Math 101 Homework 1

Name: Leonhard Euler () **Due:**
Email: **Course:**

Problem 1

(a) Prove that there is no rational number whose square is 2.
(b) Let p be a prime number and n an integer greater than 1. Prove that there is no rational number whose nth power is p.

Solution. (a) Suppose that there is a rational number whose square is 2. Then we can find coprime nonzero integers a and b such that $a^2/b^2 = 2$. Then $a^2 = 2b^2$, so a^2 is even. It follows that a is even, so $a = 2k$ for some integer k. Then $2b^2 = 4k^2$, so $b^2 = 2k^2$, which implies that b^2 is even. But then b itself is even, which is a contradiction since a is even and a and b are coprime. Thus, there is no rational number whose square is 2.

(b) If there were such a rational number, then it would be a root of the polynomial $f = x^n - p$. This polynomial is irreducible over $\mathbb{Z}$ by Eisenstein's criterion, and so it is irreducible over $\mathbb{Q}$ by Gauss's Lemma. In particular, f has no roots in $\mathbb{Q}$, there is no rational number whose nth power is p.

Problem 2

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