Bayesian Basics for True Effect Size Post

Nathan (Nat) Goodman

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Bayesian details

The snippet of math below is Bayes's formula for example at hand. An important nuance is that the "probabilities" are *probability densities* - think R's `dnorm`.

$$P(d_{true} \mid d_{obs}) = \frac{P(d_{true}) \times P(d_{obs} \mid d_{true})}{P(d_{obs})}$$

* The term on the left hand side is what we're trying to compute, namely, the posterior probability distribution of d_{true} for a given value of d_{obs} (0.5 in the running example).

- The first term on the right hand side, $P(d_{true})$, is the prior. For the normal prior in Figures 2 and 3, this is R's `dnorm` with $mean = 0.3$ and $sd = 0.2$.
- The next term, $P(d_{obs} \mid d_{true})$, is the probability distribution of d_{obs} for a given d_{true}. For the running example, this is a noncentral t-distribution.
- The denominator, $P(d_{obs})$, is the probability of a given value of d_{obs} across all values of d_{true}. To compute this, you integrate the numerator for d_{true} ranging from $-\infty$ to ∞.

Below is R code to compute the posterior for the examples in this post.

```
## Compute the posterior probability density of d.true given d.obs for the examples in this post
##   d.true, d.obs are standardized effect sizes
##   n is the sample size per group
##   prior is a function giving the prior probability density of d.true; see examples below
posterior=function(d.true,d.obs,n,prior) {
  ## probability of d.obs given d.true for the examples at hand
  ##   d_d2t is my function for probability density of the noncentral t in terms of sample size, d.true
  ##   see code in https://natgoodman.github.io/repwr/stats.stable.html
  P_obsGIVENtrue=function(d.true) d_d2t(n=n,d0=d.true,d=d.obs);
  ## numerator in Bayes formula
  numerator=function(d.true) prior(d.true)*P_obsGIVENtrue(d.true);
  ## denominator in Bayes formula
  P_obs=integrate(function(d.true) numerator(d.true),-Inf,Inf)$value;
  ## final answer
  numerator(d.true)/P_obs;
}

## example priors
## uniform centered on d0
prior_unif=function(d0,u) {
  span=u/2;
  lim=d.obs+c(-span,span);
  function(d.true) dunif(d.true,d0-span,d0+span);
}
## normal with given mean and sd
prior_norm=function(mean.prior,sd.prior)
  function(d.pop) dnorm(d.pop,mean=mean.prior,sd=sd.prior);
```

```
## Example
posterior(d.true=seq(0,1,by=0.2),d.obs=0.5,n=20,prior=prior_norm(mean=0.3,sd=0.2));
## [1] 0.258453797 1.534427917 2.274534502 0.841955050 0.077840115 0.001797645
```

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