# Introduction to Algorithms

## I. What is an Algorithm?

- A sequence of clear instructions for problem solving
  - For obtaining a required output for valid input in finite amount of time

## II. Notion of an Algorithm

Problem

↓

Algorithm

↓

Solution

## III. Euclid's Algorithm

- Based on repeated application of equality

$$\gcd(m,n) = \gcd(n, m \bmod n)$$

until the second number becomes 0, w ich makes the first number become the answer

- Euclid's Algorithm: EX 1

$\gcd(n,m)$ becomes $\gcd(m, n \bmod m)$

$\gcd(60,24)$
$= \gcd(24, 60 \bmod 24)$
$= \gcd(24, 12)$
$= \gcd(12, 24 \bmod 12)$
$= \gcd(12, 0)$ // right num = 0, other number solves original gcd
$= 12$

$\boxed{\gcd(60,24) = 12}$

- Input: 2 non-negative, non-zero integers $(m,n)$
- Output: Greatest common divisor of $m$ & $n$

```
while n≠0 do
    r ← m mod n
    m ← n
    n ← r
return m
```

- gcd w/ Middle School Approach

Prime Factorization

gcd(60, 24) = 12

$60 = \underline{2} \cdot \underline{2} \cdot \underline{3} \cdot 5$

$24 = \underline{2} \cdot \underline{2} \cdot 2 \cdot \underline{3}$

$\rightarrow 2 \cdot 2 \cdot 3 = 12$

- Faster than brute force, worse than Euclid's
- Requires second algorithm with very large numbers (Prime Factorization)

# IV. Important Points for Algorithms

- Input must be specified clearly
- The same algorithm can be represented in several different ways
- Several algorithms for solving the same problem may exist
- Algorithms for the same problem can be based on very different ideas and can solve the problem with dramatically different speeds

## V. Sieve of Eratosthenes

- An algorithm to identify prime numbers from 2 to n
- Find prime, then eliminate all multiples of said prime up to n
  - Start eliminating at square of prime