

Denoting clone sizes in prevalence (i.e. absolute number of cells), we obtain the following transition probabilities for a single clone:

$$\begin{cases} \mathbb{P}\{m+1, t+\Delta t \mid m, t\} = \frac{m}{N(t)} \left(1 - \frac{m}{N(t)}\right) \rho N(t) \Delta t + \frac{m}{N(t)} \gamma N(t) \Delta t \\ \mathbb{P}\{m-1, t+\Delta t \mid m, t\} = \frac{m}{N(t)} \left(1 - \frac{m}{N(t)}\right) \rho N(t) \Delta t \\ \mathbb{P}\{m, t+\Delta t \mid m, t\} = 1 - \mathbb{P}\{m+1, t+\Delta t \mid m, t\} - \mathbb{P}\{m-1, t+\Delta t \mid m, t\} \end{cases} \quad (1)$$

Moving to a continuous prevalence picture, the FP equation is given by

$$\partial_t p(m, t) = -\partial_m A(m, t) p(m, t) + \partial_m^2 B(m, t) p(m, t) / 2 \quad (2)$$

where $A(m, t)$ and $B(m, t)$ are given by

$$A(m, t) = \lim_{\Delta t \rightarrow 0} \langle \Delta m \rangle_{\Delta t} / \Delta t \quad (3)$$

$$B(m, t) = \lim_{\Delta t \rightarrow 0} \langle \Delta m^2 \rangle_{\Delta t} / \Delta t \quad (4)$$

Using the above defined transition probabilities these become

$$A(m, t) = m\gamma \quad (5)$$

and

$$B(m, t) = 2m[1 - m/N(t)]\rho + m\gamma \quad (6)$$

Thus the FP equation becomes

$$\partial_t p(m, t) = -\partial_m \gamma m p(m, t) + \partial_m^2 [m(1 - m/N)\rho + m\gamma/2] p(m, t). \quad (7)$$

Note that if there is no growth, i.e. $\gamma = 0$, this reduces to

$$\frac{\partial}{\partial t} p(m, t) = \rho N \frac{\partial^2}{\partial m^2} \frac{m}{N} \left(1 - \frac{m}{N}\right) p(m, t) \quad (8)$$

which reduces to the standard Moran Fokker-Planck equation upon performing the transformation $x(m) = m/N$.

To obtain the expression for the expected prevalence spectrum $v(m, t)$ we must still add the flux of incoming variants, so that finally we have

$$\begin{aligned} \partial_t v(m, t) = & -\partial_m \gamma m v(m, t) + \partial_m^2 [m(1 - m/N)\rho + m\gamma/2] v(m, t) \\ & + 2\mu(\rho + \gamma + \phi/2)\delta(m-1) \end{aligned} \quad (9)$$

While the solution to (8) is known, it is rather unwieldy, which does not bode well for the more complex expressions (7) and (9). We will here use numerical approximations of their solution, though to obtain these we must be cautious about the singularity at $\partial_t v(m=1, t)$ introduced by the incoming flux. We applied a method of lines approach

with a finite difference discretization in the frequency coordinate. In order to optimize performance, we opted for a variable stepsize which is smallest near the singularity, with the distances in prevalence space given by

$$\Delta m_i = 1 + (i - 1) \cdot \alpha_i \quad (10)$$

where $i \in 1, \dots, l - 1$, with l the number of discretized points, and

$$\alpha_i = 2 \frac{N - (l - 1)}{(l - 1)(l - 2)} \quad (11)$$

In this formalism the delta function in (9) is approximated as a step function with height $2/(\Delta m_1 + \Delta m_2)$.