:

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(will always hold as we can redefine z as $z = -w$).

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- Even in this case, the IV "V(1)" improves upon OLS

New Bounds

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Our Contribution

We Improve Upon these Bounds Further

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- **Transitivity in Correlations**

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- **Transitivity in Correlations**
- Our simulations show that this leads to improved bound and as the IV becomes weaker and more correlated with the unobservables

Transitivity in Correlations

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Two publications in the Statistics Literature establish this property.

Transitivity in Correlations

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We use these properties, which require no additional assumptions, to derive a new set of bounds.

Intuition

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Consider Case 2

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- $\rho_{xu} > 0$, $\rho_{zu} > 0$, $\rho_{xz} < 0$

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- $\rho_{xu} > 0,\ \rho_{zu} > 0,\ \rho_{xz} < 0$
- u is positively correlated with x
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- *transitivity* leads us to believe that u will be negatively correlated with z

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- But $\rho_{zu} > 0$ in this case

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- u is positively correlated with x
- x is negatively correlated with z
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- But $\rho_{zu} > 0$ in this case
- So this is a case of *intransitive correlations*

Intuition

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- But $\rho_{zu} < 0$ in this case

Intuition

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Consider Case 4

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- u is negatively correlated with x
- x is negatively correlated with z
- *transitivity* leads us to believe that u will be positively correlated with z
- But $\rho_{zu} < 0$ in this case
- So this is also a case of *intransitive correlations*

Transitivity in Correlations
Theoretical Background

Theorem 1

Transitivity in Correlations

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Langford et al (2001, *American Statistician*) introduces the concept of transitivity in correlations:

Transitivity in Correlations
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Theorem

A sufficient condition for positive correlation between A and C ($\rho_{AC} > 0$), when $\rho_{AB}\rho_{BC} > 0$ can be stated as follows: $\rho_{AB}^2 + \rho_{BC}^2 > 1 \implies \rho_{AC} > 0$.

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A sufficient condition for positive correlation between A and C ($\rho_{AC} > 0$), when $\rho_{AB}\rho_{BC} > 0$ can be stated as follows: $\rho_{AB}^2 + \rho_{BC}^2 > 1 \implies \rho_{AC} > 0$.

Or: if two ρs are big enough, the 3rd one must go the way we expected!

Transitivity in Correlations

Theoretical Background

Corollary 1

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We provide the following corollary of *intransitivity in correlations*:

Transitivity in Correlations

Theoretical Background

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Corollary

A necessary condition for negative correlation between A and C ($\rho_{AC} < 0$), when $\rho_{AB}\rho_{BC} > 0$, is as follows:
$\rho_{AC} < 0 \implies \rho_{AB}^2 + \rho_{BC}^2 < 1$.

Transitivity in Correlations
Theoretical Background

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Or: if the 3rd one didn't go the way we expected, the first two ρs must not have been big enough!

Transitivity in Correlations
Theoretical Background

Theorem 2

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Lepovotsky and Conklin (2004) extend this research with a second theorem

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Transitivity in Correlations

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Same intuition, just in the "negative" case.

Transitivity in Correlations
Theoretical Background

Corollary 2

Transitivity in Correlations
Theoretical Background

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We again provide a corollary:

Transitivity in Correlations

Theoretical Background

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A necessary condition for positive correlation between A and C ($\rho_{AC} > 0$), when $\rho_{AB}\rho_{BC} < 0$, is as follows:
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Transitivity in Correlations

Application

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Transitivity in Correlations

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Transitivity in Correlations
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- $\rho_{xu} < 0$, $\rho_{zu} < 0$, $\rho_{xz} < 0$
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- So we use Corrolary 1

Transitivity in Correlations
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- Correspondence A=Z, B=X, C=U tells us that
- $\rho_{zu} < 0$ implies $\rho_{xz}^2 + \rho_{xu}^2 < 1$
- Because we would expect $\rho_{zu} > 0$, so there must not be "*too much*" information in the other two ρ terms.

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Transitivity in Correlations

Application

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To Case 2:

- $\rho_{xu} > 0$, $\rho_{zu} > 0$, $\rho_{xz} < 0$
- $\rho_{xz}\rho_{xu} < 0$
- So we use Corollary 2
- Correspondence A=Z, B=X, C=U tells us that
- $\rho_{zu} < 0$ implies $\rho_{xz}^2 + \rho_{xu}^2 < 1$
- Because we would expect $\rho_{zu} < 0$, so there must not be "*too much*" information in the other two ρ terms.

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Solving a quadratic inequality for ρ_{xu}

$$\begin{aligned} \rho_{xu}^2 &< 1 - \rho_{xz}^2 \\ \frac{\sigma_{xu}}{\sigma_x \sigma_u} = \rho_{xu} &\in \pm\sqrt{1 - \rho_{xz}^2} \end{aligned}$$

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Multiplying by $\frac{\sigma_u}{\sigma_x}$

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$$\frac{\sigma_{xu}}{\sigma_x^2} = (\beta_{OLS} - \beta) \quad \in \quad \pm \frac{\sigma_u}{\sigma_x}\sqrt{1 - \rho_{xz}^2}$$

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- We now turn to simulations

Simulation
DGP

DGP

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- $y = .5x + u$

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- $\alpha_2,\ \alpha_4$ are uniform(0,1)
- Other terms are N(0,1)

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- Dataset: N=1,000,000

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- Rows: midpoint of ρ_{xz} bin

Simulation
Counts

Number of Iterations in Each Bin

Simulation

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Number of Iterations in Each Bin

.	.05	.15	.25	.35	.45	.55	.65	.75	.85	.95
-0.75	480	361	329	250	220	182	171	141	139	102
-0.65	1441	1353	1139	1142	991	822	694	672	574	553
-0.55	1538	1459	1495	1442	1403	1325	1214	1147	1078	957
-0.45	1474	1483	1470	1413	1436	1455	1390	1396	1412	1226
-0.35	1471	1526	1408	1447	1447	1413	1448	1465	1361	1281
-0.25	1427	1551	1469	1491	1416	1469	1432	1461	1376	1338
-0.15	1590	1630	1590	1497	1540	1617	1581	1522	1464	1288
-0.05	1991	1791	1761	1740	1656	1779	1707	1663	1511	1416

Simulation
IV

IV

Simulation
IV

IV

.	.05	.15	.25	.35	.45	.55	.65	.75
-0.75	0.492	0.479	0.469	0.458	0.449	0.445	0.439	0.435
-0.65	0.488	0.464	0.443	0.425	0.408	0.393	0.380	0.368
-0.55	0.481	0.446	0.412	0.383	0.355	0.331	0.307	0.286
-0.45	0.471	0.419	0.370	0.323	0.285	0.246	0.209	0.181
-0.35	0.457	0.378	0.302	0.234	0.173	0.114	0.062	0.010
-0.25	0.432	0.305	0.183	0.078	-0.024	-0.126	-0.213	-0.298
-0.15	0.374	0.138	-0.093	-0.298	-0.502	-0.698	-0.862	-1.018
-0.05	-2.940	-3.786	-8.123	-15.890	-11.365	-16.597	-18.315	-22.781

Simulation
Our Lower Bound

Our Lower Bound

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.	.05	.15	.25	.35	.45	.55	.65	.75	.85
-0.75	0.036	0.025	0.006	-0.004	-0.006	-0.024	-0.026	-0.033	-0.03
-0.65	0.074	0.066	0.058	0.053	0.040	0.030	0.015	0.006	-0.00
-0.55	0.095	0.078	0.078	0.074	0.071	0.062	0.056	0.054	0.043
-0.45	0.123	0.107	0.097	0.094	0.081	0.090	0.079	0.069	0.070
-0.35	0.142	0.128	0.129	0.119	0.103	0.097	0.094	0.087	0.084
-0.25	0.179	0.162	0.153	0.139	0.126	0.119	0.116	0.098	0.094
-0.15	0.202	0.183	0.178	0.160	0.150	0.138	0.119	0.121	0.088
-0.05	0.208	0.199	0.199	0.172	0.167	0.150	0.137	0.111	0.089

Simulation
Share with Improvement

Share with Improvement

Simulation

Share with Improvement

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.	.05	.15	.25	.35	.45	.55	.65	.75	.85
-0.75	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
-0.65	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
-0.55	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.007
-0.45	0.000	0.000	0.005	0.035	0.086	0.160	0.261	0.353	0.431
-0.35	0.000	0.056	0.205	0.340	0.459	0.565	0.649	0.706	0.766
-0.25	0.062	0.318	0.521	0.628	0.720	0.799	0.847	0.872	0.900
-0.15	0.242	0.590	0.760	0.846	0.880	0.920	0.937	0.961	0.958
-0.05	0.627	0.894	0.949	0.967	0.982	0.987	0.985	0.996	0.992

Conclusions

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- These are two sided in two of four cases
- Nevo and Rosen improve either lower or upper bound in this case

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Our Contribution

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