

The solution provided is a Python script that solves a shortest path problem using the Bellman-Ford algorithm combined with dynamic programming. The problem aims to find the minimum cost to visit all bathhouses in a grid. Each cell of the grid has a height, and the moving cost between cells depends on the height difference.

First, the script reads the grid from a file using the `read_grid` function. It then creates a graph representation of the grid using the `create_graph` function. This graph is a dictionary where each key is a tuple representing a cell's coordinates, and the value is a list of tuples. These tuples represent the neighboring cells and the costs to move to them.

The `bellman_ford` function implements the Bellman-Ford algorithm to find the shortest path from a source cell to a target cell in the graph. The `shortest_paths` function utilizes the Bellman-Ford algorithm to determine the shortest paths from a source cell to all other cells in the grid. The `shortest_path_to_baths` function uses these shortest paths to find routes from a source cell to all bathhouses.

The `min_cost_to_visit_all_baths` function employs dynamic programming to calculate the minimum cost to visit all bathhouses. It uses a bit mask to represent the state of visited bathhouses and a 2D array `dp` to store the minimum costs. The recursive equation is $dp[visited | (1 \ll next)][next] = \min(dp[visited | (1 \ll next)][next], dp[visited][current] + shortest[current][baths[next]])$. Here, `visited` is the bit mask of visited bathhouses, `current` is the index of the current bathhouse, `next` is the index of the next bathhouse to visit, and `shortest` is a dictionary that stores the shortest paths from each bathhouse to all others.

The time complexity of this algorithm is $O(n^2 * 2^n)$, where n is the number of bathhouses. This complexity comes from the 2^n possible states for the visited bathhouses, and for each state, checking all pairs of bathhouses is necessary.

I also used matplotlib to help me visualize the bathhouses and the connections between the nodes with their respective weights. If you have matplotlib installed feel free to try it out. Here is the test case example that was in the paper:

