



Locality and Error Mitigation of Quantum Circuits

Quantum Wednesday

Nate Stemen

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Locality and Error Mitigation of Quantum Circuits

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In this work, we study and improve two leading error mitigation techniques, namely Probabilistic Error Cancellation (PEC) and Zero-Noise Extrapolation (ZNE), for estimating the expectation value of local observables. For PEC, we introduce a new estimator that takes into account the light cone of the unitary circuit with respect to a target local observable. Given a fixed error tolerance, the sampling overhead for the new estimator can be several orders of magnitude smaller than the standard PEC estimators. For ZNE, we also use light-cone arguments to establish an error bound that closely captures the behavior of the bias that remains after extrapolation.

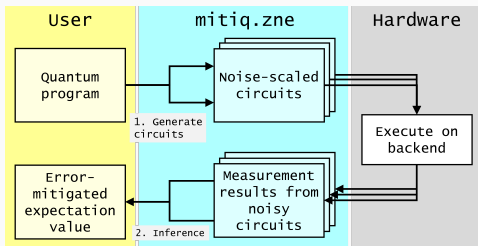
<https://arxiv.org/abs/2303.06496>

Zero-Noise Extrapolation

¹McDonough et al., “Automated quantum error mitigation based on probabilistic error reduction”.

²Temme, Bravyi, and Gambetta, “Error Mitigation for Short-Depth Quantum Circuits”.

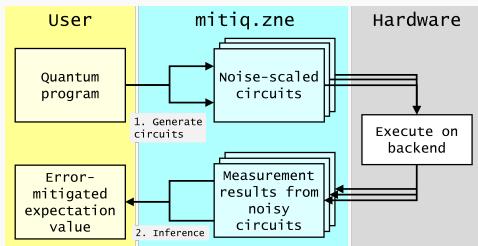
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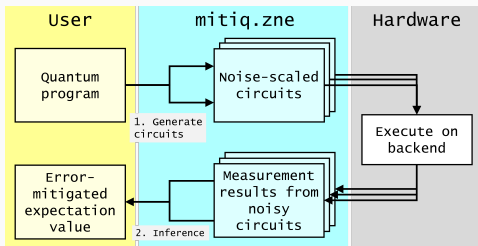
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- biased estimator, but *no knowledge* of noise model needed
- error bounded by (Richardson) extrapolation error

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Probabilistic Error Cancellation

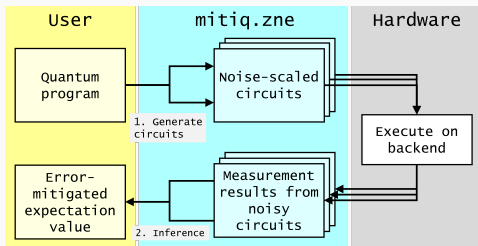


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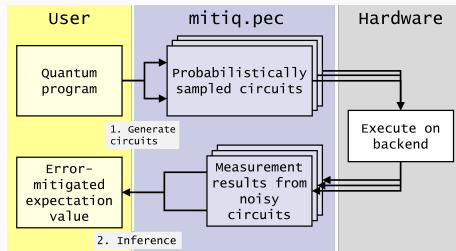
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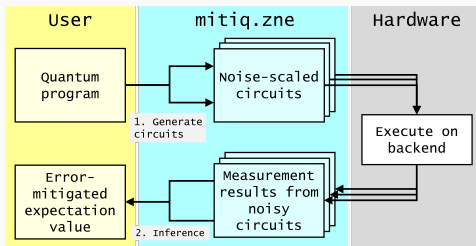


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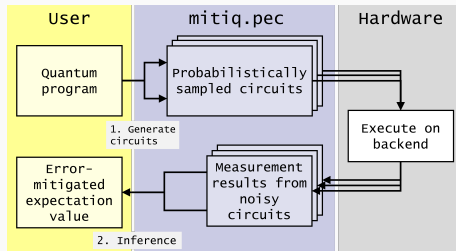
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Probabilistic Error Cancellation



- $\langle A \rangle = \text{tr}[A\mathcal{U}(\rho)] = \sum_{\vec{\alpha}} \eta_{\vec{\alpha}} \langle A_{\vec{\alpha}} \rangle_{\text{noisy}}$
- *unbiased* estimator, but requires detailed knowledge of noise model
- more efficient methods exist¹

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Me: Can we have $\mathcal{U} = \mathcal{U}_d \cdots \mathcal{U}_1$

QC: We have unitaries at home

Unitaries at home: $\tilde{\mathcal{U}} = \tilde{\mathcal{U}}_d \cdots \tilde{\mathcal{U}}_1$

$$\tilde{\mathcal{U}}_i = \mathcal{U}_i \mathcal{N}_i$$

1. $\tilde{\mathcal{U}} = \tilde{\mathcal{U}}_d \cdots \tilde{\mathcal{U}}_1$ with $\tilde{\mathcal{U}}_i = \mathcal{U}_i \mathcal{N}_i$
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6. Probabilistically implement \mathcal{N}_{ij} by applying P_{ij} before $\tilde{\mathcal{U}}_i$ with probability p_{ij}

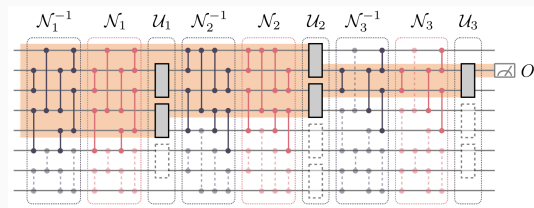
Local Observable

1. An *observable* is a thing that we can design an experiment to measure.
2. Mathematically, this means a self-adjoint, or Hermitian, operator $A = A^\dagger$.
3. **Local** means A is supported on a subset of the quantum systems at hand.
4. Examples:

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4. Examples:

Light Cone



1. $\mu_i = \text{supp}(\mathcal{U}_i^\dagger \mathcal{U}_{i+1}^\dagger \cdots \mathcal{U}_d^\dagger(O))$

Unlocal	Local
$\hat{o}_z^{\text{PEC}}(\boldsymbol{\sigma}) = o_z(\boldsymbol{\sigma}) \prod_{i,j} \gamma_{ij} \sigma_{ij}$	$\hat{o}_z^{\text{LoPEC}}(\boldsymbol{\sigma}) = o_z(\boldsymbol{\sigma}) \prod_{(i,j) \in \mu} \gamma_{i,j} \sigma_{i,j}$
$\text{Var}[\hat{o}_z^{\text{PEC}}] = \mathcal{O}\left(\exp\left[4 \sum_{i,j} \lambda_{ij}\right]\right)$	$\text{Var}[\hat{o}_z^{\text{LoPEC}}] = \mathcal{O}\left(\exp\left[4 \sum_{(i,j) \in \mu} \lambda_{i,j}\right]\right)$

PEC Results

Unlocal

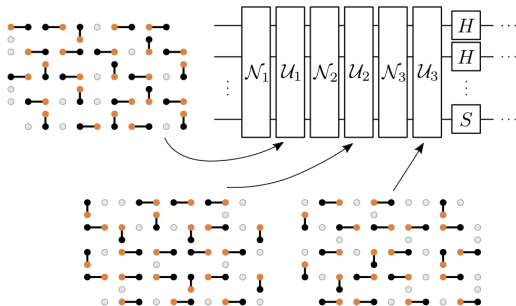
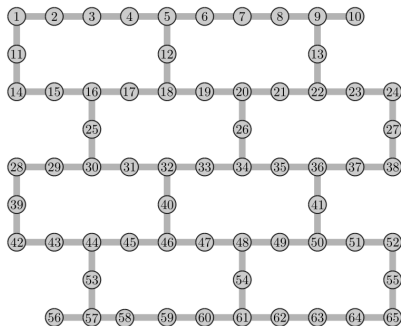
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Local

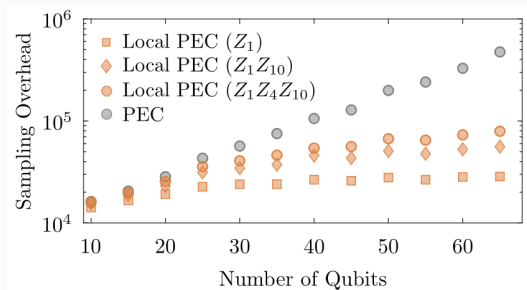
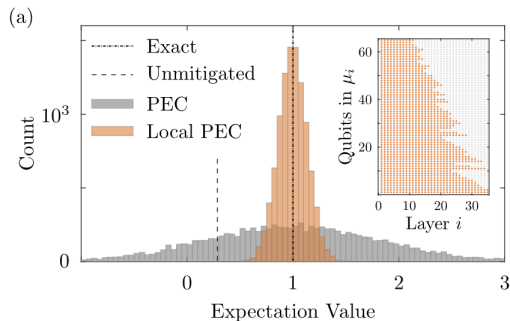
$$\hat{o}_z^{\text{LoPEC}}(\boldsymbol{\sigma}) = o_z(\boldsymbol{\sigma}) \prod_{(i,j) \in \mu} \gamma_{i,j} \sigma_{i,j}$$

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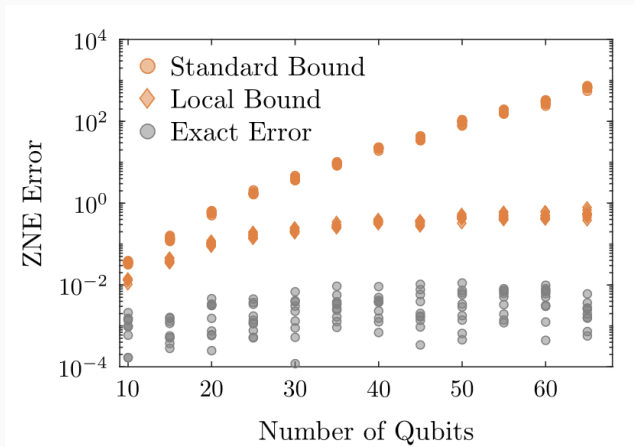


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$\text{Var}[\hat{o}_z^{\text{PEC}}] = \mathcal{O}\left(\exp\left[4 \sum_{i,j} \lambda_{ij}\right]\right)$	$\text{Var}[\hat{o}_z^{\text{LoPEC}}] = \mathcal{O}\left(\exp\left[4 \sum_{(i,j) \in \mu} \lambda_{i,j}\right]\right)$



ZNE Results



1. Does this slot into our existing `execute_with_pec` function?
2. How does this perform as an observable O go from local to “unlocal”.
3. Are the assumptions (efficiently learned)

V. CONCLUSIONS

In this paper, we analyzed and improved error-mitigation techniques for measuring expectation values of local observables. Our results directly improve the performance of near-term algorithms, which heavily rely on substantial mitigation of errors in quantum devices. In particular, one can use our results and similar light-cone arguments to combine error mitigation with other techniques, including circuit knitting and classical shadow tomography.

Thank you!