(A + B)'s 2 days' work $= \frac{1}{9}+\frac{1}{7}=\frac{7}{36}$. Evidently, the work done by A and B during 5 pairs of days $= 5\times\frac{7}{36}=\frac{35}{36}$

Remaining work $= 1-\frac{35}{36}=\frac{1}{36}$

Now, $\frac{1}{9}$ work is done by A in 1 day

$\therefore$ $\frac{1}{36}$ work, will be done by A in

$9\times\frac{1}{36}=\frac{1}{4}$ day

So, total time taken $= 10\frac{1}{4}$ days.

19. Ans. (c)

B's 23 days' work $=\frac{23}{40}$

Remaining work $=1-\frac{23}{40}=\frac{17}{40}$

Now, (A+B)'s 1 day's work

$=\frac{1}{45}+\frac{1}{40}=\frac{17}{360}$

$\frac{17}{360}$ work is done by A and B in 1 day

$\therefore$ $\frac{17}{40}$ work is done by A and B in

$\frac{360\times17}{17\times40}=9$ days

so, A left after 9 days.

20. Ans. (a)

(A+B)'s 1 day's work $=\frac{1}{30}+\frac{1}{50}=\frac{8}{150}$

Also (A + C)'s 1 day's work $=\frac{1}{30}+\frac{1}{40}=\frac{7}{120}$

$\therefore$ Work done in first 2 days

$=\frac{8}{150}+\frac{7}{120}=\frac{67}{600}$

Work done in $8\times2=16$ days

$=\frac{67\times8}{600}=\frac{67}{75}$

Work left $=1-\frac{67}{75}=\frac{8}{75}$

On 17 th day(A + B) will work and they will finish $\frac{8}{150}$

$\therefore$ Work left $=\frac{8}{75}-\frac{8}{150}=\frac{8}{150}=\frac{4}{75}$

On 18th day (A + C) will work and they will finish it in $\frac{120}{7}\times\frac{4}{75}=\frac{32}{35}$ days

$\therefore$ Whole work will be done in $17\frac{32}{35}$ days

□□□□

Pipes & Cisterns

I Practice Exercise: I

1. One tap can fill a eastern in 2 hours and another can empty the cistern in 3 hours. How long will they take to fill the cistern if both the taps are opened?
   (a) 6 hours (b) 7 hours
   (c) 6.30 hours (d) None of these

2. A tap can fill a tank in 25 minutes and another can empty it in 50 minutes. Find whether the tank will be filled up or emptied and in how many minutes?
   (a) Tank is filled up in 50 minutes
   (b) Tank is emptied in 25 minutes
   (c) Tank is filled up in 25 minutes
   (d) None of these

3. Two taps A and B can fill a tank in 10 hours and 15 hours, respectively. If both the taps are opened together the tank will be full in:
   (a) 5 hours (b) 6 hours
   (c) 8 hours (d) None of these

4. Two pipes can fill a tank in 10 hours and 12 hours, respectively. While a third pipe emptied the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time the tank will be filled?
   (a) 7 hours 30 minutes
   (b) 8 hours 40 minutes
   (c) 8 hours 30 minutes
   (d) None of these

5. A cistern is normally filled in 8 hours but takes 2 hours longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in
   (a) 38 hours (b) 48 hours
   (c) 40 hours (d) None of these

6. If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?
   (a) 36 hours (b) 30 hours
   (c) 40 hours (d) None of these

7. One inlet pipe A is 3 times faster than second inlet pipe B and takes 32 minutes less than the inlet pipe B. When will the cistern be full if both pipes are opened together?
   (a) 28 minutes (b) 12 minutes
   (c) 30 minutes (d) Data inadequate

and 3 minutes, respectively. They are turned on alternately for 1 minute each, how long will it take for the cistern to [illegible]
   (a) 4 min 40 sec (b) 3 min 20 sec
   (c) 4 min 50 sec (d) 3 min 30 sec

9. A cistern is provided by two taps A and B. A can fill it in 20 minutes and B in 25 minutes. Both the taps are kept open for 5 minutes and then the second is turned off. The cistern will be completely filled in another
   (a) 11 minutes (b) 10 minutes
   (c) 15 minutes (d) 12 minutes

10. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty a full cistern?
   (a) 8 minutes (b) 10 minutes
   (c) 12 minutes (d) 16 minutes

❑❑❑❑

Solutions

1. Ans. (a)

   Part of the cistern filled in 1 hour

   $$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

   ∴ Total time taken to fill the cistern = [illegible]

2. Ans. (a)

   Part of the tank filled or emptied in 1 min

   $$= \frac{1}{25} - \frac{1}{50} = \frac{1}{50}$$

   Which is positive, therefore the tank will be filled.

   ∴ Total time taken to fill the tank = 50 minutes

3. **Ans. (b)**

Part of the tank filled in 1 hour

$= \frac{1}{10} + \frac{1}{15} = \frac{1}{16}$

$\therefore$ Total time taken to fill tank = 6 hours

4. **Ans. (a)**

Part of the tank filled in 1 hour

$= \frac{1}{10} + \frac{1}{12} - \frac{1}{20} = \frac{2}{15}$

$\therefore$ Total time taken to fill the tank

$= \frac{15}{2}$ = 7 hours 30 minutes

5. **Ans. (c)**

Let the leak can empty the full cistern is x hours, then

$$\left(\frac{1}{8} - \frac{1}{x}\right) = \frac{1}{10}$$

$$\Rightarrow \frac{1}{8} - \frac{1}{10} = \frac{1}{x}$$

$\therefore$ $x = 40$ hours

6. **Ans. (b)**

Let one pipe takes x hours to fill the reservoirs.

Then, another pipe takes $(x - 10)$ hours.

$$\therefore \frac{1}{x} + \frac{1}{x-10} = \frac{1}{12}$$

$$\Rightarrow x(x-10) = 12(x + x - 10)$$

$$\Rightarrow x^2 - 34x + 120 = 0$$

or $(x-30)(x-4) = 0$

$\therefore$ $x = 30$ or $x = 4$

$\therefore$ The faster pipe takes 30 hours to fill the reservoir.

7. **Ans. (b)**

Let A requires x minutes to fill the tank then

B requires 3x minutes

It is given that $3x - x = 32$

So $2x = 32$

$\Rightarrow$ $x = 16$

When both A and B are opened then

$\frac{1}{16} + \frac{1}{48} = \frac{1}{12}$ part in 1 minute

So total 12 minutes.

8. **Ans. (a)**

As the pipes are operating alternately, thus their 2 minutes job is $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

In the next 2 minutes the pipes can fill another $\frac{5}{12}$ part of cistern. Therefore, in 4 minutes the two pipes which are operating alternately will fill $\frac{5}{12} + \frac{5}{12} = \frac{10}{12} = \frac{5}{6}$ part.

The part of the cistern left unfilled = $1 - \frac{5}{6} = \frac{1}{6}$

Pipe A can fill $\frac{1}{4}$ of the cistern in 1 minutes.

Pipe A can fill $\frac{1}{6}$ of the cistern in

$4 \times \frac{1}{6} = \frac{2}{3}$ minutes

Total time taken to fill the cistern

$4 + \frac{2}{3} = 4\frac{2}{3}$ min.

Or, 4 minutes 40 seconds.

9. **Ans. (a)**

Part filled in 1 minute $\frac{1}{20} + \frac{1}{25} = \frac{9}{100}$

Part filled in 5 minutes = $\frac{9}{100} \times 5 = \frac{9}{20}$

Unfilled part = $1 - \frac{9}{20} = \frac{11}{20}$

This is to be filled by A alone and hence will be filled in $20 \times \frac{11}{20}$ = 11 minutes.

10. Ans. (b)

Work done by waste pipe in 1 minutes

$$= \left(\frac{1}{12}+\frac{1}{15}\right)-\frac{1}{20}$$

$$= \left(\frac{3}{20}-\frac{1}{20}\right)=\frac{1}{10}$$

∴ Waste pipe can empty the cistern in 10 minutes.

□□□□

Pipes & Cisterns

II Practice Exercise: II

1. Two taps A and B can fill a tank in 10 hours and 15 hours respectively. If both the taps are opened together, the tank will be full in:
   (a) 5 hrs. (b) 6 hrs.
   (c) $12\frac{1}{2}$ hrs (d) $7\frac{1}{2}$ hrs.

2. Two pipes can fill a tank in 10 hours and 12 hours respectively while a third pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time the tank will be filled?
   (a) 7 hrs (b) 8 hrs
   (c) 7 hrs 30 min. (d) 8 hrs 30 min.

3. An electric pump can fill a tank in 3 hours. Because of a leak in the tank, it took 3½ hours to fill the tank. The leak can drain out all the water of the tank in:
   (a) 10½ hrs (b) 12 hrs
   (c) 21 hrs. (d) 24 hrs

4. Taps A and B can fill a bucket in 12 minutes and 15 minutes respectively. If both are opened [illegible] further time would it take [illegible] bucket?
   (a) 7 min. 45 sec (b) 7 min 15 sec
   (c) 8 min. 5 sec (d) 8 min 15 sec

5. If two pipes function simultaneously reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the second one. How many hours it takes the second one to fill the reservoir?
   (a) 25 hrs (b) 28 hrs
   (c) 30 hrs (d) 35 hrs

6. 12 buckets of water fill a tank when the capacity of each bucket is 13.5 litres, how many buckets will be needed to fill the same tank, if the capacity of each bucket is
   (a) 8 (b) 16
   (c) 15 (d) 18

7. Bucket P has thrice the capacity as bucket Q. It takes 60 turns for bucket P to fill an empty drum. How many turns it will take for both the buckets P and Q, having each to fill the empty drum?
   (a) 30 (b) 40
   (c) 45 (d) 90

8. Two pipes A and B can fill a cistern in 12 minutes and 15 minutes respectively and a third pipe C can empty the full tank in 5 minutes, A and B are kept open for 5 minutes in the begining and then C is also opened what time is the cistern emptied?
   (a) 30 min. (b) 33 min.
   (c) $37\frac{1}{2}$ min. (d) 45 min.

9. Three pipes A, B and C can fill a tank in 6 hours. After working at it together for 2 hours C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the cistern, is:

(a) 12
(c) 14
(b) 12
(d) 16

12. Two pipes A and B can fill a cistern in 12 minutes and 16 minutes respectively. If both the pipes are opened together then after how much time B should be closed so that the tank is full in 9 minutes?

(a) $3\frac{1}{2}$ min
(b) 4 min
(c) $4\frac{1}{2}$ min
(d) $4\frac{3}{4}$ min

Solutions

1. **Ans. (b)**

A's 1 hour's work = $\frac{1}{2}$ B's hour's work = $\frac{1}{15}$

(A + B)'s 1 hour's work

$$=\left(\frac{1}{10}+\frac{1}{15}\right)=\frac{5}{30}=\frac{1}{6}$$

∴ Both the taps can fill the tank in 6 hours.

2. **Ans. (c)**

Net part filled in 1 hour

$$=\left(\frac{1}{10}+\frac{1}{12}-\frac{1}{20}\right)=\frac{6}{60}=\frac{2}{15}$$

∴ The tank will be full in $\frac{15}{2}$ hrs.

= 7 hrs. 30 min.

3. **Ans. (c)**

Let the leak can empty the tank in x hours.

then

$$\frac{1}{3}-\frac{1}{x}=\frac{2}{7}$$

$$\Rightarrow \frac{1}{x}=\left(\frac{1}{3}-\frac{2}{7}\right)=\frac{1}{21}$$

4. **Ans. (d)**

Part filled in 3 min

$$=3\left(\frac{1}{12}+\frac{1}{15}\right)=\left(3\times\frac{9}{60}\right)=\frac{9}{20}$$

Remaining part $=\left(1-\frac{9}{20}\right)=\frac{11}{20}$

Part filled by B in 1 min $=\frac{1}{15}$

∵ $\frac{1}{15}$ part in 1 minute

then 1 part in 15 minute

∴ $\frac{11}{20}$ part in $\frac{11}{20}\times 15=\frac{33}{4}$ minute

= 8 minute 15 second

5. **Ans. (c)**

Let the reservoir be filled by first pipe in x hours. The second pipe will fill in (x + 10) hours.

$$\therefore \frac{1}{x}+\frac{1}{x+10}=\frac{1}{12}\Leftrightarrow\frac{x+10+x}{x(x+10)}=\frac{1}{12}$$

$$\Rightarrow x^2-14x-120=0$$

$$\Rightarrow (x-20)(x+6)=0 \Rightarrow x=20$$

∴ Second pipe takes 30 hrs to fill the reservoir.

6. **Ans. (d)**

Capacity of the tank = (12 × 13.5) litres

= 162 litres

Capacity of each bucket = 9 litres

Number of buckets needed $=\left(\frac{162}{9}\right)=18$

7. **Ans. (c)**

Let capacity of P be x litres

Then capacity of Q = $\frac{x}{3}$ litres

Capacity of the drum = 60x litres

Required number of cans

8 Time, Speed & Distance

ıe term **'Time' and 'Distance'** are related to speed or velocity of a moving object.

ɔeed

define the speed of an object as the distance velled by it in per unit time interval. It is tained by dividing the distance covered by the ject, by the time it takes to cover that distance. us,

$$\text{Speed/velocity} = \frac{\text{Distance Travelled}}{\text{Time taken}}$$

ɔtes:

If the time taken is constant, the distance travelled is proportional to the speed, that is more the speed; more the distance travelled in the same time.

If the speed is constant, the distance travelled is proportional to the time taken that is more the distance travelled; more the time taken at the same speed.

If the distance travelled is constant, the speed is inversely proportional to the time taken, that is, more the speed; less the time taken for the same distance travelled.

1. $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$
2. $\text{Distance} = \text{Speed} \times \text{Time}$
3. $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Units of Measurement

Generally, if the distance is measured in kilometers, we measure time in hours and speed in kilometer per hour and is written as km/hr; and if distance is measured in metres then time is taken in second and speed in metre per second and is written as m/sec.

Conversion of Units

$$\text{km/hour} = \frac{1000 \text{ meter}}{60 \times 60 \text{ seconds}} = \frac{5}{18} \text{ m/sec}$$

$$\therefore \quad 1 \text{ m/sec} = \frac{18}{5} \text{ km/hr}$$

$$\text{Thus, } x \text{ km/hr} = \left(x \times \frac{5}{18}\right) \text{ m/sec}$$

$$\text{and, } x \text{ m/sec} = \left(x \times \frac{18}{5}\right) \text{km/hr}$$

Ex. 1: Raman can cover a distance of 30 km in 3 hours in his bicycle, calculate the speed of Raman.

Solution:

Speed = $\frac{\text{Distance Covered}}{\text{Time taken}}$ = [illegible] = [illegible]

Ex. [illegible] Sameer can cover a distance of [illegible] km by travelling at the speed of [illegible] km/hr. Find the time taken by him to cover the distance.

Solution:

Time = $\frac{\text{Distance}}{\text{Speed}}$ = [illegible] = [illegible] hours

Ex. [illegible] A train running at a speed of [illegible] km/hr crosses a pole on the platform in [illegible] seconds. Find the length of the train in metres.

Solution:

Speed of the train = [illegible] km/hr

= [illegible] × $\frac{5}{18}$ = [illegible] m/s

∴ Length of the train = Speed of the train × time taken crossing the pole

⇒ [illegible] × [illegible] = [illegible] m

Relative Speed

CASE I: Same Direction

When objects are moving in same direction relative speed is equal to difference between speed of two objects.

Relative speed

$= S_1 - S_2$

Case II: Opposite Direction

(face to face) (back to back)

When objects are moving in opposite direction facing each other then

Relative speed = $(S_1 + S_2)$

face to face

back to back

Concept of Acceleration

Acceleration is defined as [illegible] speed.

Acceleration can be positive [illegible] negative (speed decreases) [illegible] deceleration.

Acceleration = $\frac{\text{Final Speed} - \text{Initial Speed}}{\text{Time taken}}$

The unit of acceleration is [illegible].

For instance, if a [illegible] at [illegible] m/s and speed increases to [illegible] in [illegible] seconds then

Acceleration = [illegible] = [illegible]

Final speed = Initial speed + [illegible]

Acceleration due to gravity

This is the acceleration [illegible] of earth g = [illegible]

$v = u + gt$

Where

u = initial speed

v = final speed

g = acceleration due to gravity [illegible]

t = time taken

$S = ut + \frac{1}{2}gt^2$

where S = distance covered

Application of time speed & di[illegible]

A. Problems on Trains

Trains are a special case in questions [illegible] to time, speed and distance [illegible] have their own length and [illegible]

The basic relation for trains [illegible]

Speed × Time = Distance

The following things need to be [illegible] before solving questions [illegible]

(a) when the train is crossing a moving object, the speed has to be taken as reltive speed of the train with respect to the object. All the rules for relative speed will apply for calculating relative speed.

The distance to be covered when crossing an object, wherever a train crossess an object will be equal to :

Length of train + length of object

Thus, the following cases will yield separate equations, which will govern the crossing of the object by the train.

For each of the following situations the following rotions have been used :

S_T = Speed of train

L_T = Length of train

S_o = Speed of object

L_o = Length of object

t = time taken

Case I: Train crossing a stationary object (pole, standing human being etc.) without length

$$\text{Then, time taken} = \frac{\text{Length of train}}{\text{Speed of train}}$$

$$t = \frac{L_T}{S_T}$$

Case II: Train crossing a stationary object with length : (eg → plateform, Railway tunnel etc.)

Then, time taken =

$$\frac{\text{Sum of length of train and stationary object}}{\text{Speed of train}}$$

$$t = \frac{L_T + L_o}{S_T}$$

Case III: Train crossing a moving object without length.

Then time taken =

$$\frac{\text{Length of train}}{\text{Relative speed of train and moving object}}$$

→ In opposite direction:

$$t = \frac{L_T}{(S_T + S_o)}$$

→ In same direction:

$$t = \frac{L_T}{(S_T - S_o)}$$

Case IV: Train crossing a moving object with length:

Then, time taken =

$$\frac{\text{Sum of length of train and moving object}}{\text{Relative speed of train and moving object}}$$

In opposite direction.

$$t = \frac{L_T + L_o}{S_T + S_o}$$

→ In same direction

$$t = \frac{L_T + L_o}{S_T - S_o}$$

Ex. 1: A train crosses a pole in 15 seconds If the length of the train is 300 metres, find the speed of the train.

Solution: $t = \frac{L_T}{S_T}$

So $\frac{L_T}{t} = \frac{300}{15} = 20 \text{m/sec.}$

Ex. 2: Two trains of length 120 metres and 80 metres are running on parallel lines in opposite directions with the speed of 35 km/hr and 55 km/hr respectively. In what time will they pass each other.

Solutions: $t = \frac{L_T + L_o}{S_t + S_o}$

here $S_T + S_o = 35 + 55 = 90$ km/hr

$= 90 \times \frac{5}{18} = 25$ m/sec

So, $t = \frac{120 + 80}{25} = \frac{200}{25} = 8$ seconds

□□□□

Boats and Streams

The problems of boats and streams are also dependent on the basic equation of time, speed and distance.

Speed × Time = Distance.

Let the speed of Boat is S_B.

The speed of stream is S_S.

The speed of movement of boat is dependent on the speed of stream and direction in which it is moving.

(a) In still water speed of stream $S_S = 0$
Speed of Boat $= S_B$ (Normal speed at which the boat is moving)

(b) While moving upstream (or against the flow of the water), the speed of movement is given by $S_U = S_B - S_S$

(c) While moving down stream (or with the flow of the water), the speed of morement is given by
$S_D = S_B + S_S$

(d) Speed of boat S_B and speed of stream S_S when speed down stream and speed upstream of boat are given.

Speed of boat =

$$\frac{\text{Speed Downstream + Speed Upstream}}{2}$$

$$S_B = \frac{S_D + S_S}{2}$$

Speed of stream =

$$\frac{\text{Speed Downstream} - \text{Speed Upstream}}{2}$$

$$S_S = \frac{S_D - S_S}{2}$$

□□□□

Solved Exampls

1. Walking at 3/4 of his normal speed, Ankur is 16 minutes late in reaching his office. The usual time taken by him to cover the distance between his home and his office is
(a) 48 minutes (b) 60 minutes
(c) 42 minutes (d) 62 minutes

Ans. (a)

Speed × Time = Distance

$S \times T = D$

here S is changed to $\frac{3}{4}S$ so T changed to $\frac{4}{3}T$ to keep D constant

$$\frac{3}{4}S \times \frac{4}{3}T = D$$

$$\text{Now } \frac{3}{4}T = T + 16$$

$$\text{So } \frac{1}{3}T = 16;\ T = 48$$

2. Sanjay and Naveen travel the same distance at the rate of 6 km per hour and 10 km per hour respectively. If Sanjay takes 30 minutes longer than Naveen, the distance travelled by each is
(a) 6 km (b) 10 km
(c) 7.5 km (d) 20 km

Ans. (c)

$$\frac{D}{S_1} - \frac{D}{S_2} = \frac{1}{2} \text{ hour}$$

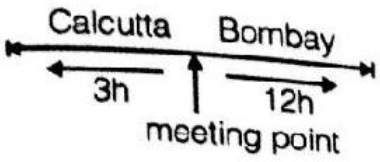
$$\frac{D}{6} - \frac{D}{10} = \frac{1}{2} \Rightarrow D\left[\frac{1}{6} - \frac{1}{10}\right] = \frac{1}{2}$$

$$D = \frac{15}{2} = 7.5 \text{ km}$$

3. Two trains, Calcutta Mail and Bombay Mail start at the same time from stations Calcutta and Bombay respectively towards each other. After passing each other, they take 4 hours and 3 hours to reach Bombay and Calcutta respectively. If the Calcutta Mail is moving at the speed of 48 km/h the speed of the Bombay Mail is

(a) 24 km/h (b) 22km/h
(c) 21 km/h (d) 96 km/h

Ans. (d)



in this case $\frac{S_1}{S_2} = \frac{\sqrt{T_2}}{\sqrt{T_1}}$

$= \frac{48}{S_2} = \frac{\sqrt{3}}{\sqrt{12}} = \frac{1}{2}$ $S_2 = 96$ km/h

4. Walking at 3/4 of his normal speed, a man takes 2(1/2)hours more than the normal time. Find the normal time.

(a) 7.5 h (b) 6 h
(c) 8 h (d) 12 h

Ans. (a)

$S \times T = D$ (costant)

$\frac{3}{4}S \times \frac{4}{3}T = D$

Now $\frac{4}{3}T = T + \frac{5}{2}$

$T = \frac{15}{2} = 7.5$ h

5. What is the time taken by Rohan to cover a distance of 360km by a motorcycle moving at a speed of 10 m/s

(a) 10 h (b) 5 h
(c) 8 h (d) 6 h

Ans. (a)

$10 \text{ m/s} = 10 \times \frac{18}{5} = 36$ km/h

$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{360}{36} = 10$ h

6. Rajdhani Express travels 650 km in 5 h and another 940 km in 10 h. What is the average speed of train?

(a) 1590 km/h (b) 63 km/h
(c) 106 km/h (d) 126 km/h

Ans. (c)

$\text{Average speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time taken}}$

$= \frac{650 + 940}{15} = 106$ km/h

7. A car travels from A to B at V_1 km/h, travels back from B to A at V_2 km/h and again goes back from A to B at V_2 km/h. The average speed of the car is :

(a) $\frac{2V_1V_2}{V_1 + 2V_2}$ (b) $\frac{2V_1V_2}{V_2 + 2V_1}$

(c) $\frac{3V_1V_2}{V_2 + 2V_1}$ (d) $\frac{3V_1V_2}{V_1 + 2V_2}$

Ans. (c)

$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}}$

$= \frac{3D}{\frac{D}{V_1} + \frac{D}{V_2} + \frac{D}{V_2}} = \frac{3}{\frac{1}{V_1} + \frac{2}{V_2}} = \frac{3V_1V_2}{2V_1 + V_2}$

8. Narayan Murthy walking at a speed of 20 km/h reaches his college 10 minutes late. Next time he increases his speed by 5 km/h. but finds that he is still late by 4 minutes. What is the distance of his college from his house.

(a) 20 km (b) 6 km
(c) 12 km (d) None of these

Ans. (d)

Let distance be D km

$\frac{D}{20} = T + \frac{10}{60}$ ...(i)

also $\frac{D}{25} = T + \frac{4}{60}$ ...(ii)

From (i) and (ii) we get

$D \times \frac{1}{100} = \frac{1}{10}$

$D = \frac{100}{10} = 10$ km

9. Jayshree goes to office at a speed of 6 km/h and returns to her home at a speed of 4 km/h. If she takes 10 hours in all, what is the distance between her office and her home?
(a) 24 km (b) 12 km
(c) 10 km (d) None of these

Ans. (a)

Let the distance be D km then

$$\frac{D}{6}+\frac{D}{4}=10 \qquad D\left[\frac{1}{6}+\frac{1}{4}\right]=10$$

$$D=\frac{10}{\left(\frac{5}{12}\right)}=24 \text{ km}$$

10. A motor car does a journey in 17.5 hours, covering the first half at 30 km/h and the second half at 40 km/h. Find the distance of the journey.
(a) 684 km (b) 600 km
(c) 120 km (d) 540 km

Ans. (b)

Here Total time = 17.5 hours

let total Distance be 2 D km

then $\frac{D}{30}+\frac{D}{40}=17.5$

$$D\left[\frac{1}{30}+\frac{1}{40}\right]=17.5$$

$$D\times\frac{7}{120}=17.5$$

$$D=\frac{17.5\times120}{7}$$

D = 300 km

Total Distance = 2D = 600 km

11. Manish travels a certain distance by car at the rate of 12km/h and walks back at the rate of 3 km/h. The whole journey took 5 hours. What is the distance he covered on the car?
(a) 12 km (b) 30km
(c) 15 km (d) 6 km

Ana. (a)

Let Distance be D km

$$\frac{D}{12}+\frac{D}{3}=5$$

$$D=\frac{5\times12}{5}=12 \text{ km}$$

12. A railway passenger counts the te... poles on the rail road as he passe... The telegraph poles are at a distanc... meters. What will be his count in 4 ... the speed of the train is 45 km per ...
(a) 600 (b) 2500
(c) 3600 (d) 5000

Ans. (c)

Total Distance covered = 45 x 4 = 18...

$$\text{Number of Poles}=\frac{180000}{50}=3600$$

13. Two tains A and B start simultaneously ... opposite direction from two points A a... and arrive at their destinations 9 and 4 ... respectively after their meeting each o... At what rate does the second train B tra... the first train travels at 80 km per hour
(a) 60 km/h (b) 100 km/h
(c) 120 km/h (d) None of these

Ans. (c)

Here $\frac{S_1}{S_2}=\frac{\sqrt{T_2}}{\sqrt{T_1}}$

$$\frac{80}{s_2}=\frac{\sqrt{4}}{\sqrt{9}} \qquad \frac{80}{S_2}=\frac{2}{3}$$

S_2 = 120 km/hour

14. A journay of 192 km takes 2 hours ... fast train than by a slow train. If the ... speed of the slow train be 16 km/...

that of fast train, what is the average speed of the faster train.

(a) 32 kmph (b) 16 kmph
(c) 12 kmph (d) 48 kmph

Ans. (d)

let speed of fast train be S km/hour then

$$\frac{192}{S-16}-\frac{192}{S}=2 \quad ...(i)$$

This type of questions can be directly solved by going through option
heare using equation (i) and puting options we get S = 48 km/hour

15. A passenger train takes 2 h less for a journey of 300 kilometres if its speed is increased by 5 kmph over its usual speed. Find the usual speed.

(a) 10 kmph (b) 12 kmph
(c) 20 kmph (d) 25 kmph

Ans. (d)

$$\frac{300}{S}-\frac{300}{S+5}=2 \quad ...(i)$$

going directly through options and using equation (i) we get
S = 25 km/hour

16. A plane left half an hour later than the scheduled time and in order to reach its destination 1500 kilometre away in time, it had to increase its speed by 33.33 per cent over its usual speed. Find its increased speed.

(a) 250 kmph (b) 500 kmph
(c) 750 kmph (d) None of these

Ans. (c)

$33.33\% = \frac{1}{3}$ of normal speed

$$\frac{D}{S+\frac{1}{3}S}-\frac{D}{S}=\frac{1}{2}$$

$$\frac{1500}{4/3S}-\frac{1500}{S}=\frac{1}{2} \quad ...(i)$$

going through option and (1) we get
S = 750 km/hour

17. A train moves at a constant speed of 120 km/h for one kilometre and at 40 kmph for the next one kilometre. What is the average speed of the train.

(a) 48 kmph (b) 50 kmph
(c) 80 kmph (d) None of these

Ans. (d)

$$\text{Average Speed} = \frac{\text{Distance covered}}{\text{Time taken}}$$

$$= \frac{1+1}{\frac{1}{120}+\frac{1}{40}} = \frac{2\times120\times40}{120+40}$$

= 60 km/hour

18. A car travels 1/3 of the distance on a straight road with a velocity of 10 km/h, the next 1/3 with a velocity of 20 km/h and the last 1/3 with a velocity of 60 km/h. What is the average velocity of the car for the whole journey?

(a) 18 km/h (b) 10 km/h
(c) 20 km/h (d) 15 km/h

Ans. (a)

$$\text{Average speed} = \frac{\text{Distance Covered}}{\text{Time taken}}$$

$$= \frac{\frac{D}{3}+\frac{D}{3}+\frac{D}{3}}{\frac{D}{3}\times\frac{1}{20}+\frac{D}{3}\times\frac{1}{10}+\frac{D}{3}\times\frac{1}{60}} = \frac{3}{\frac{1}{20}+\frac{1}{10}+\frac{1}{60}}$$

= 18 km/hour

19. Walking at 3/4 of his usual speed, a man is 16 minutes late for his office. The usual time taken by him to cover that distance is

(a) 48 minutes (b) 60 minutes
(c) 42 minutes (d) 62 minutes

Ans. (a)

S × T = Distance (constant)

$$\frac{3}{4}S\times\frac{4}{3}T=D$$

Time has become $\frac{4}{3}T$,

Now $\frac{4}{3}T - T = 16$ minutes

$T = 48$ minutes

20. Two trains for Patna leave Delhi at 6 a.m. and 6.45 a.m. and travel at 98 kmph and 136 kmph respectively. How many kilometres from Delhi will the two trains meet?

(a) 262.4 km (b) 260 km

(c) 200 km (d) None of these

Ans.(d)

Distance travelled by 1st train in 45 minutes

$= 98 \times \frac{45}{60} = 73.5 \text{ km} = \frac{147}{2} \text{ km}$

Relative speed $= S_2 - S_1 = 136 - 98 = 38$ km/hr

Time taken to cover 73.5 km is

$\frac{73.5}{38} = \frac{\frac{147}{2}}{38} = \frac{147}{76}$ hour

$\text{Distance} = \frac{147}{76} \times 98 = 189.55$

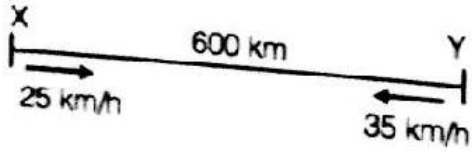
Total distance $= 73.5 + 189.55 = 263$ km approx

21. X and Y are two stations 600 km apart. A train starts from X and moves towards Y at the rate of 25 km/h. Another train starts from Y at the 35 km/h. How far from X they will cross each other

(a) 250 km (b) 300 km

(c) 450 km (d) 475 km

Ans. (a)



$= \frac{600}{35 + 25} = 10$ h.

Distance from X $= 10 \times 25 = 250$ km

22. A motorboat went downstream for 28 km immediately returned. It took the boat as long to make the return trip. If the speed of the river flow were twice as high, the downstream and back would take minutes. Find the speed of the boat water and the speed of the river flow

(a) 9 km/h, 3 km/h (b) 9 km/h, 6 km/h

(c) 8 km/h, 2 km/h (d) 12 km/h, 3 km/h

Ans.(a)

Let speed of boat be S_B

Speed of stream $= S_S$

then $\frac{28}{S_B - S_S} = 2 \times \frac{28}{S_B + S_S}$

also $\frac{28}{S_B + S_S} + \frac{28}{S_B - S_S} = 672$ minutes

going directly by option

$S_B = 9$ km

$S_S = 3$ km

23. A train requires 7 seconds to pass a pole while it requires 25 seconds to cross a stationary train which is 378 metres long. Find the speed of the train.

(a) 75.6 km/h (b) 75.4 km/h

(c) 76.2 km/h (d) 21 km/h

Ans. (a)

To cross a pole

$t = \frac{L_T}{S_T} = \frac{\text{length of train}}{\text{speed of train}}$

$7 = \frac{L_T}{S_T}$

also to cross a stationary train $t = \frac{L_T + L_S}{S_T}$

L_S = length of stationary train

$$25 = \frac{L_T + 378}{S_T} \quad ...(ii)$$

from (i) and (ii)

$$7 \times S_T = L_T$$

$$25 \times S_T = L_T + 378$$

$$18\, S_T = 378$$

$$S_T = 21 \text{ m/s} = 21 \times \frac{18}{5} = 75.6 \text{ km/h}$$

24. A boat sails down the river for 10 km and then up the river for 6 km. The speed of the river flow is 1 km/h. What should be the minimum speed of the boat for the trip to take a maximum of 4 hours?

(a) 2 kmph (b) 3 kmph
(c) 4 kmph (d) 5 kmph

Ans. (c)

$$\frac{D_1}{S_B + S_S} + \frac{D_2}{S_B - S_S} = 4 \text{ hour}$$

$$\frac{10}{S_B + 1} + \frac{6}{S_B - 1} = 4 \text{ hour} \quad ...(i)$$

going by option and (i) we get

S_B = 4 km/hour

25. Two trains are running on parallel lines in the same direction at speeds of 40 kmph and 20 kmph respectively. The faster train crosses a man in the second train in 36 seconds. The length of the faster train is

(a) 200 metres (b) 185 metres
(c) 225 metres (d) 210 metres

Ans. (a)

t = time required

$$\frac{L_T + L_o}{S_T - S_o} \quad ...(i)$$

$$S_T - S_O = 40 - 20 = 20 \text{ km/h}$$

$$= 20 \times \frac{5}{18} = \frac{50}{9} \text{ m/s}$$

$$\text{now } t = \frac{\frac{L_T}{50}}{9} \Rightarrow 36 = \frac{\frac{L_T}{50}}{9}$$

So, L_T = 200m

26. The speed of the boat in still water is 12 km/h and the speed of the stream is 2 km/h. A distance of 8 km, going upstream, is covered in

(a) 1 h (b) 1 h15 min
(c) 1 h 12 min (d) None of these

Ans. (d)

$$t = \frac{D}{S_B - S_S} \text{ (upstream)}$$

$$= \frac{8}{12-2} = \frac{8}{10} = \frac{8}{10} \times 60 = 48 \text{ min}$$

27. A boat goes 15 km upstream in 80 minutes. The speed of the stream is 5 km/h. The speed of the boat in still water is

(a) 16.25 km/h (b) 16 km/h
(c) 15 km/h (d) 17 km/h

Ans. (a)

$$t = \frac{D}{S_B - S_S} \text{ (upstream)}$$

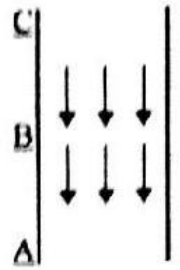
$$\frac{80}{60} = \frac{15}{S_B - 5} \Rightarrow \frac{4}{3} = \frac{15}{S_B - 5}$$

S_B = 16.25 km/h

28. In a stream, B lies in between A and C such that it is equidistant from both A and C. A boat can go from A to B and back in 6 h 30 minutes while it goes from A to C in 9 h. How long would it take to go from C to A?

(a) 3.75 h (b) 4 h
(c) 4.25 h (d) 4.5 h

Ans. (b)



let AB = D km then
BC = D km
AC = 2D km

$$\frac{D}{S_B - S_S} + \frac{D}{S_B + S_S} = \frac{13}{2} \quad ...(i)$$

$$\frac{2D}{S_B - S_S} = 9 \quad ...(ii)$$

So $$\frac{D}{S_B - S_S} = \frac{9}{2} \quad ...(iii)$$

from (1) and (3) we get

$$\frac{9}{2} + \frac{D}{S_B + S_S} = \frac{13}{2}$$

$$\frac{D}{S_B + S_S} = 2$$

to go from C to A it takes

$$\frac{2D}{S_B + S_S} = 4 \text{ hours}$$

29. Two trains are travelling in the same direction at 50 km/h and 30 km/h respectively. The faster train crosses a man in the slower train in 18 seconds. Find the length of the faster train.
(a) 0.1 km (b) 1 km
(c) 1.5 km (d) 1.4 km
Ans. (a)

$$t = \frac{L_T + L_0}{S_T - S_0}$$

$50 - 30 = 20$ km/hr $= 50/9$ m/s

$$\text{now } 18 = \frac{L_T}{50/9} \Rightarrow L_T = 100 \text{ m}$$

30. Without stoppage, a train travels with an average speed of 75 km/h and with stoppage it covers the same distance at an average speed of 60 km/h. How many minutes per hour does the train stop?
(a) 10 minutes (b) 12 minutes
(c) 14 minutes (d) 18 minutes
Ans. (b)
Speed of trains are 75 km/h and 60 km/h and to cover a distance of 60 km it will take 60 minutes in second case (with stoppage) while without stoppage it

$$\frac{60}{75} = 48 \text{ minuts}$$

So, stoppage = 12 minutes

31. A boat rows 16 km up the stream and 30 km down stream taking 5 h each time. Find the velocity of the current
(a) 1.1 km/h (b) 1.2 km/h
(c) 1.4 km/h (d) 1.5 km/h
Ans. (c)

$$\frac{16}{S_B - S_S} = 5 \text{ hour}$$

$$\frac{30}{S_B + S_S} = 5 \text{ hour}$$

$16 = 5S_B - 5S_S$

$30 = 5S_B + 5S_S$

$14 = 10 S_S$

$S_S = 1.4$ km/hr

32. A lazy man can row upstream at 16 km/h and downstream at 22 km/h. Find the man's rate in still water (in kmph.).
(a) 19 (b) 14
(c) 17 (d) 18
Ans.(a)
Let Man's rate in still water is S_B
here $S_B - S_S = 16$
$S_B + S_S = 22$ km

S_B = 45 kmph

D = 30 km

$$t = \frac{9}{2}\text{hr} = \frac{30}{15+S_S} + \frac{30}{15-S_S} = \frac{9}{2}$$

S_S = 5 kmph

□□□□

Time, Speed & Distance

I Practice Exercise: I

1. Ramesh crosses a street 600 m long in 5 minutes. His speed in km/hr is
   (a) 8.2 (b) 7.2
   (c) 9.2 (d) None of these

2. A train 100 metres long passes a bridge at the rate of 72 km/hr in 25 seconds. What is the length of the bridge?
   (a) 400 m (b) 17 m
   (c) 600 m (d) None of these

3. A train passes a railway bridge 150 m long in 18 seconds. If the train is running at a speed of 60 km/hr., then the length of the train in metres is
   (a) 160 m (b) 150 m
   (c) 180 m (d) None of these

4. Sound travels 330 metres a second. If the sound of a thunder-cloud follows the flash after 10 seconds. The thunder-could is at a distance of
   (a) 3.7 km (b) 3.5 km
   (c) 3.3 km (d) None of these

5. The wheel of an engine is $3\frac{3}{4}$ metres in circumference and makes 4 revolutions in 2 seconds The speed of the train is

6. A person covers half of his journey at ... and the remainning half 20 km/hr. ... speed for the whole journey is
   (a) 25 km/hr (b) 28 km/hr
   (c) 32 km/hr (d) None of these

7. A and B are two towns. A car goes ... B at a speed of 64 km/hr and returns ... a slower speed. If its average speed ... whole journey is 56 km/hr, it returns ... speed
   (a) 52.54 km/hr (b) 47.74 km/hr
   (c) 49.78 km/hr (d) None of these

8. On a tour a man travels at the rate ... an hour for the first 160 km, then travels ... next 160 km at the rate of 80 km an hour. ... average speed in km per hour for the ... 320 km of the tour is
   (a) 81.13 km/hr (b) 73.11 km/hr
   (c) 71.11 km/hr (d) None of these

9. Rakesh sets out to cycle from Delhi to ... and at the same time Suresh starts ... Mathura to Delhi. After passing each ... they complete their journeys in 9 and ... respectively. At what speed does ... cycle if Rakesh cycles at 16 km per ...
   (a) 12 km/hr (b) 16 km/hr
   (c) 14 km/hr (d) None of these

10. A car during its journey travels 30 ... a speed of 40 km/hr, another 45 ... speed of 60 km/hr, and 2 hours at ... 70 km/hr. The average speed of the ...
   (a) 63 km/hr (b) 65 km/hr
   (c) 70 km/hr (d) None of these

11. By walking at $\frac{3}{4}$ of his usual speed ... reaches office 20 minutes later ... His usual time is

(c) 70 minutes (d) None of these

12. A car start from A for B travelling 20 km an hour. $1\frac{1}{2}$ hours later another car starts from A and travelling at the rate of 30 km an hour reaches B $2\frac{1}{2}$ hours before the first car. Find the distance from A to B
(a) 280 km (b) 260 km
(c) 240 km (d) None of these

13. A train does a journey without stopping in 8 hours. If it had travelled 5 km an hour faster, it would have done the journey in 6 hours 40 min. What is its slower speed?
(a) 35 km/hr (b) 25 km/hr
(c) 40 km/hr (d) None of these

14. Without any stoppage a person travels a certain distance at an average speed of 42 km/hr and with stoppages he covers the same distance at an average speed of 28 km/hr. How many minutes per hour does he stop?
(a) 25 minutes (b) 30 minutes
(c) 20 minutes (d) None of these

15. A train passes through a telegraph post in 9 seconds moving with a speed of 54 km per hour. The length of the train is
(a) 135 metres (b) 145 metres
(c) 125 metres (d) None of these

16. A train 50 m long passes a platfkorm 100 m long in 10 seconds. The speed of the train in m/sec. is
(a) 25 (b) 15
(c) 35 (d) None of these

17. A train 300 metres long is running at a speed of 90 km/hr. How many seconds will it take to cross a 200 metres long train running in the same direction at a speed of 60 km/hr?
(a) 70 sec (b) 60 sec
(c) 50 sec (d) None of these

18. Two trains are running in opposite directions with the same speed. If the length of each train is 135 metres and they cross each other in 18 seconds, the speed of each train is
(a) 29 km/hr (b) 35 km/hr
(c) 27 km/hr (d) None of these

19. Two trains are moving in the same direction at 50 km/hr and 30 km/hr. The faster train crosses a man in the slower train in 18 seconds. Find the length of the faster train.
(a) 120 m (b) 110 m
(c) 100 m (d) None of these

20. Two trains, 130 m and 110 m long, while going in the same direction, the faster traintakes one minutes to pass the other completely. If they are moving in opposite direction, they pass each other completely in 3 seconds. Find the speed of each train.
(a) 42m/sec., 38m/sec.
(b) 38m/sec., 36m/sec.
(c) 36m/sec., 42m/sec.
(d) None of these

21. Two stations A and B are 100 km apart on a straight line. One train starts from A at 7 A.M. and travels towards B at 20 km/hr speed. another train starts from B at 8 A.M. and travels towards A at 25 km/hr. speed. At what time will they meet?
(a) 10.30 A.M. (b) 11 A.M
(c) 10 A.M (d) None of these

22. Two trains start at the same time from Mumbai and Pune and proceed towards each other at the rate of 60 km and 40 km per hour respectively. When they meet, it is found that one train has travelled 20 km more than the other. Find the distance between Mumbai and Pune.

(a) 150 km (b) 100 km
(c) 120 km (d) None of these

23. A boy takes as much time in running 12 metres as a car takes in covering 36 metres. The ratio of the speeds of the boy and the car is
(a) 1 : 3 (b) 1 : 2
(c) 2 : 3 (d) 2 : 5

24. A and B are two stations. A train goes from A to B at 64 km/hr and returns to A at a slower speed. If its average speed for the whole journey is 56 km/hr, at what speed did it return?
(a) 48 km/hr (b) 49.77 km/hr
(c) 30 km/hr (d) 47.46 km/hr

25. Excluding stoppages, the speed of a bus is 54 km/hr and including stoppages, it is 45 km/hr. For how many minutes does the bus stop per hour?
(a) 9 (b) 10
(c) 12 (d) 20

26. Ramesh sees a train passing over 1 km long bridge. The length of the train is half that of bridge. If the train clears the bridge in 2 minutes, the speed of the train is
(a) 45 km/hr (b) 43 km/hr
(c) 50 km/hr (d) None of these

27. Amit started cycling along the boundaries of a square field from cover point A. After half an hour, he reached the corner point C, diagonally opposite to A. If his speed was 8 km/hr, what is the area of the field in square km
(a) 64
(b) 8
(c) 4
(d) Cannot be determined

28. A motor cyclist goes from Mumbai to Pune, a distance of 192 kms, at an average speed of 32 kmph. Another man starts from Mumbai by car, $2\frac{1}{2}$ hours after the [illegible] Pune half an hour earlier. What is [illegible] the speed of the motorcycle and [illegible]
(a) 1 : 2 (b) 1 : 3
(c) 10 : 27 (d) 5 : 4

29. A person sets to cover a distance of [illegible] 45 minutes. If he covers $\frac{3}{4}$ of the distance [illegible] $\frac{2}{3}$rd time, what should be his speed [illegible] the remaining distance in the remaining
(a) 16 km/hr (b) 8 km/hr
(c) 12 km/hr (d) 14 km/hr

30. A train 110 metres in length passes a man walking at the speed of 6 km/hr against it in 6 seconds. The speed of the train in km per hour is
(a) 60 km/hr (b) 45 km/hr
(c) 50 km/hr (d) 55 km/hr

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Solutions

1. **Ans. (b)**

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$= \left(\frac{600}{5 \times 60}\right) \text{m/sec}$$

$$= \left(\frac{600}{5 \times 60} \times \frac{18}{5}\right) \text{km/hr} = 7.2 \text{ km/hr}$$

2. **Ans. (a)**
Distance travelled by the train in 25 sec at 72 km/hr.

$$= 72 \times \frac{5}{18} \times 25 = 500 \text{ m}$$

train
= 500 – 100 = 400 m.

Ans. (b)

Let the length of the train be x m.

∴ Total distance covered by the train
$= (x + 150)$m

Speed of the train = 60 km/hr

$$= 60 \times \frac{5}{18} = \frac{50}{3} \text{ m/sec}$$

Since, Distance = Speed × time

$$\therefore \quad x + 150 = \frac{50}{3} \times 18 = 300$$

or, $x = 300 - 150 = 150$ m.

∴ Length of the train = 150 m.

4. **Ans. (c)**

Distance of thunder-cloud
= distance travelled by sound in 10 seconds
= (330 × 10) metres
= 3.3 km.

5. **Ans. (a)**

Distance covered in 2 seconds

$$= \frac{15}{4} \times 4 = 15 \text{ m.}$$

$$\therefore \text{ Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{15}{2} \text{ m/sec}$$

$$= \left(\frac{15}{2} \times \frac{18}{5}\right) \text{km/hr} = 27 \text{ km/hr.}$$

6. **Ans. (a)**

Let D be the total distance then,

$$\text{Average speed} = \frac{D/2 + D/2}{\frac{D/2}{30} + \frac{D/2}{20}} = 25 \text{ km/hr}$$

7. **Ans. (c)**

Let the speed on the return journey be x km/hr.

$$\text{Then, } 56 = \left(\frac{2s_1s_2}{s_1 + s_2}\right) = \frac{2 \times 64 \times x}{64 + x}$$

$\therefore$ $(64 + x) = 16x$ or $9x = 448$

$$\therefore \quad x = \frac{448}{9} = 49.78 \text{ km/hr.}$$

8. **Ans. (c)**

Here, $s_1 = 64$ and $S_2 = 80$.

∴ Average speed

$$= \left(\frac{2s_1s_2}{s_1 + s_2}\right) = \frac{2 \times 64 \times 80}{64 + 80}$$

$= 71.11$ km/hr.

9. **Ans. (a)**

$$\frac{\text{Rakesh's speed}}{\text{Suresh's speed}} = \frac{\sqrt{T_2}}{\sqrt{T_1}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$$

$$\therefore \text{ Suresh's speed} = \frac{3}{4} \text{ Rakesh's speed}$$

$$= \frac{3}{4} \times 16 = 12 \text{km/hr}$$

10. **Ans. (a)**

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

$$\text{Here, } T_1 = \frac{30}{60}, T_2 = \frac{45}{60}, T_3 = 2, s_1 = 40$$

$s_2 = 60$ and $s_3 = 70$.

∴ The average speed of the car

$$= \frac{s_1T_1 + s_2T_2 + s_3T_3}{T_1 + T_2 + T_3}$$

$$= \frac{40 \times \frac{30}{60} + 60 \times \frac{45}{60} + 70 \times 2}{\frac{30}{60} + \frac{45}{60} + 2}$$

$= 63$ km/hr.

11. **Ans. (b)**

$S \times T = D$ ...a constant

$$\frac{3}{4}S \times \frac{4}{3}T = D$$

Now, $\frac{4}{3}T - T = \frac{1}{3}T = 20$ minutes

$\Rightarrow$ $T = 60$ minutes

12. Ans. (c)

It is given that

$$\frac{D}{20} = \frac{D}{30} + 1\frac{1}{2} + 2\frac{1}{2}$$

$$\Rightarrow \frac{D}{20} = \frac{D}{30} + 4$$

So $D = 240$ km

13. Ans. (b)

Let the slower speed = s km/hr.

Since the distance travelled is same in both the cases therefore,

$$s_1 \times T_1 = s_2 \times T_2 = D$$

$$\Rightarrow s \times 8 = (s+5) \times \frac{20}{3}$$

$$\Rightarrow 24s = 20(s+5)$$

$\therefore$ $s = 25$ km/hr.

14. Ans. (c)

Here, $s_1 = 42$ and $s_2 = 28$.

$$\therefore \text{ Stoppage time/hr} = \frac{s_1 - s_2}{s_1} = \frac{42-28}{42}$$

$$= \frac{1}{3} \text{ hour} = 20 \text{ minutes}$$

15. Ans. (a)

Length of the train = Speed of the train x Time taken in crossing the post

$$= \left(54 \times \frac{5}{18}\right) \times 9 = 135 \text{ m}$$

16. Ans. (b)

We have, speed of the train

$$\frac{\text{Length of the train + Length of the platform}}{\text{Time taken in crossing the platform}}$$

$$= \frac{50+100}{10} = 15 \text{ sec.}$$

17. Ans. (b)

Here, $L_1 = 300$ m, $L_2 = 200$ m.

$s_1 = 90$ km/hr and $s_2 = 60$ km/hr

$$\therefore \quad s_1 - s_2 = 90 - 60 = 30 \text{ km/hr} = 30 \times \frac{5}{18}$$

$$\therefore \text{ Time taken} = \frac{L_1 + L_2}{s_1 - s_2} = \frac{300+200}{30 \times \frac{5}{18}}$$

$$= \frac{500 \times 18}{30 \times 5} = 60 \text{ sec.}$$

18. Ans. (c)

Let the speed of each train be x m/sec

We have, $L_1 = L_2 = 135$ m

and $S_1 = S_2 = x$ m/sec

$$\therefore \text{ Therefore time taken } = \frac{L_1 + L_2}{S_1 + S_2}$$

$$\Rightarrow 18 = \frac{135+135}{x+x}$$

$$\text{or, } x = \frac{270}{2 \times 18} \text{ m/s}$$

$$= \frac{270}{2 \times 18} \times \frac{18}{5} \text{ km/hr} = 27 \text{ km/hr.}$$

19. Ans. (c)

Relative speed = (50 – 30)km/hr = 20 km/hr

$$= \left(20 \times \frac{5}{18}\right) = \left(\frac{50}{9}\right) \text{m/sec.}$$

Distance covered in 18 sec at this speed

$$= \left(18 \times \frac{50}{9}\right) \text{m} = 100 \text{ m.}$$

$\therefore$ Length of faster train = 100 m.

20. Ans. (a)

Time taken when they are moving in same direction i.e.

$$60 = \frac{130+110}{s_1 - s_2}$$

$\therefore\ s_1 - s_2 = \frac{240}{60} = 4$

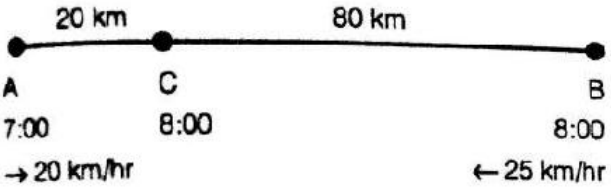
Time taken when they are moving opposite direction i.e.

$$3 = \frac{130+110}{s_1+s_2}$$

$$\Rightarrow\ s_1 + s_2 = \frac{240}{3} = 80$$

$\therefore\ s_1 = 42,\ s_2 = 38$

. **Ans. (d)**



From B to C

$$t = \left(\frac{80}{20+25} \times 60\right) \text{ minutes}$$

$$\Rightarrow\ t = \frac{320}{3} = 106\frac{2}{3} \text{ minutes}$$

= 1 hour 46 minutes 40 sec

2. **Ans. (b)**

Both will meet at time $t = \frac{D}{100}$

also $(60t - 40t) = 20t = 20$ km

$\Rightarrow\ t = 1$ hour

So $D = 100t = 100$ km

3. **Ans. (a)**

Let speeds of the boy and the car by x km/hr and y km/hr. respectively

Then, $\frac{12/1000}{x} = \frac{36/1000}{y}$

$$\therefore\quad \frac{x}{y} = \frac{1}{3}$$

24. **Ans. (b)**

Let the required speed by x km/hr.

Then, $\frac{2 \times 64 \times x}{64+x} = 56$

$\therefore\ 128x = 64 \times 56 + 56x$

$$\therefore\quad x = \frac{64 \times 56}{72} = 49.77 \text{ km/hr}$$

25. **Ans. (b)**

Due to stoppages, it covers 9 km less per hour.

Time taken to cover 9 km $= \left(\frac{9}{54} \times 60\right)$ min

= 10 min.

So, the bus stops for 10 min. per hr.

26. **Ans. (a)**

Distance travelled in 2 minutes

$= \left(1 + \frac{1}{2}\right)$ km i.e. $\frac{3}{2}$ kms.

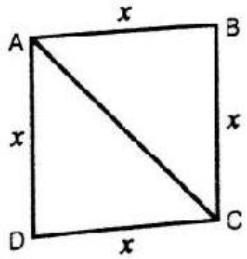
$\therefore$ Distance covered in 1 min. $= \frac{3}{2} \times \frac{1}{2}$ km

$\therefore$ Distance covered in 1 hr. $= \left(\frac{3}{2} \times \frac{60}{2}\right)$ km

= 45 km

$\therefore$ Speed of the train = 45 km/hr

27. **Ans. (c)**



Distance covered in $\frac{1}{2}$ hours $= 2x$

$$= \frac{1}{2} \times 8 = 4$$

$\Rightarrow$ area $x = 4$

28. Ans. (a)

Speed of the first man = 32 km/hr.

Time taken = 192 ÷ 32 = 6 hr

Second man covers 192 km in 3 hr

$\therefore$ Speed of the second man

= 192 ÷ 3 = 64 km/hr

Ratio = 32 : 64 or 1 : 2

29. Ans. (c)

$$\text{Distance already covered} = \frac{3}{4} \times 12 = 9 \text{ km}$$

$$\text{Time spent} = \frac{2}{3} \times 45 \text{ min} = 30 \text{ min}$$

Distance left = (12 – 9) km = 3 km

Time left = (45 – 30)min = 15 min

$$\therefore \text{ Required speed} = \frac{3}{15/60} \text{ km/hr}$$

= 12 km/hr

30. Ans. (a)

Let the speed of the train in km/hr = x

Then, relative speed = $(x + 6)$ km/hr

$$= (x+6) \times \frac{5}{18} \text{ m/sec}$$

it is given that $(x+6) \times \frac{5}{18} \times 6 = 110$

$\therefore$ $x = 60$

$\therefore$ Speed of the train = 60 km/hr.

□□□□

Boats & Streams

1. If a man can swim downstream at 6 kmph and upstream at 2 kmph, his speed in still water is:
   (a) 4 km/hr (b) 2 km/hr
   (c) 3 km/hr (d) 2.5 km/hr

...peed of the stream is
(a) 1 km/hr (b) 1.5 km/hr
(c) 2 km/hr (d) 12 km/hr

3. If a boat goes 7 km upstream in 42 ... and the speed of the stream is 3 kmp... the speed of the boat in still water is
(a) 4.2 km/hr (b) 9 km/hr
(c) 13 km/hr (d) 21 km/hr

4. Raman can row $9\frac{1}{3}$ kmph in still wate... finds that it takes him thrice as much ti... row up than as to row down the same dis... in the river. The speed of the current is
(a) $3\frac{1}{3}$ km/hr (b) $3\frac{1}{9}$ km/hr
(c) $4\frac{2}{3}$ km/hr (d) 14 km/hr

5. A man can row a boat at 10 kmph in still w... If the speed of the stream is 6 kmph... time taken to row a distance of 80 km ... the stream is
(a) 8 hours (b) 5 hours
(c) 10 hours (d) 20 hours

6. A boat takes 4 hours for trav... downstream from point A to point B ... coming back to point A upstream ... velocity of the stream is 2 kmph a... speed of the boat in still water is 4 ... what is the distance between A and B
(a) 4 kms (b) 6 kms
(c) 8 km (d) 9 km

7. Speed of a boat in standing water is ... and the speed of the stream is 1.5 ... man rows to a distance of 10.5 km ... back to the starting point. The total ... by him is:

(c) 24 hours

A man rows to a place 48 km distant and back in 14 hours. He finds that he can row 4 km with the steam in the same time as 3 km against the stream. The rate of the steam is :

(a) 1 km/hr (b) 1.8 km/hr
(c) 3.5 km/hr (d) 1.5 km/hr

A man can row three-quarters of a kilometre against the stream in $11\frac{1}{4}$ minutes and return in $7\frac{1}{2}$ minutes. The speed of the man in still water is:

(a) 2 km/hr (b) 3 km/hr
(c) 4 km/hr (d) 5 km/hr

). A man can row 5 kmph in still water. If the river is running at 1 kmph, it takes him 75 minutes to row to a place and back. How far is the place?

(a) 3km (b) 2.5 km
(c) 4 km (d) 5 km

❑❑❑❑

Solutions

. **Ans. (a)**

Speed in still water $S_B = \left(\frac{S_D + S_U}{2}\right)$

$= \frac{1}{2}(6+2)$ kmph

$= 4$ kmph.

2. **Ans. (a)**

Speed Upstream $S_U = \left(\frac{15}{3}\right)$ kmph $= 5$ kmph.

Speed Downstream $S_D = \left(\frac{21}{3}\right)$ kmph

$= 7$ kmph.

$\therefore$ Speed of stream $S_S = \frac{1}{2}(7-5)$ kmph

$= 1$ kmph.

3. **Ans. (c)**

Speed Upstream $S_U = \left(\frac{1}{42} \times 60\right)$ kmph

$= 10$ kmph

Speed of Stream $S_S = 3$ kmph

$\because S_U = S_B - S_S$

$\therefore S_B = S_U + S_S$

$\Rightarrow S_B = 10 + 3 = 13$ kmph

4. **Ans. (c)**

Let Speed Upstream S_U be x kmph.

Then, Speed Downstream $S_D = 3x$ kmph.

$\therefore$ Speed in Still water $S_B = \frac{1}{2}(3x + x)$ kmph

$= 2x$ kmph

$\therefore 2x = \frac{28}{3} \quad \Rightarrow x = \frac{14}{3}$

$\therefore$ Speed upstream $= \frac{14}{3}$ km/hr

Speed downstream $= 14$ km/hr

$\therefore$ Speed of the current $S_S = \frac{1}{2}S_D - S_U$

$= \frac{1}{2}\left(14 - \frac{14}{3}\right)$ km/hr

$= \frac{14}{3}$ km/hr $= 4\frac{2}{3}$ km/hr

5. **Ans. (b)**

Speed Downstream $S_D = (10 + 6)$ km/hr

$= 16$ km/hr.

Time taken to cover 80 km downstream

$= \left(\frac{80}{16}\right)$ hrs $= 5$ hrs.

6. **Ans. (b)**

Let the distance between A and B be x km

Speed Downstream $S_D = 6$ kmph.

Speed Upstream S_U = 4 kmph

$\Rightarrow \frac{x}{6}+\frac{x}{2}=4$

$\Rightarrow \frac{4x}{6}=4$

$\Rightarrow x = 6$ km

$\therefore$ Distance of A and B = 6 km.

7. **Ans. (d)**
Speed Upstream S_U = 7.5 kmph.
Speed Downstream S_D = 10.5 kmph.

$\therefore$ Total time taken $= \left(\frac{105}{7.5}+\frac{105}{10.5}\right)$ hrs

= 24 hours

8. **Ans. (a)**
Suppose he moves 4 km downstream in x hours. Then, speed downstream $= \left(\frac{4}{x}\right)$ km/hr,

speed upstream $= \left(\frac{3}{x}\right)$ km/hr.

$\Rightarrow \frac{48}{(4/x)}+\frac{48}{(3/x)}=14$ or $x=\frac{1}{2}$

$\therefore$ Speed downstream = 8 km/hr.
Speed upstream = 6 km/hr

Speed of the stream $=\frac{1}{2}(8-6)$ km/hr

= 1 km/hr.

9. **Ans. (d)**

Speed Upstream $S_U = \left(\frac{3}{4}\times\frac{4}{45}\times 60\right)$ kmph

= 4 kmph.

Speed Downstream $S_D = \left(\frac{3}{4}\times\frac{2}{15}\times 60\right)$ kmph

= 6 kmph

Speed in still water $S_B = \frac{1}{2}(6+4)$ kmph

= 5 kmph.

Speed Downstream S_D = 6km/hr.

Speed Upstream S_U = (5 – 1) kmph = 4 km/hr.

Let the required distance be x km.

Then, $\frac{x}{6}+\frac{x}{4}=\frac{75}{60}$

or $10x + 15x = 75$

$\Rightarrow 2x + 3x = 15$ or $x = 3$

$\therefore$ Required distance = 3 km.

□□□□

Problems on Trains

1. A train moves with a speed of 108 speed in metres per second is
(a) 38.8 (b) 18
(c) 30 (d) 10.8

2. A speed of 14 metres per second is as
(a) 50.4 km/hr (b) 28 km/hr
(c) 70 km/hr (d) 46.6 km/hr

3. A man on riding crosses a bridge in when riding is being done at 15 k length of the bridge is
(a) 125m (b) 250 m
(c) 1250 m (d) 2500 m

4. A train 280 m long, running with a 63 km/hr will pass an electric pole
(a) 20 sec (b) 16 sec.
(c) 15 sec (d) 18 sec

5. A train is moving at a speed of 13 the length of the train is 110 metres will it take to cross a railway platfo long?
(a) 5 sec (b) 7.5 sec
(c) 10 sec (d) 15 sec

6. A train 700 m crosses a tunn the tunnel is
(a) 700 m
(c) 550 m

7. If 200 m long tr same length as then the speed
(a) 50 km/hr
(c) 72 km/hr

8. A train 300 m lo long in 1 minu the train (in km
(a) 45
(c) 54

9. A train of leng to cross a tunn of the train (in
(a) 13.33
(c) 40

10. A train speed and a platform length is
(a) 200 m
(c) 50 m

11. If a train 110 in 3 second cross a railw
(a) 3 sec
(c) 5 sec

12. A train 150 25 metres moving at direction. Th
(a) 5 sec
(c) $4\frac{2}{7}$ sec

13. Two trains on parallel

5. A train 700 m long is running at 72 kmph. If it crosses a tunnel in 1 minute, the length of the tunnel is
(a) 700 m (b) 600 m
(c) 550 m (d) 500 m

7. If 200 m long train crosses a platform of the same length as that of the train in 20 seconds, then the speed of the train is
(a) 50 km/hr (b) 60 km/hr
(c) 72 km/hr (d) 80 km/hr

8. A train 300 m long crossed a platform 900 m long in 1 minute 12 seconds. The speed of the train (in km/hr) is
(a) 45 (b) 50
(c) 54 (d) 60

9. A train of length 150 m takes 40.5 seconds to cross a tunnel of length 300 m. The speed of the train (in km/hr) is:
(a) 13.33 (b) 26.67
(c) 40 (d) 400

10. A train speeds past a pole in 15 seconds and a platform 100 m long in 25 seconds. Its length is
(a) 200 m (b) 150 m
(c) 50 m (d) Data inadequate

11. If a train 110 m long passes a telegraph pole in 3 seconds, then the time taken by it to cross a railway platform 165 m long is
(a) 3 sec (b) 4 sec
(c) 5 sec (d) 7.5 sec

12. A train 150 m long moving at a speed of 25 metres per second overtakes a man moving at 5 metres/sec in opposite direction. The train will pass the man in
(a) 5 sec (b) 6 sec
(c) $4\frac{2}{7}$ sec (d) 8 sec

13. Two trains 200 m and 150 m long are running on parallel rails at the rate of 40 kmph and 45 kmph respectively. In how much time will they cross each other, if they are running in the same direction?
(a) 72 sec (b) 132 sec
(c) 192 sec (d) 252 sec

14. Two train 126 m and 114 m long are running in opposite directions, one at the rate of 30 kmph and another one at 42 kmph. From the moment they meet will cross each other in:
(a) 10 sec (b) 11 sec
(c) 12 sec (d) 13 sec

15. A train 110 m long passes a man, running at 6 kmph in the direction opposite to that of the train, in 6 seconds. The speed of the train is
(a) 60 km/hr (b) 66 km/hr
(c) 54 km/hr (d) 72 km/hr

16. A train 108 m long moving at a speed of 50 km/hr crosses another train 112 m long coming from opposite direction in 6 seconds. The speed of the second train is
(a) 48 kmph (b) 54 kmph
(c) 66 kmph (d) 82 kmph

17. A train B speeding with 120 kmph crosses another train C, running in the same direction in 2 minutes. If the lengths of the trains B and C be 100 m and 200m respectively, what is the speed of the train C?
(a) 111 kmph (b) 127 kmph
(c) 123 kmph (d) 129 kmph

18. Two trains travel in opposite directions at 36 kmph and 45 kmph and a man sitting in slower train passes the faster train in 8 seconds. The length of the faster train is
(a) 80 m (b) 100 m
(c) 120 m (d) 180m

19. A train running at certain speed crosses a stationary engine in 20 seconds. To find out the speed of the train, which of the following information is necessary

(a) Only the length of the train
(b) Only the length of the engine
(c) Either the length of the train or the length of the engine
(d) Both the length of the train and the length of the engine

20. A train overtakes two persons who are walking in the same direction in which the train is going, at the rate of 2 kmph and 4 kmph and passes them completely in 9 and 10 seconds. respectively. The length of the train is
(a) 72 m (b) 54 m
(c) 50 m (d) 45 m

21. Two stations A and B are 110 km apart on a straight line. One train starts from A at 7 a.m. and travels towards B at 20 kmph. Another train starts from B at 8 a.m. and travels towards A at a speed of 25 kmph. At what time will they meet?
(a) 9 a.m. (b) 10 a.m.
(c) 11 a.m. (d) 10.30 a.m

22. Two train are running in opposite directions towards each other with speeds of 54 kmph and 48 kmph respectively. If the length of the train is 250 m and they cross each other in 18 seconds, the length of the other trains is
(a) 145 m (b) 230m
(c) 260 m (d) 180 m

23. A train travelling at 48 kmph completely crosses another train having half its length and travelling in opposite direction at 42 kmph, in 12 seconds. It also passes a railway platform in 45 seconds. The length of the platform is
(a) 560 m (b) 400 m
(c) 600 m (d) 450 m

24. A train of length 150 m takes 10 seconds to pass over another train 100 m long coming from the opposite direction. If the speed of the first train is 30 kmph, the speed of second train is
(a) 36 kmph (b) 54 kmph
(c) 60 kmph (d) 72 kmph

25. A man sees a train passing over a bridge 1 km long. The length of the train is half of the bridge. If the train clears the bridge in 2 minutes, the speed of the train is
(a) 30 km/hr (b) 45 km/hr
(c) 50 km/hr (d) 60 km/hr

□□□□

Solutions

1. **Ans. (c)**

$$108 \text{ kmph} = \left(108 \times \frac{5}{18}\right) \text{m/sec} = 30 \text{ m}$$

2. **Ans. (a)**

$$14 \text{ m/sec.} = \left(14 \times \frac{18}{5}\right) \text{km/hr} = 50.4 \text{ k}$$

3. **Ans. (c)**
Length of bridge = (speed × Time)

$$15 \times \frac{5}{60} = \frac{5}{4} \text{km} = 1250 \text{ m}$$

4. **Ans. (b)**

$$\text{Speed} = \left(63 \times \frac{5}{18}\right) \text{m/sec} = \frac{35}{2} \text{ m/sec}$$

$$\text{Time taken} = \left(280 \times \frac{2}{35}\right) \text{sec} = 16 \text{ sec}$$

5. **Ans. (b)**

$$\text{Speed} = \left(132 \times \frac{5}{18}\right) \text{m/sec} = \frac{110}{3} \text{ m/s}$$

Total distance covered = (110 + 165) m

$$\therefore \text{ Required time } \left(275 \times \frac{3}{110}\right) \text{sec}$$

= 7.5 seconds.

6. Ans. (d)

Speed $\left(72 \times \frac{5}{18}\right)$ m/sec = 20 m/sec

Time = 50 sec

$50 \times 20 = 700 + x$

$\Rightarrow x = 500$ m

7. Ans. (c)

Total distance covered = (200 + 200) m

= 400 m

Time taken = 20 sec

$\therefore$ Speed $= \left(\frac{400}{20}\right)$ m/sec

$= \left(20 \times \frac{18}{5}\right)$ km/hr = 72 km/hr

8. Ans. (d)

Total distance covered = (300 + 900) m

= 1200 m

Time taken = 1 min 12 sec = 72 sec

$\therefore$ Speed $= \left(\frac{1200}{72}\right)$ m/sec

$= \left(\frac{1200}{72} \times \frac{18}{5}\right)$ km/hr = 60 km/hr

9. Ans. (c)

Total distance covered = (150 + 300) m

= 450 m

Time taken $= \left(\frac{81}{2}\right)$ seconds

Speed $= \left(450 \times \frac{2}{81}\right)$ m/sec

$= \left(450 \times \frac{2}{81} \times \frac{18}{5}\right)$ km/hr = 40 km/hr

10. Ans. (b)

Let the length of train be x metres and its speed be y metres/sec

Then $\frac{x}{y} = 15 \Rightarrow y = \frac{x}{15}$ also,

Speed of the train

$y = \frac{x + 120}{25} = \frac{x}{15} \Rightarrow x = 180$ m

11. Ans. (d)

Speed $= \left(\frac{110}{3}\right)$ m/sec

Time taken to cross railway platform

$= \left[\frac{110 + 165}{\frac{110}{3}}\right]$ sec

$= \left(275 \times \frac{3}{110}\right)$ sec = 7.5 sec

12. Ans. (a)

Speed of train relative to man

= (125 + 5) m/sec

= 30 m/sec

$\therefore$ Time taken to pass the man

$= \left(\frac{150}{30}\right)$ sec = 5 sec

13. Ans. (d)

Relative speed = (45 − 40) km/hr = 5 km/hr

$= \left(5 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{25}{18}\right)$ m/sec

Total distance covered = Sum of length of trains = 360 m

$\therefore$ Time taken $= \left(360 \times \frac{18}{25}\right)$ sec = 252 sec

14. Ans. (c)

Relative speed = (30 + 42) km/hr = 72 km/hr

$= \left(72 \times \frac{5}{18}\right)$ m/sec = 20 m/sec

Distance covered in crossing each other

= (120 + 120) m = 240 m

Required time $= \left(\frac{240}{20}\right)$ sec = 12 Seconds

15. Ans. (a)

Speed of the train relative to man

$= \left(\frac{110}{6}\right)$ m/sec.

$= \left(\frac{110}{6} \times \frac{18}{5}\right)$ kmph = 66 kmph.

Let the speed of the train be x kmph.

Then, relative speed = $(x + 6)$ kmph.

$x + 6 = 66$ or $x = 60$ kmph.

16. Ans. (d)

Let the speed of the second train be x kmph

Relative speed = $(x + 50)$ kmph

$= \left[(x+50) \times \frac{5}{18}\right]$ m/sec.

$= \left(\frac{250+5x}{18}\right)$ m/sec.

Distance covered = (108 + 112) = 220 m.

$\therefore \quad \frac{220}{\left(\frac{250+5x}{18}\right)} = 6$ or $250 + 5x = 660$

or $x = 82$ kmph

17. Ans. (a)

Let the speed of train C be x kmph.

Speed of B relative to C = $(120 - x)$

$= \left[(120-x) \times \frac{5}{18}\right]$ m/sec

$= \left(\frac{600-5x}{18}\right)$ m/sec

Distance covered = (100 + 200)m = 300 m.

$\therefore \quad \frac{300}{\left(\frac{600-5x}{18}\right)} = 120 \quad \Rightarrow 5400$

$= 120(600 - 5x) \quad \Rightarrow x = 111.$

18. Ans. (d)

Relative speed = (36 + 54) km/hr

$= \left(81 \times \frac{5}{18}\right)$ m/sec $= \left(\frac{45}{2}\right)$ m/sec.

Length of train = $\left(\frac{45}{2} \times 8\right)$ m = 180 m.

19. Ans. (d)

Since the sum of the lengths of the train a the engine is needed, so both the leng must be known

20. Ans. (c)

2 kmph = $\left(2 \times \frac{5}{18}\right)$ m/sec

$= \frac{5}{9}$ m/sec & 4 kmph = $\frac{10}{9}$ m/sec.

Let the length of the train be x metres & speed by y m/sec.

Then, $\frac{x}{\left(y - \frac{5}{9}\right)} = 9$ and $\frac{x}{\left(y - \frac{10}{9}\right)} = 10.$

$\therefore \quad 9y - 5 = x$ and $10(9y - 10) = 9x$

$\therefore \quad 9y - x = 5$ and $90y - 9x = 100$

On solving we get $x = 50$.

$\therefore$ Length of the train is 50 m.

21. Ans. (b)

Suppose they meet x hours after 7 a.m.

Distance covered by A in x hours

$= 20x$ km.

Distance covered by B in $(x - 1)$ hours

$= 25(x - 1)$km

$\therefore \quad 20x + 25(x - 1) = 110$

or $45x = 135$ or $x = 3$.

So, they meet at 10 a.m.

22. Ans. (c)

Relative speed = (54 + 48) kmph

$= \left(102 \times \frac{5}{18}\right)$m/sec $= \left(\frac{85}{3}\right)$m/sec.

Let the length of the other train be x metres.

Then, $(250 + x) \times \frac{3}{85} = 18$

or $750 + 3x = 1530$ or $x = 260$m

$\therefore$ The length of the other train is 260 m.

23. Ans. (b)

Let the length of first train be x metres.

Then, the length of second train is $(x/2)$ metres.

Relative speed = (48 + 42) kmph

$= \left(90 \times \frac{5}{18}\right)$ m/sec $= 25$ m/sec.

$\therefore \quad \frac{\left(x + \frac{x}{2}\right)}{25} = 12$ or $\frac{3x}{2} = 300$

or $x = 200$.

$\therefore$ Length of first train = 200 m.

Let the length of platform be y metres.

Speed of the first train

$= \left(48 \times \frac{5}{18}\right)$m/sec $= \frac{25}{3}$m/sec.

$\therefore \quad (200 + y) \times \frac{3}{40} = 45 \Rightarrow 600 + 3y = 1800 \Rightarrow y = 400$m.

24. Ans. (c)

Let the speed of second train be x kmph.

Relative speed = $(30 + x)$ kmph

$= (30 + x) \times \frac{5}{18}$ m/sec

$= \frac{150 + 100}{10} = 25$ m/s

$\Rightarrow \quad 30 + x = \frac{25 \times 18}{5} = 90$

$\Rightarrow \quad x = 60$

25. Ans. (b)

Lengtyh of bridge = 1000 m.

Length of train = 500 m.

Toal distance covered in clearing the bridge = 1500 m.

Time taken = 120 seconds.

$\therefore$ Speed $= \left(\frac{1500}{120}\right)$m/sec

$= \left(\frac{25}{2} \times \frac{18}{5}\right)$kmph = 45 kmph.

Section B

Algebra & Geometry

LAWS OF INDICES

(1) $a^m \times a^n = a^{m+n}$

Example

→ $2^4 \times 2^{13} = 2^{17}$

→ $3^8 \times 3^7 = 3^{15}$

(2) $\frac{a^m}{a^n} = a^{m-n}$

Example

(i) $\frac{7^5}{7^2} = 7^{5-2} = 7^3$

(ii) $\frac{27^5}{27^2} = 27^{5-2} = 27^3$

(3) $(a^m)^n = a^{m*n}$

Example

(i) $(3^2)^4 = 3^{2\times4} = 3^8$

(ii) $(5^3)^9 = 5^{3\times9} = 5^{27}$

4) $(a \times b)^n = a^n \times b^n$

Example

(i) $(3\times5)^2 = 3^2 \times 5^2$

(ii) $(4\times5)^3 = 4^3 \times 5^3$

) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example

(i) $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$

(ii) $\left(\frac{5}{8}\right)^3 = \frac{5^3}{8^3} = \frac{125}{512}$

(6) $a^0 = 1$

here a may be any number

Example

(i) $5^0 = 1$

(ii) $1000^0 = 1$

(7) $a^x = a^y$

If and only if

(i) $x = y$ or

(ii) $x = 0, y = 0$

(8) $a^{-m} = \frac{1}{a^m}$

Example

(i) $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(ii) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

(9) $a^{-1} = \frac{1}{a}$

Example $10^{-1} = \frac{1}{10}$

We write $\sqrt[n]{a} = a^{1/n}$ and it is called a surd of order 'n'

(1) $\left(\sqrt[n]{a}\right)^n = \left(a^{\frac{1}{n}}\right)^n = a$

(2) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Example

(i) $\sqrt[3]{2\times5} = \sqrt[3]{2} \times \sqrt[3]{5}$

(ii) $\sqrt[5]{7\times13} = \sqrt[5]{7} \times \sqrt[5]{13}$

(3) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example

(i) $\sqrt[3]{\frac{2}{5}} = \frac{\sqrt[3]{2}}{\sqrt[3]{5}}$

(ii) $\sqrt[5]{\frac{7}{13}} = \frac{\sqrt[5]{7}}{\sqrt[5]{13}}$

(4) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Example

(i) $\left(\sqrt[3]{2}\right)^5 = \sqrt[3]{2^5} = \sqrt[3]{32}$

(ii) $\left(\sqrt[5]{4}\right)^3 = \sqrt[5]{4^3} = \sqrt[5]{64}$

(5) $\sqrt[m]{\sqrt[n]{\sqrt[p]{a}}} = \sqrt[mnp]{a} = a^{\frac{1}{mnp}}$

Example

(i) $\sqrt[2]{\sqrt[3]{\sqrt[4]{10}}} = 10^{\frac{1}{2\times3\times4}} = 10^{\frac{1}{24}}$

(ii) $\sqrt[3]{\sqrt[5]{25}} = 25^{\frac{1}{3\times5}} = 25^{\frac{1}{15}}$

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Solved Examples

1. Find the value of $\left(\frac{32}{243}\right)^{-3/5}$

Sol.: $\left(\frac{32}{243}\right)^{-3/5} = \left(\frac{2^5}{3^5}\right)^{-3/5} = \left(\frac{2}{3}\right)^{5\times\frac{-3}{5}}$

$= \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

2. Solve $4^{3x-2}\times8 = 1$

Sol.: $4^{3x-2}\times8 = \left(2^2\right)^{3x-2}\times2^3 = 1$

$= 2^{6x-4+3} = 1$

$= 2^{6x-1} = 2^0 \quad [\because 2^0 = 1]$

$= 2^{6x-1} = 2^0 \Rightarrow x = \frac{1}{6}$

3. Find the value of x if $\left(\frac{a}{b}\right)^{3x-5} = \left(\frac{b}{a}\right)^{x+3}$

Sol.: $= \left(\frac{a}{b}\right)^{3x-5} = \left(\left(\frac{a}{b}\right)^{-1}\right)^{x+3}$

$= \left(\frac{a}{b}\right)^{3x-5} = \left(\frac{a}{b}\right)^{-x-3}$

$= 3x - 5 = -x - 3$

$= 4x = 2 \Rightarrow x = \frac{1}{2}$

4. Which is greater $\sqrt[4]{4}$ or $\sqrt[5]{5}$

Sol.: $\sqrt[4]{4} = 4^{\frac{1}{4}}$ and $\sqrt[5]{5} = 5^{\frac{1}{5}}$

Now the LCM of 4 & 5 is 20, so to compare two surds raise the power by 20.

$\left(4^{1/4}\right)^{20} = 4^5 = 1024$

$$\left(5^{\frac{1}{5}}\right)^{20} = 5^4 = 625$$

hence, $\sqrt[4]{4} > \sqrt[5]{5}$

5. Simplify, $\dfrac{\sqrt{8}+\sqrt{2}}{\sqrt{8}-\sqrt{2}}$

Sol.: $\dfrac{\sqrt{8}+\sqrt{2}}{\sqrt{8}-\sqrt{2}} \times \dfrac{\sqrt{8}+\sqrt{2}}{\sqrt{8}+\sqrt{2}}$

(Multiply numerator & denominator by conjugate of denominator)

$$= \frac{\left(\sqrt{8}+\sqrt{2}\right)^2}{\left(\sqrt{8}\right)^2-\left(\sqrt{2}\right)^2} = \frac{8+2+2\sqrt{16}}{8-2}$$

$$= \frac{10+8}{6} = 3 \text{ Ans.}$$

6. Simplify $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Sol.: Multiplying numerator & denominator of above equation by $\sqrt{5}-\sqrt{3}$ we get

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{5+3-2\sqrt{15}}{5-3} = \frac{8-2\sqrt{15}}{2} = 4-\sqrt{15}$$

7. Simplify $\dfrac{\sqrt{a^2+1}+\sqrt{a^2-1}}{\sqrt{a^2+1}-\sqrt{a^2-1}}$

Sol.:

$$\frac{\sqrt{a^2+1}+\sqrt{a^2-1}}{\sqrt{a^2+1}-\sqrt{a^2-1}} \times \frac{\sqrt{a^2+1}+\sqrt{a^2-1}}{\sqrt{a^2+1}+\sqrt{a^2-1}}$$

$$= \frac{a^2+1+a^2-1+2\left(\sqrt{a^2+1}\right)\left(\sqrt{a^2-1}\right)}{a^2+1-\left(a^2-1\right)}$$

$$= \frac{2a^2+2\sqrt{a^4-1}}{2} = a^2+\sqrt{a^4-1}$$

8. Which is greater $\sqrt[3]{4}$ or $\sqrt[4]{5}$

Sol.: $\sqrt[3]{4} = 4^{\frac{1}{3}}$ and $\sqrt[4]{5} = 5^{\frac{1}{4}}$

raise the power of both the surds by 12(LCM of 3 and 4)

$$\left(4^{\frac{1}{3}}\right)^{12} = 4^4 = 256$$

$$\left(5^{\frac{1}{4}}\right)^{12} = 5^3 = 125$$

Hence, $\sqrt[3]{4} > \sqrt[4]{5}$

9. If $2^{2x-1} = \dfrac{1}{8^{x-3}}$ then find value of x

Sol.: $2^{2x-1} = \dfrac{1}{8^{x-3}}$

$$= 2^{2x-1} = \frac{1}{\left(2^3\right)^{x-3}} = \frac{1}{2^{3x-9}}$$

$$= 2^{2x-1} = 2^{-3x+9} \quad \left[\because \frac{1}{a^m} = a^{-m}\right]$$

$\Rightarrow 2x - 1 = -3x + 9$

$= 5x = 10, \; x = 2$

10. Find the value of x if $\sqrt[3]{32} = 2^x$

Sol.: $\sqrt[3]{32} = 32^{1/3} = \left(2^5\right)^{1/3}$

$$= 2^{5/3} = 2^x, \; x = 5/3$$

11. If $2^x = 3^y = 6^{-z}$ then $\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)$ is equal to?

Sol.: let $2^x = 3^y = 6^{-z} = k$

So, $2 = k^{1/x}$, $3 = k^{1/y}$, $6 = k^{-1/z}$

We know that $2 \times 3 = 6$ this gives

$$k^{1/x} \times k^{1/y} = k^{-1/z} = \frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$$

$$= \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

12. Arrange $\sqrt{2}$, $\sqrt[3]{4}$ and $\sqrt[4]{6}$ in ascending order

Sol.: $\sqrt{2} = 2^{1/2}$

$\sqrt[3]{4} = 4^{1/3}$, $\sqrt[4]{6} = 6^{1/4}$

Now raise the power of given surds by LCM of 2, 3, 4 that is 12

$$\left(2^{1/2}\right)^{12} = 2^6 = 64$$

$$\left(4^{1/3}\right)^{12} = 4^4 = 256$$

$$\left(6^{1/4}\right)^{12} = 6^3 = 216$$

Ans: $\sqrt{2}$, $\sqrt[4]{6}$, $\sqrt[3]{4}$

Logarithms

Definition

The logarithm of any number of a given base is equal to the index to which the base should be raised to obtain the given number.

For example if

$a^x = c$, then x is called logarithm of a number c to the base a. It is written as $\log_a c = x$

Similarly

$2^3 = 8$ is similar as $\log_2 8 = 3$

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ is written

as $\log_4 \frac{1}{16} = -2$

$10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

$3^5 = 243 \Rightarrow \log_3 243 = 5$

PROPERTIES OF LOGARITHMS

(a) $\log_a (m \times n) = \log_a m + \log_a n$

Example:

$\log_{10} (15) = \log_{10} 3 + \log_{10} 5$

(b) $\log_a \left(\frac{m}{n}\right) = \log_a (m) - \log_a (n)$

Example:

$\log_2 \left(\frac{6}{5}\right) = \log_2 6 - \log_2 5$

(c) $\log_a \left(m^n\right) = n \log_a m$

Example:

$\log_5 625 = \log_5 5^4 = 4 \log_5 5$

(d) $\log_{a^n} (m) = \frac{1}{n} \log_a (m)$

Example:

$\log_{2^4} (8) = \frac{1}{4} \log_2 8$

(2) $\log_{10^2} 1000 = \frac{1}{2} \log_{10} 1000$

(e) $\log_a b = \frac{\log_n b}{\log_n a}$

[here n may be any natural number)

Example:

$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2}$

$\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$

(f) $\log_a x = \frac{\log_n x}{\log_n a} = \frac{1}{\log_x a}$

Example:

$\log_2 5 = \frac{1}{\log_5 2}$

$\log_3 8 = \frac{1}{\log_8 3}$

(g) $\log_a b \times \log_b a = \frac{\log_n b}{\log_n a} \times \frac{\log_n a}{\log_n b} = 1$

Example:

$\log_3 4 \times \log_4 3 = 1$

$= \log_2 10 \times \log_{10} 2 = 1$

Remark:

(1) When base is not mentioned it is taken as 10.

(2) Logarithms to the base 10 is known as common logarithms.

(3) $\log_a a = 1$

Example:

$\log_5 5 = 1$

$\log_2 2 = 1$

(i) $\log_a 1 = 0$

[a may be any natural number.]

(j) $a^{\log_a x} = x$

$5^{\log_5 3} = 3$

$3^{\log_3 7} = 7$

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Solved Examples

1. Find the logarithm of 32 to the base 2

Sol.: $\log_2 32 = \log_2 (2^5)$

$= 5\log_2 2 = 5 \times 1 = 5$

2. If $\log_5 a = 3$ find value of a

Sol.: $\log_5 a = 3$

$a = 5^3 = 125$

3. Find the value of $2^{\log_2 5}$

Sol.: $\therefore a^{\log_a x} = x$

So $2^{\log_2 5} = 5$

4. Find the value of $3^{2+\log_3 5}$

Sol.: $3^{2+\log_3 5} = 3^2 \times 3^{\log_3 5}$

$= 3^2 \times 5 = 45$

5. Find the value of $3^{2-\log_3 5}$

Sol.: $3^{2-\log_3 5} = \frac{3^2}{3^{\log_3 5}} = \frac{9}{5}$

6. Find the value of $\log_5 125 - \log_4 16$

Sol.: $\log_5 125 - \log_4 16$

$= \log_5 (5^3) - \log_4 (4^2) = 3 - 2 = 1$

7. Find the value of $\log x^3 + \log x$

Sol.: $\log x^3 + \log x = 3\log x + \log x = 3$

8. If $\log_{16} a = \frac{1}{2}$, Find a

Sol.: $\log_{16} a = \frac{1}{2}$

$a = 16^{1/2}$

$a = 4$

9. $\log_a \sqrt{3} = \frac{1}{6}$, find the value of a

Sol.: $\log_a \sqrt{3} = \frac{1}{6}$

$= \log_a 3^{1/2} = \frac{1}{6}$

or $a^{1/6} = 3^{1/2}$, $a = 3^3 = 27$

10. If $\log_{27} x + \log_9 x + \log_3 x = 11$, find the value of x

Sol.: $\log_{27} x + \log_9 x + \log_3 x = 11$

$= \frac{1}{3}\log_3 x + \frac{1}{2}\log_3 x + \log_3 x = 11$

$= \frac{11}{6}\log_3 x = 11$

MADE EA...

$= \frac{1}{6}\log_3 x = 1$

$= \log_3 x = 6$

$= x = 3^6 = 729$

11. Solve $\log_3 n - \log_3 4 = 2$

Sol.: $\log_3 n - \log_3 4 = \log_3 \frac{n}{4} = 2$

$\log_3 \frac{n}{4} = 2$

or $\frac{n}{4} = 3^2,\ n = 3^2 \times 4 = 36$

12. Find the value of $\log\frac{x^2}{yz} + \log\frac{y^2}{zx} + \log\frac{z^2}{xy}$

Sol.: $\log\frac{x^2}{yz} + \log\frac{y^2}{zx} + \log\frac{z^2}{xy}$

$= \log\frac{x^2}{yz} \times \frac{y^2}{zx} \times \frac{z^2}{xy}$

$= \log\frac{x^2y^2z^2}{y^2z^2x^2} = \log 1 = 0$

13. Evaluate $\log_7\left(\frac{1}{343}\right)$

Sol.: $\log_7\left(\frac{1}{343}\right) = \log_7 \frac{1}{7^3}$

$= \log_7 7^{-3} = -3\log_7 7 = -3$

14. Evaluate $= \log_{100}(0.01)$

Sol.: $\log_{100}(100)^{-1} = -1\log_{100} 100 = -1$

15. If $\log_{\sqrt{8}} x = 3\frac{1}{3}$, find the value of x

Sol.: $\log_{\sqrt{8}} x = \frac{10}{3} = \left(2^{3/2}\right)^{10/3}$

$= 2^{3/2 \times 10/3} = 2^5 = 32$

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Surds, Indices & Logarithms

I Practice Exercise: I

1. The value of $\left(\sqrt{8}\right)^{1/3}$ is:
   (a) 2 (b) 4
   (c) $\sqrt{2}$ (d) 8

2. $\left(\frac{1}{216}\right)^{-2/3} + \left(\frac{1}{27}\right)^{-4/3} = ?$
   (a) $\frac{3}{4}$ (b) $\frac{2}{3}$
   (c) $\frac{4}{9}$ (d) $\frac{1}{8}$

3. If $\sqrt{2^n} = 64$, then the value of n is:
   (a) 8 (b) 4
   (c) 6 (d) 12

4. If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then n equals
   (a) 0 (b) 2
   (c) 3 (d) 4

5. If $\left(\sqrt{3}\right)^5 \times 9^2 = 3^\alpha \times 3\sqrt{3}$, then α equals
   (a) 2 (b) 3
   (c) 4 (d) 5

6. If x, y, z are real numbers, then the value of $\sqrt{x^{-1}y}.\sqrt{y^{-1}z}.\sqrt{z^{-1}x}$ is
   (a) xyz (b) $\sqrt{xyz}$
   (c) $\frac{1}{xyz}$ (d) 1

7. $\left(\frac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a-b)} \cdot \left(\frac{x^a}{x^b}\right)^{(a+b-c)} =$

(a) x^{abc} (b) x^{a+b+c}

(c) $x^{ab+bc+ca}$ (d) 1

8. If $2^{x+4} - 2^{x+2} = 3$, then x is equal to:

(a) 0 (b) 2

(c) −1 (d) −2

9. If $2^{2x-1} = \frac{1}{8^{x-3}}$, the value of x is:

(a) 3 (b) 2

(c) 0 (d) −2

10. If $\sqrt{5+\sqrt[3]{x}} = 3$, then x is equal to:

(a) 125 (b) 64

(c) 27 (d) 9

11. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals:

(a) $\frac{xz}{x+z}$ (b) $\frac{xz}{2(x-z)}$

(c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(x+z)}$

12. If $x = y^a$, $y = z^b$ and $z = x^c$, then the value of abc is:

(a) 4 (b) 3

(c) 2 (d) 1

Solutions

1. **Ans. (c)**

$$\left(\sqrt{8}\right)^{1/3} = \left(8^{1/2}\right)^{1/3} = 8^{\left(\frac{1}{2}\times\frac{1}{3}\right)} = 8^{1/6}$$

$$= \left(2^3\right)^{1/6} = 2^{\left(3\times\frac{1}{6}\right)} = 2^{1/2} = \sqrt{2}.$$

2. **Ans. (c)**

$$\left(\frac{1}{216}\right)^{-2/3} + \left(\frac{1}{27}\right)^{-4/3}$$

$$= (216)^{2/3} + (27)^{4/3} = \left(6^3\right)^{2/3} + \left(3^3\right)^{4/3}$$

$$= 6^{\left(3\times\frac{2}{3}\right)} + 3^{\left(3\times\frac{4}{3}\right)} = 6^2 + 3^4 = \frac{36}{81} = \frac{4}{9}.$$

3. **Ans. (d)**

$\sqrt{2^n} = 64 \Rightarrow 2^{n/2} = 64 = 2^6$.

$\therefore \frac{n}{2} = 6$ or $n = 12$.

4. **Ans. (c)**

$$\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$$

$$\Rightarrow \frac{3^{2n} \times 3^5 \times \left(3^3\right)^3}{3 \times \left(3^4\right)^4} = 3^3.$$

or $\frac{3^{2n} \times 3^5 \times 3^9}{3^1 \times 3^{16}} = 3^3$

or $3^{2n+5+9} = 3^3 \times 3^1 \times 3^{16}$

or $3^{2n+14} = 3^{20}$

$\therefore 2n + 14 = 20$

or $2n = 6$ or $n = 3$.

5. **Ans. (d)**

$$\left(\sqrt{3}\right)^5 \times 9^2 = 3^\alpha \times 3\sqrt{3}$$

$$\Rightarrow \left(3^{1/2}\right)^5 \times \left(3^2\right)^2 = 3^\alpha \times 3^1 \times 3^{1/2}$$

$\therefore 3^{5/2} \times 3^4 = 3^\alpha \times 3^1 \times 3^{1/2}$

or $3^{\left(\frac{5}{2}+4\right)} = 3^{\left(\alpha+1+\frac{1}{2}\right)}$

or, $3^{13/2} = 3^{\alpha+\frac{3}{2}}$ So, $\alpha + \frac{3}{2} = \frac{13}{2}$

or $\alpha = \left(\frac{13}{2} - \frac{3}{2}\right) = 5.$

6. Ans. (d)

$$\sqrt{x^{-1}y}.\sqrt{y^{-1}z}.\sqrt{z^{-1}x}$$

$$=\sqrt{\frac{y}{x}}.\sqrt{\frac{z}{y}}.\sqrt{\frac{x}{z}}=\frac{\sqrt{y}}{\sqrt{x}}\times\frac{\sqrt{z}}{\sqrt{y}}\times\frac{\sqrt{x}}{\sqrt{z}}=1.$$

7. Ans. (d)

Given Exp.

$$=x^{(b-c)(b+c-a)}.x^{(c-a)(c+a-b)}.x^{(a-b)(a+b-c)}$$

$$=x^{(b^2-c^2)+(c^2-a^2)+(a^2-b^2)}.x^{-a(b-c)-b(c-a)-c(a-b)}$$

$$=x^0.x^0=1$$

8. Ans. (d)

$$2^{x+4}-2^{x+2}=3$$

$$\Rightarrow 2^{x+2}\left(2^2-1\right)=3$$

$$\Rightarrow 2^{x+2}=1=2^0.$$

$$\therefore \quad x+2=0 \text{ or } x=-2.$$

9. Ans. (b)

$$2^{2x-1}=\frac{1}{8^{x-3}} \qquad \Rightarrow 2^{2x-1}=\frac{1}{(2^3)^{(x-3)}}$$

$$\Rightarrow 2^{2x-1}=\frac{1}{2^{3x-9}} \qquad \Rightarrow 2^{2x-1}=2^{9-3x}$$

$$\therefore \quad 2x-1=9-3x \text{ or } 5x=10 \text{ or } x=2.$$

10. Ans. (b)

On squaring both sides, we get :

$5+\sqrt[3]{x}=9$ or $\sqrt[3]{x}=4$.

Cubing both sides, we get

$x=(4\times4\times4)=64$.

11. Ans. (d)

Let $a^x=b^y=c^z=k$.

Then $a=k^{1/x}$, $b=k^{1/y}$ and $c=k^{1/z}$.

$$b^2=ac \Rightarrow k^{2/y}=k^{1/x}.k^{1/z}$$

$$\Rightarrow \quad k^{2/y}=k^{\left(\frac{1}{x}+\frac{1}{z}\right)}$$

$$\Rightarrow y=2\frac{(zx)}{x+z}$$

12. Ans. (d)

$$x=y^a=\left(z^b\right)^a=z^{ab}=\left(x^c\right)^{ab}=x^{abc}$$

$$\therefore \quad abc=1.$$

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Surds, Indices & Logarithm

II Practice Exercise: II

1. The value of $\log_{343}7$ is:

(a) $\frac{1}{3}$ (b) -3

(c) $-\frac{1}{3}$ (d) 3

2. The value of $\log_{10}(.0001)$ is:

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$

(c) -4 (d) 4

3. If $\log_8 x=\frac{2}{3}$, then the value of x is :

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) 4 (d) 3

4. If $\log_x 4=\frac{1}{4}$, then x is equal to:

(a) 16 (b) 64

(c) 128 (d) 256

5. $\log_{32}x=0.8$, then x is equal to:

(a) 25.6 (b) 16

(c) 10 (d) 12.8

6. If $\log_4 x+\log_2 x=6$, then x is equal to

(a) 2 (b) 4

(c) 8 (d) 16

7. If $\log 2 = 0.30103$, then the number of digits in 5^{20} is:
(a) 14 (b) 16
(c) 18 (d) 25

8. The value of $\log_2(\log_5 625)$ is:
(a) 2 (b) 5
(c) 10 (d) 15

9. $(\log_b a \times \log_c b \times \log_a c)$ is equal to:
(a) 0 (b) 1
(c) abc (d) a + b + c

10. $\left[\frac{1}{(\log_a bc)+1}+\frac{1}{(\log_b ca)+1}+\frac{1}{(\log_c ab)+1}\right]$ is equal to:
(a) 1 (b) 2
(c) 3 (d) $\frac{3}{2}$

11. If $\log_2[\log_3(\log_2 x)]=1$, then x is equal to:
(a) 512 (b) 128
(c) 12 (d) 0

12. If $\log_{10}2=0.3010$, then $\log_2 10$ is :
(a) 0.3322 (b) 3.2320
(c) 3.3222 (d) 5

13. If $\log_{10}2=0.3010$, the value of $\log_{10}5$ is:
(a) 0.3241 (b) 0.6911
(c) 0.6990 (d) 0.7525

14. The value of $(\log_9 27 + \log_8 32)$ is
(a) 4 (b) 7
(c) $\frac{7}{2}$ (d) $\frac{19}{6}$

15. The value of $16^{\log_4 5}$ is
(a) 5 (b) 16
(c) 25 (d) $\frac{5}{64}$

16. If $\log_5(x^2+x)-\log_5(x+1)=2$, then the value of x is
(a) 5
(c) 25 (b) 32
(d) 10

17. If $\log x + \log y = \log(x+y)$, then:
(a) $x = y$ (b) $xy = 1$
(c) $y=\frac{x-1}{x}$ (d) $y=\frac{x}{x-1}$

18. The value of
$\left[\frac{1}{\log_{(p/q)} x}+\frac{1}{\log_{(q/r)} x}+\frac{1}{\log_{(r/p)} x}\right]$ is:
(a) 3 (b) 2
(c) 1 (d) 0

19. If log2 = 0.3010 and log3 = 0.4771 then the value of log 4.5 is:
(a) 0.6532 (b) 0.7727
(c) 0.3266 (d) None of these

Solutions

1. **Ans. (a)**
Let $\log_{343}7=m$.
Then, $(343)^m = 7 \Rightarrow (7^3)^m = 7$
$\Rightarrow 7^{3m} = 7$
$\Rightarrow 3m = 1 \Rightarrow m = \frac{1}{3}$.
$\therefore \log_{343}7 = \frac{1}{3}$.

2. **Ans. (c)**
Let $\log_{10}(.0001) = m$. Then,
$10^m = .0001 \Rightarrow 10^m = \frac{1}{10000} \Rightarrow 10^m$
$= \frac{1}{10^4} \Rightarrow 10^m = 10^{-4} \Rightarrow m = -4$
$\therefore \log_{10}(.0001) = -4$.

3. Ans. (c)

$$\log_8 x = \frac{2}{3} \Rightarrow x = 8^{2/3} = \left(2^3\right)^{2/3}$$

$$= 2^{\left(3\times\frac{2}{3}\right)} = 2^2 = 4.$$

4. Ans. (d)

$$\log_x 4 = \frac{1}{4} \Rightarrow x^{1/4} = 4 \Rightarrow x = 4^4 = 256.$$

5. Ans. (b)

$$\log_{32} x = 0.8 \Rightarrow x = (32)^{0.8}$$

$$= \left(2^5\right)^{4/5} = 2^4 = 16.$$

6. Ans. (d)

$\log_4 x + \log_2 x = 6$

$$\Rightarrow \frac{1}{2}\log_2 x + \log_2 x = 6$$

$$\Rightarrow \frac{3}{2}\log_2 x = 6$$

$\Rightarrow \log_2 x = 4$

$\Rightarrow x = 2^4 = 16$

7. Ans. (a)

$$\log 5^{20} = 20\log 5 = 20\times\left[\log\left(\frac{10}{2}\right)\right]$$

$$= 20\times[\log 10 - \log 2] = 20\times[1 - 0.3010]$$

$= 20\times 0.6990 = 13.9800$

$\therefore$ Characteristic $= 13$.

$\therefore$ Number of digits in 5^{20} is 14.

8. Ans. (a)

Let $\log_5 625 = x$. Then $5^x = 625 = 5^4$ or $x = 4$

Let $\log_2(\log_5 625) = y$. Then $\log_2(4) = y$

or $2^y = 4 = 2^2$. So, $y = 2$.

$\therefore \log_2(\log_5 625) = 2$.

9. Ans. (b)

$$\text{Given Exp.} = \left(\frac{\log a}{\log b}\times\frac{\log b}{\log c}\times\frac{\log c}{\log a}\right)^{\log_a}$$

10. Ans. (a)

$$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b}$$

$$+ \frac{1}{\log_c ab + \log_c c}$$

$$= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)}$$

$$= \log_{abc} a + \log_{abc} b + \log_{abc} c$$

$$= \log_{abc} (abc) = 1.$$

11. Ans. (a)

$$\log_2\left[\log_3(\log_2 x)\right] = 1$$

$$\Rightarrow \log_3(\log_2 x) = 2$$

$$\Rightarrow \log_2 x = 3^2 = 9$$

$$\Rightarrow x = 2^9 = 512$$

12. Ans. (c)

$$\log_2 10 = \frac{1}{\log_{10} 2} = \frac{1}{.3010} = 3.3222$$

13. Ans. (c)

$$\log_{10} 5 = \log_{10}\left(\frac{10}{2}\right) = \log_{10} 10 - \log_{10} 2$$

$$= 1 - \log_{10} 2 = (1 - 0.3010) = 0.6990$$

14. Ans. (d)

$\log_9 27 + \log_8 32$

$\log_{3^2} 3^3 + \log_{2^3} 2^5$

$$\frac{3}{2}\log_3 3 + \frac{5}{3}\log_2 2$$

So, $\frac{3}{2} + \frac{5}{3} = \frac{19}{6}$

15. Ans. (c)

Remember that $a^{\log_a x} = x$.

$\therefore \quad 16^{\log_4 5} = \left(4^2\right)^{\log_4 5} = 4^{2\log_4 5}$

$= 4^{\log(5^2)} = 4^{\log_4(25)} = 25.$

16. Ans. (c)

$\log_5\left(x^2+x\right) - \log_5(x+1) = 2$

$\Rightarrow \log_5\left(\frac{x^2+x}{x+1}\right) = 2$

$\therefore \log_5\left[\frac{x(x+1)}{x+1}\right] = 2$

or $\log_5 x = 2$ or $x = 5^2 = 25.$

17. Ans. (d)

$\log x + \log y = \log(x+y)$

$\Rightarrow \log(x+y) = \log(xy)$

$\therefore x+y = xy$ or $x = y(x-1)$

$\Rightarrow y = \frac{x}{x-1}$

18. Ans. (d)

Given Expression

$= \log_x\left(\frac{p}{q}\right) + \log_x\left(\frac{q}{r}\right) + \log_x\left(\frac{r}{p}\right)$

$\log_x\left(\frac{p}{q} \times \frac{q}{r} \times \frac{r}{p}\right) = \log 1 = 0.$

19. Ans. (a)

$\log 4.5 = \log\left(\frac{9}{2}\right) = \log 9 - \log 2$

$= \log\left(3^2\right) - \log 2 = 2\log 3 - \log 2$

$= (2 \times 0.4771 = 0.3010) = 0.6532$

□□□□

2 Progression

The chapter on progression is very interesting as it deals real life problem in mathematical manner. Lots of question on this topics are asked in competitive Examination.

Questions in Public Sector, Banking and various Management Entrance Exam mostly appear from either Arithmetic progression or from Geometric progression.

Arithmetic Progression (AP)

Quantities are said to be in AP when they increase or decrease by a common difference.
For Example.

1. All natural numbers are in AP with common difference 1.
   1, 2, 3, 4, 5 .....
2. All Even & odd numbers are in AP having common difference 2.
   Even numbers = 2, 4, 6, 8, ...
   Odd numbers = 1, 3, 5, 7, ...
3. 3, 8, 13, 18 .....
   Arithmetic Progression can be represented as
   a, a + d, a + 2d, a + 3d .....
   Where a is the first term and
   d is the common difference of series given.
   n^{th} term of series = a + (n – 1) d

Sum of the given number of a series in AP
let a be the first term,
d be the common difference, and
n the total number of term then

Sum of series $S = \frac{n}{2}[2a + (n-1)d]$

also if L be the last term then

$L = a + (n-1)d$ then

$$S = \frac{n(a+L)}{2}$$

To find arithmetic mean between any two given quantities
Arithmetic mean between two numbers a and b can be calculated by

$$AM = \frac{a+b}{2}$$

Let's have a glimpse of few examples on Arithmetic Progression.

1. Find the arithmetic mean between 3 & 13

Sol.: $AM = \frac{a+b}{2}$

$= \frac{3+13}{2} = 8$ Ans.

3. Find 15th term of series
$-8\ -5\ -2\ 1\ 4$ .....
Sol.: here a= –8, d=3
So 15th term will be $-8 + 3(15-1) = 34$ Ans.

4. Find the sum of 20 term of given series
3, 10, 17, 24 .....
Sol.: Sum of n term S is given by

$$S = \frac{n}{2}[2a + (n-1)d]$$

Here a = 3, d = 7, n = 20

$$\text{So, } S = \frac{20}{2}[2\times 3 + (20-1)\times 7] = 1390 \text{ Ans.}$$

5. Calculate sum of 50 first positive even numbers.
Sol.: Even number = 2, 4, 6, 8, ...
here a = 2, d = 2, n = 50 So,

$$S = \frac{50}{2}[2\times 2 + (50-1)\times 2] = 2550 \text{ Ans.}$$

To insert n Arithmetic means between two given Quantities

Let a and b be numbers, then to insert n AMs between a and b we have to calculate common difference between them common difference,

$$d = \frac{b-a}{n+1}$$

Now n arithmetic means can be calculated as

$$a + \frac{b-a}{n+1}; a + \frac{2(b-a)}{n+1}; ..a + \frac{n(b-a)}{n+1}$$

For Example:

6. Insert 4 arithmetic means between 17 & 42

Sol.: Common difference $d = \frac{b-a}{n+1}$

(GP)

Quantities are said to be in GP when the
or decrease by a constant factor

The constant factor is called comm

3, 6, 12, 24, ...

2, 6, 18, 54, ...

$5, 5^2, 5^3, 5^4$..... etc are in GP

Geometric Progression can be repre
as

a, ar, ar^2, ar^3 ....

where a is the first term and r is the
ratio.

n^{th} term of a GP = ar^{n-1}

Sum of n term of a GP

$$S_n = \frac{a(r^n - 1)}{r-1} \text{ [when } r > 1\text{]}$$

$$S_n = \frac{a(1-r^n)}{(1-r)} \text{ [when } r < 1\text{]}$$

To find Geometric mean (GM)
two numbers

GM between two numbers a and
calculated by

$$GM = \sqrt{ab}$$

7. What is common ratio of given seri

$$-3, 1, \frac{-1}{3}, \frac{1}{9} \ldots$$

Sol.: Common ratio $= \frac{1}{-3}$ Ans

8. Find 6th term of given series 3
Sol.: here a = 3, r = 4

So 6th term can be given by $a \times r^{(n-1)}$

$= 3 \times 4^5 = 3072$ Ans.

9. Find sum of 10 term of given series

2, 4, 8, 16 .....

Sol.: $S = \frac{a(r^n - 1)}{(r-1)}$ [when $r > 1$]

$S = \frac{2 \times (2^{10} - 1)}{(2-1)} = 2046$ Ans.

10. Calculate sum of six term of given series

$1, \frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3} \ldots$

Sol.: $S = \frac{a(1-r^n)}{(1-r)}$ [when $r < 1$]

$S = \frac{1[1-(\frac{1}{3})^6]}{1-\frac{1}{3}} = \frac{364}{243}$ Ans.

Sum of Infinite terms of a series

Sum of infinite terms of a series can be calculated when common ratio is less then 1.

$S_\infty = \frac{a}{1-r}$ [$r < 1$]

Otherwise S_∞ will be equal to ∞.

11. Find sum of infinite terms of series

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$

Sol.: $S_\infty = \frac{a}{1-r}$ [$\because r < 1$] $= \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$ Ans.

12. Find sum of infinite terms of given series

$1, \frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3} \ldots$

Sol.: $S_\infty = \frac{a}{1-r}$ [$\because r < 1$] $= \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$ Ans.

Harmonic Progression (HP)

If a, b, c, d are in Arithmetic progression then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ are in Harmonic Progression

Harmonic mean HM of a and b

$HM = \frac{2ab}{a+b}$

13. Find next 3 terms of given series

$\frac{1}{3}, \frac{1}{8}, \frac{1}{13} \ldots$

Sol.: Here term are in HP because 3, 8, 13 are in AP.

Now next 3 term in AP are 18, 23, 28 so next 3 terms in HP are $\frac{1}{18}, \frac{1}{23}, \frac{1}{28}$ now series become $\frac{1}{3}, \frac{1}{18}, \frac{1}{13}, \frac{1}{18}, \frac{1}{23}, \frac{1}{28}$ Ans.

14. Find harmonic mean between 40 & 60.

Sol.: $HM = \frac{2ab}{a+b}$

$HM = \frac{2 \times 40 \times 60}{40+60} = 48$ Ans.

Some Useful Results

1. Sum of n natural numbers

$S = 1+2+3+4+\ldots\ldots n$

$S = \frac{n(n+1)}{2}$

2. Sum of squares of first n natural numbers

$S = 1^2 + 2^2 + 3^2 + \ldots\ldots n^2$

$S = \left\{\frac{n(n+1)(2n+1)}{6}\right\}$

3. Sum of cubes of first ... terms
$S = 1^3 + 2^3 + 3^3 + 4^3 + \ldots$

$$S = \left\{\frac{n(n+1)}{2}\right\}^2$$

4. Sum of first n odd natural numbers
$S = 1 + 3 + 5 + \ldots\ldots (2n-1)$
$S = n^2$

5. Sum of first n even natural numbers
$S = 2 + 4 + 6 + \ldots 2n$
$= n(n+1) = n^2 + n$

□□□□

Progression

Solved Examples

1. How many terms are there in the AP 20, 25, 30, ...130.
(a) 22 (b) 23
(c) 21 (d) 24

Ans. (b)
In series 20, 25, 30 .....130.
$a = 20, d = 5$
n^{th} term is 130
$\Rightarrow 20 + (n-1)5 = 130$
$\Rightarrow 5n = 115, n = 23$

2. Find the 1st term of an AP whose 8th and 12th terms are respecityvely 39 and 59.
(a) 5 (b) 6
(c) 4 (d) 3

Ans. (c)
Here $a + 7d = 39$ ...(i)
$a + 11d = 59$ ...(ii)
So, $4d = 20$
$d = 5$
$a + (7 \times 5) = 39$
So, $a = 4$

And. (d)
A P is given 1, 3, 5, ...
here $a = 1$, $d = 2$
$a + (n-1)d = 55$
$1 + (n-1)2 = 55$
$n = 28$

4. Find the lowest number in an AP such that the sum of all the terms is 105 and greatest term is 6 times the least.
(a) 5 (b) 10
(c) 15 (d) All of these

Ans. (d)
Here $a + (n-1)d = 6a$
or $(n-1)d = 5a$
also it is given that

$$\frac{n}{2}[2a + (n-1)d] = 105$$

also $n/2*[a + l] = 105$
[where l is last term]
Now $n/2*[a + 6a] = 105$
$n/2*[7a] = 105$ or $n*a = 30$
now, we will go by options
if $a = 5$ then $l = 30$
then $d = 5$
we will check also for 10 & 15. In this case the options will satisfy the given condition

5. Find the 15th term of the sequence 20, 10, ...
(a) –45 (b) –55
(c) –50 (d) 0

Ans. (c)
AP 20, 15, 10
here $a = 20$
$d = -5$
15^{th} term will be $a + 14d$ which is equal to
$20 + -5 \times 14 = -50$

6. A sum of money kept in a bank amounts to Rs. 1240 in 4 years and Rs. 1600 in 10 years at simple Interst. Find sum.
(a) Rs. 800 (b) Rs. 900
(c) Rs. 1150 (d) Rs. 1000

Ans. (d)
This is case of simple interst, simple interest follows *AP*
$a + 4d = 1240$
$a + 10d = 1600$ ...(i)
from (i) and (ii) we get ...(ii)
$a = 1000$ and $d = 60$

7. A number 15 is divided into three parts which are in *AP* and the sum of their squares is 83. Find the smallest number.
(a) 5 (b) 3
(c) 6 (d) 8

Ans. (b)
Let numbers are
$a - d$, a and $a + d$
now it is given that
sum = 15 & sum of there square is 83 i.e.
$3a = 15 \Rightarrow a = 5$ ...(i)
$(a - d)^2 + a^2 + (a + d)^2 = 83$ ...(ii)
$3a^2 + 2d^2 = 83$
$75 + 2d^2 = 83$
$2d^2 = 8$
$d^2 = 4$, $d = 2$
So, least term is $a - d = 3$

8. The sum of the first 16 terms of an AP whose first term and third are 5 and 15 respectively is
(a) 600 (b) 765
(c) 640 (d) 680

Ans. (d)
$a = 5$, $a + 2d = 15$
So $d = 5$
now sum of 16 term will be
$16/2\ [10 + 15 \times 5] = 680$

9. How many terms are there in the GP 5, 20, 80, ... 20480?
(a) 6 (b) 5
(c) 7 (d) 8

Ans. (c)
GP is given: 5, 20, 80, ... 20 480
here $a = 5$
$r = 20/5 = 4$
now $20480 = ar^{n-1} = 5 \times 4^{n-1}$
$5 \times 4^{n-1}$
$\Rightarrow 4096 = 4^{n-1}$
$n - 1 = 6$
$n = 7$

10. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and so on. How much will the boy get if he starts working on the 1st of February and finishes on the 20th of February?
(a) 2^{20} (b) $2^{20} - 1$
(c) $2^{19} - 1$ (d) 2^{19}

Ans. (b)
This sequence is in GP 1, 2, 4, 8, ...
Now sum of first 20 term will be

$$\frac{a(r^n - 1)}{r - 1} = 2^{20} - 1$$

11. The seventh term of as *GP* is 8 times the fourth term. What will be the first term when it's fifth term is 48?
(a) 4 (b) 3
(c) 5 (d) 2

Ans. (b)
It is given that
$ar^6 = 8\,ar^3$
$r^3 = 8$, $r = 2$
Now $ar^4 = 48$
$a \times 2^4 = 48$, $a = 3$

12. The sum of three numbers of a GP is 14 and the sum of their squares is 84. Find the largest number.
(a) 8 (b) 6
(c) 4 (d) None of these

Ans. (a)

a, ar, ar^2

now $a + ar + ar^2 = 14$ ...(i)

also $a^2 + a^2r^2 + a^2r^4 = 84$ ...(ii)

only suitable combination is 2, 4, 8

So largest term is 8

13. How many natural numbers between 300 to 500 are multiples of 7?

(a) 29 (b) 28

(c) 27 (d) None of these

Ans. (a)

Between 300 and 500 the first term divisible by 7 is 301 and last term divisible by 7 is 497

So, here a = 301, d = 7

$L = a + (n - 1)d$

$497 = 301 + (n-1) \times 7$

$\Rightarrow 7n = 203$

$\Rightarrow n = 29$

14. The 4th and 10^{th} term of an GP are 1/3 and 243 respectively. Find the 2nd term.

(a) 3 (b) 1

(c) 1/27 (d) 1/9

Ans. (c)

It is given that

$ar^3 = 1/3$ ...(i)

$ar^9 = 243$ ...(ii)

from (i) and (ii) we get

$r^6 = 729$

$r = 3$ ...(iii)

from (i) and (ii) we get

$$a \times 3^3 = \frac{1}{3},\ a = \frac{1}{81}$$

second term will be $\frac{1}{27}$

15. The 7th and 21st terms of an AP are 6 and –22 respectively. Find the 26th term.

(a) –34 (b) –32

(c) –12 (d) –10

Ans. (b)

It is given that

$a + 6d = 6$ and (i)

$a + 20d = -22$

$\Rightarrow 14d = -28,\ d = -2,$

$\Rightarrow a = 18$

26th term will be -32

16. Find the number of terms of the GP

1/81, –1/27, 1/9, ... 729

(a) 11 (b) 12

(c) 10 (d) 13

Ans. (a)

It is given that

GP is $\frac{1}{81}, \frac{-1}{27}, \frac{1}{9}, \ldots\ldots\ldots = 729$

Here $a = \frac{1}{81}$, $r = -3$

now $ar^{n-1} = 729$

$$\frac{1}{81} \times (-3)^{n-1} = -729$$

$(-3)^{n-1} = 3^{10}$, n = 11

17. 'a' and 'b' are two number whose AM is 25 and GM is 7. Which of the following may be value of 'a'?

(a) 10 (b) 20

(c) 49 (d) 25

Ans. (c)

$$\frac{a+b}{2} = 25$$

$a + b = 50$

also $\sqrt{ab} = 7$

$ab = 49$

here a = 1 & b = 49 or vice verca

18. Two number A and B are such that their GM is 20% lower than their AM. Find the ratio between the numbers.

(a) 3 : 2 (b) 4 : 1

(c) 2 : 1 (d) 3 : 1

$\frac{a+b}{2} = AM$

$\sqrt{ab} = GM$

Here it is given that

GM = 20% less than AM, that is

$GM = \frac{4}{5} AM$

$\Rightarrow \sqrt{ab} = \frac{4}{5} \times \frac{a+b}{2}$

only possibility is a = 1 & b = 4

or a = 4, b = 1

So, ratio is 4 : 1

□□□□

Progression

I Practice Exercise: I

1. Determine 25th term of an A.P. whose 9th term is –6 and common difference is $\frac{5}{4}$.

   (a) 16 (b) 18
   (c) 12 (d) 14

2. Which term of the A.P. 5, 13, 21, ... is 181?

   (a) 21st (b) 22nd
   (c) 23rd (d) 24th

3. Find the n^{th} term of the series

   $\frac{1}{n} + \frac{n+1}{n} + \frac{2n+1}{n} + ...$

   (a) $\frac{3+n^2+n}{n}$ (b) $\frac{1+n^2-n}{n}$

   (c) $\frac{2+n^2-n}{n}$ (d) None of these

4. Determine k so that $\frac{2}{3}$, k, and $\frac{5}{8}$ k are three consecutive terms of an A.P.

   (a) 16/33 (b) 14/33
   (c) 12/33 (d) 18/33

5. If 7 time the 7th term of an A.P. is equal to 11 times its, 11th term, then the 18th term of the A.P. is

   (a) 1 (b) 2
   (c) 0 (d) 3

6. If the pth, qth and rth terms of an A.P. are a, b, c, respectively, find the value of $a(q-r) + b(r-p) + c(p-q)$

   (a) 2 (b) 1
   (c) 0 (d) 3

7. A ball rolling up an incline covers 36 metres during the first second, 32 metres during the second, 28 metres during the next and so on. How much distance will it travel during the 8th second?

   (a) 8 metres (b) 6 metres
   (c) 7 metres (d) 9 metres

8. If the 5th and the 12th terms of an A.P. are 30 and 65, respectively, what is the sum of the first 20 terms?

   (a) 1175 (b) 1250
   (c) 1150 (d) 1350

9. The sum of a series in A.P. is 525. Its first term is 3 and last term is 39. Find the common difference.

   (a) 3/2 (b) 3/3
   (c) 2/3 (d) 1/3

10. The sum of p terms of an A.P. is $3p^2+4p$. Find the n^{th} term?

    (a) 5n + 2 (b) 6n + 1
    (c) 8n + 3 (d) 7n + 3

11. Find the sum of all integer between 50 and 500 which are divisible by 7.

(c) 17766 (d) 17696

12. Which term of the gemoetric sequence

$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\ldots$ is $\frac{1}{19683}$?

(a) 9 (b) 7
(c) 11 (d) 13

13. The 5th term of a G.P. is 2, find the product of first 9 terms.

(a) 508 (b) 512
(c) 504 (d) 516

14. The 3rd term of a G.P. is the square of the first term. If the second term is 8, determine the 6th term.

(a) 136 (b) 132
(c) 128 (d) 124

15. For what value of x, the number $\frac{-2}{7}, x, -\frac{7}{2}$ are in G.P.?

(a) 1, –2 (b) 1, –3
(c) 1, –5 (d) 1, –1

16. The sum of first three terms of a G.P. is to the sum of first six terms is 125 : 152. Find the common ration of G.P.

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

17. Evaluate $\sum_{i=1}^{11}(2+3^i)$

(a) $22+\frac{3}{2}(3^{11}-1)$ (b) $11+\frac{3}{2}(3^{11}-1)$

(c) $22+\frac{3}{2}\frac{3}{2}(3^{10}-1)$ (d) None of these

18. The common ratio of a G.P. is $\frac{3}{4}$ and sum to infinity is $\frac{80}{9}$. Find the first term

(a) 14 (b) 16
(c) 14 (d) 10

19. Sum the series to infinity

$\frac{3}{4}-\frac{5}{4^2}+\frac{3}{4^3}-\frac{5}{4^4}+\frac{3}{4^5}-\frac{5}{4^6}+\ldots\ldots$

(a) $\frac{8}{15}$ (b) $\frac{7}{17}$

(c) $\frac{7}{15}$ (d) $\frac{8}{17}$

20. Find the 9th term of the H.P. 6, 4, 3, ...

(a) 7/5 (b) 6/5
(c) 5/6 (d) None of these

21. If $x > 1$, $y > 1$, $z > 1$ are in G.P., th

$\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in

(a) A.P. (b) H.P.
(c) G.P. (d) None of these

22. If a be the first term of a G.P., l the nth te and P the product of first n terms then P =

(a) $(al)^{n/2}$ (b) $(a-l)^{n/2}$
(c) $(a+l)^{n/2}$ (d) None of these

Solutions

1. **Ans. (d)**
Let a be the first term and d the comm difference of an A.P.
Then, $a_n = a + (n-1)d$

$a_9 = a + (9-1)\left(\frac{5}{4}\right)$

$\Rightarrow -6 = a + 10$

$\Rightarrow a = -6 - 10 = -16$

$\therefore a_{25} = -16 + (25-1)\frac{5}{4}$

$= -16 + 30 = 14$

2. **Ans. (c)**

Here, first term $a = 5$

Common difference $d = 8$

Let 181 be the nth term. i.e. $a_n = 181$.

$\therefore 181 = 5 + (n-1)8$ or, $176 = (n-1)8$

$\therefore n - 1 = 176 \div 8 = 22$

$\therefore n = 23$

Hence, 181 is the 23rd term.

3. **Ans. (b)**

Here $a = \frac{1}{n}$, $d = \frac{n+1}{n} - \frac{1}{n} = 1$

$\therefore a_n = a + (n-1)d$

$\therefore a_n = \frac{1}{n} + (n-1)(1) = \frac{1}{n} + n - 1 = \frac{1 + n^2 - n}{n}$

4. **Ans. (a)**

$\because \frac{2}{3}, k, \frac{5}{8}k$ are in A.P.

$\therefore k - \frac{2}{3} = \frac{5}{8}k - k \Rightarrow \frac{5k}{8} - 2k = \frac{-2}{3}$

$\Rightarrow \frac{-11k}{8} = \frac{-2}{3} \Rightarrow k = \frac{16}{33}$.

5. **Ans. (c)**

Let a be the first and d, the common difference of an A.P.

$\therefore a_7 = a + 6d$

$a_{11} = a + 10d$

$\therefore 7a_7 = 11a_{11}$

$7(a + 6d) = 11(a + 10d)$

$\Rightarrow 7a + 42d = 11a + 110d$

$\Rightarrow 4a + 68d = 0$

$\Rightarrow a + 17d = 0$

$\Rightarrow a_{18} = 0$

6. **Ans. (c)**

Let A be the first term and D, the common difference of A.P.

$a_p = A + (p-1)D = a$ ...(i)

$a_q = A + (q-1)D = b$ ...(ii)

$a_r = A + (r-1)D = c$ ...(iii)

$\therefore a(q-r) + b(r-p) + c(p-q)$

$= [A + (p-1)D](q-r) + [A + (q-1)D](r-p)$

$+ [A + (r-1)D](p-q)$

$= (q - r + r - p + p - q)A + [(p-1)(q-r)$

$+ (q-1)(r-p) + (r-1)(p-q)]D$

$= 0 \cdot A + 0 \cdot D = 0$

7. **Ans. (a)**

Distance covered during the Ist second = 36 m

Distance covered during the 2nd second = 32 m

Distance covered during the 3rd second = 28 m.

The distance covered form an A.P.

= 36 + 32 + 28 ..... in which

$a = 36$, $d = -4$

$\therefore$ Distance covered in 8th second

= 8th term of the A.P.

$= a + 7d = 36 + 7(-4)$

$= 36 - 28 = 8$ metres.

8. **Ans. (c)**

Let a be the first term and d the common difference of an AP., then

$a_5 = a + 4d = 30$ ...(i)

$a_{12} = a + 11d = 65$ ...(ii)

Subtracting (i) from (ii), we get

$7d = 35 \Rightarrow d = 5$

$\therefore$ From (i)

$a + 4(5) = 30$

$\Rightarrow a = 30 - 20 = 10$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{20} = \frac{20}{2}[2\times 10 + (20-1)5]$

$= 10[20 + 95] = 1150.$

9. Ans. (a)

If n be the number of terms, then

$a_n = a + (n-1)d$

where a is the first term and d the common difference.

$\therefore 39 = 3 + (n-1)d.$

Also $s_n = \frac{n}{2}[a_1 + a_n]$

$\therefore 525 = \frac{n}{2}[3+39] \Rightarrow 1050 = n(42)$

or, $n = \frac{1050}{42} = 25$

putting n = 25 in (i), we get

$(25-1)d = 36 \Rightarrow d = 36 \div 24 = \frac{3}{2}$

10. Ans. (b)

Here, $S_p = 3p^2 + 4p$

Putting p = n, we have

$S_n = 3n^2 + 4n$

Changing n to $(n-1)$, we get

$S_{n-1} = 3(n-1)^2 + 4(n-1)$

$= 3(n^2 - 2n + 1) + 4n - 4$

$= 3n^2 - 2n - 1$

$\therefore a_n = s_n - S_{n-1}$

$= 3n^2 + 4n - 3n^2 + 2n + 1 = 6n + 1.$

11. Ans. (d)

The first integer, after 50 which is divisible by 7 is 56 and the last integer before 500 which is divisible by 7 is 497.

$\therefore$ The sequence of integers between 50 and 500 which are divisible by 7 is

56, 63, 70, ..., 497

It is an A.P. with

$a = 56,\ d = 7$

$a_n = 497 = a + (n-1)d$

$\therefore 497 = 56 + (n-1)\times 7$

$\therefore 7n = 497 + 7 - 56$

or $7n = 448$

or $n = 488 \div 7 = 64$

Required sum $= \frac{n}{2}(a_1 + a_n)$

$= \frac{64}{2}(56 + 497) = 32 \times (553) =$

12. Ans. (a)

Let n^{th} term of the given sequence be

Then,

$a_n = ar^{n-1} \Rightarrow \frac{1}{19683} = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1}$

$\Rightarrow \left(\frac{1}{3}\right)^8 = \left(\frac{1}{3}\right)^{n-1}$

$\Rightarrow n - 1 = 8 \Rightarrow n = 9.$

13. Ans. (b)

Let a be the first term and r the common

$\therefore a_5 = 2 \Rightarrow ar^4 = 2$

Now, product of first 9 terms

$= a \times ar \times ar^2 \times \ldots\ldots ar^8$

$= a^9 r^{1+2+\ldots\ldots+8} = a^9 r^{36}$

$= (ar^4)^9 = 2^9 = 512.$

14. Ans. (c)

Let a be the first term and r be the co

ration of G.P.

We have $a_3 = (a_1)^2 \Rightarrow ar^2 = a^2$

$\Rightarrow r^2 = a$

Also, $a_2 = 8$

$\Rightarrow ar = 8$

Mulitiplying (i) and (ii) we get

$ar^3 = 8 \times a$

$\therefore r^3 = 8 \Rightarrow r = 2$

From (i) $a = (2)^2 = 4$

Hance, $a_6 = ar^5 = (4)(2)^5 = 4 \times 32$

15. Ans. (d)

16. Ans. (c)

Here,

17. Ans. (a)

18. Ans. (b

19. Ans.

$\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.

$\Rightarrow \frac{x}{-2/7} = \frac{-7/2}{x} \quad \Rightarrow x^2 = \frac{-7}{2} \times \frac{-2}{7}$

$\therefore \quad x^2 = 1 \Rightarrow x = \pm 1.$

16. Ans. (c)

Here, $\frac{S_3}{S_6} = \frac{125}{152}, \frac{a(r^3-1)/(r-1)}{a(r^6-1)/(r-1)} = \frac{125}{152}$

$\Rightarrow \frac{r^3-1}{r^6-1} = \frac{125}{152}, \quad \therefore \quad \frac{r^3-1}{(r^3-1)(r^3+1)} = \frac{125}{152}$

$\Rightarrow \frac{1}{r^3+1} = \frac{125}{152}, \quad \therefore \quad 152 = 125r^3 + 125$

$\Rightarrow 125r^3 = 27, \Rightarrow r^3 = \frac{27}{125}.$

or, $r^3 = \left(\frac{3}{5}\right)^3, \quad \therefore r = \frac{3}{5}.$

Hence, the common ratio of G. P. is $\frac{3}{5}$.

17. Ans. (a)

$(2+3^1) + (2+3^2) + (2+3^3) + ..+ (2+3^{11})$

$= 2 + 2 + 2 + \ldots\ldots$ up to 11 terms

$+(3 + 3^2 + 3^3 + \ldots\ldots$ up to 11 terms)

$= 11 \times 2 + \frac{3(3^{11}-1)}{3-1} = 22 + \frac{3}{2}(3^{11}-1)$

18. Ans. (b)

$S_\infty = \frac{a}{1-r} \Rightarrow \frac{80}{9} = \frac{a}{1-\left(\frac{-4}{5}\right)} \Rightarrow \frac{80}{9} = \frac{a}{9/5}$

$\Rightarrow a = \frac{80}{9} \times \frac{9}{5} = 16$

Hence, the first term is 16.

19. Ans. (c)

$\left(\frac{3}{4} + \frac{3}{4^3} + \frac{3}{4^5} + \ldots\text{to } \infty\right)$

$= \frac{3/4}{1-\left(\frac{1}{4}\right)^2} - \frac{5/4^2}{1-\left(\frac{1}{4}\right)^2}$

$= \frac{3}{4} \times \frac{16}{15} - \frac{5}{16} \times \frac{16}{15} = \frac{4}{5} - \frac{1}{3}$

$= \frac{12-5}{15} = \frac{7}{15}$

20. Ans. (b)

The given sequence is 6, 4, 3, ... are in H.P.

The sequence of reciprocals of the terms is

$\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ ..... which is an A.P.

Here, $a = \frac{1}{6}, \ d = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

$\therefore \ a_9$ of A.P. $= a + 8d$

$= \frac{1}{6} + 8 \times \frac{1}{12} = \frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

$\therefore$ 9th term of H.P. is $\frac{6}{5}$.

21. Ans. (b)

$\because$ x, y, z are in G.P.

$\because \ y^2 = xz$

Taking log on both sides

$2 \log y = \log x + \log z$

$\Rightarrow 2 + 2 \log y = (1+\log x) + (1+\log z)$

$\Rightarrow 2(1+\log y) = (1+\log x) + (1+\log z)$

$\Rightarrow 1+\log x, \ 1+\log y, \ 1+\log z$ are in A.P.

$\Rightarrow \frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in H.P.

22. Ans. (a)

If r is the common ratio of G.P. then

$l = a r^{n-1}$

The first n terms of the G.P. are

$a, ar, ar^2, ar^3, \ldots\ldots ar^{n-1}$

$P = a \times ar \times ar^2 \times \ldots \times ar^{n-1}$

$= a^n \times r^{1+2+3+\ldots+(n-1)}$

$= a^n \times r^{\frac{(n-1)n}{2}} = \left(a^2\right)^{n/2} \times \left(r^{n-1}\right)^{n/2}$

$= \left(a^2 r^{n-1}\right)^{n/2} = \left(a \cdot ar^{n-1}\right)^{n/2} = (al)^{n/2}$

□□□□

Progression

II Practice Exercise: II

1. If the sum of the 6th and the 15th elements of an arithmetic progression is equal to the sum of the 7th, 10th and 12th elements of the same progression, then which element of the series should necessary be equal to zero?
   (a) 10th (b) 8th
   (c) 1st (d) None of these

2. What is the eighth term of the sequence 1, 4, 9, 16, 25, ... ?
   (a) 8 (b) 64
   (c) 128 (d) 200

3. If p, q, r, s are in harmonic progression and p>s, then
   (a) $\frac{1}{ps} < \frac{1}{qr}$ (b) q + r = p + s
   (c) $\frac{1}{q} + \frac{1}{r} = \frac{1}{p} + \frac{1}{s}$ (d) None of these

4. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then x is
   (a) $\log_a(\log_b a)$
   (b) $\log_a(\log_e a) + \log_a(\log_e b)$
   (c) $-\log_a(\log_e b)$
   (d) $\log_a(\log_e b) - \log_a(\log_e a)$

5. A person pays Rs. 975 in monthly installments, each monthly installment being less than the former by Rs. 5. If the first installment is Rs. [illegible], in how many months will the entire amount be paid?
   (a) 12 months (b) 26 months
   (c) 15 months (d) 18 months

6. Let S_n denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$. Then the ratio $\frac{S_{3n}}{S_n}$ is equal to
   (a) 4 (b) 6
   (c) 8 (d) 10

7. Three numbers are in G.P. Their sum is 19 and their product is 512. The numbers are
   (a) 6, 9 and 13 (b) 4, 8 and 13
   (c) 2, 8 and 18 (d) 2, 6 and 13

8. The sum of the series:
   $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 15^2$ is
   (a) 1080 (b) 1240
   (c) 1460 (d) 1620

9. If $\frac{1}{b} - \frac{1}{c} = \frac{1}{a} - \frac{1}{b}$, then a, b, c form a/an
   (a) Arithmetic progression
   (b) Geometric Progression
   (c) Harmonic Progression
   (d) None of these

10. In G. P., the first term is 5 and the common ratio is 2. The eigthth term is
    (a) 640 (b) 1280
    (c) 256 (d) 160

11. If the arithmetic mean of two numbers is 5 and geometric mean is 4, then the numbers are
    (a) 4, 6 (b) 4, 7
    (c) 3, 8 (d) 2, 8

12. The value of
    $(1^3 + 2^3 + 3^3 + \ldots 15^3) - (1 + 2 + 3 + \ldots 15)$
    (a) 14280 (b) 14400
    (c) 12280 (d) 13280

13. The mean of the cubes of the first n natural numbers is

(a) $\frac{n(n+1)^3}{4}$ (b) n^2

(c) $\frac{n(n+1)(n+2)}{8}$ (d) (n^2+n+1)

14. If $\frac{3+5+7+....+n \text{ terms}}{5+8+11+...+10 \text{ terms}} = 7$, then the value of n is

(a) 35 (b) 36

(c) 37 (d) 40

15. If the sum of first *n* natural numbers is one-fifth of the sum of their squares, then *n* is

(a) 5 (b) 6

(c) 7 (d) 8

16. What is the least value of *n* such that $(1 + 3 + 3^2 + ... + 3^n)$ exceeds 2000?

(a) 7 (b) 5

(c) 8 (d) 6

17. The sum of 12 terms of an A.P., whose first term is 4, is 252. What is the last term?

(a) 35 (b) 36

(c) 37 (d) 38

18. If log 2, $\log(2^x - 1)$ and $\log(2^x + 3)$ (all to the base 10) be three consecutive terms of an Arithmetic Progression, then the value of *x* is equal to

(a) 0 (b) 1

(c) $\log_2 5$ (d) $\log_{10} 2$

19. The third term of a Geometric Progression is 4. The product of the first five terms is

(a) 4^3 (b) 4^5

(c) 4^4 (d) None of these

Solutions

1. Ans. (b)
Let a be the first term and d be the common ratio of an A.P.

$\therefore (a+5d)+(a+14d)$
$= (a + 6d) + (a + 9d) + (a + 11d)$
$\Rightarrow a + 7d = 0$
$\Rightarrow$ 8th term = 0.

2. Ans. (b)
$8^2 = 64$.

3. Ans. (c)
p, q, r, s are in harmonical progression
$\Rightarrow \frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ and $\frac{1}{s}$ are in A.P.

$\Rightarrow \frac{1}{q} - \frac{1}{p} = \frac{1}{s} - \frac{1}{r}$

$\Rightarrow \frac{1}{q} + \frac{1}{r} = \frac{1}{p} + \frac{1}{s}$.

4. Ans. (a), (c)
Given statement

$\Rightarrow \left(a^{x/2}\right)^2 = (\log_b x) \times (\log_x a)$

$= \frac{1}{\log_x b} \times \log_x a$

$\Rightarrow a^x = \log_b a$

$\Rightarrow x \log_a a = \log_a [\log_b a]$

$\Rightarrow x = \log_a [\log_b a]$.

Also $x = -\log_a(\log_a b)$

5. Ans. (c)
Let n be the number of months in which all the installments can be paid
First Installment = Rs. 100 = a
Common Difference = –5 = d
$\Rightarrow$ Sum of the series with *n* terms whose first term is 100 or common difference is (–5) = 975

i.e. $\frac{n}{2}[2a + (n-1)d] = 975$

i.e. $\frac{n}{2}[2 \times 100 + (n-1)(-5)] = 975$

i.e. $n^2 - 41n + 390 = 0$

i.e. $n = 26$ or $n = 15$

For $n=15$, total amount paid

$$= \frac{15}{2}[2\times100 + (15-1)(-5)]$$

$$= \frac{15}{2}[200-70] = 975.$$

6. Ans.(b)

$$S_n = \frac{n}{2}[a+(n-1)d]$$

[where a is the first term and d is the common difference]

$$S_{2n} = \frac{2n}{2}[a+(2n-1)d]$$

$$S_{3n} = \frac{3n}{2}[a+(3n-1)d]$$

Given, $S_{2n} = 3S_n$

$$\Rightarrow n[a+2nd-d] = 3\left[\frac{n}{2}(a+nd-d)\right]$$

$$\Rightarrow 2a + 4nd - 2d = 3a + 3nd - 3d$$

$$\Rightarrow nd + d = a$$

$$\Rightarrow d = \frac{a}{n+1}$$

$$\therefore \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[a+3nd-d]}{\frac{n}{2}[a+nd-d]}$$

$$= \frac{3\left[a+\frac{3na}{1+n}-\frac{a}{1+n}\right]}{a+\frac{na}{1+n}-\frac{a}{1+n}}$$

$$= 3\left[\frac{a+an+3na-a}{a+an+na-a}\right]$$

$$= \frac{12an}{2an} = 6$$

7. Ans. (b)

Let the three numbers be a, ar, ar^2, where r common ratio.

$\therefore$ $a+ar+ar^2 = 28$ and $a^3r^3 = 512$

$\therefore$ $ar = 8 \Rightarrow a+ar^2 = 20$

$\Rightarrow$ $ar + ar*r^2 - 20n = 0$

$\Rightarrow$ $8r^2 - 20r + 8 = 0$

$\Rightarrow$ $r = 2,\ r = \frac{1}{2}$.

if $r = 2$, $a = 4$. Therefore, the three numbers are 4, 8, 16

8. Ans. (b)

The sum of the squares of the first n natural numbers is

$$\frac{n(n+1)(2n+1)}{6}$$

Put $n = 15$, we have, $1^2+2^2+3^2+4^2+\ldots+15^2$

$$= \frac{15(15+1)(30+1)}{6} = 1240.$$

9. Ans. (c)

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$\Rightarrow$ $b = \frac{2ac}{a+c} \Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow$ a, b, c are in H.P.

10. Ans. (a)

n^{th} term of a G.P. $= ar^{n-1}$

where a = first term and r is the common ratio

$\therefore$ 8th term $= 5\times(2)^7 = 5\times128 = 640$.

11. Ans. (d)

Let x, y be the numbers

$\therefore$ $\frac{x+y}{2} = 5$ and $\sqrt{xy} = 4 \Rightarrow xy = 16$

$\therefore$ $x + y = 10$, $xy = 16$

$\Rightarrow$ $(x-y)^2 = (x+y)^2 - 4xy = 100 - 64 = 36$

$\Rightarrow$ $x-y = 6$ $\quad\therefore\quad x = 8,\ y = 2$.

12. Ans. (a)
Given expression

$$=\left[\frac{15\times16}{2}\right]^2-\frac{15\times16}{2}=(120)^2-120$$

$$=120\times119=14280.$$

13. Ans. (a)
Sum of the cubes of first n natural numbers

$$=\left[\frac{n(n+1)}{2}\right]^2=\frac{n^2(n+1)^2}{4}$$

$$\therefore \text{ Mean}=\frac{n(n+1)^2}{4}.$$

14. Ans. (a)
S_n = Sum of n terms of an A.P.

$$=\frac{n}{2}[2a+(n-1)d]$$

where a = first term,
d = common difference

$$\therefore \frac{3+5+7+\ldots+n \text{ terms}}{5+8+11+\ldots+10 \text{ terms}}=7$$

$$\Rightarrow \frac{\frac{n}{2}[2\times3+(n-1)\times2]}{\frac{10}{2}[2\times5+(10-1)\times3]}=7$$

$$\Rightarrow \frac{n(2n+4)}{370}=7$$

$$\Rightarrow 2n^2+4n-2590=0$$

$$\Rightarrow n^2+2n-1295=0$$

$$\Rightarrow n^2+37n-35n-1295=0$$

$$\Rightarrow n(n+37)-35(n+37)=0$$

$$\Rightarrow (n-35)(n+37)=0$$

$$\Rightarrow n=35.$$

15. Ans. (c)

Sum of the first n natural numbers $=\frac{n(n+1)}{2}$

Sum of the squares of the first n natural numbers

$$=\frac{n(n+1)(2n+1)}{6}$$

$$\therefore \frac{n(n+1)}{2}=\frac{1}{5}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$\Rightarrow 2n+1=15\Rightarrow n=7.$$

16. Ans. (c)

$$\frac{3^n-1}{3-1}>2000\Rightarrow 3^n>4001\Rightarrow n=8.$$

17. Ans. (d)

$$S_n=\frac{n}{2}[\text{First term + Last term}]$$

Where a = 4, n=12, L = ?

$$\therefore S_{12}=256=\frac{12}{2}[4+L]$$

$$\Rightarrow 4+L=42$$

$$\Rightarrow L=38$$

18. Ans. (c)
log 2, $\log(2^x-1)$ and
$\log(2^x+3)$ are in A.P.

$$\Rightarrow 2\left[\log\left(2^x-1\right)\right]=\log2+\log\left(2^x+3\right)$$

$$=\log\left[2\times\left(2^x+3\right)\right]$$

$$\Rightarrow \log\left(2^x-1\right)^2=\log\left[2^{x+1}+6\right]$$

$$\Rightarrow \left(2^x-1\right)^2=2^{x+1}+6=2^x.2+6$$

Let $2^x=y$

$$\therefore (y-1)^2=2y+6\Rightarrow y^2-2y+1=2y+6$$

$\Rightarrow y^2 - 4y - 5 = 0$

$\Rightarrow (y-5)(y+1) = 0$

$\Rightarrow y = 5, -1$

f $y = 5 \Rightarrow 2^x = 5 \Rightarrow x\log 2 = \log 5$

$\Rightarrow x = \dfrac{\log 5}{\log 2} \Rightarrow x = \log_2 5.$

19. Ans.(b)

Let a be the first term of a G.P. and r be common ratio

$\therefore$ First five terms of a G.P. are a, ar, ar^3, ar^4.

$\therefore$ Third term = $ar^2 = 4$

product of first five terms

$= a \times ar \times ar^2 \times ar^3 \times ar^4 = (ar^2)^5 = 4^5$

❑❑❑❑

al life situations we come across various lems of selections and arrangements. For nple selection of Indian cricket team, and deciding their batting order. Similarly ation of a group of 5-6 members out of 40 lents in a class for college tech festival and iding their order of arrangement for their stage formance etc. All these problems are very eresting and comes under the topic mutations and combinations.

rmutation

rmutation is defined as arrangement of ject in an ordered manner.

r example, let there are 3 chairs for 3 persons , B and C then sitting arrangement can be made 3! ways like

ABC
ACB
BAC
BCA
CAB
CBA

Permutation is represented by nPr here nPr is total number of ways in which r things at a time can be selected and arranged at a time from amongs n things.

$$nPr = \frac{n!}{(n-r)!}$$

For example, If we have to make a sitting arrange for 5 students and if only three chairs are available, then this can be done in $5P_3$ i.e.

$$5P_3 = \frac{5!}{(5-3)!} = 60 \text{ way}$$

THEORY

1. Factorial notion n!
   $n! = n(n-1)(n-2)\ldots\ldots 3.2.1$ product of all n con-sequetive integer from n to 1
2. $0! = 1$
3. $1! = 1$
4. $(n-1)! = (n-1)(n-2)(n-3)\ldots\ldots 3.2.1$
5. $n! = n.(n-1)!$
6. only the factorial of natural numbers are defined.
   n! is defined for only $n > 0$
   n! is not defined for $n < 0$

Combination

Combination is defined as selection of objects in which order does not matter.

For example let a team of 2 players is to be formed out of 3 players A, B and C. Then this can be done in three ways AB, BC and CA. Here order of AB is not important whether B comes first or second. AB is similar to BA because we are selecting two players A and B.

Now, if one player is captain and other is vice captain then this can be done in 6 ways.

Captain	Vice Captain
A	B
B	A
B	C
C	B
C	A
A	C

In all these 6 cases we are selecting two players and arranging them into order manner in which there is one captain and one vice captain. So, this is a case of arrangement and called permutation.

Combination is represented by nC_r

$$nC_r = \frac{n!}{(n-r)! \times r!}$$

Here nC_r can be defined as number of ways of selection of r things at a time from among n things.

Example: Selection of 5 person from among 10 can be done by $10C_5$ way

which is equal to $\frac{10!}{5! \times 5!} = 252$

Permutation is a special case of selection and arrangement

We know that $nP_r = \frac{n!}{(n-r)!}$ and also

$$nC_r = \frac{n!}{(n-r)! \times r!}$$

So, $nP_r = nC_r \times r!$

$= nC_r \times rP_r$

Thus, in words it can be defined as

The permutation or arrangement of r things out of n is nothing but the selection of r things out of n things followed by the arrangement of selected things amongst themselves.

For example a sitting arrangment for 5 players from among 11 players of Indian cricket team can be done by first selecting 5 from among 11 players & then arranging them in different order

$11P_5 = 11C_5 \times 5P_5$

= (Selection & then arrangement)

Binomial Theorem

According to binomial theorem

$$nC_0 + nC_1 + nC_2 \ldots\ldots\ldots nC_n = 2^n$$

This is a very powerful theorem having wider application in various sets of problems.

It is nothing but total number of selection of zero or more things out of n different things =

$$nC_0 + nC_1 + nC_2 \ldots\ldots\ldots nC_n = 2^n$$

For Example: Rahul has 5 friends, he wants to invite them into a New Year Party. He can invite either

(1) No one into a party. i.e. $5C_0 = 1$
(2) One into a party, i.e. $5C_1 = 5$
(3) Two into a party, i.e. $5C_2 = 10$
(4) Three into a party i.e. $5C_3 = 10$
(5) Four into a party i.e. $5C_4 = 5$
(6) All 5 into the pore i.e. $5C_5 = 1$

Thus, according to Binomial theorem total number of ways comes out is 32 which is equal to 2^5.

Important Results with Explanation

1. **Number of permutation of n different things taken all at a time = n!**

   Note: This is a case of non-repetition i.e. when repetition is not allowed

   Ex. 1(a). How many five digit numbers can be formed by 1, 2, 3, 4 and 5 if repetition is not allowed.

   Sol. This can be done in 5! ways.

Ex. 1(b). How many ways of sitting arrangements are possible if there are six chairs for six persons A, B, C, D, E and F.
Sol. This can done in 6! ways.

2. **Number of permutation of n things out of which p are of one type**
q are alike of second type and
r are alike of third type and rest all different

$$= \frac{n!}{p!q!r!}$$

Ex. 2(a). Out of 20 balls, if 10 are red in colour, 6 are blue in colour and 4 are green in colour. Now these balls can be arranged in linear fashion in $\frac{20!}{10! \times 6! \times 4!}$ number of ways.

Ex. 2(b). How may different words can be formed by using letters of word MISSISSIPPI?
Sol. MISSISSIPPI we have
1 - M
4 - I's
4 - S's
2 - P's
So total number of words that can be formed are $\frac{11!}{4! \times 4! \times 2!}$

Ex. 2(c). How many different words can be formed by using letters of ALLAHABAD?
Sol. In ALLAHABAD we have
4 - A's
2 - L's
1 - H
1 - B
1 - D

So $\frac{9!}{4! \times 2!}$ words.

3. **Number of permutation of n different things taken r at a time when repetetion is allowed**
$= n \times n \times n \times \ldots\ldots$ r times $= n^r$

Ex. 3(a): How many three digit number can be formed by 1, 3 and 5 if repetetion is allowed?
Sol. This can be done in $3^3 = 27$ ways.
Ex. 3(b): How many four letter word with or without meaning can be formed by using vowels a, e, i, o and u if repetition is allowed.
Sol. This can be done in 5^4 ways.

4. **Number of selection of r things out of n indetical things = 1**
Ex.4(a). In how many ways 3 balls pens can be chosen out of 50 indentical balls pens?
Sol. Since, all the 50 ball pens are indentical then number of ways to select 5 balls pens = 1.

5. **Selection of r things out of n things if k things are always selected = $(n - k)C_{(r-k)}$**
Ex. 5(a). If out of 11 football players 6 are to be invited into a party such that Captain & Goalkeeper will always be invited, then this can be done in $(11-2)C_{(6-2)} = 9C_4$ ways.

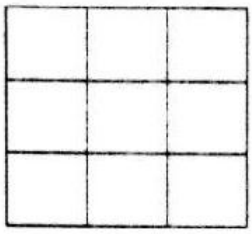
Ex. 5(b). If out of 7 handball players in a team 4 are to be invited into a party such that Goal Keepers will always be invited then this can be done in $(7-1)C_{(4-1)} = 6C_3$
$\Rightarrow$ 20 ways.

6. **Selection of r things out of n things if k things are never be selected = $(n - k)C_r$**
Ex. 6(a). If out of 11 football players 6 are to be invited into a party such that Captain and Goal Keeper will never be invited,
then this can be done in $(11-2)C_6 = 9C_6$ ways
Ex.6(b). If out of 15 Rugby players, 8 players are to be invited into a party such that captain & vice captain will never be invited,
then this can be done in $(15-2)C_8$ ways.

7. $nC_r = nC_{n-r}$
Ex. 7(a). $8C_3 = 8C_{(8-3)} = 8C_5$
Ex. 7(b). $18C_{10} = 18C_{(18-10)} = 18C_8$

8. **Number of selection of k consecutive things out of n things in a row**
= n – k + 1 ways

9. **Number of squares in a square of n x n side = $1^2 + 2^2 + 3^2$ ..... n^2**
Number of rectangles in a square of n x n sides = $1^3 + 2^3 + 3^3$ .... n^3
Thus the number of squares and rectangles in the following figures are given by



Number of square

$$= 1^2 + 2^2 + 3^2 = 14 = \sum_1^n n^2$$

Number of rectangles

$$= 1^3 + 2^3 + 3^3 = 36 = \sum_1^n n^3$$

Circular Permutation

In case of ciruclar permutation the total number of ways of arrangement is (n – 1)!
For example: Sitting arrangements for 6 people around a dinning table in 6 chairs will be (6– 1)! = 120 ways

Flowers in a garland and beads in a neckless can be arranged in $\frac{(n-1)!}{2}$ ways.

Note: Here clockwise and anticlock-wise observations are not different.

□□□□

Lets have a glimps of few examples from Permutations and Combinations

Ex.1: Find the number of ways of arrangement of 10 person in four chairs?

Sol. : $^{10}P_4 = \frac{10!}{6!} = 5040$

Ex.2: How many 2 digit numbers can be formed out of digit 2, 4, 6, 8 when repetition is not allowed?
Sol. : This can be done in $^4P_2$ ways i.e. 12 number can be formed.

Ex.3: In the above question how many numbers can be formed when repetition is allowed?
Sol. : We have to form two digit number i.e., we have to fill two place □□ out of 4 numbers & any number can be used any number of times. This can be done in 4 × 4 = 16 ways.

Ex.4: How many words can be formed by using letters of word MADAM?
Sol. : In MADAM we have
2 – M's
2 – A's
So total number of ways of forming word are $\frac{5!}{2! \times 2!} = 30$

Ex.5: Of the different words that can be formed from letters of word MOBILE, how many begins with M and ends with E?
Sol. : M□□□□E
M and E are fixed at the start and end positions. Hence, we have to arrange the I, L, O among themselves (ie in four places), this can be done in 4! ways

Ex.6: Six boys and 4 girls wanted to enjoy a movie How many ways of sitting arrangement can be possible for them if girls want to sit together?

Sol. : $B_1B_2B_3B_4B_5B_6G_1G_2G_3G_4$ ...(i)
Now if all girls are sitting together then
$B_1B_2B_3B_4B_5B_6X$ ...(ii)
(here X is symbolic notation that all girls are sitting together)
equation (ii) Can be arranged in 7! ways
Now girls can be arranged among themselves i.e., 4! ways so total number of sitting arrangement will be $7! \times 4!$ ways

Ex.7: In the above question find number of ways of arrangement if no two girls sit together.
Sol. : This can be done in following manner – B_1–B_2–B_3–B_4–B_5–B_6–
First of all, boys can be arranged in 6! ways, now vacant seats between them will be filled by girls. This can be done in $7P_4$ ways so total arrangement = $7P_4 \times 6!$

Ex.8: In the question number (6) find total arrangement if no two boys sit together
Sol. : first of all above we have to make arrangement for girls –G_1–G_2–G_3–G_4– this can be done in 4! ways. Now, since only 5 places are available for boys, then difinitely two boys will sit together. So this is impossible condition.

Ex.9: How many words can be formed from letters of ENGINEER?
Sol. : Here Engineer contains
3 E's and 2 N's, one each G, I & R
So total number of words

$$= \frac{8!}{3! \times 2!} = 3360$$

Ex.10: How many 5 digit number can be formed by using 0, 1, 2, 3, 4, 5, 6, 7 only once?

Sol. : $\square_{d_1}\square_{d_2}\square_{d_3}\square_{d_4}\square_{d_5}$

box d_1 cannot be filled with zero so only 7 numbers can be filled in first box rest can be done in
$7 \times 7 \times 6 \times 5 \times 4 = 5880$ ways

Ex.11: How many different sums can be formed with the following coins?
5 rupees, 1 rupee, 50 paisa, 25 paisa, 10 paisa, 1 paisa.
Sol. : A distinct sum will be formed by selecting either 1 or 2 or 3 or 4 or 5 or all 6 coins.
$6C_1 + 6C_2 + 6C_3 + 6C_4 + 6C_5 + 6C_6$ ways.
By binomial theorem this can be done in $2^6 - 1$ way.
Answer = 63.

Ex.12: Out of 15 points in a plane, no three are in straight line except 5 which are collinear. How many straight lines can be formed?
Sol. : If all 15 points were non collinear then the answer would have been $15C_2$. However, in this case, $15C_2$ has double counting since the 5 collinear points are also among the 15. These would have been counted as $5C_2$ whereas they should have been counted as 1. Thus to remove the double counting and get the correct answer, we need to adjust by reducing the count by $(5C_2 - 1)$.
Hence, the answer $15C_2 - (5C_2 - 1) = 96$

Ex.13: How many triangle can be formed by 18 points if all are non collinear.
Sol. : To form a triangle we have to connect any three points. i.e., in this case select any three points out of 18. This can be done in $18C_3$ ways = 816

Ex.14: In the above situation how many triangle can be formed if 5 points are collinear.
Sol. : The triangles will be given by
$18C_3 - 5C_3 = 806$ ways

□□□□

Permutations & Combinations

Solved Examples

1. How many number of 3 digits can be formed with the digits 1, 2, 3, 4, 5 (repetition of digits not allowed)?

(a) 125 (b) 120

(c) 60 (d) 150

Ans. (c)

Three digits can be formed in $5P_3$ ways that is $\frac{5!}{(5-3)!} = 60$

2. How many numbers between 2000 and 3000 can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7 (repetition of digits not allowed)?

(a) 42 (b) 210

(c) 336 (d) 440

Ans. (b)

Number between 2000 and 3000 are four digit numbers in which 2 is the first digit

d_1 d_2 d_3 d_4

2 □ □ □

So, 2 must be taken in box d_1

Now rest box can be filled in $7P_3$ ways

$\Rightarrow \frac{7!}{(7-3)!} = 210$ ways

3. In how many ways can a person send invitation cards to 6 of his friends if he has four servants to distribute the cards?

(a) 6^4 (b) 4^6

(c) 24 (d) None of these

Ans. (b)

Person has six invitation cards 1 card can be given to any of 4 servants so six cards can be distributed in

$4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$ ways

4. In how many ways can 7 Indians, 5 Pakistanis and 6 Dutch be seated in a row so that persons of the same nationality sit together?

(a) 3! (b) 7!5!6!

(c) 3! 7! 5! 6! (d) 18!

Ans. (c)

We have

→ 7 Indian

→ 5 Pakistanis and

→ 6 Dutch

7 Indias can sit in 7! ways

5 Pakistanis can sit in 5! ways

6 Dutch can sit in 6! ways

Also they can sit together in 3! different ways $7! \times 5! \times 6! \times 3!$

5. How many straight lines can be formed by 8 non-collinear points on the X-Y plane?

(a) 28 (b) 56

(c) 18 (d) 19860

Ans. (a)

To draw a line we have to choose two points

So, $8C_2 = 28$ ways

6. If ${}^nC_3 = {}^nC_8$ then find n.

(a) 11 (b) 12

(c) 14 (d) 10

Ans. (a)

${}^nC_r = {}^nC_{(n-r)}$

$r + (n-r) = n$

So, $3 + 8 = n = 11$

7. In how many ways can the letters of the word PATNA be rearranged?

(a) 60 (b) 120

(c) 119 (d) 59

Ans. (a)

This can be done in $\frac{5!}{2!}$ ways $\frac{5!}{2!} = 60$

8. In the above question, how many words would be there which would start with the letter P?
(a) 24 (b) 12
(c) 60 (d) 18

Ans. (b)
In PATNA P is fixed so we have only four letter ATNA in which there are two A's.

So, $\frac{4!}{2!} = 12$

9. How many numbers of four digits can be formed with the digits, 0, 1, 2, 3 (repetition of digits being allowed)?
(a) 12 (b) 108
(c) 256 (d) 192

Ans. (d)

d_1 d_2 d_3 d_4
□□□□

We have given four digits.
'0' cannot be placed in box d_1
So total digits $3 \times 4 \times 4 \times 4 \Rightarrow 192$ Ways.

10. A captain and a vice-captain are to be chosen out of a team having 11 players. How many ways are there to achieve this?
(a) 10×9 (b) ${}^{11}C_2$
(c) 110 (d) 109!

Ans. (c)
${}^{11}C_2 \times 2!$ Ways which is equal to 110

11. In how many ways can Ram choose a vowel and a consonant from the letter ALLAHABAD?
(a) 4 (b) 6
(c) 9 (d) 5

Ans. (a)
In Allahabad we have 4 A's that is only one vowel & 4 consonants (B, D, H, L)
So any four combination
AB, AD, AH, AL are only 4 possibilities.
Thus 4 ways.

12. There are three rooms in a motel : one single, one double and one for four persons. How many ways are there to house seven persons in these rooms?
(a) $7! / 1! 2! 4!$ (b) $7!$
(c) $7! / 3$ (d) $7! / 3!$

Ans. (a)

This can be done in $\frac{7!}{1! \times 2! \times 4!}$ ways

13. How many new words are possible from the letters of the word PERMUTATION?
(a) 11!/2! (b) (11!/2!) – 1
(c) 11! – 1 (d) None of these

Ans. (b)
In PERMUTATION we have 2T's, and one each P, E, R, M, U, A, I, O, N. So total new words formed will be $\frac{11!}{2!} - 1$ ways

(Note: here PERMUTATION is not counted)

14. In how many ways can the letters of the word EQUATION be arranged so that all the vowels come together?
(a) ${}^9C_4 \; {}^9C_5$ (b) $4! \; 5!$
(c) $9! / 5!$ (d) $9! - 4!5!$

Ans. (b)
In EQUATION there are five Vowels A, E, I, O and U and three consonants N,Q, T so the possible arrangements will be $4! \times 5!$ ways

15. A man has 3 shirts, 4 trousers and 6 ties. What are the number of ways in which he can dress himself with a combination of all the three?
(a) 13 (b) 72
(c) $13! / 3! \; 4! \; 6!$ (d) $3! \; 4! \; 6!$

Ans. (b)
We have

Item	Shirts	Trousers	Ties
Number	3	4	6

So total number of dressing will be
$3 \times 4 \times 6 = 72$ ways

Direction for questions 16-17 : There are 25 points on a plane of which 7 are collinear. Now solve the following :

16. How many straight lines can be formed?
(a) 7 (b) 300
(c) 280 (d) None of these

Ans. (c)

Total number of straights lines

$= 25C_2 - 7C_2 + 1 = 280$

17. How many triangles can be formed from these points?
(a) 453 (b) 2265
(c) 755 (d) None of these

Ans. (b)

Total triangle formed will be equal to $25C_3 - 7C_3 = 2265$

18. How many batting orders are possible for the Indian cricket team if there is a squad of 15 to choose from such that Sachin Tendulkar is always chosen?
(a) $1001 \times 11!$ (b) $364 \times 11!$
(c) $11!$ (d) $15 \times 11!$

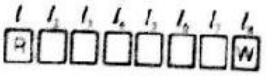
Ans. (a)

$^{15-1}C_{11-1} \times 11!$

$\Rightarrow {}^{14}C_{10} \times 11! = 1001 \times 11!$

19. How many distinct words can be formed out of the word PROWLING that start with R and end with W?
(a) 8!/2! (b) 6!2!
(c) 6! (d) None of these

Ans. (c)

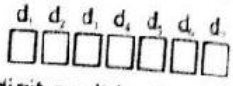


l_1 is R & l_8 is W rest boxes can be filled in 6! ways

20. How many 7-digit numbers are there having the digit 3 three times and the digit 0 four times?
(a) 15 (b) $3^3 \times 4^4$
(c) 30 (d) None of these

Ans. (a)

We have given 333 and 0000



First digit can't be 0 so d_1 is 3 because we have to form 7 digit number. Now we have rest six boxes d_2, d_3, d_4, d_5, d_6 d_7 to be filled. This can be done

$\frac{6!}{2! \times 4!} = 15$ ways

□□□□

Permutations & Combinations

Practice Exercise: I

1. There are 6 candidates for 3 posts. In how many ways can the posts be filled?
(a) 120 (b) 130
(c) 100 (d) 110

2. There are 15 buses running between Delhi & Mumbai. In how many ways can a man go to mumbai and return by a different bus?
(a) 280 (b) 310
(c) 240 (d) 210

3. A teacher of a class wants to set one question from each of two exercises in a book. If there are 15 and 12 questions in the two exercises respectively, then in how many ways can the two questions be selected?
(a) 160 (b) 140
(c) 180 (d) 120

4. A code word is to consist of two English alphabets followed by two distinct numbers between 1 and 9. For example, CA23 is a code word. How many such code words are there?

4 Probability

Introduction

The probability has its origin in the problems dealing with games of chance such as gambling, coin tossing, die throwing and playing cards. In all these cases the outcome of a trial is uncertain. These days probability is widely used in business and economics in the field of prediction for future.

Lots of questions based on coins, die, playing cards and alphabetical arrangements are asked in various Competitive Examinations.

Theory

Probability means the chance of occurrence of an event. In layman's term, we can say that, it is likelihood that something that is defined as an event-will or will not occur mathematically,

$$\text{Probability (P)} = \frac{\text{Number of favourable event}}{\text{Total Number of event}}$$

for example probability of getting an even number after throwing a die can be calculated as.

$$(P) = \frac{\{2, 4, 6\}}{\{1, 2, 3, 4, 5, 6\}} = \frac{3}{6} = \frac{1}{2}$$

Were {2, 4, 6} are even numbers & hence favorable outcomes and {1,2,3,4,5,6} etc. all 6 possible outcomes.

Similarly, we can calculate probability of getting a number divisible by 3 out of 9 single digit number in following manner Probability

$$= \frac{\text{Number divisible by 3}}{\text{All nine single digit number}}$$

$$= \frac{\{3, 6, 9\}}{\{1, 2, 3, 4, 5, 6, 7, 8, 9\}} = \frac{3}{9} \Rightarrow \frac{1}{3}$$

The followings remark may be important for learning this chapter on probability

1. **Die**: A die is small cube used in games of chance. On its six faces dots are marked numbering (1, 2, 3, 4, 5, 6)
2. **Playing card**: A pack (or deck) of playing cards has 52 cards, divided into four suits:
   (1) Spades (2) Clubs
   (3) Hearts (4) Diamonds
   Each suit has 13 cards
   (a) Nine number cards 2, 3, 4, 5, 6, 7, 8, 9, 10
   (b) An Ace, a king, Queen and a jack or knave known as face cards
   Ace - A King - K
   Queen - Q Jack -J
   Spade and clubs are black faced cards while Hearts and Diamonds are red- faced cards. The King, Queens and Jacks are called court cards.
3. Unbiased coin - coin having two faces head (H) and Tail (T)

Some Basic Concepts

Random experiment

An experiment whose out come has to be among a set of events that are completely known but whose exact out come is unknown is random experiment.

Eg: (Throwing of a dice, tossing of a coin)

Sample Space

This is defined in the context of a random experiment and denotes the set representing all the possible outcomes of the random experiment.

Eg: 1. Sample Space when a coin is tossed is Head (H) or Tail (T)
2. Sample space when a dice is thrown is (1, 2, 3, 4, 5, 6)

Event

The set representing the desired out come of a random experiment is called event. Event is subset of a sample space for example: Probability of getting a number divisible by 5 in a single throw of a die if odd numbers are obtained.

Here sample space is (1, 2, 3, 4, 5, 6), event is (1, 3, 5) favourable outcome is (5)only.

$$\text{So probability} = \frac{\{5\}}{\{1, 3, 5\}} = \frac{1}{3}$$

Non Event

The outcome that is opposite of the desired event is the non-event.

Note: If the event occurs, the non-event does not ocur and vice versa.

Impossible Event

An event that can never occur is an impossible event. The probability of an impossible event is 0.

Eg. (Probability of occurance of 7 when a dice with 6 faces number 1-6 is thrown).

Mutually Exclusive Events

A set of event is mutually exclusive when the occurence of any one of them means that the other events cannot occur.

Eg: 1. If head appears on a coin tail will not appear and vice versa
2. If 5 occurs in a single throw of a die then (1, 2, 3, 4, 6) will never appear.

Equally likely Events

If two events have the same probability or chance of occurence they are called equally likely events.

Example: In a throw of a dice, the chance of showing on the dice is equal to 2, is equal to 3 is equal to 4, is equal to 5, is equal to 6 appearing on the dice.

Exhaustive Set of Event

A set of events that includes all the possibilities of a sample space is said to be an exhaustive set of an events.

Example: In a throw of a dice the number is less than four or more than or equal to four.

Independent Events

An event is described as such if the occurence of an event has no effect on the probability of ocurence of an ther event.

Example: If the first child of a couple is girl there is no effect on the chances of the second child being a girl.

Conditional Probability

Probability of the occurence of an event A given that event B has already ocured.

This is denoted by P (A/B)

Example: The probability that in two throws of a dice we get a total of 7 or more given that in the first throw of the dices number 4 had occured.

Concept of AND and OR

Whenever we use AND as the natural conjunction joining two separate parts of even definitions, we replace the AND by the multiplication sign.

Example: If probability of passing in an exam is 1/2 for A and probability of passing the same is 1/3 for B then probability of passing of A and B both is P(A)×P (B)

i.e.: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Whenever we use OR as the natural conjunction joing two separate parts of the event definition, we replace the OR by their addition sign.

Example: If we have the probability of A winning a race as 2/3 and that of B as 1/6 then probability that either A or B wins a race is given by

$$P(A) + P(B) = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}.$$

□□□□

Solved Example

1. In throwing a fair dice, what is the probability of getting the number '3'?

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{9}$ (d) $\frac{1}{12}$

Ans. (b)

2. Find the chance of throwing at least one ace in a simple throw with two dice.

(a) $\frac{1}{12}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{11}{36}$

Ans. (d)

Ace is nothing but 6. we can get it in 11 different ways (1,6), (6,1), (2, 6), (6, 2), (3, 6), (6, 3), (4, 6), (6, 4), (5, 6), (6, 5), (6, 6)

$\Rightarrow \frac{11}{36}$ (Probability)

3. Find the chance of drawing 2 blue balls in succession from a bag containing 5 red and 7 blue balls, if the balls are not being replaced.

(a) $\frac{3}{13}$ (b) $\frac{21}{64}$

(c) $\frac{7}{22}$ (d) $\frac{21}{61}$

Ans. (c)

Probability of getting blue ball in single draw is $\frac{7}{12}$ and probability of getting blue ball in second draw is $\frac{6}{11}$ so $\frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$

4. If a card is picked up at random from a pack of 52 cards. Find the probability that it is

(i) a spade.

(a) $\frac{1}{9}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

(ii) a king or queen.

(a) $\frac{3}{13}$ (b) $\frac{2}{13}$

(c) $\frac{7}{52}$ (d) $\frac{1}{169}$

(iii) 'a spade' or 'a king' or 'a queen'

(a) $\frac{21}{52}$ (b) $\frac{5}{13}$

(c) $\frac{19}{52}$ (d) None of these

Ans. (i) (c)

Ans. (ii) (b)

Ans. (iii) (c)

(i) $\frac{13}{52} = \frac{1}{4}$

(ii) 4 king or 4 Queen = 8

$\Rightarrow \frac{8}{52} = \frac{2}{13}$

(iii) Spade = 13, King = 4 Queen = 4 but there will be 1 King & 1 Queen of spade, thus, we have 19 to choose from

$\Rightarrow \frac{19}{52}$ (Probability)

5. Three coins are tossed. What is the probability of getting 2 Tails and 1 Head

(a) $\frac{1}{2}$ (b) $\frac{3}{8}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Ans. (b)

When three coins are tossed we get

HHH	or	(1)
HHT	or	(2)
HTT	or	(3)
HTH	or	(4)
THH	or	(5)
THT	or	(6)
TTT	or	(7)
TTH	or	(8)

We get 2T and one H in 3 cases so probability $= \frac{3}{8}$

6. For the above question, the probability that there is at least one tail is :

(a) $\frac{2}{3}$ (b) $\frac{7}{8}$

(c) $\frac{3}{8}$ (d) $\frac{1}{2}$

Ans. (b)

For at least one tail, we have to exclude the case of all Heads {H, H, H} and rest 7 are favourable.

So $P = \frac{7}{8}$

7. A bag contains 3 green and 7 white balls. Two balls are drawn from the bag in succession without replacement.

(i) What is the probability that b

(a) $\frac{1}{7}$ (b) $\frac{5}{11}$

(c) $\frac{7}{11}$ (d) $\frac{7}{15}$

(ii) What is the probability tha different colour?

(a) $\frac{1}{7}$ (b) $\frac{5}{11}$

(c) $\frac{7}{11}$ (d) $\frac{7}{15}$

Ans. (i) (d)

Ans. (ii) (d)

(i) To get both white

$$\frac{7C_2}{10C_2} = \frac{21}{45} = \frac{7}{15}$$

(ii) To get balls of different colour

$$P = \frac{3C_1 \times 7C_1}{10C_2} = \frac{3 \times 7}{45} = \frac{7}{15}$$

8. What is the probability of throwing a greater than 2 with a fair dice?

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) 1 (d) $\frac{3}{5}$

Ans. (a)

Number greater than 2 = {3, 4, 5, 6}

So $P = \frac{4}{6} = \frac{2}{3}$

9. Three cards numbered 2, 4 and 8 a into a box. If a card is drawn at random is the probability that the card drawn (i) a prime number ?

(a) 1 (b) $\frac{1}{3}$

(c) $\frac{4}{5}$ (d) $\frac{5}{7}$

(ii) an even number?

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{5}$ and

(iii) an odd number?

(a) 1 (b) 0

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

Ans. (i) (b)
Ans. (ii) (a)
Ans. (iii) (b)

(i) In (2, 4, 8) 2 is the only prime number

So $P = \frac{1}{3}$

(ii) Probability for even number $\frac{\{2,4,8\}}{\{2,4,8\}} = 1$

(iii) No odd number is given in the sample
So, $P = 0$

10. Two fair coins are tossed. Find the probability of obtaining

(i) 2 Heads

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(ii) 1 Head and 1 Tail

(a) $\frac{1}{4}$ (b) 1

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(iii) 2 Tails

(a) 1 (b) $\frac{1}{4}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

Ans. (i) (d)
Ans. (ii) (c)
Ans. (iii) (b)

When two coins are tossed then four outcomes are

HH, HT, TH, or TT

(i) $P = \frac{1}{4}$

(ii) $P = \frac{2}{4} = \frac{1}{2}$

(iii) $P = \frac{1}{4}$

11. In rolling two dices, find the probability that

(i) there is at least one '6'

(a) $\frac{11}{36}$ (b) $\frac{22}{36}$

(c) $\frac{15}{36}$ (d) $\frac{29}{36}$

(ii) the sum is 5

(a) $\frac{1}{4}$ (b) $\frac{1}{9}$

(c) $\frac{1}{2}$ (d) $\frac{1}{6}$

Ans. (i) (a)
Ans. (ii) (b)

Rolling two dice gives following sums

1. Two (1, 1)
2. Three (1, 2), (2, 1)
3. Four (1, 3), (3, 1), (2, 2)
4. Five (1, 4), (2, 3), (3, 2), (4, 1)
5. Six (1, 5), (5, 1), (2, 4), (4, 2), (3, 3)
6. Seven (1, 6), (6, 1), (5, 2), (2, 5), (4, 3), (3, 4),
7. Eight (6, 2), (2, 6), (5, 3), (3, 5), (4, 4)
8. Nine (6, 3), (3, 6), (5, 4), (4, 5)
9. Ten (4, 6), (4, 6), (5, 5)
10. Eleven (6, 5), (5, 6)
11. Twelve (6, 6)

(i) $P = \frac{11}{36}$

(ii) $P = \frac{4}{36} = \frac{1}{9}$

12. From a bag containing 4 white and 5 black balls a man draws 3 at random. What are the odds against these being all black?

(a) $\frac{5}{37}$ (b) $\frac{37}{5}$

(c) $\frac{11}{13}$ (d) $\frac{13}{37}$

Ans. (b)

Probability that all 3 balls are black

$\frac{5C_3}{9C_3} = \frac{10}{84} = \frac{5}{42}$ out of 42 cases only 5 are favourable and 37 are not favourable so odd against these being all black is $\frac{37}{5}$

13. Two balls are to be drawn from a bag containing 8 grey and 3 blue balls. Find the chance that they will both be blue.

(a) $\frac{1}{5}$ (b) $\frac{3}{55}$

(c) $\frac{11}{15}$ (d) $\frac{14}{45}$

Ans. (b)

8 grey and 3 blue balls

probability that all are blue $= \frac{3C_2}{11C_2} = \frac{3}{55}$

14. Two fair dice are thrown. What is the probability of

(i) throwing a double?

(a) $\frac{1}{6}$ (b) 1

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

(ii) the sum is greater than 10

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) $\frac{1}{6}$ (d) $\frac{1}{12}$

(iii) the sum is less than 10?

(a) $\frac{5}{6}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

Ans. (i) (a)

Ans. (ii) (d)

Ans. (iii) (a)

(i) Cases of double are

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6)

$P = \frac{6}{36} = \frac{1}{6}$

(ii) 11 & 12 are only favourable sums so

$P = \frac{3}{36} = \frac{1}{12}$

(iii) $P = \frac{30}{36} = \frac{5}{6}$

15. A bag contains four black and five red balls. If three balls from the bag are chosen at radom, what is the chance that they are all black?

(a) $\frac{1}{21}$ (b) $\frac{1}{20}$

(c) $\frac{2}{23}$ (d) $\frac{1}{9}$

Ans. (a)

Bag contains 4 black and 5 red balls,

to get all black $P = \frac{4C_3}{9C_3} = \frac{4}{84} = \frac{1}{21}$

16. If a number of two digits is formed with the digits 2, 3, 5, 7, 9 without repetition of digits what is the probability that the number formed is 35?

(a) $\frac{1}{10}$ (b) $\frac{1}{20}$

(c) $\frac{2}{11}$ (d) $\frac{1}{11}$

Ans. (b)
Total two digit numbers formed = $5P_2 = 20$,

So probability this number being 35 is $\frac{1}{20}$

17. From a pack of 52 playing cards, three cards are drawn at random. Find the probability of drawing a king, a queen and jack.

(a) $\frac{16}{5525}$ (b) $\frac{1}{13^3}$

(c) $\frac{1}{14^3}$ (d) $\frac{1}{15^3}$

Ans. (a)

$$P = \frac{4C_1 \times 4C_1 \times 4C_1}{52C_3} = \frac{64}{22100} = \frac{16}{5525}$$

18. A bag contains 3 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both are black?

(a) $\frac{1}{8}$ (b) $\frac{7}{40}$

(c) $\frac{12}{40}$ (d) $\frac{13}{40}$

Ans. (b)
Here 3 red, 6 white & 7 black balls

$$P = \frac{7C_2}{16C_2} = \frac{21}{120} = \frac{7}{40}$$

19. A bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. If one ball is drawn from each bag, find the probability that

(i) both are white.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

(ii) both are black.

(a) $\frac{3}{24}$ (b) $\frac{1}{24}$

(c) $\frac{3}{12}$ (d) $\frac{5}{24}$

(iii) one is white and one is black.

(a) $\frac{13}{24}$ (b) $\frac{15}{24}$

(c) $\frac{11}{21}$ (d) $\frac{1}{2}$

Ans. (i) (c)
Ans. (ii) (d)
Ans. (iii) (a)

Bag 1 → 4 white , 2 black and

Bag 2 → 3 white & 5 black ball

(i) Probability of getting both white

$$\frac{4}{6} \times \frac{3}{8} = \frac{1}{4}$$

(ii) Probability of geting both black

$$= \frac{2}{6} \times \frac{5}{8} = \frac{5}{24}$$

(iii) $\frac{4}{6} \times \frac{5}{8} + \frac{2}{6} \times \frac{3}{8} = \frac{13}{24}$

Probability

I Practice Exercise: I

1. In a simultaneous toss of two coins, find the probability of 2 tails.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

2. Three coins are tossed. Find the probability of all heads.

(a) $\frac{1}{6}$ (b) 1/8

(c) $\frac{1}{4}$ (d) None of these.

3. Three coins are tossed. Find the probability of no heads.

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$

(c) $\frac{1}{2}$ (d) None of these

4. A coin is tossed three times. Find the chance that head and tail show alternately.

(a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) None of these

5. In a single throw of two dice, find the probability of getting a total of 3 or 5.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

6. In a single throw of two dice, find the probability of getting a total of 12.

(a) $\frac{1}{36}$ (b) $\frac{1}{9}$

(c) $\frac{1}{18}$ (d) $\frac{35}{36}$

(For Question No. 7-11)
In a single throw of two dice, what is the probability of

7. A doublet?

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{1}{9}$ (d) $\frac{1}{18}$

8. A sum less then 6 in throw of two dice is probability

(a) $\frac{7}{18}$ (b) $\frac{5}{18}$

(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

9. An even number as the sum.

(a) $\frac{1}{36}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

10. A multiple of 3 as the sum

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{9}$ (d) $\frac{5}{36}$

11. Sum as a prime number

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$

(c) $\frac{7}{12}$ (d) $\frac{3}{4}$

(For Question No. 12-14)
In a single throw of three dice, find the probability of getting.

12. A total of 5?

(a) $\frac{1}{4}$ (b) $\frac{1}{18}$

(c) $\frac{1}{36}$ (d) $\frac{1}{9}$

13. A total of at most 5.

(a) $\frac{5}{108}$ (b) $\frac{103}{108}$

(c) $\frac{1}{18}$ (d) None of these

14. A total of at least 5.

(a) $\frac{7}{54}$ (b) $\frac{1}{54}$

(c) $\frac{53}{54}$ (d) None of these

15. What is the chance that a leap year, selected at random will contain 53 Sunday?

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

16. The letters of word "SOCIETY" are placed in a row. What is the probability that three vowels come together ?

(a) $\frac{3}{7}$ (b) $\frac{2}{7}$

(c) $\frac{1}{7}$ (d) None of these

17. Find the probability that in a random arrangement of letter of the words "UNIVERSITY" two I's do not come together.

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

18. Two dice are thrown. Find the odds in favour of getting the sum 5.

(a) 8:1 (b) 1:8

(c) 7 : 8 (d) 8 : 7

19. An interger is chosen at random from first two hundred natural numbers. What is the probability that the integer chosen is divisible by 6 or 8?

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) None of these.

20. In a simultaneous throw of two dice, find P (A or B) if A denotes the event 'a total of 11 and B denotes the event' 'an odd number on each die'.

(a) $\frac{11}{36}$ (b) $\frac{1}{4}$

(c) $\frac{5}{18}$ (d) $\frac{1}{6}$

21. Two dice are thrown together. What is the probability that the sum of number on two faces is neither 9 nor 11?

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

22. *A* and *B* are mutually exclusive events of an experiment. If P ('not A') = 0.65, P(AUB) = 0.65 and P(B) = P, find the value of p.

(a) 0.70 (b) 0.30

(c) 0.63 (d) 0.35

23. The probabilities that a student will recive an A, B, C or D grade are 0.30, 0.38, 0.22 and 0.01, respectively. What is the probability that the student will receive at least B grade?

(a) 0.38 (b) 0.42

(c) 0.68 (d) None of these.

24. A card is drawn from an ordinary pack and a gambler bets that it is a spade or an ace. What are the odds against his winning the bet?

(a) 9 : 4 (b) 4 : 9

(c) 5 : 9 (d) 9 : 5

25. A problem in Statistics is given to four students A, B, C and D. Their chances of solving it are $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ respectively. What is the probability that the problem will be solved?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{5}$ (d) None of these.

(For Question No. 26-28)
An urn contains 25 balls numbered 1 to 25. Suppose an odd number is considered a 'success'. Two balls are drawn from the urn with replacement.

26. Find the probability of getting two successes.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

27. Find the probability of getting no success.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

28. Find the probability of getting at most 2 successes.

(a) $\frac{1}{27}$ (b) $\frac{2}{9}$

(c) 1 (d) $\frac{7}{27}$

(For Question No. 29-32)
A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's is 1/5.

29. What is the probability that only one of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{25}$ (d) $\frac{11}{35}$

30. What is the probability that both of them be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

31. What is the probability that none of them be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

32. What is the probability that at least one of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

33. A man speaks truth in 80% of the cases and another in 90% of the cases. While stating the same fact, what is the probability that they contradict?

(a) $\frac{37}{50}$ (b) $\frac{13}{50}$

(c) $\frac{16}{50}$ (d) None of these

34. *A* Can solve 90% of the problems given in a book and *B* solve 70%. What is the probability that atleast one of them will solve a problem selected at random from the book?

(a) $\frac{3}{100}$ (b) $\frac{97}{100}$

(c) $\frac{83}{100}$ (d) $\frac{17}{100}$

35. Find the probability that in a random arrangement of the letters of the word DAUGHTER, the letter D occupies the first place

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{1}{2}$

Solutions

1. **Ans. (b)**
Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4
There is only one favourable case TT.

$\therefore$ P(2 tails) = $\frac{1}{4}$.

2. **Ans. (b)**
Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}
Number of exhaustive cases = 8
There is only one favourable case HHH.

$\therefore$ P (all heads) = $\frac{1}{8}$.

3. **Ans. (b)**
Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}
Number of exhaustive cases = 8
P (no heads) = P (all tails) = $\frac{1}{8}$
($\because$ there is only favourable case TTT).

4. **Ans. (b)**
Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}
Number of exhaustive cases = 8
Favourable case are HTH, THT
Number of favourable = 2.

$\therefore$ Required probability = $\frac{2}{8} = \frac{1}{4}$.

5. **Ans. (c)**
A total of 3 or 5 may be obtained in 6 ways, viz, (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1).
No. of exhaustive cases = $6 \times 6 = 36$.
$\therefore$ Probability of getting a total of 3 or 5 = $\frac{6}{36} = \frac{1}{6}$.

6. **Ans. (a)**
A total of 12 may be obtained in 1 way, viz, (6, 6).

$\therefore$ Required probability = $\frac{1}{36}$.

7. **Ans. (a)**
A 'doublet' means that both the dice show the same number on the upper most faces. Therefore, the outcomes, favourable to this event are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)
Thus, the number of favourable cases = 6.

Hence, P(doublet) = $\frac{6}{36} = \frac{1}{6}$

8. **Ans. (b)**
A : Getting total less than 6
A = [(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (1, 3), (4, 1), (1, 4), (3, 2), (2, 3)]
n(A) = 10, n (S) = 36
$\therefore$ Required probability
= P (A) = $\frac{n(A)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

9. **Ans. (c)**
A = Getting even number as the sum
A = [(1, 1), (1, 3), (3,1), (2, 2), (3, 3), (4, 2),

(2, 4), (5, 1), (1, 5), (6, 2), (2, 6), (5, 3), (3, 5), (4, 4), (5, 5), (6, 4), (4, 6), (6, 6)]

$n(A) = 18, n(S) = 36$

∴ Required probability

$$= P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}.$$

10. Ans. (b)

A : Getting a multiple of 3 as the sum

(1, 2), (2, 1), (3, 3), (5, 1), (1, 5), (4, 2), (2, 4), (6, 3), (3, 6), (4, 5), (5, 4), (6, 6)

$n(A) = 12, n(S) = 36$

∴ Required probability

$$= P(A) = \frac{n(A)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

11. Ans. (a)

A : Getting sum as a prime number

A = [(1, 1), (1, 2), (2, 1), (2, 3), (3, 2), (4, 1), (1, 4), (4, 3), (3, 4), (6, 1), (1, 6), (5, 2), (2, 5), (6, 5), (5, 6)

$n(A) = 11, \quad n(S) = 36$

∴ Required probability

$$= p(A) = \frac{n(A)}{n(S)} = \frac{15}{36} = \frac{5}{12}.$$

12. Ans. (c)

Number of exhaustive cases in a single throw of three dice = 6 ×6 ×6 = 216.

Cases favourable to a total of 5 are (1,2,2), (2,1,2), (2,2,1), (1,1,3), (1,3,1), (3,1,1).

$$\therefore \quad P(\text{a total of 5}) = \frac{6}{216} = \frac{1}{36}.$$

13. Ans. (a)

A total of at most 5 means a total 3, 4 or 5.

Cases favourable to a total of 3 are (1,1,1)

Cases favourable to a total of 4 are (1,1, 2), (1,2,1), (2,1,1)

Cases favourable to a total of 5 are (1, 2, 2), (2, 1, 2), (2, 2, 1), (1, 1, 3), (1, 3, 1), (3, 1,1).

Number of cases favourable to a total of 3 or 4 or 5 is 10.

$$\therefore \quad P(\text{a total of at most 5}) = \frac{10}{216} = \frac{5}{108}$$

14. Ans. (c)

A total of at least 5 means not a total of 3 or 4. number of cases favourable to a total of 3 or 4 is 4.

$$P(\text{a total of 3 or 4}) = \frac{4}{216} = \frac{1}{54}$$

∴ P (a total of at least 5)

= P (not a total of 3 or 4)

= 1–P (a total of 3 or 4)

$$= 1 - \frac{1}{54} = \frac{53}{54}.$$

15. Ans. (b)

We know that a leap year has 366 days and thus a leap year has 52 weeks and 2 days over.

The two over (successive days have the following likely cases:

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

∴ Number of exhaustive cases 'n' = 7.

Out of these, the favourable cases are (i) and (vii)

∴ Number of favourable cases 'm' = 2

∴ Probability of having 53 Sunday = $\frac{2}{7}$

16. Ans. (c)

There are 7 letters in the word 'SOCIETY' which can be arranged in 71 ways. Considering the three vowels in the word 'SOCIETY' as one letter, we can arrange 5 letters in a row in 5! ways. Also, three vowels can themselves be arranged in 3! ways.

∴ Thetotal number of arrangements in which three vowels come together are $5! \times 3!$

Hence, the required probability $= \frac{5! \times 3!}{7!}$

$= \frac{3 \times 2 \times 1}{7 \times 6} = \frac{1}{7}$.

17. Ans. (a)

Out of the letters in the word 'UNIVERSITY' two letters I's are alike.

∴ Number of permutations $= \frac{10!}{2}$ ...(i)

Number of words in which two 'I' are never together = Total number of words – Number of words in which two I's are together

$\frac{10!}{2} - 9! = \frac{10! - 2.9!}{2} = \frac{9![10-2]}{2} = \frac{9!.8}{2} = 9!.4$

∴ Required probability $= \frac{9!.4}{10!/2} = \frac{9!.8}{10!}$

$= \frac{8}{10} = \frac{4}{5}$.

18. Ans. (b)

Let A be the event of "getting the sum 5".

Then, A = [(1, 4), (4, 1), (2, 3), (3, 2)].

There are 4 favourable outcomes and (36 – 4) = 32 outcomes are unfavourable.

∴ Odds in favour of sum 5 $= \frac{4}{32} = \frac{1}{8}$

19. Ans. (a)

A : Integer chosen is divisible by 6B : Integer chosen is divisible by 8

$n(A) = 33,\ n(B) = 25,\ n(A \cap B) = 8,\ n(S) = 200$

$P(A) = \frac{33}{200},\ P(B) = \frac{25}{200}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{50}{200} = \frac{1}{4}$.

20. Ans. (a)

A:Getting total of 11 B: Getting odd number one each die

A = [(6, 5), (5, 6)]

B = [(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5 ,3), (5, 5)]

$P(A) = \frac{2}{36}, P(B) = \frac{9}{36}, P(A \cap B) = 0$

∴ Required probability

$= P(A) + P(B) - P(A \cap B)$

$= \frac{2}{36} + \frac{9}{36} - 0 = \frac{11}{36}$.

21. Ans. (b)

A:Getting a total of 9, B:getting a total of 11

A = [(5, 4), (4, 5), (6, 3), (3, 6)] B = [(6,5), (5,6)]

$P(A) = \frac{4}{36}, P(B) = \frac{2}{36}, P(A \cap B) = \frac{0}{36}$

∴ Required probability $= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - \left(\frac{4}{36} + \frac{2}{36} - 0\right)$

$= 1 - \frac{1}{6} = \frac{5}{6}$.

22. Ans. (b)

We know $P(A) = 1 - P(\bar{A})$

$= 1 - 0.65 = 0.35$ and

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.65 = 0.35 + P - 0$

[∵ A and B are mutually exclusive events

∴ $P(A \cap B) = 0$]

$\Rightarrow P = 0.65 - 0.35 = 0.30$.

23. Ans. (c)

P (at least B grade) = P(B grade) + P (A grade) = 0.38+0.30=0.68.

24. Ans. (a)

Let A : a sapde is drawn and B: an ace is drawn

Probability of winning the bet = P(A or B)

$= P(A) + P(B) - P(A \text{ and } B)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{52}$

Probability of losing the bet $= 1 - \frac{4}{13} = \frac{9}{13}$

Odds against winning the bet

$= \frac{9}{13} : \frac{4}{13} = 9 : 4.$

25. Ans. (b)

Probability that A fails to solve the problem is $1 - \frac{1}{3} = \frac{2}{3}$

Probability that B fails to solve the problem is $1 - \frac{1}{4} = \frac{3}{4}$

Probability that C fails to solve the problem is $1 - \frac{1}{5} = \frac{4}{5}$

Probability that D fails to solve the problem is $1 - \frac{1}{6} = \frac{5}{6}$

Since the events are independent, the probability that all the four students fail to solve the problem is

$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{1}{3}.$

$\therefore$ The probability that the problem will be solved $= 1 - \frac{1}{3} = \frac{2}{3}.$

26. Ans. (a)

Success: Getting odd number $P = \frac{13}{25}$

$\Rightarrow q = 1 - P = 1 - \frac{13}{25} = \frac{12}{25}$

P (two successes) $= P \times P$

$= \frac{13}{25} \times \frac{13}{25} = \frac{169}{625}.$

27. Ans. (d)

P (no success) $= qq = \frac{12}{25}\left(\frac{12}{25}\right) = \frac{144}{625}$

28. Ans. (c)

29. Ans. (a)

P(only one of them will be selected)

$= P(A) \times P(\bar{B}) + P(B)P(\bar{A})$

$= \frac{1}{7}\left(\frac{4}{5}\right) + \frac{1}{5}\left(\frac{6}{7}\right) = \frac{4+6}{35} = \frac{10}{35} = \frac{2}{7}.$

30. Ans. (b)

P(both of them will be selected)

$= P(A) \times P(B) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}.$

31. Ans. (c)

P(none of them will be selected)

$= P(\bar{A})P(\bar{B}) = \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}.$

32. Ans. (d)

P(at least one of them will be selected)

$= 1 - P(\bar{A}) \times P(\bar{B})$

$= 1 - \frac{6}{7} \times \frac{4}{5} = 1 - \frac{24}{35} = \frac{11}{35}.$

33. Ans. (b)

Probability that they contradict

$= P(A)P(\bar{B}) + P(B)P(\bar{A})$

$= \frac{80}{100} \times \frac{10}{100} + \frac{90}{100} \times \frac{20}{100}$

$= \frac{8+18}{100} = \frac{26}{100} = \frac{13}{50}.$

34. Ans. (b)

Required probability $= 1 - P(\bar{A})P(\bar{B})$

$$= 1 - \frac{10}{100} \times \frac{30}{100} = 1 - \frac{3}{100} = \frac{97}{100}$$

35. **Ans. (a)**

A : D occupies the first place

$n(A) = 7!$, $n(S) = 8!$

∴ Required probability

$$= P(A) = \frac{n(A)}{n(S)} = \frac{7!}{8!} = \frac{1}{8}.$$

□□□□

Probability

II Practice Exercise: II

1. The Probability of raining on day 1 is 0.2 and on day 2 is 0.3. What is the probability of raining on both the days?
   (a) 0.2 (b) 0.1
   (c) 0.06 (d) 0.25

2. A bag contains 5 red balls and 8 blue balls. It also contains 4 green and 7 black balls. If a ball is drawn at random, then find the probability that it is not green.
   (a) 5/6 (b) 1/4
   (c) 1/6 (d) 7/4

3. If the probability that A will live 15 years is $\frac{7}{8}$ and that B will live 15 years is $\frac{9}{10}$, then what is the probability that both will live after 15 years?
   (a) $\frac{1}{20}$ (b) $\frac{63}{80}$
   (c) $\frac{1}{5}$ (d) None of these

4. Suppose six coins are tossed. Then the probability of getting at least one tail is
   (a) $\frac{71}{72}$ (b) $\frac{53}{54}$
   (c) $\frac{63}{64}$ (d) $\frac{1}{12}$

5. The probability that a student is not a swimmer is 1/5. Then the probability that out of the five students, four are swimmers, is
   (a) ${}^5C_4\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)$ (b) $\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$
   (c) ${}^5C_1\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$ (d) None of these

6. If events A and B are independent and $P(A) = 0.15$, $P(A \cup B) = 0.45$ then $P(B) =$
   (a) $\frac{6}{13}$ (b) $\frac{6}{17}$
   (c) $\frac{6}{19}$ (d) $\frac{6}{23}$

7. The probability that a marksman will hit a target in given time is 1/5. Then, his probability of atleast one hit in 10 shots is
   (a) $\frac{1}{6^{10}}$ (b) $1-\left(\frac{4}{5}\right)^{10}$
   (c) $1-\frac{1}{5^{10}}$ (d) $1-\frac{1}{5^{19}}$

8. Two dice are tossed. The probability that the total score is a prime number is
   (a) $\frac{1}{6}$ (b) $\frac{5}{12}$
   (c) $\frac{1}{2}$ (d) $\frac{7}{9}$

9. Four different objects 1, 2, 3, 4 are distributed at random in four places marked 1, 2, 3, 4. What is the pobability that none of the objects occupy the place corresponding to its number?
   (a) 17/24 (b) 3/8
   (c) 1/2 (d) 5/8

10. If the probability of rain on any given day in Pune city is 50%, then what is the probability that it rains on exactly 3 days in a 5-day period?
   (a) 8/125 (b) 5/16
   (c) 8/25 (d) 2/25

11. The probability that an event A happens in one trial of an experiment is 0.4. Three independent trial of the experiment are formed. The probability that the event A happens at least once is
   (a) 0.934 (b) 0.784
   (c) 0.548 (d) 0.343

12. A number is chosen at random among the first 120 natural numbers. The probability of the number chosen being a multiple of 5 or 15 is
   (a) 1/5 (b) 1/6
   (c) 1/7 (d) 1/9

13. From a pack of 52 playing cards, two cards are drawn together at random. Find the probability of both the cards being Kings.
   (a) 1/15 (b) 25/57
   (c) 35/256 (d) None of these

14. What is the possibility of getting at least 6 heads if eight coins are tossed simultaneously?
   (a) 37/256 (b) 25/57
   (c) 1/13 (d) None of these

15. From a box containing 60 standard and 40 substandard article, two articles are chosen at random. What is the probability that one of them is standard and the other substandard?
   (a) $\frac{60}{100}\times\frac{40}{100}$ (b) [illegible]
   (c) $\frac{16}{33}$ (d) 24%
   (e) None of these

Solutions

1. **Ans. (c)**
   $0.2 \times 0.3 = 0.06$.

2. **Ans. (a)**
   The probability that the ball drawn is of gre
   colour $= \frac{4}{24} = \frac{1}{6}$
   Probability that the ball drawn is not of gre
   colour $= 1 - \frac{1}{6} = \frac{5}{6}$.

3. **Ans. (b)**
   $P(A) \times P(B) = \frac{7}{8} \times \frac{9}{10} = \frac{63}{80}$.

4. **Ans. (c)**
   Total number of events that would occur o
   tossing six coins $= 2^6 = 64$.
   Probability that no tail occurs $= \frac{1}{64}$
   $\therefore$ Probability of occuring at least one tai
   $= 1 - \frac{1}{64} = \frac{63}{64}$.

5. **Ans. (a)**
   $\left(\frac{4}{5} + \frac{1}{5}\right)^5$
   Probability that out of five students four are
   swimmers $= {}^5C_4 \times \left(\frac{4}{5}\right)^4 \times \frac{1}{5}$

6. **Ans. (b)**
   $P(A \cap B) = P(A) \times P(B) = 0.15 \times P(B)$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$0.45=0.15+P(B)-(0.15)\times P(B)$$

$$=0.15+P(B)(1-0.15)$$

$$=0.15+0.85P(B)$$

$$\therefore\ 0.85P(B)=0.45-0.15=0.30$$

$$\therefore\ P(B)=\frac{0.30}{0.85}=\frac{30}{85}=\frac{6}{17}.$$

7. **Ans. (b)**

$$P(A)=\frac{1}{5}, P(\bar{A})=1-\frac{1}{5}=\frac{4}{5}$$

The probability that he will not hit the target in 10 shots is $\left(\frac{4}{5}\right)^{10}$. So, Probability that atleast once target will be hit = $1-\left(\frac{4}{5}\right)^{10}$.

8. **Ans. (b)**
Sum should be either 2, 3, 5, 7 or 11

$$\therefore\ \text{Required probability}=\frac{15}{36}=\frac{5}{12}.$$

9. **Ans. (c)**

Let the four places be [1] [2] [3] [4]

Now object i cannot occupy the place i..... (A)

Suppose object 2 occpuies the place [1].

Then other placements can be done in 6 ways as follows:

(1)	2	1	3	4
(2)	2	1	4	3
(3)	2	3	1	4
(4)	2	3	4	1
(5)	2	4	1	3
(6)	2	4	3	1

Here out of the six ways, only three are permissible, because (1), (3) and (6) are not permissible because of the non-fulfilment of condition (A). Hence, required probability is $\frac{3}{6}=\frac{1}{2}$. Similarly you can allow objects 3 and 4 to occupy place 1 and in each case you can find that the probability is 1/2.

10. **Ans. (b)**

Prob. that it rains on the 1st day = $\frac{1}{2}$

Prob. that it rains on the 2nd day = $\frac{1}{2}$

Prob. that it rains on the 3rd day = $\frac{1}{2}$

Prob. that it rains on the 4th day = $\frac{1}{2}$

Prob. that it rains on the 5th day = $\frac{1}{2}$

Prob. that it rains on the any day in a 5-day period = $\frac{1}{32}$

Prob. that it it rains on exactly 3 days in a 5-day period.

$$={}^5C_3\times\frac{1}{32}={}^5C_2\times\frac{1}{32}=\frac{10}{32}=\frac{5}{16}.$$

11. **Ans. (b)**
Required probability

$$={}^3C_1(0.4)(0.6)^2+{}^3C_2(0.4)^2(0.6)+{}^3C_3(0.4)^3$$

$$=3(0.144)+3(0.096)+1(0.064)=0.784$$

12. **Ans. (a)**

13. **Ans. (d)**
Two cards can be drawnfrom a pack of 52 playing cards in ${}^{52}C_2$ ways,

i.e. $\frac{52\times51}{2}=1326$ ways.

The event that two kings appear in a single draw can appear in ${}^4C_2$ ways, i.e. 6 ways.

∴ The probability that the two cards drawn from a pack of 52 cards are kings

$$= \frac{6}{1326} = \frac{1}{221}.$$

14. Ans. (a)

Probability of getting 6 Heads + Prob. of getting 7 Heads + Prob. of getting all Heads

$$= {}^8C_6 \times \frac{1}{2^8} + {}^8C_7 \times \frac{1}{2^8} + {}^8C_8 \times \frac{1}{2^8}$$

$$= \frac{37}{256}.$$

15. Ans. (c)

Required probability

$$= \frac{60}{100} \times \frac{40}{99} + \frac{40}{100} \times \frac{60}{99} = \frac{4800}{100 \times 99} = \frac{16}{33}$$

□□□□

The chapter of Geometry and mensuration have had their share in various competition examinations. For doing well in questions based on this topic, student should be familiar with the very basics of various two dimentional and three dimensional solid figures.

To grasp easily the given topic of Geometry and mensuration, we have divided the theory in five parts.

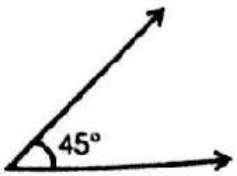
- Angles, Parallel lines & Transverse.
- Triangles and Quadrilaterals
- Mensuration and Solid Geometry
- Circles and its properties
- Coordinate Geometry and Trigonometry

Angles, Parallel Lines and Transverse

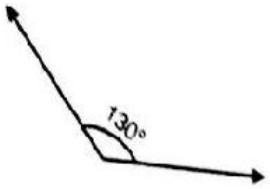
When two lines meet at common point they form angle.

Types

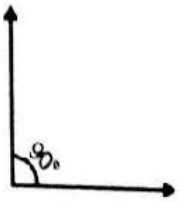
1. **Acute angle** → Angle less than 90°.



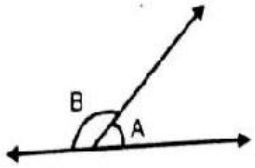
2. **Obtuse Angle** → Angle more than 90° but less than 180°.



3. **Right Angle** → Angle equal to 90°.

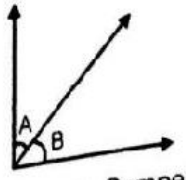


4. **Supplementary angle** → when sum of two angles is equal to 180° then angles are said to be supplementary.



$\angle A + \angle B = 180°$

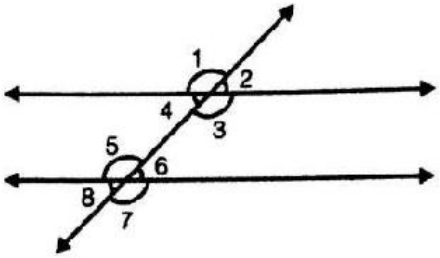
5. **Complementary Angle** → when sum of two angles is equal to 90° then angles are said to be complementary.



$\angle A$ and $\angle B$ are Compelmentary

Parallel Lines: Two straight lines are parallel if they lie on the same plane and do not intersect however far produced.

Transversal: It is a straight line that intersect two parallel lines. When a transversal intersect two parallel lines then



1. **Corresponding angle are equal**, (that is: for above figure)

   $\angle 1 = \angle 5$; $\angle 2 = \angle 6$; $\angle 4 = \angle 8$; $\angle 3 = \angle 7$

2. **Alternate interior angles are equal**, (that is : for above figure)

   $\angle 4 = \angle 6$; $\angle 5 = \angle 3$

3. **Alternate exterior angles are equal**, (that is : for above figure)

   $\angle 2 = \angle 8$; $\angle 1 = \angle 7$

4. **Interior angles on the same side of transversal add upto 180°**, (that is : for above figure).

   $\angle 4 + \angle 5 = \angle 3 + \angle 6 = 180°$

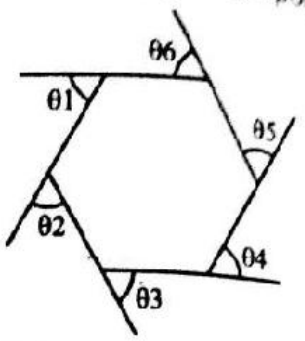
Polygons

Polygons are plane figures formed by a closed series of straight segments Examples: Triangles, Rectangles, Pentagon etc.

There are two types of Polygon:

1. **Regular Polygons:** Polygons with all the sides and angles equal.
2. **Irregular Polygons:** Polygons in which all the sides or angles are not of same measure.

- Sum of all interior angles of a polygon with n sides = $(n-2)\,180°$
- Sum of all exterior angles of a polygon

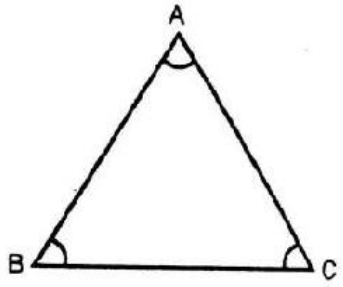


In the figure

$\theta 1 + \theta 2 + \theta 3 + \theta 4 + \theta 5 + \theta 6 = 360°$

Triangles

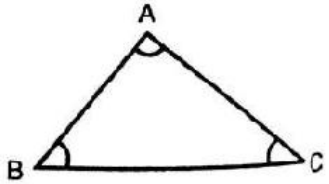
A triangle is a polygon having three sides. Sum of all the interior angles of a triangle is equal to 180^0



$\angle A + \angle B + \angle C = 180°$

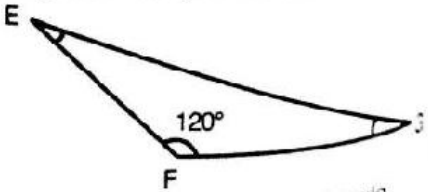
Types of Triangles:

1. **Acute angle triangle:** Triangles having all three angles less than 90°.



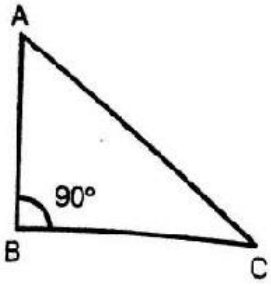
Δ ABC is Acute angle triangle

2. **Obtuse angle triangle:** Triangle with one of the angle obtuse, i.e. more than 90°.

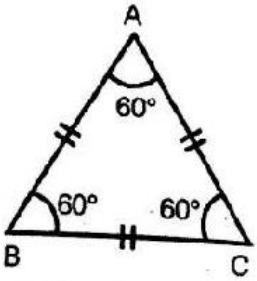


Δ EFG is obtuse angle triangle.

3. **Right angle triangle:** Triangle with one of the angle equal to 90°.



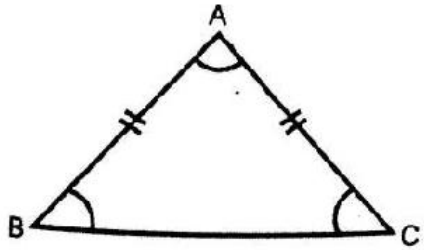
4. **Equilateral triangle:** Triangle in which all sides are equal. All the angles are also equal, and is equal to 60°.



Sides AB = BC = CA

also, $\angle A = \angle B = \angle C = 60°$

5. **Isoscles triangle:** Triangle with two of its sides equal and consequently the angles, opposite the equal sides are also equal.



$\angle B = \angle C$ and sides AB = AC

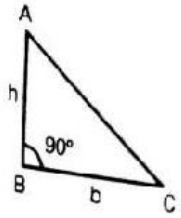
Properties of Triangles

- Sum of the lengths of any two sides of a triangle has to be always greater than the third side.
- Difference between the lengths of any two sides of a triangle has to be always lesser than the third side.
- Side opposite to the greatest angle will be the greatest and the side opposite to the smallest angle will be smallest.

Area of Triangle

1. Area = $\frac{1}{2}$ base × height

$\Delta = \frac{1}{2} \times b \times h$

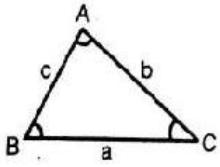


2. Area = $\sqrt{S(S-a)(S-b)(S-c)}$

This is called Hero's formula

Where $S = \frac{a+b+c}{2}$, and a, b and c are the sides of triangle.

3. Area = $\frac{1}{2}$ × product of two sides × sine of the included angle



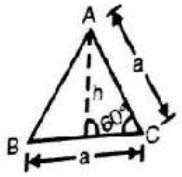
Area of $\Delta ABC = \frac{1}{2} ac \sin B$

$= \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A$

Where a, b, c denotes sides BC, CA and AB and A, B and C denotes angle $\angle A$, $\angle B$ and $\angle C$ respectively.

Equilateral Triangles

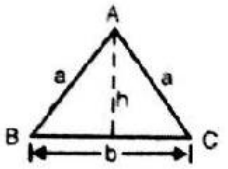
1. Altitude $h = \frac{a\sqrt{3}}{2}$



2. Area = $\frac{1}{2}$(base) × height

$$= \frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4} a^2$$

Isosceles Triangles



1. Altitude $h = \frac{1}{2} \times \sqrt{4a^2 - b^2}$,

2. Area $= \frac{1}{2} \times$ base $\times$ height

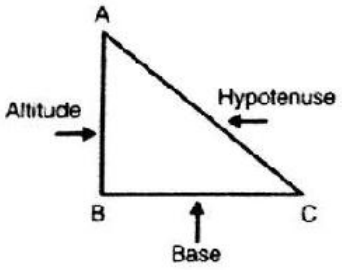
$$\frac{1}{2} \times b \times h = \frac{1}{2} \times b \times \frac{1}{2}\sqrt{4a^2 - b^2}$$

$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

Right Angled Triangle

Pythagoras Theorem

According to Pythagoras Theorem Square of hypotenuse of a right angled triangle is equal to sum of squares of altitude and base.



$$(AC)^2 = (AB)^2 + (BC)^2$$

Some Basic Pythyagorean Triplets

3, 4, 5	→	$3^2+4^2=5^2$
5, 12, 13	→	$5^2+12^2=13^2$
7, 24, 25	→	$7^2+24^2=25^2$
8, 15, 17	→	$8^2+15^2=17^2$
9, 40, 41	→	$9^2+40^2=41^2$
11, 60, 61	→	$11^2+60^2=61^2$
12, 35, 37	→	$12^2+35^2=37^2$
16, 63, 65	→	$16^2+63^2=65^2$
20, 21, 29	→	$20^2+21^2=29^2$
28, 45, 53	→	$28^2+45^2=53^2$

These triplets are very important since large number of questions are based on them. Remembering these teiplets can save lots of valuable time.

Multiplying or Dividing the triplet by any real number gives new triplets. For example

3, 4, 5 will give 6, 8, 10
9, 12, 15 and
12, 16, 20 etc.

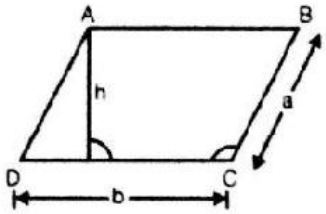
Quadrilaterals

Polygons having four sides are called quadrilaterals. Sum of all internal angles in a quadrilateral is equal to 360°.

Types of Quadrilaterals

1. **Parallelogram**

   A quadrilateral in which opposite sides are parallel (as shown in figure)



   Area = Base $\times$ height = $b * h$

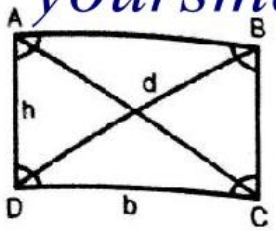
   Area = Product of any two adjacent sides $\times \sin\theta$ of the included angle i.e.

   $ab \sin C$ (here $\angle C$ is $\angle DCB$)

   Perimeter = 2(a + b)
   where a and b are two adjacent sides

2. **Rectangles**

   A rectangle is a parallelogram in which all anges are equal to 90°

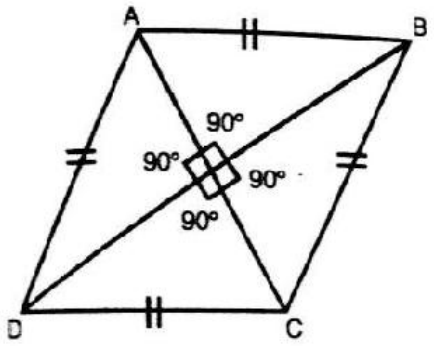


(a) Area = Base × height = b × h

(b) Diagonal (d) = $\sqrt{h^2 + b^2}$

3. **Rhombus**

A Parallelogram having all sides equal is a rhombus.

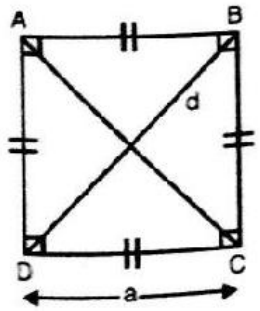


Area = $\frac{1}{2}$ × product of diagonal × sin of angle between them

- In rhombus diagonals bisect each other at right angle.

4. **Square**

Square is a parallelogram in which all the sides are equal and also all the angles are equal to 90°.



Area = $(\text{Side})^2 = a^2$

$= \frac{1}{2}(\text{diagonal})^2 = \frac{1}{2}d^2$

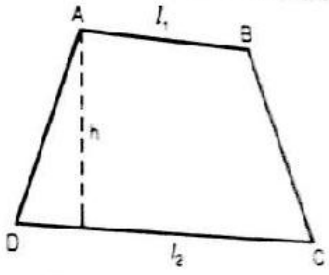
Diagonal = $\sqrt{2} \cdot a$

Perimeter = 4a

In squares, diagonal bisect each other at 90°, and also diagonals are equal to each other.

5. **Trapezium**

A quadrilateral in which only two sides are parallel to each other is called trapezium.



Area = $\frac{1}{2}$ × sum of parallel sides × distance between them

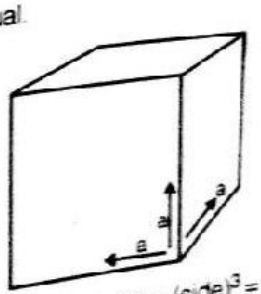
$= \frac{1}{2} \times (l_1 + l_2) \times h$

Mensuration & Solid Geometry

Mensuration is a branch of science which deals with measurements especially of surface area, Volume, curved surface area etc of plane and solid figures. So far we have discussed the plane figure like triangles and quadrilateral, now we will proceed on solid figures.

1. **Cube**

Cube is a three dimentional box in which all the three side that is length breath & height are equal.



Volume of a cube = $(\text{side})^3 = a^3$

Total surface area of a cube

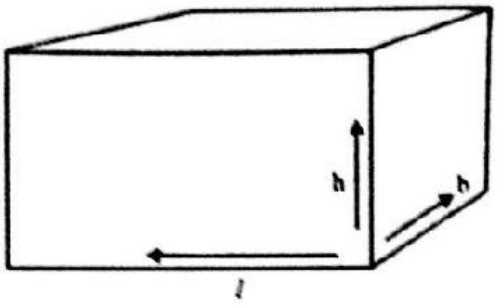
$= 6 \times (\text{side})^2 = 6a^2$

Surface area of four walls

$= 4 \times (\text{side})^2 = 4a^2$

2. **Cuboid**

Cuboid is a three dimensional solid figure in which length breadth & heights are not equal.



This is the most common figure.

Volume = length × breadth × height

$= l \times b \times h$

Total surface area $= 2(lb + bh + hl)$

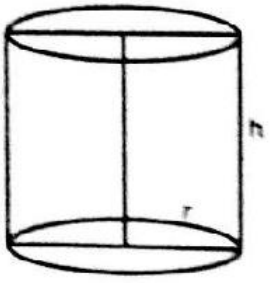
Area of the four walls of cuboid shaped room $= 2(bh + hl) = 2h(b + l)$

Diagonal of cuboid $= \sqrt{l^2 + b^2 + h^2}$

Diagonal of cube $d = \sqrt{3} \times a$

Where a is the side of cube.

3. **Cylinder**



Volume of cylinder

$= \pi \times (\text{radius})^2 \times \text{height} = \pi r^2 h$

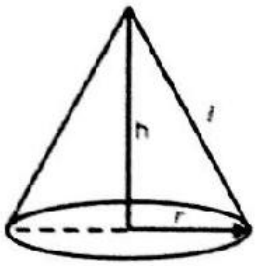
Curved surface area

$= 2 \times \pi \times \text{radius} \times \text{height} = 2\pi rh$

Total surface area = Curved surface area + Area of upper & lower circles.

$= 2\pi rh + 2\pi r^2 = 2\pi r(r + h)$

4. **Cone**



Volume of a cone

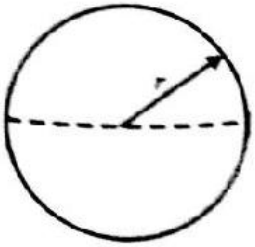
$= \frac{1}{3} \times \pi \times (\text{radius})^2 \times \text{height} = \frac{1}{3}\pi r^2 h$

Curved surface area $= \pi rl$

(where l is lateral height which is equal to $l = \sqrt{r^2 + h^2}$)

Total surface Area $= \pi rl + \pi r^2 = \pi r(l + r)$

5. **Sphere**

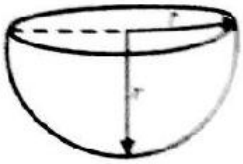


Voulme of sphere $= \frac{4}{3}\pi r^3$

Total surface Area $= 4\pi r^2$

6. **Hemisphere**

Volume $= \frac{2}{3}\pi r^3$



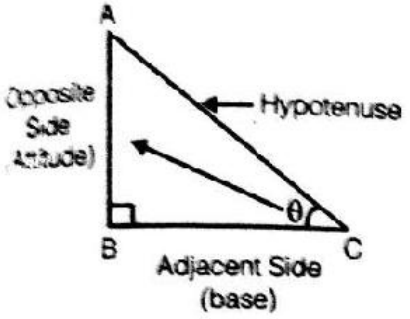
Curved Surface Area $= 2\pi r^2$

Total Surface Area $= 3\pi r^2$

Now, T.S.A = curved surface area of the flat surface = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Trigonometry

This is one of the most important part of geometry. Questions of trigonometry are based mainly on the basic concept of sine, cosine & tangents. At the same time, problem on height & distance are also very common.



$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

$$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\sec\theta = \frac{1}{\text{Cos}\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent side}}$$

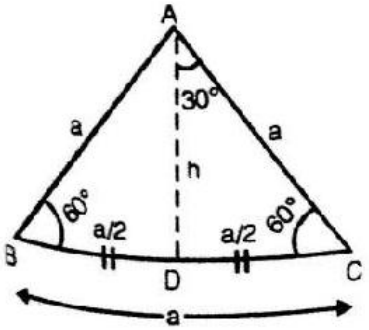
$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{Adjacent side}}{\text{Opposite side}}$$

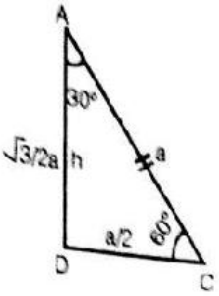
Calculation of sine, cosine & tangent of different angles.

1. **for 30° & 60°**

Construct equilateral triangle of side a.



In ΔADC



$$\text{Height } h = \frac{\sqrt{3}a}{2}$$

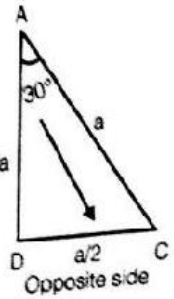
$$\sin 60° = \frac{\frac{\sqrt{3}}{2}a}{a} = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{\frac{a}{2}}{a} = \frac{1}{2}$$

$$\tan 60° = \frac{\frac{\sqrt{3}}{2}a}{\frac{a}{2}} = \sqrt{3}$$

2. **for 30°**

In Triangle ADC



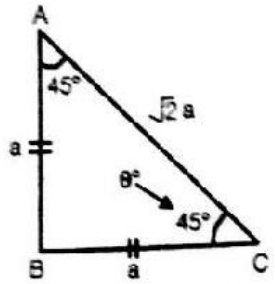
$$\sin 30° = \frac{\frac{a}{2}}{a} = \frac{1}{2}$$

$$\cos 30° = \frac{\frac{\sqrt{3}}{2}a}{a} = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{\frac{a}{2}}{\frac{\sqrt{3}}{2}a} = \frac{1}{\sqrt{3}}$$

3. For 45°

Construct a right angled triangle in which one angle is equal to 45°.



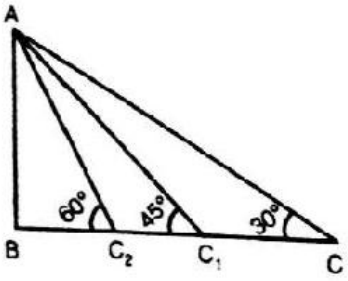
$$\sin 45° = a/\sqrt{2}\,a = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\tan 45° = \frac{a}{a} = 1$$

4. For 90°

In the above figure to make angle equal to 90° side AC should become very closer to AB. Its closeness of AC to AB is upto such an extent that AC overlapes (coincides) AB.



When ∠C becomes 90° and AC overlapes (coinsides) AB then

AC = AB (∵ AB is opposite side)

$$\sin 90° = \frac{AB}{AC} = 1$$

$$\cos 90° = \frac{AB}{AC} = 0\ (\therefore BC = 0)$$

$$\tan 90° = \frac{AB}{BC} = \infty (\therefore BC = 0)$$

5. For 0°

In the above figure angle ∠A will be equal to 0° if angle ∠C is equal to 90°

So, in 0°

AC is Hypotenuse = AB(Adjacent side) & opposite side BC=0

$$\text{So, } \sin 0° = \frac{BC}{AC} = 0$$

$$\cos 0° = \frac{AB}{AC} = 1$$

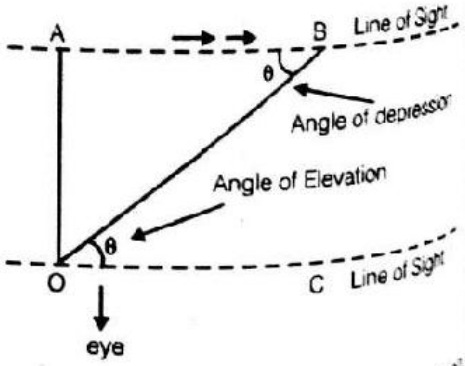
$$\tan 0° = \frac{BC}{AB} = 0$$

Trigonometric Table

S.N.	Angle(θ)	sinθ	cosθ	tanθ	cosecθ	secθ	cotθ
1.	0°	0	1	0	∞	1	∞
2.	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{\sqrt{2}}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
3.	45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
4.	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
5.	90°	1	0	∞	1	∞	0

Angle of Elevation & angle of Depression

1. **Angle of Elevation:** It is that angle which our eye makes with that of horizontal line of sight when observing objects at height above the line of sight.



In the above figure ∠BOC is Angle of Elevation

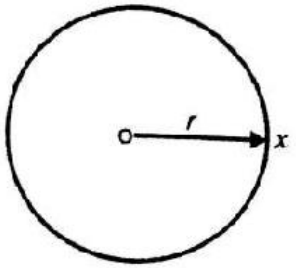
2. **Angle of Depression**: It is that angle which our eye makes with that of line of sight while observing object below it ex:- when pilot of an aircraft observe the base station angle formed is the angle of depression. Here ∠ABO is angle of depression.

Note: Angle of Elevation is always equal to angle of depression.

Circles and its properties

Circle

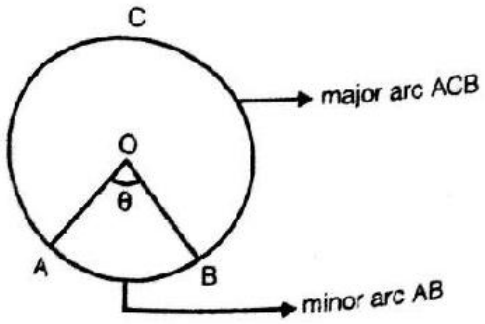
A circle is a set of all those points in a plane, each one of which is at given constant distance from a fixed point in the plane



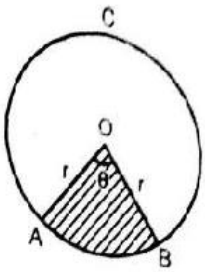
The fixed point is called the centre and the given constant distance is called the radius of the circle.

- Area $= \pi r^2$
- Circumference $= 2\pi r$
- Length of Arc of given circle i.e.

$$AB = \frac{\theta}{360} \times 2\pi r$$



- Sector of a Circle: It is a part of the area of a cirlce between two radii.

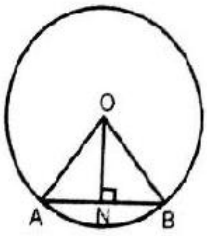


Area of Sector $= \pi r^2 \times \frac{\theta}{360^\circ}$

- in the above figure perimeter of sector is equal to $= 2\pi r \times \frac{\theta}{360^\circ} + 2r$

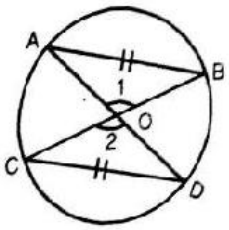
Properties

1. The perpendicular from the centre of a cirlce to a chord bisect the chord



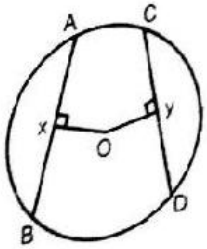
If ON ⊥ AB, then AN = NB

2. Equal chord of a circle subtends equal angle at the centre



If AB = CD then ∠1 = ∠2

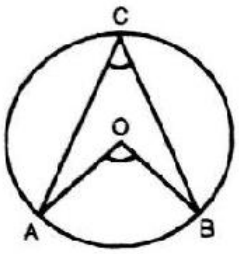
3. Equal chords of a circle are equidistance from the centre



If the chord AB and CD of a circle are equal and if OX ⊥ CD then

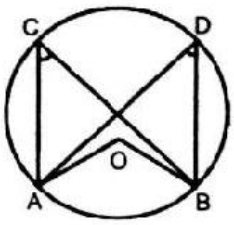
$$OX = OY$$

4. The angle substended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the cirlce.



In the given fig: $\angle AOB = 2 \times \angle ACB$

5. Angle in the same segment of a circle are equal

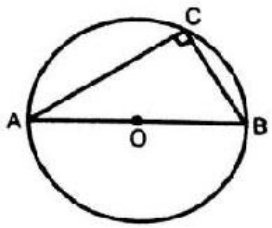


In fig $\angle AOB = 2\angle ACB$

also $\angle AOB = 2\angle ADB$

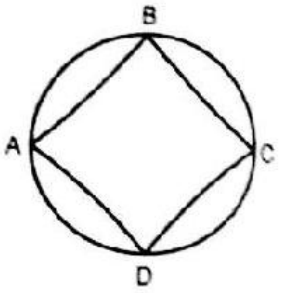
So $\angle ACB = \angle ADB$

6. The angle in a semi circle is a right angle



In the figure above $\angle ACB = 90°$

7. The opposite pairs of angles of a cyclic quadrilateral is supplementry to each other

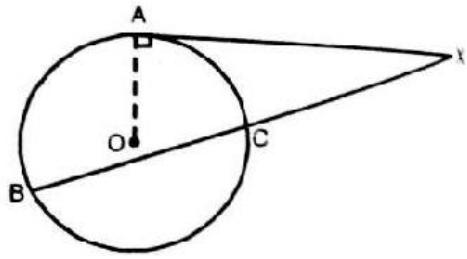


In the figure $\angle A + \angle C = 180°$ & also

$\angle B + \angle D = 180°$

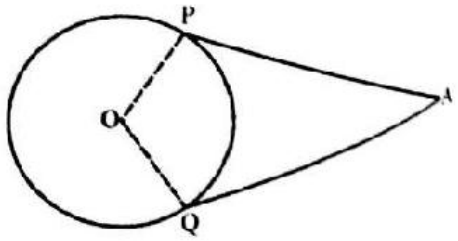
Tangents and Secants

Tangent touch the circle at one point, while secant touch the circle at two points.



In the figure AX is tangent while BCX is a secant

- A tangent at any point of a circle is perpendicular to the radius through the point of contact
- The length of two tangents drawn from an external points to a circle are equal



In the above figure AP = AQ

- If two chords AB and CD intersect internally or externally at a point P then

$PA \times PB = PC \times PD$

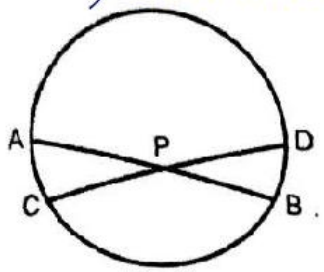


Fig (i)

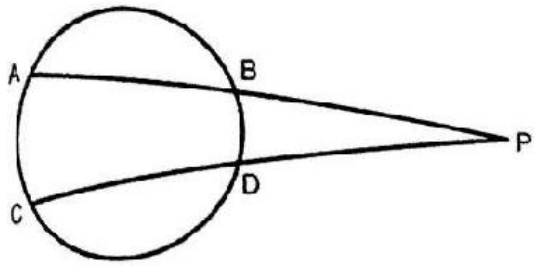
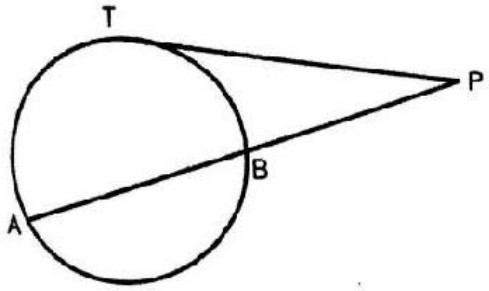


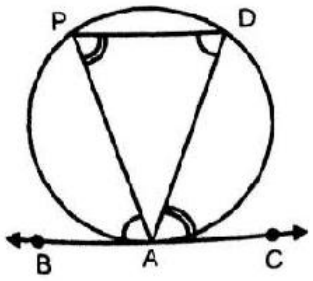
Fig (ii)

- If PAB is a secant to a circle intersecting the circle at A and B, and PT is tangent segment then $PA \times PB = PT^2$

(Popularly known as **Tangent -Secant theorem**)



- **Alternate segment Theorem**



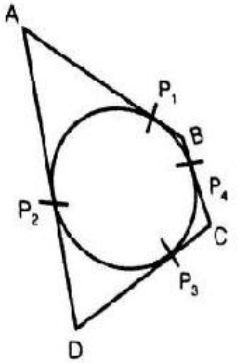
In the figure above, if BAC is the tangent touching at a point A to a circle and if AD is any chord then

$\angle DAC = \angle APD$ and

$\angle PAB = \angle PDA$

(Angles in alternate segment of a circle are always equal)

- If a circle touches all the four sides of a quadrilateral then the sum of opposite pair sides are equal



If ABCD is a circumscribed quadrilateral, then AB + CD = AD+ BC.

□□□□

Triangles & Quadrilaterals

Solved Examples

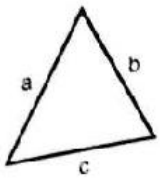
1. The area of a triangle whose sides are 15 m, 16 m and 17 m

(a) $24\sqrt{4}$ sqm (b) $24\sqrt{3}$ sqm

(c) $24\sqrt{21}$ sqm (d) None of these

Ans: (c)

Let a =15 m, b = 16 m, c = 17 m.



Then

$$s = \frac{a+b+c}{2} = \frac{15+16+17}{2} = 24m$$

$$\therefore \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{24(24-15)(24-16)(24-17)}$$

$$= \sqrt{24\times9\times8\times7} = \sqrt{24\times3\times8\times3\times7}$$

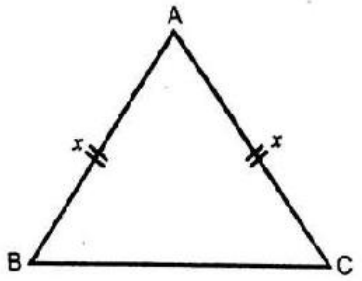
$$= 24\sqrt{21} \text{ sqm}$$

2. The area of a triangular lawn is 1600 Sq m. If one side is 64 m long and the other two sides are equal in length, the length of each equal side is

(a) 60.37 m (b) 59.36 m
(c) 60.36 cm (d) None of these

Ans. (b)

Let the length of equal sides be x.



Then, $s = \frac{x+x+64}{2} = x+32$

Area = 1600 sqm.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(x+32)(x+32-x)(x+32-x)(x+32-64)}$$

$$= \sqrt{(x+32)\times32\times32\times(x-32)}$$

or $1600 = 32\sqrt{x^2-32^2}$

$\Rightarrow \sqrt{x^2-32^2} = 50$

or $x^2 = 32^2 + 50^2 = 1024 + 2500 = 3524$

$\therefore$ x = 59.36 m

3. The sides of a triangle are in the ratio 3 : 4 : 5. If its perimeter is 36 cm, area of the triangle is

(a) 57 sq cm (b) 54 sq cm
(c) 56.5 sq cm (d) None of these

Ans. (b)

Let the sides of the triangle a, b & c are $3x$, $4x$ & $5x$

then $12x = 36 \Rightarrow x = 3$

$\therefore$ a = 9; b = 12; c = 15 cm

$$s = \frac{a+b+c}{2} = \frac{36}{2} = 18 \text{ cm}$$

$$\therefore \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-9)(18-12)(18-15)}$$

$$= \sqrt{18\times9\times6\times3}$$

$$= \sqrt{9^2\times2^2\times3^2} = 9\times2\times3$$

= 54 sq m

4. A ladder is placed so as to reach a window 63 cm high. The ladder is then turned over to the opposite side of the street and is found to reach a point 56 cm high. If the ladder is 65 cm long, the width of the street is

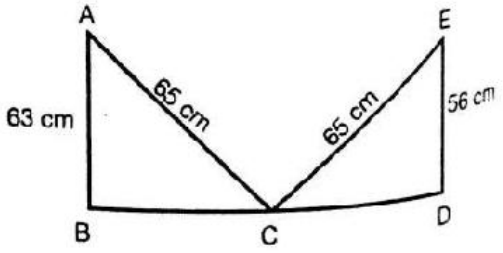
(a) 59 cm (b) 39 cm
(c) 49 cm (d) None of these

Ans. (c)

$$BC = \sqrt{(65)^2 - (63)^2} = 16$$

$$CD = \sqrt{(65)^2 - (56)^2} = 33$$

$\therefore$ Width of the street = 16 + 33 = 49 cm



5. If the area of a triangle is 150 sq m and base : height is 3:4, find its height.
(a) 25 m (b) 35 m
(c) 20 m (d) None of these

Ans. (c)
Let the base be $3x$. Then, the height is $4x$.

Given : $\frac{1}{2} \times 3x \times 4x = 150$

$\Rightarrow x^2 = 25 \Rightarrow x = 5$

$\therefore$ Base $= 3 \times 5 = 15$ m and
height $= 4 \times 5 = 20$ m

6. Two sides of a triangular field are 85 m and 154, respectively, and its perimeter is 324 m. The cost of ploughing the field at the rate of Rs. 10 per sq m is
(a) Rs. 27720 (b) Rs. 37620
(c) Rs. 26750 (d) None of these

Ans. (a)
The third side of the triangle
$= 324 - (85 + 154) = 85$m

Also, $s = \frac{a+b+c}{2} = \frac{324}{2} = 162$

$\therefore$ Area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{162(162-85)(162-85)(162-154)}$

$= \sqrt{162 \times 77 \times 77 \times 8} = 2772$ sq.m

$\therefore$ The cost of ploughing the field
$= 2772 \times 10 =$ Rs. 27720

7. If the perimeter of an equilateral triangle is 12 m, find its area.
(a) $3\sqrt{4}$ m (b) $4\sqrt{3}$ m
(c) $5\sqrt{3}$ m (d) None of these

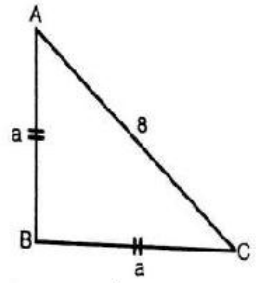
Ans. (b)
$3 \times (\text{side}) = 12 \Rightarrow \text{side} = 4$m

$\therefore$ Area of equilateral triangle

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times 16 = 4\sqrt{3}$ m

8. Find the area of an isosceles right-angled triangle whose hypotenuse is 8 cm.
(a) 32 sq m (b) 24 sq m
(c) 16 sq m (d) None of these

Ans. (c)



We have, hypotenuse

$= \sqrt{2}a = 8 \Rightarrow a = 8/\sqrt{2}$

$\therefore$ Area of isosceles right angled triangle

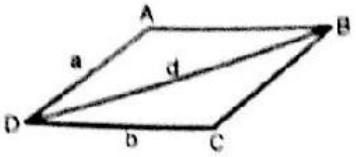
$= \frac{1}{2}a^2 = \frac{1}{2} \times \frac{64}{2} = 16$ sq. m

9. If all sides of a triangle are increased by 200% the area of the triangle will increase by
(a) 400% (b) 600%
(c) 800% (d) None of these

Ans. (c)
Area of a triangle is directly proportional to square of its sides.
So if side is increased by 200% or become three times area will become 9 times i.e. increased by 800%.

10. The two adjacent sides of a parallelogram are 60 m and 40 m and one of the diagonals is 80 m long. The area of the parallelogram is
(a) $600\sqrt{15}$ sq.m (b) $800\sqrt{15}$ sq.m
(c) $700\sqrt{15}$ sq.m (d) None of these

Ans. (a)



Here, a = 60, b = 40 and d = 80
Area of parallelogram = 2 × area of ΔABD

Here $s = \frac{a+b+d}{2} = \frac{60+40+80}{2} = 90$

∴ Area of the parallelogram

$= 2\sqrt{s(s-a)(s-b)(s-d)}$

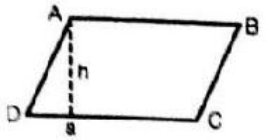
$= 2\sqrt{90(90-60)(90-40)(90-80)}$

$= 2\sqrt{90\times30\times50\times10}$

$= 600\sqrt{15}$ sqm

11. One side of a parallelogram is 10 m and the corresponding altitude is 7 m. The area of the parallelogram is
(a) 70 sq m (b) 60 sq m
(c) 80 sq m (d) None of these

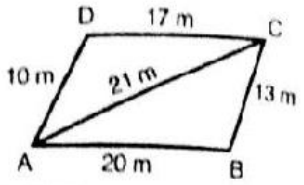
Ans. (a)



The area of the parallelogram = a × h
= 10 × 7 = 70 sq m

12. In a quadrilateral ABCD, the sides AB, BC, CD, DA measure 20 m, 13 m, 17 m and 10 m, respectively and the diagonal AC is 21m. The area of the quadrilateral is
(a) 210 sq m (b) 220 sq m
(c) 240 sq m (d) None of these

Ans. (a)
Area of quadrilateral ABCD
= Area of Δ ADC+Area of Δ ABC,
Where,



Area of ΔABC

$= \sqrt{s(s-AB)(s-BC)(s-d)}$

$\left(s = \frac{AB+BC+d}{2} = \frac{20+13+21}{2} = 27\right)$

$= \sqrt{27(27-20)(27-13)(27-21)}$

$= \sqrt{27\times7\times14\times6} = 126$ sq m

Area of ΔACD

$= \sqrt{s(s-AD)(s-DC)(s-d)}$

$= \left(s = \frac{AD+DC+d}{2} = \frac{10+17+21}{2} = 24\right)$

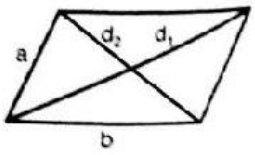
$= \sqrt{24(24-10)(24-17)(24-21)}$

$= \sqrt{24\times14\times7\times3} = 84$ sq m

Here, the area of quadrilateral ABCD
= 126 + 84 = 210 sq m.

13. In a parallelogram, the length of adjacent sides are 12 cm and 14 cm respectively. If the length of one diagonal is 16 cm, find the length of the other diagonal.
(a) 24.8 cm (b) 20.6 cm
(c) 22.4 cm (d) None of these

Ans. (b)



In a parallelogram

$$d_1^2 + d_2^2 = 2\left(a^2 + b^2\right)$$

$$\Rightarrow (16)^2 + d_2^2 = 2\left(12^2 + 14^2\right)$$

$$\Rightarrow d_2^2 = 2(144 + 196) - 256 = 424$$

$$\therefore d_2^2 = \sqrt{424} = 20.6 \text{ cm}$$

14. The perimeter of a circle is equal to that of a square. Compare their areas.

(a) 14 : 11 (b) 25 : 12
(c) 24 : 7 (d) 22 : 7

Ans. (a)

If x be the side of the square and r be the radius of the cirlce, then

$$4x = 2\pi r \text{ or } x = \frac{\pi r}{2}$$

Now, $x^2 : x^2 = \pi r^2 : \frac{\pi^2 r^2}{4}$

or $4 : \pi = 4 : \frac{22}{7}$ or $14 : 11$

15. The wheel of a scooter has diameter 70 cm. How many revolutions per minute must the wheel make so that the speed of the scooter is kept 66 km per hour?

(a) 400 (b) 600
(c) 500 (d) 800

Ans. (c)

The distance covered by the wheel in one minute

$$= \frac{66 \times 1000 \times 100}{60} = 110000 \text{ cm}$$

The distance covered by the wheel in one revolution

= The circumference of the wheel

$$= 2\pi r = 2 \times \frac{22}{7} \times \frac{70}{2} = 220 \text{ cm}$$

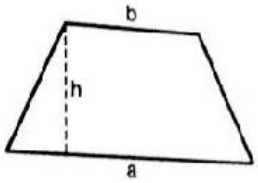
$\therefore$ Number of the revolution of the wheel

$$= \frac{110000}{220} = 500$$

16. The area of a trapezium is 2500 sq m. One of its parallel sides is 75m. If the distance between the two parallel sides is 40 m, find the other parallel side.

(a) 20 m (b) 30 m
(c) 40 m (d) 50 m

Ans. (d)



Area of trapezium $= \frac{1}{2}[a+b] \times h$

$$\Rightarrow \frac{1}{2}(75 + x) + 40 = 2400$$

$$\Rightarrow 75 + x = 125$$

$$\Rightarrow x = 50$$

$\therefore$ The other parallel side = 50 m

17. If the ratio of the areas of two squares is 9 : 1, the ratio of their perimeters is

(a) 9 : 1 (b) 3 : 4
(c) 3 : 1 (c) 1 : 3

Ans. (c)

Let x, y be the sides of squares

$$\frac{x^2}{y^2} = \frac{9}{1} \Rightarrow \frac{x}{y} = \frac{3}{1}$$

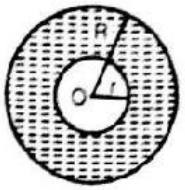
$\therefore$ The ratio of perimeters is $4x : 4y$

i.e. $x : y = 3 : 1$

18. A rope by which a calf is tied is increased from 12 m to 23 m. How much additional grassy ground shall it graze?

(a) 1120 m^2 (b) 1250 m^2
(c) 1210 m^2 (d) 1200 m^2

Ans. (c)



Additional grassy ground grazed
$= \pi[R^2 - r^2]$
$= \pi(23^2 - 12^2)m^2 = \pi \cdot 385$
$= \frac{22}{7} \times 35 \times 11 = 1210\ m^2$

19. The length of a rectangular field is twice its breadth. If the rent of the field at Rs. 3500 a hectare is Rs. 28000, find the cost of surrounding it with fence at Rs. 5 per metre.
(a) Rs. 6000 (b) Rs. 7000
(c) Rs. 6500 (d) Rs. 8000

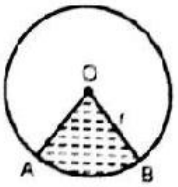
Ans. (a)
Area of the rectangular field

$= \frac{28000}{3500} = 8$ square hectare

[$\because$ 1 sq. hectare = 10000 sq m]
Area = $l \times b = 2x \times x = 80000$
$\therefore\ x^2 = 40000 \Rightarrow x = 200$
$\therefore$ Breadth = 200 m
Length = 400 m
Perimeter = 2(400 + 200) = 1200 m
$\therefore$ Cost of fencing
= 1200 × 5 = Rs. 6000

20. The area of a sector of a circle of radius 5 cm, formed by an arc of length 3.5 cms, is
(a) 35 cm^2 (b) 17.5 cm^2
(c) 8.75 cm^2 (d) 55 cm^2

Ans. (c)



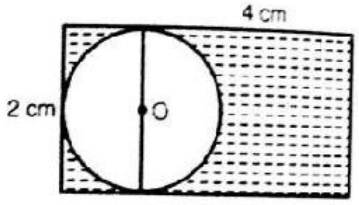
Let r be the radius O be the centre & AB the arc of circle then,
Area of Sector OAB

$= \left(\frac{1}{2} \times \text{arc length} \times \text{radius}\right) cm^2$

$= \left(\frac{1}{2} \times 3.5 \times 5\right) cm^2 = 8.75 cm^2$

21. A rectangular sheet of cardboad is of 4 cm × 2 cm, If a circle of greatest possible area is cut from it, then the area of remaining portion is
(a) $(2 - \pi)cm^2$ (b) $(4 - \pi)cm^2$
(c) $(8 - \pi)cm^2$ (d) $(16 - \pi)cm^2$

Ans. (c)



Area of remaining portion
= Area of rectangle – Area of circle
$= 4 \times 2 - \pi \times (1)^2 = (8 - \pi)cm^2$

22. If the area of a square is equal to the area of a rectangle 6.4 m long and 2.5 m wide, then each side of the square measures
(a) 8 m (b) 5.4 m
(c) 3.8 m (d) 4 m

Ans. (d)
Area of rectangle = $6.4 \times 2.5 = 16\ m^2$
According to question:
Area of square = Area of the rectangle
$\therefore$ Area of square = 16 m^2
$\therefore$ Side of the square = 4 m

23. A rectangular carpet has an area of 120 sq m and a perimeter of 46 m. The length of its diagonal is

(a) 15 m (b) 16 m
(c) 17 m (d) 20 m

Ans. (c)

Let length = a and breadth = b

Then, $2(a + b) = 46$

or, $a + b = 23$ and $ab = 120$

$$\therefore \text{Diagonal} = \sqrt{a^2 + b^2} = \sqrt{(a+b)^2 - 2ab}$$

$$= \sqrt{(23)^2 - 2 \times 120} = \sqrt{289} = 17 \text{ m}$$

24. A piece of wire 132 cm long is bent successively in the shape of an equilateral triangle, a square, a regular hexagon, and a circle. Then, which has the largest surface area?
(a) equilateral triangle
(b) Square
(c) circle
(d) regular hexagon

Ans. (c)

Area of an equilateral $\Delta = \frac{\sqrt{3}}{4}a^2$

[3a = 132 θ a = 44]

$$= \frac{\sqrt{3}}{4} \times 44 \times 44 = 838.312 \text{ sq. m}$$

Area of square $= a^2 = 33 \times 33$

$$= 1089 \text{ sq. m} \left[a = \frac{132}{4} = 33\right]$$

Area of regular hexagon

$$= \frac{3\sqrt{3}a^2}{2} = \frac{3\sqrt{3} \times 22 \times 22}{2}$$

$$\left[a = \frac{132}{6} = 22\right]$$

Area of circle $= \pi r^2$

$$\frac{22}{7} \times 21 \times 21 \left[r = \frac{132}{2\pi} = 21\right]$$

Circle has largest surface area.

25. The radius of the wheel of a vehicle is 70 cm. The wheel makes 10 revolutions in 5 seconds. The speed of the vehicle is
(a) 29.46 km/hr (b) 31.68 km/hr
(c) 36.25 km/hr (d) 32.72 km/hr

Ans. (b)

Circumference

$$= \left(2 \times \frac{22}{7} \times 70\right) \text{cm} = 440 \text{ cm}$$

Distance travelled in 10 revolutions
= 4400 cm = 44 m

$$\therefore \text{Speed} = \frac{\text{Distance}}{\text{Time}} = \left(\frac{44}{5}\right) \text{m/sec}$$

$$= \left(\frac{44}{2} \times \frac{18}{5}\right) \text{km/hr} = 31.68 \text{ km/hr}$$

26. The area of a big rectangle is equal to 1/6th of a small rectangle. If the length of the big rectangle is equal to the length of the small rectangle and the width of big rectangle is 2m, what is the width of a small rectangle?
(a) 1/3 m (b) 1 m
(c) 2 m (d) None of these
(e) Cannot be determined

Ans. (a)

Let the length of the big rectangle be x m

∴ Area of the big rectangle
$= x \times 2 = 2x \text{ m}^2$

∴ Area of the small rectangle

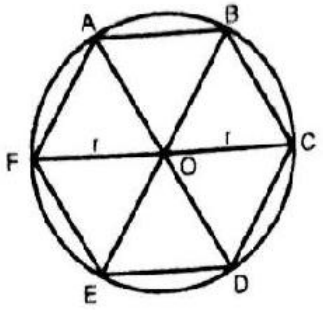
$$= \frac{1}{6} \times 2x = \frac{x}{3} \text{m}^2$$

∴ Breadth of the small rectangle

$$= \frac{x}{3} + x = \frac{1}{3} \text{m}$$

27. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is
(a) 3r (b) 6r
(c) 9r (d) 12r

Ans. (b)



Length of each side of hexagon = r

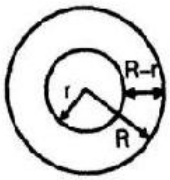
∴ Its perimeter = 6r.

28. A circle road runs around a circular garden. If the difference between the circumference of the outer circle and the inner circle is 44 m, find the width of the road.

(a) 4 m (b) 7 m

(c) 3.5 m (d) 7.5 m

Ans (b)



$2\pi R - 2\pi R = 44$ m

∴ $2\pi(R - r) = 44$ m

or $R - r = 44 \div 2p$

$= 44 \div \left(2 \times \frac{22}{7}\right) = 7$m

29. A garden is 24 m long and 14 m wide. There is a path 1 m wide outside the garden along its sides. If the path is to be constructed with square marble tiles 20 cm × 20 cm find the number of tiles required to cover the path.

(a) 1800 (b) 200

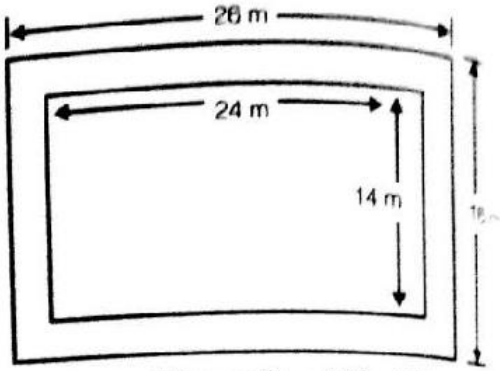
(c) 2000 (d) 2150

Ans. (c)

Area of garden = $24 \times 14 = 336$ m²

Area of the (garden + path)

$= 26 \times 16 = 416$ m²



∴ Area of the path = 416 – 336 = 80 m²

Area of 1 tile = $20 \times 20 = 400$ cm²

$= 0.04$ m²

∴ Number of tiles required

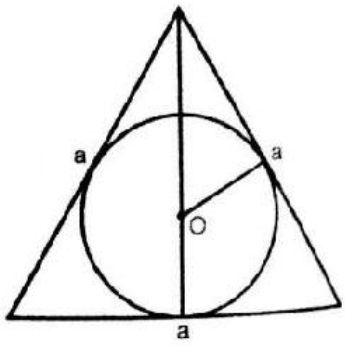
$= \frac{80}{0.04} = 2000$

30. The area of the circle inscribed in a equilateral triagle of side 24 cm is

(a) 24π cm² (b) 36π cm²

(c) 48π cm² (d) 18π cm²

Ans. (c)



$h = \frac{\sqrt{3}}{2}a$

$\Rightarrow h = \frac{\sqrt{3}}{2} \times 24$

or $h = 12\sqrt{3}$

Area of the circle

Note: The centre O of a circle inscribed in equilateral triangle devides the altitude in the ratio 2 : 1.

□□□□

Practice Exercise: I

1. The perimeter of a rectangular field is 480 meters and the ratio between the length and breadth is 5 : 3. The area of the field is
(a) 7200 m^2 (b) 15000 m^2
(c) 13500 m^2 (d) 54000 m^2

2. If each side of a square is increased by 50%, the ratio of the area of the resulting square to the area of the given square is
(a) 5 : 4 (b) 9 : 4
(c) 4 : 5 (d) 4 : 9

3. If the diagonal of a square is doubled to make the diagonal of another square, the area of the new square will
(a) becomes two folds
(b) becomes three folds
(c) becomes four folds
(d) remains the same

4. A hall 20 m long and 15 m broad is surrounded by a verandah of uniform width of 2.5 m. The cost of flooring the verandah at Rs 3.50 per square meters, is
(a) Rs. 500 (b) Rs. 600
(c) Rs. 700 (d) Rs. 800

5. A man walking at the speed of 4 kmph crosses a square field diagonally in 3 minutes. The area of the field is
(a) 18000 m^2 (b) 20000 m^2
(c) 19000 m^2 (d) 25000 m^2

6. If the perimeters of a square and a rectangle are the same, then the areas A and B enclosed by them would satisfy the condition:
(a) $A < B$ (b) $A \leq B$
(c) $A > B$ (d) $A \geq B$

7. If the perimeter of a rectangle and a square, each is equal to 80 cm and the difference of their areas is 100 sq. cm, the sides of the rectangle are:
(a) 35 cm, 15 cm (b) 30 cm, 10 cm
(c) 28 cm, 12 cm (d) 25 cm, 15 cm

8. The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. The length of the rectangle is:
(a) 20 cm (b) 30 cm
(c) 40 cm (d) 50 cm

9. Within a rectangular garden 10 m wide and 20 m long, we wish to pave a walk around the borders of uniform width so as to leave an area of 96 m^2 for flowers. How wide should the walk be?
(a) 1 m (b) 2 m
(c) 2.1 m (d) 2.5 m

10. A rectangular field has dimensions 25 m by 15 m. Two mutually perpendiculars passages, 2 m wide have been left in the central part. In rest of the field, grass has been grown. The area under the grass is :
(a) 295 m^2 (b) 299 m^2
(c) 300 m^2 (d) 375 m^2

11. The length of rectangle is twice its breadth and one of its diagonals measures $10\sqrt{5}$cm. The perimeter of the rectangle is
(a) 60 cm (b) 50 cm
(c) 250 m (d) None

12. A hall 36 m long and 15 m broad is to be paved with stones, each measuring 6 dm by 5 dm. The number of stones required is:
(a) 180 (b) 1800
(c) 18 (d) 18000

13. A rectangle has 15 cm as its length and 150 sq. cm as its area. If the area is increased to $1\frac{1}{3}$ times the original area by increasing its length only, then the new perimeter is

(a) 50 cm (b) 60 cm
(c) 70 cm (d) 80 cm

14. A room 5.44 m long and 3.74 m broad is to be paved with square tiles. The least number of square tiles required to cover the floor is
(a) 176 (b) 192
(c) 184 (d) 162

15. A man cycles round the boundary of a rectangular park at the rate of 12 kmph and completes one full round in 8 minutes. If the ratio between the length and breadth of the park be 3 : 2, then its area is
(a) 1536 m^2 (b) 15360 m^2
(c) 153600 m^2 (d) None

16. The area of a square is 0.5 hectare. Its diagonal is
(a) 50 m (b) 100 m
(c) 250 m (d) $50\sqrt{2}$ m

17. If the base of a rectangle is increased by 10% and the area is unchanged, then the corresponding altitude must be decreased by
(a) 10% (b) $9\frac{1}{11}\%$
(c) 11 % (d) $11\frac{1}{9}\%$

18. The length and breadth of a square are increased by 40% and 30% respectively. The area of the resulting rectangle exceeds the area of the square by
(a) 35% (b) 42%
(c) 62% (d) 82%

19. The height of a room to its semi-perimeter is 2:5. It costs Rs. 260 to paper the walls of the room with paper 50 cm wide at Rs. 2 per metre allowing an area of 15 sq. m. for doors and windows. The height of the room is
(a) 2.6 m (b) 3.9 m
(c) 4 m (d) 4.2 m

20. If the area of a rhombus is 15 sq. cm and length of one of its diagonals is 5 cm, the length of the other diagonal is
(a) 3 cm (b) 5 cm
(c) 6 cm (d) 7 cm

21. The length of one diagonal of a rhombus is 80% of the other diagonal. The area of the rhombus is how many times the square of the length of the longer diagonal?
(a) $\frac{4}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{4}$ (d) $\frac{1}{4}$

22. If a square and a rhombus stand on the same base, then the ratio of the areas of the square and the rhombus is
(a) greater than 1 (b) equal to 1
(c) equal to $\frac{1}{2}$ (d) equal to $\frac{1}{4}$

23. A parallelogram has sides 30 m and 14 m and one of its diagonals is 40 m long. Then its area is
(a) 336 m^2 (b) 168 m^2
(c) 480 m^2 (d) 372 m^2

24. The two parallel sides of a trapezium are 5 and 2.5 m respectively. If the perpendicular distance between them is 6.5 meters, the area of the trapezium is
(a) 26 m^2 (b) 13 m^2
(c) 20 m^2 (d) 10 m^2

25. The three sides of a triangle are 3 cm, 4 cm and 5 cm respectively. Then its area (in sq. cm) is
(a) $4\sqrt{2}$ (b) $2\sqrt{3}$
(c) $\sqrt{23}$ (d) 6

26. If the area of an equilateral triangle is $24\sqrt{3}$ sq. cm, then its perimeter is

a) 36 cm (b) $12\sqrt{6}$ cm

c) $4\sqrt{6}$ cm (d) $2\sqrt{6}$ cm

... is the length of a median of an equilateral ... then its area is

a) x^2 (b) $\frac{1}{2}x^2$

c) $\frac{x^2\sqrt{3}}{2}$ (d) $\frac{x^2\sqrt{3}}{3}$

... the area of a square with side a is equal to ... area of a triangle with base a, then the ...itude of the triangle is

a) $\frac{a}{2}$ (b) a

c) 2a (d) 4a

... the altitude of an equilateral triangle is $\sqrt{6}$ ... then its area is

a) $3\sqrt{3}$ (b) $2\sqrt{3}$

c) $2\sqrt{2}$ (d) $6\sqrt{2}$

... the circumference of a circle is 352 metres, ... then its area (in sq. m) is

a) 3966 (b) 6589

c) 9856 (d) 9856

... The circumferences of two concentric circles ... forming a ring are 88 cm and 66 cm ... respectively. The width of the ring is

a) 3.5 cm (b) 10.5 cm

c) 7 cm (d) 14 cm

... The diameter of a wheel is 1.26 m. How far ... will it travel in 500 revolutions?

a) 2530 m (b) 1980 m

c) 1492 m (d) 2880 m

... The wheel of an engine of train $4\frac{2}{7}$ m in ... circumference makes seven revolutions in ... seconds. Then the speed of the train is

(a) 25 km/hr

(c) 30 km/hr (b) 27 km/hr

(d) 35 km/hr

34. A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 15 revolutions?

(a) 18

(c) 25 (b) 27

(d) 30

35. If the diameter of a circle is increased by 100%, its area is increased by

(a) 100% (b) 200%

(c) 300% (d) 400%

36. The area of the largest circle that can be drawn inside a square of side 14cm in length is

(a) 154 cm^2 (b) 84 cm^2

(c) 204 cm^2 (d) None of these

37. The area of largest possible square inscribed in a circle of unit radius in sq. units is

(a) 2 (b) π

(c) $2\sqrt{2}\pi$ (d) $4\sqrt{2}\pi$

38. The ratio of the areas of the incircle and circumcircle of an equilateral triangle is

(a) 1 : 2 (b) 1 : 3

(c) 1 : 4 (d) 1 : 9

39. A circular wire ring of radius 42 cm is cut and bent into the form of a rectangle whose sides are in the ratio of 6 : 5. The smaller side of the rectangle is

(a) 30 cm (b) 60 cm

(c) 72 cm (d) 132 cm

40. Four horses are tied at four corners of a square plot of side 63 metres so that they just can not reach one another. The area left ungrazed is

(a) 675.5 m^2 (b) 780.5 m^2

(c) 795.5 m^2 (d) 850.5 m^2

Solutions

1. **Ans. (c)**
Let length = $5x$ and breadth = $3x$. Then,
$2(5x + 3x) = 480$ or $x = 30$.
$\therefore$ Length = 150 m & breadth = 90 m.
Area $= (150 \times 90)m^2 = 13500m^2$.

2. **Ans. (b)**
Let, each side = a. Then, original area = a^2.
New side $= \frac{150a}{100} = \frac{3a}{2}$
New area $= \frac{9a^2}{4}$
Required ratio $= \frac{9a^2}{4} : a^2 = 9 : 4$

3. **Ans. (c)**
Let the diagonal be a. Then, area $= \frac{1}{2}a^2$
New diagonal = 2a.
$\therefore$ New Area $= \frac{1}{2}(2a)^2 = 2a^2 = 4\left(\frac{1}{2}a^2\right)$
Four times the Original area.

4. **Ans. (c)**
Area of the verandah
$= (25 \times 20 - 20 \times 15)m^2 = 200\ m^2$
$\therefore$ Cost of flooring
$= \text{Rs.}\left(200 \times \frac{7}{2}\right) = \text{Rs.}700$

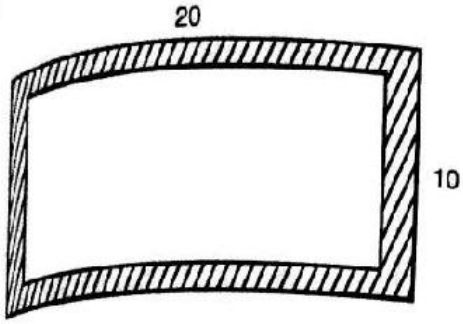
5. **Ans. (b)**
Length of diagonal = Distance covered in 3 min. at 4 km/hr.
$= \left(\frac{4000}{60} \times 3\right)m = 200\ m$
Area of the field $= \frac{1}{2}(\text{diagonal})^2$
$= \left(\frac{1}{2} \times 200 \times 200\right)m^2 = 20000\ m^2$

6. **Ans. (d)**
Let, side of square = a, length of rect. = x, breadth of rect. = y.
$4a = 2(x+y) \Rightarrow \frac{x+y}{2} = a$
Area of square $= a^2 = \left(\frac{x+y}{2}\right)^2$
So, $A = \left(\frac{x+y}{2}\right)^2$
Area of rect. = xy. So, B = xy.
$\frac{x+y}{2} > x^{1/2}y^{1/2}$ [$\because$A. M. > G. M.]
$\Rightarrow \left(\frac{x+y}{2}\right)^2 > xy$ i.e. A > B.
Also $\left(\frac{x+y}{2}\right)^2 > xy$ when $x - y$
Note: If a square & a rectangle have same perimeter, then :
(Area of square) $\geq$ (Area of rectangle).

7. **Ans. (b)**
Perimeter of square = 80 cm. So, side of square = 20 cm.
Area of the square $= (20 \times 20)cm^2 = 400\ cm^2$
With same perimeter, area of the square is larger.
$\therefore$ Area of rectangle $= (400 - 100)cm^2$
$= 300\ cm^2$.
Let, length of rect. = x & its breadth = y then,
$xy = 300$ and $x + y = 40$ ...(i)
Going through options we find that only option (b) satisfies the eq. (i)
$\therefore$ Sides are 30 cm, 10 cm.

8. **Ans.(c)**
Let, breadth = x, Then, length = $2x$.
$(2x - 5)(x + 5) - 2x \times x = 75$ or $x = 20$
$\therefore$ Length = 40 cm.

9. Ans. (b)



Let the width of walk be x metres. Then,

$(20 - 2x)(10 - 2x) = 96$

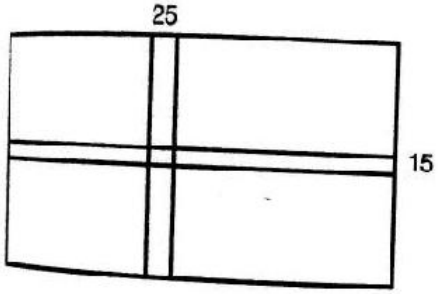
or $4x^2 + 60x - 104 = 0$

or $x^2 + 15x - 26 = 0$

or $(x - 13)(x - 2) = 0$

So, $x = 2$ $[\because x \neq 13]$

10. Ans. (b)



Area of cross roads

$= (25 \times 2 + 15 \times 2 - 2 \times 2)\ m^2 = 76\ m^2$

Area of the field $= (25 \times 15)m^2 = 375\ m^2$

Area under grass $= (375 - 76)m^2 = 299\ m^2$

11. Ans. (a)

Let breadth $= x$ cm and length $= 2x$ cm.

Then, $x^2 + (2x)^2 = \left(10\sqrt{5}\right)^2$

or $5x^2 = 500$ or $x^2 = 100$ or $x = 10$

∴ Perimeter

$= 2(2x + x)\text{cm} = 6x\text{ cm} = 60$ cm

12. Ans. (b)

Area of the hall $= (3600 \times 1500)$ sq. cm

Area of each stone $= (60 \times 50)$ sq. cm.

(Note: dm = decimeter = 10 cm)

∴ Number of stones

$= \left(\frac{3600 \times 1500}{60 \times 50}\right) = 1800$

13. Ans. (b)

Breadth $= \left(\frac{150}{15}\right)\text{cm} = 10\text{cm}$

New area $= \left(\frac{4}{3} \times 150\right)$ sq. cm $= 200$ sq. cm

∴ New length $= \left(\frac{200}{10}\right)\text{cm} = 20$ cm

New Perimeter $= 2(20 + 10)\text{cm} = 60$ cm.

14. Ans. (a)

Area of the room $= (544 \times 374)$ sq. cm

Size of largest square tile = H.C.F. of 544 cm & 374 cm = 34 cm.

Area of 1 tile $= (34 \times 34)$ sq. cm

∴ Number of tiles $= \left(\frac{544 \times 374}{34 \times 34}\right) = 176$

15. Ans. (c)

Perimeter = Distance covered in 8 min.

$= \left(\frac{12000}{60} \times 80\right)\text{m} = 1600\text{ m}$

Let, length $= 3x$ metres and breadth $= 2x$ metres.

Then, $2(3x + 2x) = 1600$ or $x = 160$.

∴ Length = 480 m and breadth = 320 m

∴ Area $= (480 \times 320)m^2 = 153600\ m^2$

16. Ans.(b)

Area $= \left(\frac{1}{2} \times 10000\right)m^2 = 5000\,m^2$

∴ $\frac{1}{2}(\text{diagonal})^2 = 5000$

or $(\text{diagonal})^2 = 5000$

or $(\text{diagonal})^2 = 10000$

∴ Diagonal $= \sqrt{10000} = 100$ m

17. Ans. (b)

Let, length = a & breadth = b.

Then, area = ab.

New length = $\frac{110}{100}a = \frac{11a}{10}$

Let new breadth = c

Then, $\frac{11a}{10} \times c = ab$ or $c = \frac{10b}{11}$

18. Ans. (d)

Percentage increase in the area of rectangle

$= \left(x + y + \frac{xy}{100}\right)\%$

$= \left(40 + 30 + \frac{40 \times 30}{100}\right)\% = 82\%$

19. Ans. (c)

Let, height = $2x$ metres &

(length + breadth) = $5x$ metres.

(∵ semi-perimeter = l + b)

Length of paper = $\left(\frac{260}{2}\right)$m = 130m

∴ Area of paper = $\left(130 \times \frac{50}{100}\right)m^2 = 65\ m^2$

Area of 4 walls = $(65 + 15)m^2 = 80\ m^2$

2(length + breadth) × height = 80

∴ $2 \times 5x \times 2x = 80$

or $x^2 = 4$

or $x = 2$

∴ Height of the room = 4m

20. Ans. (c)

Area = $\frac{1}{2}$ × (Product of diagonals)

$= \frac{1}{2} \times d_1 \times d_2$

∴ $\frac{1}{2} \times 5 \times d_2 = 15$

$\Rightarrow d_2 = \left(15 \times \frac{2}{5}\right) = 6$ cm

21. Ans. (b)

Let one diagonal = x cm.

Another diagonal = $\frac{80}{100}x = \frac{4x}{5}$ cm

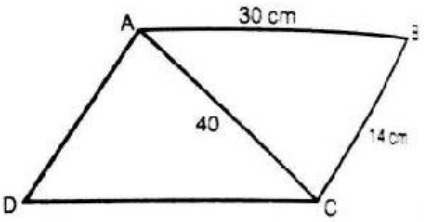
Area of rhombus = $\frac{1}{2}\left(x \times \frac{4x}{5}\right) = \frac{2}{5}x^2$

$= \frac{2}{5}$(Square of longer diagonal)

22. Ans. (b)

Area of square is always greater than or equal to area of rhombus of the same per

23. Ans. (a)



Let ABCD be a parallelogram in wh
AB = 30 m, BC = 14 m and AC = 40 m
Clearly, area of parallelogram ABCD
= 2 (area of Δ ABC).
Let a = 30, b = 14 and c = 40.

Then, $s = \frac{1}{2}(a + b + c) = 42$

∴ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-}$

$= \sqrt{40 \times 12 \times 28 \times 2} = 168\ m^2$

∴ Area of parallelogram
$= (2 \times 168)\ m^2 = 336\ m^2$

24. Ans. (b)

Area of trap. = $\frac{1}{2}$(Sum of parallel sides

distance between them)

$= \left[\frac{1}{2}(1.5 + 2.5) \times 6.5\right]m^2 = 13\ m^2$

25. Ans. (d)
$a = 3,\ b = 4$ and $c = 5$

$$\therefore\ s = \frac{1}{2}(3+4+5) = 6$$

$$\therefore\ \text{Area} = \sqrt{6\times3\times2\times1}$$

$$= \sqrt{36} = 6 \text{ sq. cm}$$

26. Ans. (a)

$$\frac{\sqrt{3}a^2}{4} = 24\sqrt{3}$$

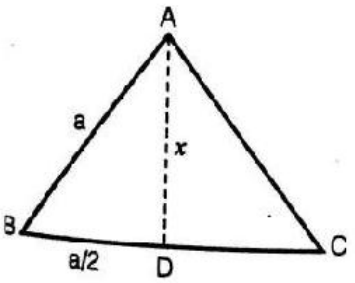
$$\Rightarrow a^2 = 96 \text{ or } a = 4\sqrt{6}$$

$$\therefore\ \text{Perimeter} = \left(3\times4\sqrt{6}\right)\text{cm} = 12\sqrt{6}\text{ cm}$$

27. Ans. (d)
Let the side of the triangle be a.

$$a^2 = \left(\frac{a}{2}\right)^2 x^2 \text{ or } \frac{3a^2}{4} = x^2$$

$$\text{or } a^2 = \frac{4x^2}{3}$$



$$\therefore\ \text{Area} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\times\frac{4}{3}x^2$$

$$= \frac{x^2}{\sqrt{3}} = \frac{x^2\sqrt{3}}{3}$$

28. Ans. (c)

$$\frac{1}{2}\times a\times\text{altitude} = a^2 \Rightarrow \text{altitude} = 2a$$

29. Ans. (b)

$$\text{Area} = \frac{1}{2}\times\text{base}\times\text{height}$$

$$\therefore\ \frac{1}{2}\times a\times\sqrt{6} = \frac{\sqrt{3}}{4}a^2$$

$$\text{or } a = \frac{2\sqrt{6}}{\sqrt{3}} = 2\sqrt{2}$$

$$\therefore\ \text{Area} = \frac{\sqrt{3}}{4}\times\left(2\sqrt{2}\right)^2 = 2\sqrt{3}$$

30. Ans. (d)

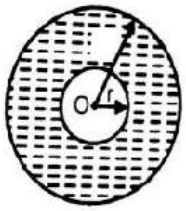
$$2\pi r = 352 \Rightarrow 2\times\frac{22}{7}\times r = 352$$

$$\therefore\ r = \left(352\times\frac{7}{44}\right) = 56 \text{ m}$$

$$\therefore\ \text{Area} = \left(\frac{22}{7}\times56\times56\right)\text{m}^2$$

$$= 9856 \text{ m}^2$$

31. Ans. (a)



Let, inner radius = r & outer radius = R cm.

Then, $2\times\frac{22}{7}\times R = 88$

$$\Rightarrow R = \left(88\times\frac{7}{44}\right) = 14 \text{ cm}$$

$$2\times\frac{22}{7}\times r = 66$$

$$\Rightarrow r = \left(66\times\frac{7}{44}\right) = \frac{21}{2}\text{cm} = 10.5 \text{ cm}$$

$$\therefore\ \text{width of the ring} = (14 - 10.5)\text{cm}$$
$$= 3.5 \text{ cm}$$

32. Ans. (b)
Radius of the wheel = 0.63 m.
Distance moved in 1 revolution = circumference

$$= \left(2\times\frac{22}{7}\times0.63\right)\text{m} = 3.96\text{m}$$

Distance moved in 500 revolutions
$= (500 \times 3.96)\text{m} = 1980$ m.

33. Ans. (b)

Distance moved in 1 sec.

$$= \left(\frac{30}{7} \times \frac{7}{4}\right)\text{m} = \frac{30}{4}\text{m}$$

$$\therefore \text{ Speed} = \left(\frac{30}{4} \times 60 \times 60 \times \frac{1}{1000}\right)\text{km/hr}$$

$= 27$ km/hr

34. Ans. (c)

Distance moved by toothed wheel in 15 revolution $= \left(15 \times 2 \times \frac{22}{7} \times 25\right)\text{cm}$

Distance moved by smaller wheel in 1 revolution

$$= \left(2 \times \frac{22}{7} \times 15\right)\text{cm}$$

$\therefore$ Required number of revolutions

$$= \left(15 \times \frac{44}{7} \times 25 \times \frac{7}{44 \times 15}\right) = 25$$

35. Ans. (c)

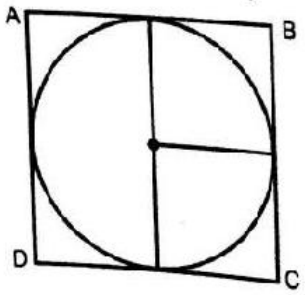
Let original radius = R

$$\text{New Radius} = \frac{200}{100}R = 2R$$

$\therefore$ Original area $= \pi(2R)^2 = 4\pi R^2$

$$\therefore \text{ Increase\%} = \left(\frac{3\pi R^2}{\pi R^2} \times 100\right)\% = 300\%$$

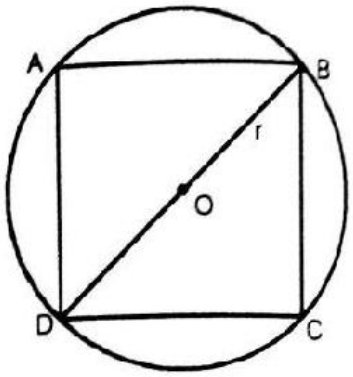
36. Ans. (a)



Radius of the required circle $= 7$ cm

$$\therefore \text{ Its area} = \left(\frac{22}{7} \times 7 \times 7\right)\text{cm}^2 = 154 \text{ cm}^2$$

37. Ans. (a)

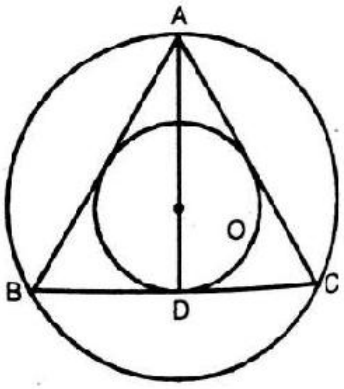


Diagonal of square = Diameter of circle = 2 units

$$\therefore \text{ Area of the square} = \frac{1}{2} \times (\text{diagonal})^2$$

$$= \left(\frac{1}{2} \times 2 \times 2\right) = 2 \text{ sq. units}$$

38. Ans. (c)



Centre O divides altitude in the ratio 2 : 1
Here AO = R, radius of circumcircle
OD = r, radius of incircle
$\therefore$ R = 2r

$$\Rightarrow \text{Required ratio} = \frac{\pi r^2}{\pi(2r)^2} = \frac{1}{4}$$

Length of a wire ring = circumference of circle of radius 42 cm

$$= \left(2 \times \frac{22}{7} \times 42\right) \text{cm} = 264 \text{cm}$$

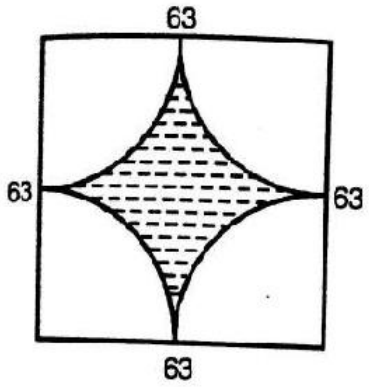
∴ Perimeter of rectangle = 264 cm

Let, length = $6x$ cm & breadth = $5x$ cm.

∴ $2(6x + 5x) = 264$ or $x = 12$

∴ Smaller side = 60 cm.

40. Ans. (d)



$$\text{Required area} = a^2 - 4 \times \frac{1}{4}\pi\left(\frac{a}{2}\right)^2$$

$$= \left(63 \times 63 - 4 \times \frac{1}{4} \times \frac{22}{7} \times \frac{63}{2} \times \frac{63}{2}\right) \text{m}^2$$

$$= 850.5 \text{ m}^2$$

□□□□

Mensuration & Solid Geometry

Solved Examples

1. The perimeter of an equilateral triangle is $72\sqrt{3}$ cm. Find its height.
(a) 63 cm (b) 24 cm
(c) 18 cm (d) 36 cm

Ans. (d)

Given, Perimeter of equilateral triangle = $72\sqrt{3}$ cm

∴ Side of equilateral triangle

$$= \frac{72\sqrt{3}}{3} \text{cm} = 24\sqrt{3} \text{cm}$$

Let the height of the given triangle be h.

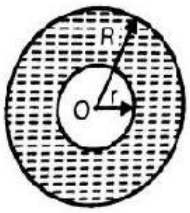
Then, $h = \frac{\sqrt{3}}{2}a$

$$\Rightarrow h = \frac{\sqrt{3}}{2} \times 24\sqrt{3}$$

$$\therefore h = 36$$

2. The inner circumference of a circular track is 440 cm. The track is 14 cm wide. Find the diameter of the outer circle of the track.
(a) 84 cm (b) 168 cm
(c) 336 cm (d) 77 cm

Ans. (b)



Let inner radius & outer radius be r & R then,
Inner circumference of the circular track

$$= 2\pi r = 440 \text{cm}$$

$$\therefore r = \frac{440}{2\pi} = 70 \text{ cm} \qquad \left(\because \pi = \frac{22}{7}\right)$$

∴ Radius of outer circle of track

$$= (70 + 14) \text{cm} = 84 \text{cm}$$

∴ Diameter of outer circle of track

$$= 2 \times 84 = 168 \text{ cm}$$

3. Find the area of a quadrant of a circle whose circumference is 44 cm.

(a) 77 cm^2 (b) 38.5 cm^2
(b) 19.25 cm^2 (d) 19.25π cm^2

Ans. (b)
Circumference of the circle
$= 2\pi r = 44$ cm

$\therefore$ Radius of the circle $= \frac{44}{2\pi}$

$= \frac{44}{2 \times \frac{22}{7}} = 7cm$

$\therefore$ Area of the circle $= \pi(7)^2 = 154\ cm^2$

$\therefore$ Area of a quadrant of circle

$= \frac{154}{4} cm^2 = 38.5 cm^2$

4. Find the length of the longest pole that can be placed in an indoor stadium 24 metre long, 18 metre wide and 16 metre high.
(a) 30 metres (b) 25 metres
(c) 34 metres (d) $\sqrt{580}$ metres

Ans. (c)
The length of the logest pole that can be placed in the stadium is equal to
Diagonal of the indoor stadium

$D = \sqrt{(24)^2 + (18)^2 + (16)^2}$ metre

$D = \sqrt{1156}$ metre $= 34$ metre

5. The length, breadth and height of a room are in the ratio of 3 : 2 : 1. If its volume be 1296 m^3, find its breadth.
(a) 12 metres (b) 18 metres
(c) 16 metres (d) 24 metres

Ans. (a)
Let the length, breadth and height of the room be $3x$, $2x$ & x metre.
Then, Volume of the room
$= (3x)(2x)(x) = 6x^3$

$\therefore$ $6x^3 = 1296$

$\therefore$ $x = \sqrt[3]{\frac{1296}{6}} = \sqrt[3]{216} = 6$ metre

$\therefore$ Breadth of room $= 2x = 12$ metre

6. The volume of a cube is 216 cm^3. Part of the cube is melted to form a cylinder of area 87[illegible] square cm. If length, breadth and height are in the ratio of 4 : 3 : 2, find length.
(a) 342 cm^3 (b) 216 cm^3
(c) 36 cm^3 (d) Data inadequate

Ans. (d)
Volume of the cylinder
= Volume of the part of the cube melted
But what part of cube is melted is not given
Hence, volume of the cylinder can not be determined.
Hence data is inadequate.

7. Three metal cubes with edges 6 cm, 8 cm and 10 cm respectively are melted together and formed into a single cube. Find the side of the resulting cube.
(a) 11 cm (b) 12 cm
(c) 13 cm (d) 24 cm

Ans. (b)
Volume of the resulting cube
= Sum of the volume of the three cubes
$= (6)^3 + (8)^3 + (10)^3$
$= 1728\ cm^3$
$\therefore$ Side of the resulting cube
$= \sqrt[3]{1728}$ cm $= 12$ cm

8. The diameters of two cones are equal. If their slant height be in the ratio 5 : 7, find the ratio of their curved surface areas.
(a) 25 : 7 (b) 25 : 49
(c) 5 : 49 (d) 5 : 7

Ans. (d)
Let the radii and slant heights of the two cones be r_1, l_1 & r_2, l_2 respectively.

Given, $r_1 = r_2$

Ratio of the curved surface areas

$$\frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{l_1}{l_2} = \frac{5}{7}$$

The ratio of radii of a cylinder to that of a cone is 1 : 2. If their heights are equal, find the ratio of their volumes?

(a) 1 : 3 (b) 2 : 3

(c) 3 : 4 (d) 3 : 2

Ans. (c)

Let the radius of cylinder = r

Then, radius of cone = 2r

Heights of cylinder = Height of cone = h

Hence, $\frac{\text{Volume of cylinder}}{\text{Volume of cone}}$

$$= \frac{\pi r^2 h}{\frac{1}{3}\pi (2r)^2 h} = \frac{3}{4}$$

A bicycle wheel makes 5000 revolutions in moving 11 km. What is the radius of the wheel?

(a) 70 cm (b) 135 cm

(c) 17.5 cm (d) 35 cm

Ans. (d)

Distance covered in one revolution

$$= \frac{11000}{5000} = 2.2 \text{ m} = 220 \text{ cm}$$

$2\pi r = 220$ cm (given)

So, $r = \frac{220 \times 7}{2 \times 22} = 35$ cm

The surface areas of two spheres are in the ratio of 1 : 4. Find the ratio of their volumes.

(a) 1 : 3 (b) 1 : 8

(c) 1 : 4 (d) 1 : 2

Ans. (b)

Let the radii of the two spheres be r_1 & r_2

Ratio of surface areas of the spheres

$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{1}{4}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{4} \Rightarrow \frac{r_1}{r_2} = \frac{1}{2}$$

$$= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

12. The radii of two spheres are in the ratio of 1 : 2. Find the ratio of their surface areas.

(a) 1 : 3 (b) 2 : 3

(c) 1 : 4 (d) 3 : 4

Ans. (c)

Let the radii of the two spheres be r_1 & r_2.

Then, ratio of the surface areas of the spheres

$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

13. A sphere of radius r has the same volumes as that of a cone with a circular base of radius r. Find the height of cone.

(a) 2r (b) r/3

(c) 4r (d) (2/3)r

Ans.(c)

Let the height of the cone be h.

Volume of the sphere = volume of the cone

$$\therefore \Rightarrow \frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$$

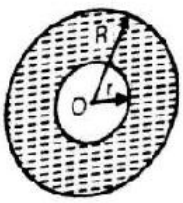
$$\therefore \quad h = 4r$$

14. A road that is 7 m wide surrounds a circular path whose circumference is 352 m. What will be the area of the road?

(a) 2618 m^2 (b) 654.5 m^2

(c) 1309 m^2 (d) 5236 m^2

Ans. (a)



Radius of the circular path,

$$r = \frac{352}{2\pi} = \frac{352}{2\times\frac{22}{7}} = 56 \text{ metre}$$

$\therefore$ outer radius of the Road, R
$= (56 + 7)\text{m} = 63$ m
Area of the Road

$$= \pi(63)^2 - \pi(56)^2 = \pi\left[(63)^2 - (56)^2\right]$$

$= 2618\ \text{m}^2$

15. How many metres of cloth 5 m wide will be required to make a conical tent, the radius of whose base is 7 m and height is 24 m?
(a) 55 m (b) 330 m
(c) 220 cm (d) 110 m

Ans. (d)
Slant Height of the cone l

$$= \sqrt{(7)^2 + (24)^2}\ \text{m} = 25\ \text{m}$$

Hence, Surface area of the tent

$$= \pi l = \pi(7)(25) = 550\text{m}^2$$

Let the length of cloth required be x m.
Then, $x \times 5 = 550 \Rightarrow x = 110$m

16. A cuboid of dimension 24 cm ×9 cm ×8 cm is melted and smaller cubes are of side 3 cm is formed. Find how many such cubes can be formed?
(a) 27 (b) 64
(c) 54 (d) 32

Ans. (b)
Volume of cuboid $= 24 \times 9 \times 8\ \text{cm}^3$
$= 24 \times 9 \times 8\ \text{cm}^3 = 1728\ \text{cm}^3$
Volume of cubes $= (3)^3\ \text{cm}^3 = 27\ \text{cm}^3$

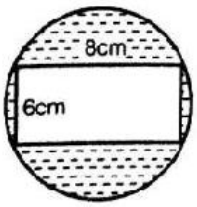
$\therefore$ No. of cubes formed $= \frac{1728}{27} = 64$

17. Three cubes each of volume of 216 m^3 are joined end to end. Find the surface area of the resulting figure.
(a) 504 m^2 (b) 216 m^2
(c) 432 m^2 (d) None of these

Ans. (a)
Side of each cube $= \sqrt[3]{216}\text{m} = 6\text{m}$
Length of the resulting figure after joining
$= 6 + 6 + 6 = 18$ m
The width & height of the resulting figure will be 6m each.
$\therefore$ Surface area of the resulting figure

$$= 2(18\times6 + 6\times6 + 18\times6) = 504\text{m}^2$$

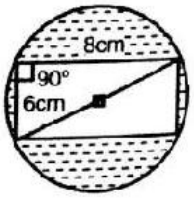
18. In the adjacant figure, find the area of the shaded region. (Use $\pi = 22/7$)



(a) 15.28 cm^2 (b) 61.14 cm^2
(c) 30.57 cm^2 (d) 40.76 cm^2

Ans. (c)
Diameter of the circle

$$= \sqrt{(8)^2 + (6)^2} = 10\ \text{cm}$$



Radius of the circle = 5 cm.

Area of circle $= \pi(5)^2\ \text{cm}^2 = 25\pi\ \text{cm}^2$

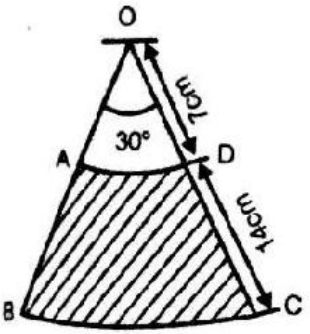
$$= 25\times\frac{22}{7}\text{cm}^2 = 78.57\text{cm}^2$$

Area of the Rectangle $= 8 \times 6\ \text{cm}^2 = 48\ \text{cm}^2$
$\therefore$ Area of the shaded region

$$= (78.57 - 48)\text{cm}^2 = 30.57\text{cm}^2$$

19. The diagram represents the area swept by the wiper of a car. With the dimensions given in the figure, calculate the shaded area swept by the wiper.

(a) 102.67 cm^2
(b) 205.34 cm^2
(c) 51.33 cm^2
(d) Cannot be determined



Ans. (a)

Area of the section OBCO

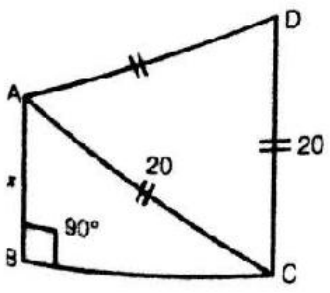
$$= \pi(21)^2 \times \left(\frac{30}{360}\right) = 115.5 cm^2$$

Area of section OADO

$$= \pi(7)^2 \left(\frac{30}{360}\right) = 12.83 cm^2$$

∴ Area swept by the wiper
= Area of the section OBCO
− Area of the section OADO
= (115.5 − 12.83) cm^2
= 102.67 cm^2

20. Find the area of quadrilateral ABCD. (Given, $\sqrt{3} = 1.73$)



(a) 452 sq. units
(b) 269 sq. units
(c) 134.5 sq. units
(d) Cannot be determined

Ans. (b)

Area of triangle ABC

$$= \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times 12 \times \sqrt{(20)^2 - (12)^2}$$

$$= \frac{1}{2} \times 12 \times 16 = 96 \text{ sq. unit}$$

Area of triangle ACD

$$= \frac{\sqrt{3}}{4}(20)^2 = 173 \text{ sq. unit}$$

∴ Area of quadrilateral ABCD
= Area of ABC + Area of ACD
= (173 + 96) sq. units = 269 sq. units

□□□□

Mensuration & Solid Geometry

I Practice Exercise: I

1. A tank 16 m long and 12 m wide contains water. How many cubic metres of water must be rushed into it to make the surface rise by $16\frac{2}{3}$ cm?

(a) 48 m^3 (b) 40 m^3
(c) 32 m^3 (d) 42 m^3

2. The area of three adjacent faces of a rectangular box are p, q and r square cm. The volume of the box is given by

(c) $(pqr)^{1/3}$ cm^3 (d) pqr cm^3

3. How many bricks are required to build a wall 15 m long, 3 m high and 50 cm thick, if each brick measures 25 cm by 12 cm by 6 cm.
(a) 16500 (b) 14500
(c) 12500 (d) 10500

4. The diagonal of a cubical box is $\sqrt{300}$ cm. Find the surface area.
(a) $600\sqrt{3}$ cm^2 (b) 600 cm^2
(b) 1200 cm^2 (d) $900\sqrt{3}$ cm^2

5. The length of a room is 12 meters, width 8 meters and height 6 metres. How many boxes will it hold if each is allowed 1.5 cubic metre of space?
(a) 864 (b) 506
(c) 384 (d) 436

6. If 210 cu m of sand be thrown into a tank 12 m long and 5 m wide, find how much the water will rise?
(a) 3.5 m (b) 4 m
(c) 7 m (d) Data inadequate

7. A bar of volume 1 cu m of metal weighing 90 kg is rolled into a square bar 9 metre long. An exact cube is cut off from the bar. How much does it weigh?
(a) $5\frac{2}{3}$ kg (b) $6\frac{1}{3}$ kg
(c) $3\frac{1}{3}$ kg (d) $4\frac{2}{3}$ kg

8. A cube of edge 6 cm is melted and smaller cubes of edge 2 cm each are formed; how many such cubes are possible?
(a) 39 (b) 24
(c) 27 (d) 21

9. A rectangular tank is 30 m long 20 m broad. Water is being flown into it through a square pipe of side 5 cm. What is the speed of water if the level of water in the tank rises by 1 m in 8 hours?
(c) 40 km/h

10. Three cubes of metal whose edges are in ratio 3 : 4 : 5 are melted into a single cube the length of whose diagonal is $48\sqrt{3}$ m. Calculate the edges of the three cubes.
(a) 24m, 32m, 40m (b) 40m, 32m, 24m
(c) 30m, 22m, 18m (d) 48m, 36m, 24m

11. Two cubes have volumes in the ratio 1 : 27. The ratio of the area of the face of one to that of the other is
(a) 1 : 2 (b) 1 : 3
(c) 1 : 6 (d) 1 : 9

12. A cube of edge 3 cm of iron weighs 12 gm. What is the weight of a similar cube of iron whose edge is 12 cm?
(a) 768 gm (b) 678 gm
(c) 964 gm (d) 864 gm

13. A 4 cm cube is cut into 1 cm cubes. What is the ratio of surface area of small cube to that of the large cube?
(a) 1 : 16 (b) 2 : 3
(c) 4 : 1 (d) 6 : 1

14. A cubical metallic tank whose each edge measures 30 cm, is completely filled with water. If 2.7 litres water is taken out of it, what will be the depth of remaining water in tank?
(a) 37 cm (b) 27 cm
(c) 17 cm (d) None of these

15. 1496 cm^3 of a metal is used to cast a pipe of length 28 cm. If the internal radius of the pipe is 8 cm, the outer radius of the pipe is
(a) 7 cm (b) 9 cm
(c) 10 cm (d) 12 cm

16. A sphere in melted to form a cylinder whose height is 4.5 times its radius, what is the ratio of radii of sphere to the cylinder?

(a) 3 : 2
(b) 3 : 5

17. A brick measure 20 cm × 10 cm × $7\frac{1}{2}$ cm. How many bricks will be required for a wall 25 m long, 2 m high and $\frac{3}{4}$ m thick?
(a) 25000 (b) 35000
(c) 20000 (d) 45000

18. The radius of the cylinder is made twice as large. How should the height be changed so that the volume remains the same?
(a) $\frac{1}{2}$ x height of original cylinder
(b) $\frac{1}{4}$ x height of original cylinder
(c) $\frac{1}{4}\pi r^2$
(d) None of these

19. A roller is 120cm long and has diameter 84 cm. If it takes 500 complete revolutions to level a playground, determine the cost of levelling at the rate of 30 paise per sq. m. $\left(\text{Use } \pi = \frac{22}{7}\right)$
(a) Rs. 475.40 (b) Rs. 375.45
(c) Rs. 375.20 (d) Rs. 475.20

20. Sum of the length, width and depth of a cuboid is s and its diagonal is d. Its surface area is
(a) s^2 (b) d^2
(c) $s^2 - d^2$ (d) $s^2 + d^2$

21. A hemisphere of lead and of radius 6 cm, is cast into a right circular cone of height 75 cm. The radius of the base of the cone is
(a) 1.4 cm (b) 2.4 cm
(c) 1.6 cm (d) 3.2 cm

22. From a solid right circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and base is removed. The volume (in cm^3) of the remaining solid is
(a) 377 (b) 754.3
(b) 1131 (d) None of these

23. A rectangular piece of paper is 22 cm long and 10 cm wide. A cylinder is formed by rolling the paper along its length. The volume of the cylinder is
(a) 225π cu cm (b) 385 cu cm
(c) 25π cu cm (d) None of these

24. If the radius of a cylinder is doubled and the height is halved, what is the ratio between the new curved surface area and the previous curved surface area of the cylinder
(a) 1 : 1 (b) 2 : 1
(c) 3 : 2 (d) 2 : 3

25. The material of solid cone is converted into the shape of solid cylinder of equal radius. If the height of the cylinder is 5 cm, what is the height of the cone?
(a) 25 cm (b) 15 cm
(c) 20 cm (d) 10 cm

26. A spherical ball of lead, 3 cm is diameter is melted and re-cast into three spherical balls. The diameter of two of these are 1.5 cm and 2 cm, respectively. The diameter of the third ball is
(a) 2.66 cm (b) 2.5 cm
(c) 3 cm (d) 3.5 cm

27. The radii of a cylinder and a cone are equal. If the height of the cylinder is equal to the slant height of the cone then the ratio of the curved surfaces of cylinder and the cone is
(a) 1 : 1 (b) 2 : 1
(c) 3 : 1 (d) 4 : 1

28. Two cm of rain has fallen on a sq km of land. Assuming that 50% of the raindrops could

have been collected and contained in a pool having a 100 m x 10 m base, by what level would the water level in the pool have increased?

(a) 15 m (b) 20 m
(c) 10 m (d) 25 m

29. A solid consists of a circular cylinder with an exact fitting right circular cone placed on the top. The height of the cone is h. If the total volume of the solid is the three times the volume of the cone, then the height of the cylinder is

(a) 2h (b) 4h
(c) $\frac{2h}{3}$ (b) $\frac{3h}{2}$

30. The curved surface of a circular cylinder of height 'h' and the slant surface of the cone of slant height '2h' having the same circular base are in the ratio of

(a) 1 : 1 (b) 1 : 2
(c) 3 : 2 (d) 1 : 3

31. Find the length of the canvas 2 m in width required to make conical tent 12 m in diameter and 6.3 m in slant height.

(a) 118.8 m (b) 62.4 m
(c) 59.4 m (d) 112.4 m

32. If the base radius of a cone is increased by 20% and its slant height is made double, then by how much percent will the area of its curved surface be increased?

(a) 140% (b) 160%
(c) 130% (d) 180%

33. A rectangular sheet of area 264 cm^2 and width 11 cm is rolled along its breadth to make a hollow cylinder. The volume of the cylinder is

(a) 231 c.c. (b) 230 c.c.
(c) 235 c.c. (d) 234 c.c.

34. If the radius of a sphere is doubled then its volume is increased by

(a) 100% (b) 200%
(c) 700% (d) 800%

35. The height of a cylinder is decreased by 8% keeping its radius unchanged. What is the percentage change in its volume?

(a) 8% increase (b) 12% decrease
(c) 8% decrease (d) None of these

Solutions

1. **Ans. (c)**

Volume of water $= 16 \times 12 \times \frac{16\frac{2}{3}}{100} m^3$

$= 16 \times 12 \times \frac{50}{3 \times 100} m^3$

$= 32\ m^3$

2. **Ans. (b)**

Let the sides of the box be x, y and z
Then $p = xy$, $q = yz$, $r = zx$
Volume of the box $= xyz$
We have,

$p \times q \times r = xy \times yz \times zx = x^2y^2z^2$

or $xyz = \sqrt{pqr}$.

3. **Ans. (c)**

Number of bricks

$= \frac{\text{Volume of the wall}}{\text{Volume of the brick}}$

$= \frac{1500\ cm \times 300\ cm \times 50\ cm}{25\ cm \times 12\ cm \times 6\ cm}$

$= 12500$ cm.

4. **Ans. (b)**

Diagonal $d = \sqrt{3}a = \sqrt{300}$

$\therefore\ a = 10$ cm

Surface area $= 6a^2 = 600\ cm^2$

5. **Ans. (c)**

Capacity of room $= 12 \times 8 \times 6$ cu m
Required number of boxes

$= \frac{12 \times 8 \times 6}{1.5} = 384.$

6. Ans. (a)
Let the initial height of the tank be h and the height after sand is thrown be H metres.
We have to find H - h.
According to question,
$12 \times 5 \times (H - h) = 210$

$\therefore H - h = \frac{210}{60} = \frac{7}{2} = 3.5$ metres.

7. Ans. (c)
(Area of the suqare end) $\times 9$ = Volume.
$= 1\ m^3$

$\therefore$ Side of the suqare end $= \sqrt{\frac{1}{9}} m = \frac{1}{3} m$

$\therefore$ Volume of this cube $= \left(\frac{1}{3}\right)^3 m^3 = \frac{1}{27} m^3$

$\therefore$ Weight of this cube $= \frac{1}{27} \times 90\ kg = 3\frac{1}{3} kg.$
($\because$ 1 cu m = 90 kg in weight)

8. Ans. (c)
Number of cubes

$$= \left(\frac{\text{Original length of edge}}{\text{New length of edge}}\right)^3$$

$\therefore$ Number of cubes $= \left(\frac{6}{2}\right)^3 = 27.$

9. Ans. (a)
Volume of water collected in the tank in 8 hours
$= 30\ m \times 20\ m \times 1\ m = 600$ cu m.
$\therefore$ Volume of water collected in the tank in

1 hour $= \frac{600}{8} = 75$ cu m.

Water comes through a pipe of cross-section

$= 5\ cm \times 5\ cm = \frac{25}{10000}$ sq. m.

The speed of water = Distance travelled by the water in the pipe in one hour

$= \frac{75 \times 10000}{25} m = 30$ km/hr.

10. Ans. (a)
Since the edges of the cubes are in the ratio 3 : 4 : 5, let these be 3k, 4k, 5k metres, respectively.
Their volumes are $27\ k^3$, $64\ k^3$, $125\ k^3$, cu m.
Thus, the volume of the single cube
$= (27+64+125)\ k^3$ cu m.
$= 216\ k^3 = (6k)^3$ cu m
We know that the length of the diagonal of a cube of side a is $\sqrt{3}a$. Therefore, the length of the diagonal of the single cube mentioned above is equal to $6k\sqrt{3}$. But the length of the diagonal of this cube is given to be $48\sqrt{3}$, hence

$6k\sqrt{3} = 48\sqrt{3}$ or $k = 8$.
Therefore, the length of the edges of the three cubes are 3×8, 4×8, 5×8 m, that is 24 m, 32 m, 40 m.

11. Ans. (d)

$a_1^3 : a_2^3 = 1 : 27$

$\Rightarrow a_1 : a_2 = 1 : 3$

$\therefore$ required ratio is $1^2 : 3^2 = 1 : 9.$

12. Ans. (a)
Ratio of the edge of cubes
$= 3 : 12 = 1 : 4$
Ratio of their volumes $= 1^3 : 4^3 = 1 : 64.$
Because volume of the new cube is 64 times the volume of the first cube, the weight of the new cube is also 64 times the weight of the first cube.
Weight of the new cube
$= 64 \times 12$ gm $= 768$ gm

13. Ans. (a)

Number of cubes $= \frac{4 \times 4 \times 4}{1 \times 1 \times 1} = 64$

Ratio of surface area $= 6 \times 1^2 : 6 \times 4^2$
$= 1 : 16$

14. Ans. (b)

Volume of the cubical metallic tank

$= l \times b \times h$

$= 30 \times 30 \times 30 = 27000\ cm^3$

$\therefore$ Volume of water in the tank

$$= \frac{27000}{1000} = 27 \text{ litre} \quad [\because 1 \text{ litre} = 1000\ cm^3]$$

$\therefore$ Volume of remaining water = 24.3 litre

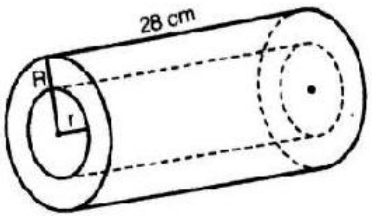
$= 243000\ cm^3$

Now, $l \times b \times h = 243000$

$\Rightarrow 30 \times 30 \times h = 243000$

$\Rightarrow h = 27$ cm.

15. Ans. (b)



Let the outer radius be R cm

Then, we have

$$1496 = \pi \times (28) \times (R^2 - 8^2)$$

$$\Rightarrow R^2 - 8^2 = \frac{1496 \times 7}{22 \times 28} = 17$$

or $R^2 = 17 + 64 = 81$

$\therefore R = 9$ cm

16. Ans. (a)

Let the radius of the sphere and cylinder be 'R' and 'r', respectively

Volume of the cylinder

$$= \pi r^2 h = \pi r^2 \left(\frac{9}{2}r\right) \quad \left(\therefore h = \frac{9}{2}r\right) = \frac{9}{2}\pi r^3$$

Volume of the sphere $= \frac{4}{3}\pi R^3$

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{9}{2}\pi r^3$$

$$\Rightarrow \frac{R^3}{r^3} = \frac{27}{8}$$

$$\Rightarrow \frac{R}{r} = \frac{3}{2}$$

17. Ans. (a)

Required number of bricks

$$= \frac{2500 \times 200 \times 75}{20 \times 10 \times \frac{15}{2}} = 25000$$

18. Ans. (b)

Volume $= \pi r^2 h$

Let $r \to 2r$, $h \to H$ then,

Volume $\pi(2r)^2 \times H = \pi r^2 h$

So $4H = h$

$$H = \frac{h}{4}$$

19. Ans. (d)

$$r = \frac{84}{2} \text{ cm} = \frac{21}{50} \text{m},$$

$$h = 120 \text{ cm} = \frac{120}{100} \text{m} = \frac{6}{5} \text{m}$$

The levelled area in one revolution of the roller = curved surface $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{21}{50} \times \frac{6}{5} = \frac{396}{125} \text{sq. m.}$$

The levelled area in 500 revolutions

$$= \frac{396}{125} \times 500 = 1584 \text{ sq. m.}$$

$\therefore$ Cost $= 0.30 \times 1584 =$ Rs. 475.2

20. Ans. (c)

$l + b + h = s$ and $\sqrt{l^2 + b^2 + h^2} = d$

So, $l^2 + b^2 + h^2 = d^2$

$\therefore (l+b+h)^2 = s^2$

$\Rightarrow l^2+b^2+h^2+2(lb+bh+hl) = s^2$

$\Rightarrow d^2+2(lb+bh+hl) = s^2$

$\Rightarrow 2(lb+bh+hl) = s^2-d^2$

$\therefore$ Surface area $= s^2-d^2$

21. Ans. (b)

Let r cm be the radius of base of cone

$$\frac{2}{3}\pi(6)^3 = \frac{1}{3}\pi r^2 \times 75$$

$$\therefore r^2 = \frac{2\times216}{75} = \frac{2\times72}{25} = \frac{144}{25}$$

$$\therefore r = \frac{12}{5} = 2.4 \text{ cm.}$$

22. Ans. (b)

Required volume

= Volume of cylinder – Volume of cone

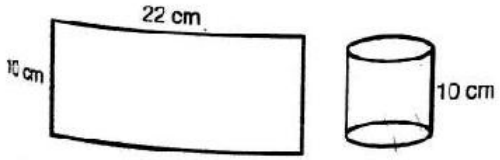
$$= \pi\times6^2\times10 - \frac{1}{3}\times\pi\times6^2\times10$$

$$= \frac{2}{3}\times\pi\times36\times10$$

$$= \frac{2}{3}\times\frac{22}{7}\times360 = \frac{5280}{7}$$

= 754.3 cu cm.

23. Ans. (b)



Rolled along with its length h=10 cm, then circumference $2\pi r = 22$

$$\therefore r = \frac{7}{2}\text{cm}$$

$$\therefore V = \pi r^2 h = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times10 = 385 \text{ cm}^3$$

24. Ans. (a)

Let the initial radius and height of the cylinder be r cm and h cm, respectively.

Then, curved surface area of the original cylinder $= 2\pi rh$ and curved surface area of the new cylinder

$$= 2\pi(2r)\times\frac{h}{2} = 2\pi rh$$

$\therefore$ Required ratio

$$= \frac{\text{New curved surface area}}{\text{Previous curved surface area}}$$

$$= \frac{2\pi rh}{2\pi rh} = 1:1.$$

25. Ans. (b)

Volume of the cylinder = 3 times volume of the cone. This is valid if base and height is the same. Radius is the same, so the height of cone is 3 times the height of the cylinder

$\therefore$ Height of the cone = 3 × 5 cm = 15 cm.

26. Ans. (b)

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\times\left\{\left[\left(\frac{3}{2}\right)^3 - \left[\left(\frac{3}{4}\right)^3 + 1^3\right]\right\}\right.$$

$$\therefore r^3 = \frac{125}{64} = \left(\frac{5}{4}\right)^3$$

$$\therefore r = \frac{5}{4}$$

$$\therefore \text{Diameter} = 2r = 2\times\frac{5}{4} = \frac{5}{2} = 2.5 \text{ cm.}$$

27. Ans. (b)

Let the height and radius of the cylinder be h and r, respectively.

Curved surface of the cylinder $= 2\pi rh$

and, curved surface of the cone

$= \pi rl = \pi rh$ $\quad (h = l)$

$\therefore$ Required ratio = 2 : 1