Bayesian Basics for Estimating True Effect Size

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This working paper shows an R implementation of Bayes's formula as used in the blog post What's the True Effect Size? It Depends What You Think to estimate true effect size in two group difference-of-mean studies. The main point is that the implementation is really, really simple. With all the fuss and bother about Bayesian methods, I imagined it would be incredibly hard to code. Happily not.

Bayes's Formula

The line of math below is Bayes's formula for the example at hand. An important nuance is that the "probabilities" are *probability densities* - think R's `dnorm`.

$$P(d_{pop} \mid d_{obs}) = \frac{P(d_{pop}) \times P(d_{obs} \mid d_{pop})}{P(d_{obs})}$$

1. The term on the left hand side is what we're trying to compute, namely, the posterior probability distribution of d_{pop} for a given value of d_{obs} (0.5 in the blog post's running example).
2. The first term on the right hand side, $P(d_{pop})$, is the prior. For the normal prior in Figures 1 and 2 of the post, this is R's `dnorm` with $mean = 0.3$ and $sd = 0.1$.
3. The next term, $P(d_{obs} \mid d_{pop})$, is the probability distribution of d_{obs} for a given d_{pop}. For the two group difference-of-mean studies in the post, this is a noncentral t-distribution centered on d_{obs}.
4. The denominator, $P(d_{obs})$, is the probability of a given value of d_{obs} across all values of d_{pop}. This is the integral from $-\infty$ to ∞ of the numerator in Bayes's formula.

R Implementation

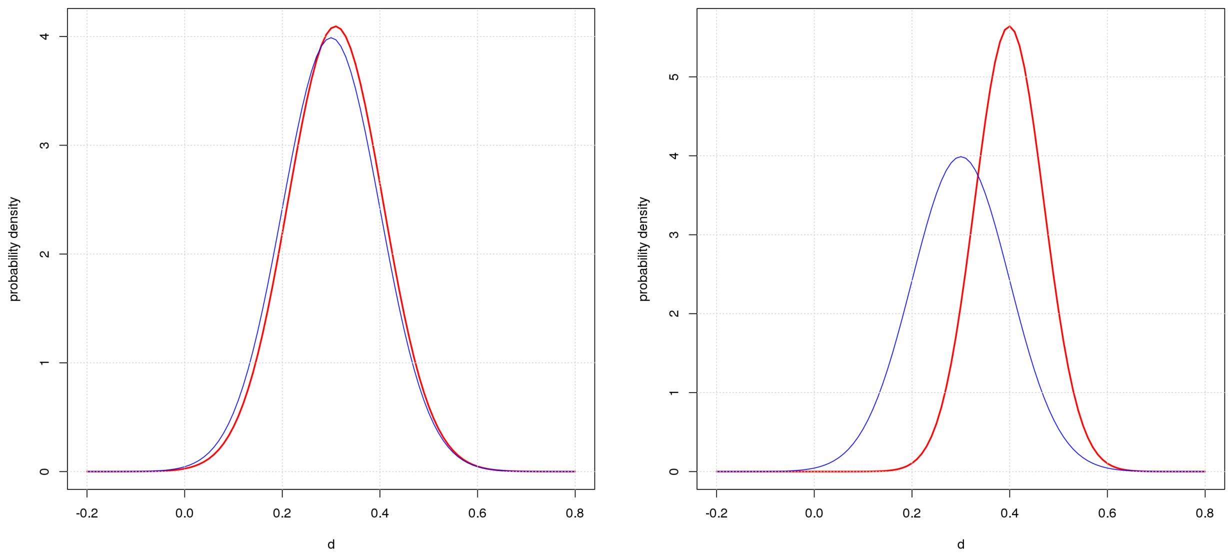
Below is R code to compute the posterior for the examples in this post. The code works: just copy-and-paste into an R session and it'll plot two graphs similar to Figures 1 and 2 in the blog post. The code is also available in the file `R/baysx.R` in my GitHub repo.

```
## Return function for posterior probability of d.pop given d.obs for two group
## difference-of-mean studies with equal sample size and unit standard deviation
##   n is sample size per group
##   d.obs is standardized observed effect size
##   prior is function of d.pop, eg, function(d.pop) dnorm(d.pop,mean=0.3,sd=0.1)
##
posterior=function(n,d.obs,prior) {
  ## probability of d.obs given d.pop for examples at hand. dd2t defined below
  P_obsGIVENpop=function(d.pop) dd2t(n,d.pop,d.obs);
  ## numerator in Bayes formula
  numerator=function(d.pop) prior(d.pop)*P_obsGIVENpop(d.pop);
  ## denominator in Bayes formula
  P_obs=integrate(function(d.pop) numerator(d.pop),-Inf,Inf)$value;
  ## final answer
  P_popGIVENobs=function(d.pop) numerator(d.pop)/P_obs;
}
```

```
## probability density of noncentral t in terms of n, d.pop, d.obs
##   adapted from https://natgoodman.github.io/repwr/stats.stable.html
dd2t=function(n,d.pop,d.obs) {
  df=2*(n-1);
  t=d.pop*sqrt(n/2);
  sqrt(n/2)*suppressWarnings(dt(t,df=df,ncp=sqrt(n/2)*d.obs));
}

## examples with normal prior as in Figures 1 and 2 of the post
##   each graph plots the posterior for the given value of n,
##   as well as the prior (dnorm(d.pop,mean=0.3,sd=0.1)) for reference
## n=10
dev.new();
dpost=posterior(n=10,d.obs=0.5,prior=function(d.pop) dnorm(d.pop,mean=0.3,sd=0.1));
curve(dpost,from=-0.2,to=0.8,col='red',lwd=2,xlab='d',ylab='probability density');
curve(dnorm(x,mean=0.3,sd=0.1),col='blue',add=T);
grid();
## n=200;
dev.new();
dpost=posterior(n=200,d.obs=0.5,prior=function(d.pop) dnorm(d.pop,mean=0.3,sd=0.1));
curve(dpost,from=-0.2,to=0.8,col='red',lwd=2,xlab='d',ylab='probability density');
curve(dnorm(x,mean=0.3,sd=0.1),col='blue',add=T);
grid();
```

The resulting graphs are



The code exploits an advanced R feature that may be unfamiliar to some readers, namely, the ability to treat functions like other kinds of data. You can store a function in a variable, pass a function as an argument to another function, create a new function on-the-fly and store it in a variable, call the newly created function via the variable, and return a function as the result of a function.

I use this feature repeatedly.

1. I pass the prior probability function (`dnorm(d.pop,mean=0.3,sd=0.1)`) to `posterior` as the argument `prior`.

2. Each line of posterior creates a function: e.g., the line `P_obsGIVENpop=function(d.pop) dd2t(n,d.pop,d.obs);` creates a function of `d.pop` that calls the function `dd2t` on variables `n`, `d.pop`, and `d.obs`. A further detail is that variables `n` and `d.obs` are *bound* to the values passed into `posterior` as arguments, while `d.pop` is *free* and gets its value later when the new function `P_obsGIVENpop` is called.
3. The final result is likewise a function of `d.pop`: `function(d.pop) numerator(d.pop)/P_obs;`
4. The examples store the function in the variable `dpost`, then use `curve` (a base R function) to call `dpost` and plot the result.

Web Resources I Found Helpful

The web is awash in Bayesian material, but I found most of it inscrutable. Here are web pages that were simple enough to get me started on the path to understanding. Thanks! I appreciate the helping hand!

- Bayesian models in R by Francisco Lima, posted on R-bloggers
- Probability concepts explained: Bayesian inference for parameter estimation by Jonny Brooks-Bartlett, posted on Medium.

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