Locality and Error Mitigation of Quantum Circuits

Quantum Wednesday

Nate Stemen

Apr 19, 2023

Today's Paper

Locality and Error Mitigation of Quantum Circuits

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(Dated: March 14, 2023)

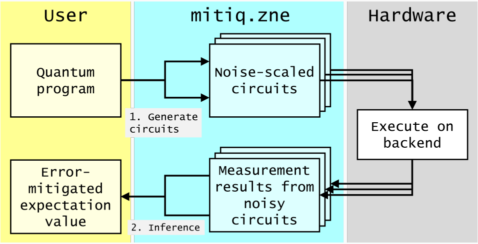
In this work, we study and improve two leading error mitigation techniques, namely Probabilistic Error Cancellation (PEC) and Zero-Noise Extrapolation (ZNE), for estimating the expectation value of local observables. For PEC, we introduce a new estimator that takes into account the light cone of the unitary circuit with respect to a target local observable. Given a fixed error tolerance, the sampling overhead for the new estimator can be several orders of magnitude smaller than the standard PEC estimators. For ZNE, we also use light-cone arguments to establish an error bound that closely captures the behavior of the bias that remains after extrapolation.

`https://arxiv.org/abs/2303.06496`

Zero-Noise Extrapolation

[1]McDonough et al., "Automated quantum error mitigation based on probabilistic error reduction".
[2]Temme, Bravyi, and Gambetta, "Error Mitigation for Short-Depth Quantum Circuits".

Zero-Noise Extrapolation



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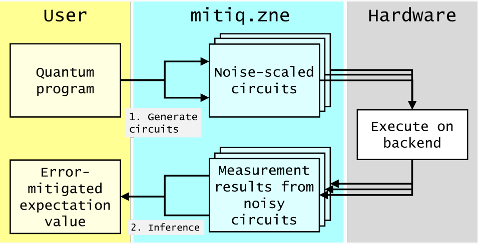

- $\langle A(\lambda)\rangle = \mathrm{tr}[A\rho(\lambda)]$
- biased estimator, but *no knowledge* of noise model needed
- error bounded by (Richardson) extrapolation error

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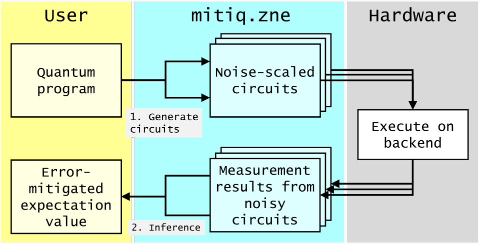

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Probabilistic Error Cancellation

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Review[2]

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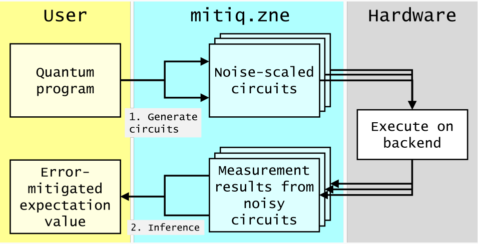

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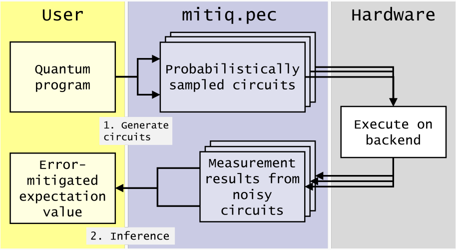

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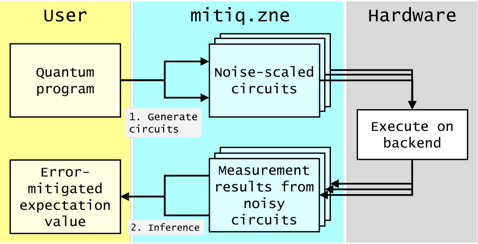

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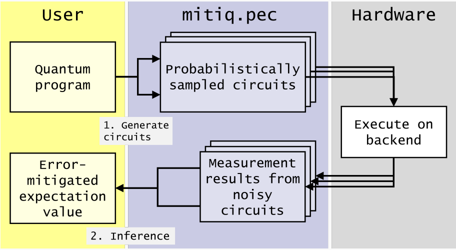
Probabilistic Error Cancellation



- $\langle A \rangle = \mathrm{tr}[A\mathcal{U}(\rho)] = \sum_{\vec{\alpha}} \eta_{\vec{\alpha}} \langle A_{\vec{\alpha}} \rangle_{\text{noisy}}$
- *unbiased* estimator, but requires detailed knowledge of noise model
- more efficient methods exist[1]

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Probabilistic Error Cancellation

Me: Can we have $\mathcal{U} = \mathcal{U}_d \cdots \mathcal{U}_1$

QC: We have unitaries at home

Unitaries at home: $\tilde{\mathcal{U}} = \tilde{\mathcal{U}}_d \cdots \tilde{\mathcal{U}}_1$

$$\tilde{\mathcal{U}}_i = \mathcal{U}_i \mathcal{N}_i$$

Probabilistic Error Cancellation

1. $\tilde{\mathcal{U}} = \tilde{\mathcal{U}}_d \cdots \tilde{\mathcal{U}}_1$ with $\tilde{\mathcal{U}}_i = \mathcal{U}_i \mathcal{N}_i$
2. Write noise as $\mathcal{N}_i = \mathrm{e}^{\mathcal{L}_i}$ where $\mathcal{L}_i(\rho) = \sum_{j=1}^{J} \lambda_{ij} \Big(P_{ij} \rho P_{ij}^\dagger - \rho \Big)$

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5. $\tilde{\mathcal{U}}_{\mathsf{PEC}} = \prod_{i=1}^{d} \tilde{\mathcal{U}}_i \Big(\prod_{j=1}^{J} \mathcal{N}_{ij}^{-1} \Big)$

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6. Probabilistically implement $\mathcal{N}_{ij}$ by applying P_{ij} before $\tilde{\mathcal{U}}_i$ with probability p_{ij}

Local Observable

1. An *observable* is a thing that we can design an experiment to measure.
2. Mathematically, this means a self-adjoint, or Hermitian, operator $A = A^\dagger$.
3. **Local** means A is supported one a subset of the quantum systems at hand.
4. Examples:

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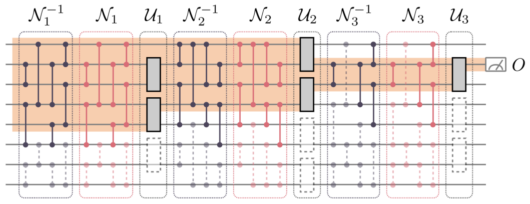
Light Cone



1. $\mu_i = \operatorname{supp}\left(\mathcal{U}_i^\dagger \mathcal{U}_{i+1}^\dagger \cdots \mathcal{U}_d^\dagger (O)\right)$

PEC Results

Unlocal	Local
$\hat{o}_z^{\mathsf{PEC}}(\boldsymbol{\sigma}) = o_z(\boldsymbol{\sigma}) \prod_{i,j} \gamma_{ij}\sigma_{ij}$	$\hat{o}_z^{\mathsf{LoPEC}}(\boldsymbol{\sigma}) = o_z(\boldsymbol{\sigma}) \prod_{(i,j)\in\mu} \gamma_{i,j}\sigma_{i,j}$
$\mathrm{Var}\left[\hat{o}_z^{\mathsf{PEC}}\right] = \mathcal{O}\left(\exp\left[4\sum_{i,j}\lambda_{ij}\right]\right)$	$\mathrm{Var}\left[\hat{o}_z^{\mathsf{LoPEC}}\right] = \mathcal{O}\left(\exp\left[4\sum_{(i,j)\in\mu}\lambda_{i,j}\right]\right)$

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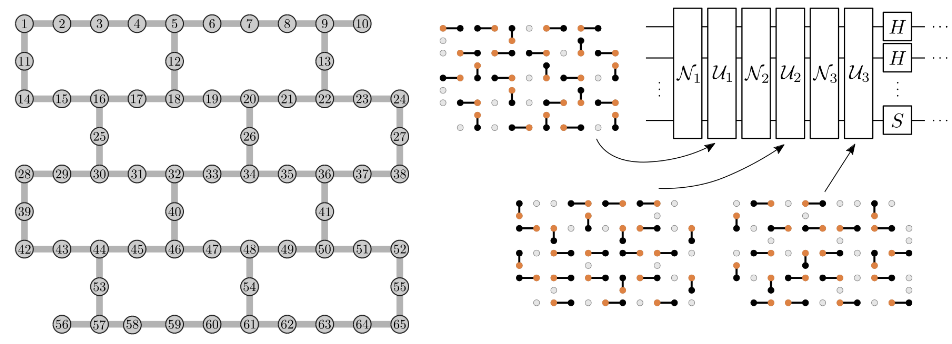

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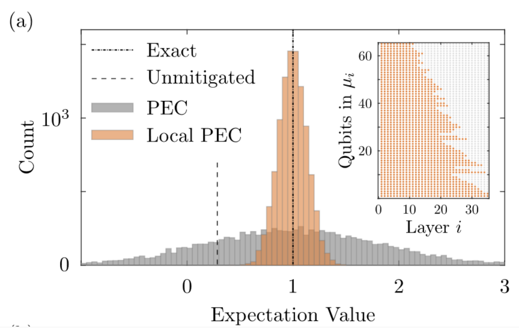
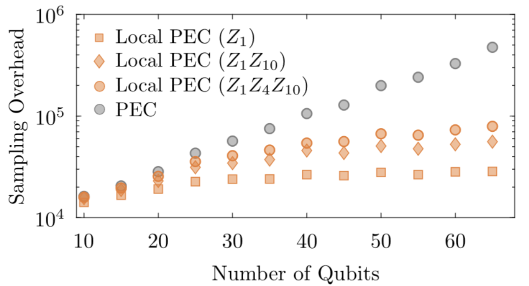
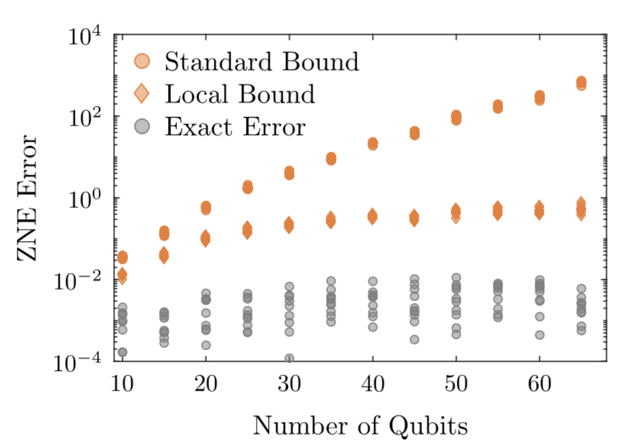




ZNE Results



Standard Bound
Local Bound
Exact Error
ZNE Error
Number of Qubits
10^4
10^2
10^0
10^{-2}
10^{-4}
10
20
30
40
50
60

Questions

1. Does this slot into our existing `execute_with_pec` function?
2. How does this perform as an observable O go from local to “unlocal”.
3. Are the assumptions (efficiently learned)

and then...

V. CONCLUSIONS

In this paper, we analyzed and improved error-mitigation techniques for measuring expectation values of local observables. Our results directly improve the performance of near-term algorithms, which heavily rely on substantial mitigation of errors in quantum devices. In particular, one can use our results and similar light-cone arguments to combine error mitigation with other techniques, including circuit knitting and classical shadow tomography.

Thank you!