

$$\boxed{W \xrightarrow{h} S \xrightarrow{g} T}$$

1. $h: W \rightarrow S$

$$\text{Hom}(h, T): \text{Hom}(S, T) \rightarrow \text{Hom}(W, T)$$

$$\text{Hom}(h, T)(g) \mapsto g \circ h$$

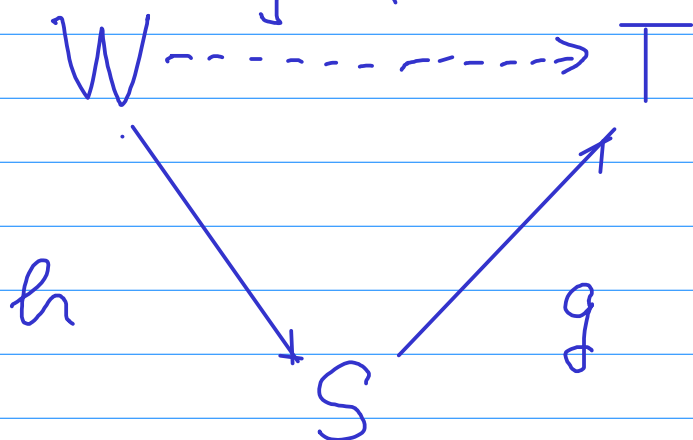
$$\text{Hom}(h, T)(g)(w) = g(h(w))$$

Why
 $|T| \geq 2$?

Show: h is surjective iff

$\text{Hom}(h, T)$ is injective

$$g \circ h = \text{Hom}(h, T)(g)$$



Assume h surjective and that $g_1 \neq g_2$:
 $S \rightarrow T$. To show $g_1 \circ h \neq g_2 \circ h$.

$$\exists x \in S \text{ s.t. } g_1(x) \neq g_2(x)$$

$$\text{Say } g_1(x) = t_1 \text{ and } g_2(x) = t_2$$

$$t_1 \neq t_2$$

Since h is surjective, $\exists w \in W$ s.t.

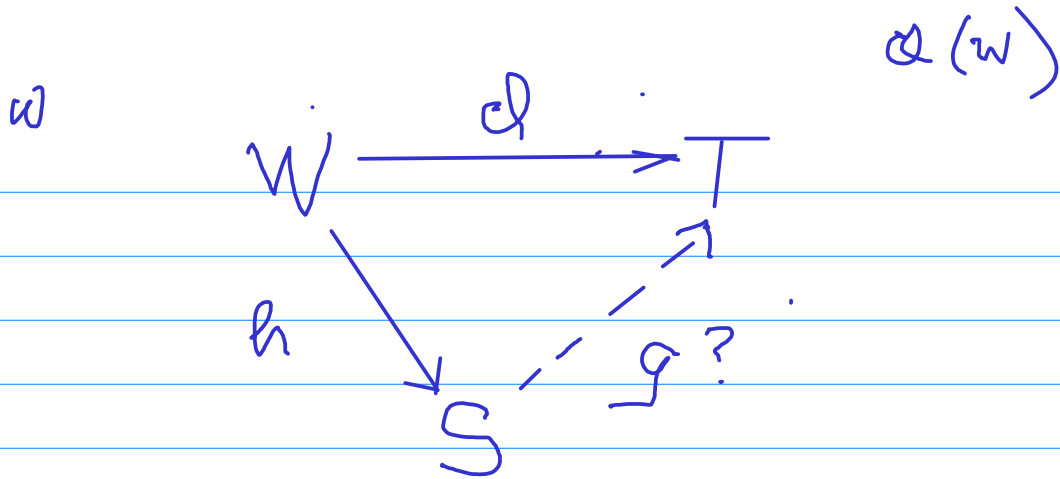
$$h(w) = x. \quad \text{Since}$$

$$\text{Hom}(h, T)(g_1) = g_1 \circ h = t_1$$

$$\text{Hom}(h, T)(g_2) = g_2 \circ h = t_2$$

We see that $g_1 \circ h \neq g_2 \circ h$, so $\text{Hom}(h, T)$ is injective.

Aside: if we had a function $q: W \rightarrow T$ (so $q \in \text{Hom}(W, T)$), then is surjectivity of $h: W \rightarrow S$ enough to induce a function $g: S \rightarrow T$?



The idea: can $g(x)$ be defined by this data for all $x \in S$?

You might think so: since h is onto, $\exists w \in W \Rightarrow h(w) = x$.
So define $g(x) = \alpha(w)$.

BUT NO! there is a huge gap in this argument!

Problem is: g is not

well-defined, because x might have multiple pre-images!

If $h(w_1) = x = h(w_2)$
with $w_1 \neq w_2$, we
don't know that $Q(w_1) =$
 $Q(w_2)$. So what then
is the value of $g(x)$?
 $Q(w_1)$ or $Q(w_2)$?

[If, however, Q is constant on
all h -pre-images of x
(all $x \in S$) Then this would
define g]