$$\frac{d^2 x}{dt} = -\gamma(\rho(t))\,\frac{dx}{dt} + \sqrt{2D(\rho)}\,\xi(t)$$

$\dot{v} = -\gamma v \ldots \xi$

Assume :

- $*$ slowly varying ρ ; local equilibrium
- $*$ detailed balance $\rightarrow$ fluctuation dissipation (locally ?)

$$\gamma(\rho) = \frac{\langle 1 \rangle}{\langle c \rangle} = \sqrt{E}\sqrt{\frac{\pi}{2}} \cdot \sqrt{2}\,\sigma_c\,\rho = \sqrt{E\pi}\,\sigma_c\,\rho$$

$$D(\rho) = E\,\gamma(\rho)^{-1} =$$

Adiabatic approximation

$\left(dx^2/dt^2 = 0 \right)$

$$\frac{dx}{dt} = \underbrace{\frac{\sqrt{2D(\rho)}}{\gamma(\rho)}}_{A(\rho) = \alpha \cdot \frac{1}{\sqrt{\rho}}}\,\xi(t) = A(\rho)\,\xi(t)$$

$$\boxed{\frac{dx}{dt} = \frac{\alpha}{\sqrt{\rho(x)}}\,\xi(t)}$$

Single particle:

$$\frac{d\vec{x}_i}{dt} = \underbrace{\frac{\cancel{\alpha}}{\sqrt{\rho(\vec{x}_i)}}}_{A(x_i)} \vec{\xi}_i(t)$$

Many body: (follow Dean 1996)

Ito's Lemma:

$$\dot{f}(x_i) = A(x_i)\,\xi_i(t)\,\partial f/\partial x_i + \frac{A^2(x_i)}{2}\partial^2 f/\partial x_i^2$$

$$\left[\text{Introduce} \quad \rho(x,t) = \sum_i \rho_i(x,t) = \sum_{i=1}^{N(t)} \underbrace{\delta(x-x_i)}_{\rho_i(x,t)}\right]$$

$$\dot{f}(x_i) = \int \rho_i(x,t)\left[A(x)\,\xi_i\, f' + \frac{A^2(x)}{2} f''\right]dx$$

$$\int f(x)\,\dot{\rho}_i(x,t)\,dx \overset{\text{P. I.}}{=} \int f(x)\left[-\left(\rho_i(x,t)\,A(x)\,\xi_i\right)' + \left(\rho_i(x,t)\frac{A^2(x)}{2}\right)''\right]dx$$

$$\Rightarrow \quad \dot{\rho}_i(x,t) = \frac{\partial^2}{\partial x^2}\left[\rho_i(x,t)\frac{A^2(x)}{2}\right] - \frac{\partial}{\partial x}\left[\rho_i(x,t)\,A(x)\,\xi_i\right]$$

$\sum_i$

$$\Rightarrow \quad \dot{\rho}(x,t) = \frac{\partial^2}{\partial x^2}\left[\rho(x,t)\frac{A^2(x)}{2}\right] - \sum_i \frac{\partial}{\partial x}\left[\rho_i(x,t)\,A(x)\,\xi_i\right]$$

recast noise field: (Dean 1996)

$$\Rightarrow \quad \boxed{\dot{\rho}(x,t) = \frac{\partial^2}{\partial x^2}\left[\rho(x,t)\frac{A^2(x)}{2}\right] + \frac{\partial}{\partial x}\left[\sqrt{\rho(x,t)}\cdot A(x)\cdot \xi(x,t)\right]}$$

$$\dot{\rho}(x,t) = \frac{\partial^2}{\partial x^2}\left[\rho(x,t)\underbrace{\frac{A^2(x)}{2}}_{\frac{\alpha}{2\rho(x,t)}}\right] + \frac{\partial}{\partial x}\left[\sqrt{\rho(x,t)}\,\underbrace{A(x)}_{\frac{\alpha}{\sqrt{\rho(x,t)}}}\,\xi(x,t)\right]$$

$$= \cancel{\frac{\partial^2}{\partial x^2}\left(\frac{\alpha}{2}\right)} + \frac{\partial}{\partial x}\left[\alpha\,\xi(x,t)\right]$$

$\Downarrow$

$$\dot{\rho}(x,t) = \alpha\frac{\partial}{\partial x}\xi(x,t)$$