CSCI 411

Advanced Algorithms and Complexity

CSU Chico, Fall 2024

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Today

- Bellman-Ford algorithm

Shortest Path Algorithms

- Given a graph $G = (V, E)$ and a source node $s \in V$, how can we find distances from s to all other nodes?

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  - Dijkstra's algorithm - $O((|E| + |V|) \log(|V|))$
- How can we find distances between all pairs of vertices?

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- How can we find distances between all pairs of vertices?
  - Johnson's algorithm - $O(|V||E| + |V|^2 \log(|V|))$
  - Floyd-Warshall algorithm

Bellman-Ford Algorithm

- Given a graph $G = (V, E)$ with edge weights and $s \in V$, find shortest path distances from s to all $t \in V$
- Iteratively relax distances
  - If $v.dist > u.dist + (u, v).weight$, then there is a shorter path using (u, v)
  - Update $v.dist$ and repeat

Bellman-Ford Pseudocode

```
function Bellman-Ford(G, s)
  for v ∈ V
    v.dist = ∞
  s.dist = 0
  for i from 1 to ITERATIONS
    for (u,v) ∈ E
      if v.dist > u.dist + (u,v).weight
        v.dist = u.dist + (u,v).weight
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- What should ITERATIONS be? Why?
- What kinds of graphs will this work on?
- Is anything missing?

Bellman-Ford Example