CS 170 Cheat Sheet

Big O notation

$f, g \in \mathbb{N}$, $f = O(g)$ means that f grows no faster than g if $\exists c > 0$ s.t. $F(n) \leq cg(n)$
$f = \Theta(g)$ means $g = O(f)$
$f = \Theta(g)$ IFF $f = O(g)$ & $g = \Theta(g)$

Master Theorem

Given: $T(n) = a \times T(\frac{n}{b}) + O(n^d)$

a) $O(n^d)$ if $d > log_b(a)$

b) $O(n^d log(n))$ if $d = log_b(a)$

c) $O(n^{log_b(a)})$ if $d < log_b(a)$

Graph Algorithms

DFS: O(V + E)

Guaranteed to visit every node reachable by v before returning from v. Can create topological sort of DAG.

```
procedure explore(G, v)
Input:    G = (V, E) is a graph; v ∈ V
Output:   visited(u) is set to true for all nodes

visited(v) = true
previsit(v)
for each edge (v, u) ∈ E:
   if not visited(u): explore(u)
postvisit(v)
```

BFS: O(V + E)

Used to find shortest path through an unweighted graph.

```
for all u ∈ V:
   dist(u) = ∞

dist(s) = 0
Q = [s] (queue containing just s)
while Q is not empty:
   u = eject(Q)
   for all edges (u, v) ∈ E:
      if dist(v) = ∞:
         inject(Q, v)
         dist(v) = dist(u) + 1
```

Dijkstras: O((V + E) log V)

Like BFS but with priority queue, used to find shortest path between two nodes on a weighted graph.

```
for all u ∈ V:
   dist(u) = ∞
   prev(u) = nil
dist(s) = 0

H = makequeue(V)  (using dist-values as keys)
while H is not empty:
   u = deletemin(H)
   for all edges (u, v) ∈ E:
      if dist(v) > dist(u) + l(u, v):
         dist(v) = dist(u) + l(u, v)
         prev(v) = u
         decreasekey(H, v)
```

Bellman Ford: O((V E)

Find shortest paths with negative edges as long as there are no negative cycles. Runs $V - 1$ updates on all E edges.

```
procedure update((u, v) ∈ E)
dist(v) = min{dist(v), dist(u) + l(u, v)}
for all u ∈ V:
   dist(u) = ∞
   prev(u) = nil

dist(s) = 0
repeat |V| − 1 times:
   for all e ∈ E:
      update(e)
```

Floyd-Warshall: $O((V^3)$

Find the shortest path between all pairs of vertexes in a graph.

Kruskal: O((E log(V))

Use the disjoint set trees to add edges in ascending order that don't complete a cycle. Used to find MST.

```
for all u ∈ V:
   makeset(u)

X = {}
Sort the edges E by weight
for all edges {u, v} ∈ E, in increasing order of weight:
   if find(u) ≠ find(v):
      add edge {u, v} to X
      union(u, v)
```

Prim's: O((E log(V))

On each iteration, the subtree defined by X grows by one edge, namely, the lightest edge between a vertex in S and a vertex outside S

```
X = { } (edges picked so far)
repeat until |X| = |V| − 1:
  pick a set S ⊂ V for which X has no edges between S and V − S
  let e ∈ E be the minimum-weight edge between S and V − S
  X = X ∪ {e}
```

Cut Property: Suppose edges X are part of a minimum spanning tree of $G = (V, E)$. Pick any subset of nodes S for which X does not cross between S and $V - S$, and let e be the lightest edge across the partition. Then $X \cup e$ is part of some Minimum Spanning Tree.

Huffman Encoding O(n log n)

Make a tree of decisions of whether to pick a 0 or a 1, make all the leaves values. Then we can follow the tree down to decode a Huffman encoding of values.

FFT

It is a black box which represents 2 polynomials as a list of points and then multiplies them together to create a new polynomial. Takes **O(N log N)** time. Uses roots of unity to determine where to multiply two polynomials together.

N^{th} **Roots of Unity** can be found by: $cos(\frac{2\pi j}{n}) + i \cdot sin(\frac{2\pi j}{n})$

Dynamic Programming

These are normally straight forward, mainly we just need to find a recursive relationship for sub problems and then figure out the order in which we need to solve them. Normally runtime can easily be deduced by the number of for loops we have to run to. Following are some basic examples of dynamic programming in case we see something like these on the final.

Edit Distance: $O(n^2)$

```
for i = 0, 1, 2, ..., m:
   E(i, 0) = i
for j = 1, 2, ..., n:
   E(0, j) = j
for i = 1, 2, ..., m:
   for j = 1, 2, ..., n:
      E(i, j) = min{E(i − 1, j) + 1, E(i, j − 1) + 1, E(i − 1, j − 1) + diff(i, j)}
return E(m, n)
```

Knapsack with repetition: $O(nW)$

K(w) is the maximum value we can achieve with weight w.

```
K(0) = 0
for w = 1 to W:
   K(w) = max{K(w − w_i) + v_i : w_i ≤ w}
return K(W)
```

Knapsack without repetition: $O(nW)$

$K(w, j)$ = maximum value achievable using a knapsack of capacity w and items $1, ..., j$.

```
Initialize all K(0, j) = 0 and all K(w, 0) = 0
for j = 1 to n:
   for w = 1 to W:
      if w_j > w:   K(w, j) = K(w, j − 1)
      else:   K(w, j) = max{K(w, j − 1), K(w − w_j, j − 1) + v_j}
return K(W, n)
```

Linear Programming

Linear Programming is a way to state problems as systems of equations and then we can solve them in polynomial time as long as we don't need Integer Linear Programming, which is NP Complete. It can be solved with the simplex method which we don't need to know, can just treat it like a black box.

Remember: when writing out your solutions make sure you add the non-negative constraints.

Max Flow

This is bounded by $O(C \cdot E)$ or $O(V \cdot E^2)$

Max-flow min-cut theorem: the size of the maximum flow in a network equals the capacity of the smallest (s,t) cut.

Maximum flow property: if all edge capacities are integers, then the optimal flow found by our algorithm is integral. We can see this directly from the algorithm, which in such cases would increment the flow by an integer amount on each iteration.

When drawing the residuals make sure you draw the flow you've already used backwards, so some paths with have an arror going forward and an arror going backward.

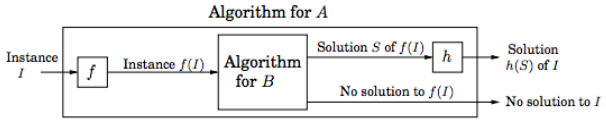
P/NP

P: search problem that can be solved in polynomial time.
NP: search problem that can be checked to be correct in polynomial time.
NP complete: search problem that is at least as hard as every other NP complete problem. Basically it reduces to circuit sat, could possibly be solved in polynomial time but we doubt it.
NP hard: there exists NP complete Y such that Y is reducible to X but can't go the other way around. Not actually in NP.
Here are some examples of **P** and **NP** algorithms:

NP	P
3+ SAT	2 SAT
TSP	MST
Longest Path	Shortest Path
3D Matching	Bipartide matching
Knapsack	Unary Knapsack
Independent Set	Independent Set on Trees
Integer LP	Linear Programming
Rudrata Path	Euler Path
Balanced Cut	Minimum cut

Reductions

A search problem is NP-complete if all other search problems reduce to it.
Make sure that the conversions between the two algorithms are of polynomial size and time, otherwise the reduction does not work.
To reduce X to Y means to find a solution for X using Y.



Dealing with NP

There are different ways to deal with NP, one is to do intelligent but exhaustive searching. This basically involves stopping looking at a branch as soon as we know that what we've selected won't work. This is basically just pruning on a tree.

Approximations

These would be like the example where we use the MST tree to find an approximation which is at most 2 times worse than the optimal solution to the TSP problem.
There are certain classes of approximations which are as follows:

- Those for which, like the TSP, no finite approximation ratio is possible.
- Those for which an approximation ratio is possible, but there are limits to how small this can be. VERTEX COVER, k-CLUSTER, and the TSP with triangle inequality belong here
- Approximability has no limits, and polynomial approximation algorithms with error ratios arbitrarily close to zero exist. KNAPSACK resides here.
- Finally, there is another class of problems, between the first two given here, for which the approximation ratio is about log n. SET COVER is an example

Local Search Heuristics

These can often be useful in practice but provide no guarantees for how well or how fast they will approximate the actual value.