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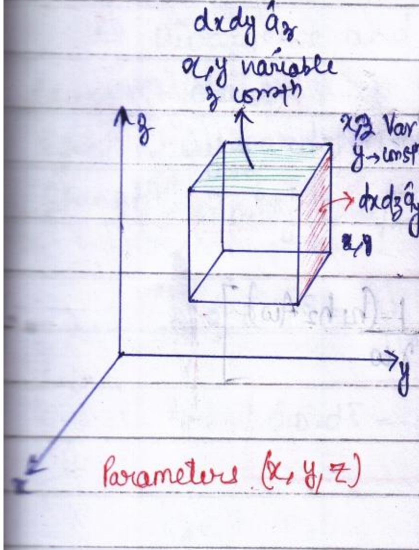
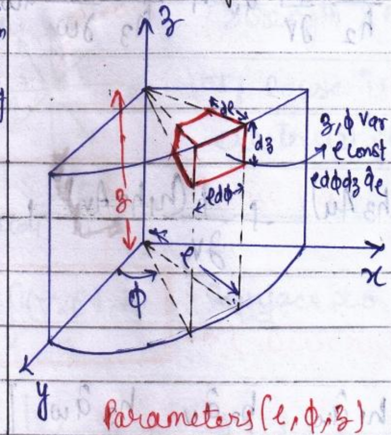
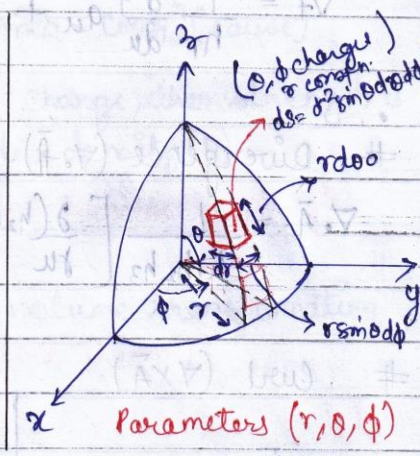
Coordinate Systems







∇ operator / Directional Derivative / vector Diffⁿ operator

∇ → operation on scalar → ∇f → Gradient of scalar [Results in vector Quantity]

∇ → operation on Vector → $(\nabla\cdot\bar{A})$ → Divergence [scalar Quantity]

∇ → operation on Vector → $(\nabla\times\bar{A})$ → Curl [vector Quantity]

$$\boxed{\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z}$$ In Cartesian Cordinate

Line Volume Surface Integral

Co-ordinate S/m	Parameter			Scaling factor		
	u	v	w	h_1	h_2	h_3
Cartesian S/m	x	y	z	1	1	1
Cylindrical S/m	ρ	φ	z	1	ρ	1
Spherical S/m	r	θ	φ	1	r	rsinθ

Line Integral (vector)

$$d\bar{l} = h_1 du\,\hat{a}_u + h_2 dv\,\hat{a}_v + h_3 dw\,\hat{a}_w$$

Surface Integral (vector)

$$d\bar{s} = h_2 h_3\, dv\, dw\, \hat{a}_u + h_3 h_1\, du\, dw\, \hat{a}_v + h_1 h_2\, du\, dv\, \hat{a}_w$$

Volume Integral (scalar)

$$d\bar{s} = h_1 h_2 h_3\, du\, dv\, dw$$

Gradient (∇f)

$$\nabla f = \frac{1}{h_1}\frac{\partial f}{\partial u}\hat{a}_u + \frac{1}{h_2}\frac{\partial f}{\partial v}\hat{a}_v + \frac{1}{h_3}\frac{\partial f}{\partial w}\hat{a}_w$$

Divergence $(\nabla \cdot \bar{A})$

$$\nabla \cdot \bar{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial (h_2 h_3 A_u)}{\partial u} + \frac{\partial (h_1 h_3 A_v)}{\partial v} + \frac{\partial (h_1 h_2 A_w)}{\partial w}\right]$$

Curl $(\nabla \times \bar{A})$

$$\nabla \times \bar{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1 \hat{a}_u & h_2 \hat{a}_v & h_3 \hat{a}_w \\ \partial/\partial u & \partial/\partial v & \partial/\partial w \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

Laplacian Operator $(\nabla^2 f)$

$$\nabla^2 f = \nabla \cdot (\nabla f) = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial f}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_1 h_3}{h_2}\frac{\partial f}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial f}{\partial w}\right)\right]$$

Note :-

Curl of Gradient $[\nabla \times (\nabla f)] = 0$
Divergence of Curl $[\nabla \cdot (\nabla \times A)] = 0$
$E = -\nabla V$ where $V \rightarrow$ potential and $E \rightarrow$ electric field
A field is said to be irrotational / vortex / conservative field if $\boxed{\nabla \times \bar{A} = 0} \rightarrow$ Irrotational field

eg :- $\bar{e}$ field is a conservative field.

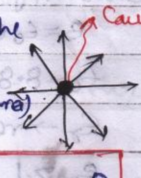
A field is said to be Solenoidal / Divergence less if $\boxed{\nabla \cdot \bar{A} = 0} \rightarrow$ Solenoidal field

eg $\rightarrow$ Mag^n filed is always solenoidal

Divergence and Divergence theorm :-

Cause :- $\boxed{\text{Charge}}$

Effect :- Outward from the Cause



Stength :- $\boxed{\text{Density}}$ (Cause/Area)

Strength X Area = Constⁿ (Cause)

If cause is charge, then strength is electric flux density (D).

$$\boxed{\oint \bar{D}\cdot d\bar{s} = \psi_{total} = Q} \rightarrow \text{Gauss Law OR Maxwell's } I^{st} eq^{n}.$$

$$\boxed{\oint_s \bar{D}\cdot d\bar{s} = \int_v (\nabla\cdot\bar{D})\, dv}$$

Surface to volume transformation → (Divergence theorm)

Positive Divergence   -ve divergence   zero divergence

Curl and Stock's theorm

Cause:- $\boxed{\text{Current}}$

Effect :- ~~Cir~~ Circulating arround the cause

Strength:- $\boxed{\text{Intensity}}$ (cause/length)

If current (I) is flowing in wire, then effect arround it called Magnetic field Intensity

$$\boxed{\oint_l \bar{H}\cdot d\bar{l} = I_{total}} \rightarrow \text{Ampere's law OR Maxwell's } IV^{th} eq^{n}.$$

$$\boxed{\oint_l \bar{H}\cdot d\bar{l} = \oint_s (\nabla\times\bar{H})\cdot ds}$$

line to surface transformation (Stock's theorm)

EM Fields :-
- Static fields
  - Electric field $E, D \neq f(t)$
  - Magnetic field $B, H \neq f(t)$
- Time Varing fields
  - Electric field $E, D = f(t)$
  - Magnetic field $B, H = f(t)$

static electric field

$D = \epsilon E$

$D \rightarrow$ Electric flux Density (C/m^2)

$E \rightarrow$ Electric field intensity (Volt/m)

$\epsilon \rightarrow$ Permitivity of the medium (F/m)

$\epsilon = \epsilon_0 \epsilon_r$

$\epsilon_0 = 8.85 \times 10^{-12}$ OR $\frac{1}{36\pi} \times 10^{-9}$ F/m

$\epsilon_r \rightarrow$ relative permitivity (unitless)

$\epsilon_r \geq 1$ (for air $\epsilon_r = 1$)

Gauss Law :-

the net flow of electric flux leaving from any closed surface is equal to the charge enclosed by that surface.

$$\boxed{\psi_{total} = Q}$$

$$\boxed{\oint \bar{D}.d\bar{s} = \psi_{total} = Q}$$

Note :- $\because$ $Q = \int_V \rho_v dv$

$$\therefore \oint \bar{D}.d\bar{s} = Q = \underbrace{\int_V (\nabla.D) dv}_{\text{divergence theorm.}} = \int_V \rho_v dv \quad \therefore \boxed{\nabla.\bar{D} = \rho_v}$$

Gauss law in point form.

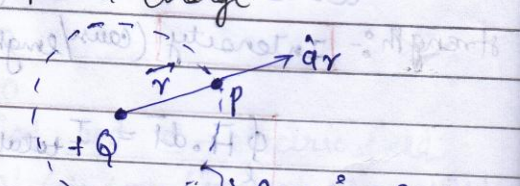
Maxwell's I st eqn.

Field Intensity (E) (Volt/m) Due to point charge

$\oint \bar{D}.d\bar{s} = Q$

$D_r 4\pi r^2 = Q \Rightarrow \boxed{D_r = \frac{Q}{4\pi r^2}}$

(Independent of Medm)

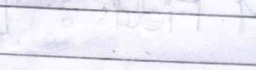


$\because D = \epsilon E$

$$\boxed{E_r = \frac{Q}{4\pi\epsilon r^2}} \text{ OR } \boxed{\vec{E} = \frac{Q}{4\pi\epsilon r^2}\hat{a}_r}$$ ← depends on medium

Electric potential and Potential Gradient

Potential diffn is the work done in moving a charge from one pt to another pt in an electric field.

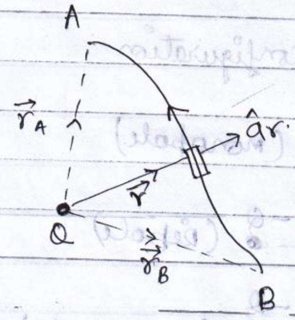


$$\boxed{V_{AB} = (V_A - V_B) = \frac{W}{Q} = -\int_B^A \bar{E}.d\bar{l}}$$ Joule/Coulamb OR Volts.

potential diffn

work done [work is done only when charge is moving against the field.]

Potential Due to Point charge



$E = \frac{Q}{4\pi r^2}\hat{a}_r$ (radially outward)

$V_{AB} = -\int_{r_B}^{r_A} \bar{E}\cdot d\bar{l}$ Here $dl = dr\,\hat{a}_r$

$V_{AB} = -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon r^2}\,dr$

$$\boxed{V_{AB} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]}$$ Volts

$$V_{AB} = \underbrace{\frac{Q}{4\pi\epsilon r_A}}_{V_A} - \underbrace{\frac{Q}{4\pi\epsilon r_B}}_{V_B}$$

V

If $r_A \to r$ and $r_B \to \infty$ $(V_B = 0)$

$$\boxed{V_{AB} = V_A = \frac{Q}{4\pi\epsilon r}}$$ → Absolute Potential

Note Potⁿ diffⁿ (V_{AB}) is Path independent $V_{AB} = -V_{BA}$.

$\therefore -\int_B^A \bar{E}\cdot d\bar{l} = -\left(-\int_A^B \bar{E}\cdot d\bar{l}\right)$

$\int_A^B \bar{E}\cdot d\bar{l} + \int_B^A \bar{E}\cdot d\bar{l} = 0$

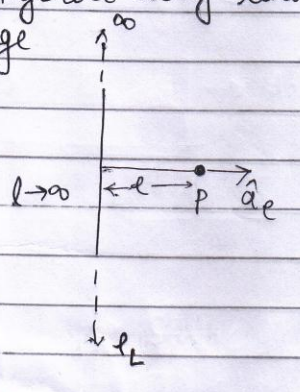
$$\boxed{\oint \bar{E}\cdot d\bar{l} = 0}$$ Maxwell's 2nd Eqⁿ.

$\because \oint_l \bar{E}\cdot d\bar{l} = \int(\nabla\times\bar{E})\cdot d\bar{s} = 0$

$\therefore \boxed{\nabla\times E = 0}$ Maxwell's 2nd eqⁿ in pt form

$\therefore$ $\bar{e}$ field is conservative field.

Potential due to infinite long line charge



at P, $\bar{e}$ field is given by

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon\rho}\hat{a}_\rho}$$ V/m

$\rho_L \to$ line charge.

$$\boxed{V_{AB} = \frac{\rho_L}{2\pi\epsilon}\ln\left(\frac{\rho_B}{\rho_A}\right)}$$

Equipotential Surface

Surface at which potential diffⁿ b/w two points is zero.

$$\boxed{V_{AB} = 0}.$$

$\therefore$ Work done is zero in moving a charge around an equipotⁿ surface.

(i) $\because V_{AB} = 0 \Rightarrow -\int_B^A \bar{E}\cdot dl = 0$

$\therefore$ flux lines are always normal to the equipotential surface.

(ii) For perfect conductor ($\sigma = \infty$)

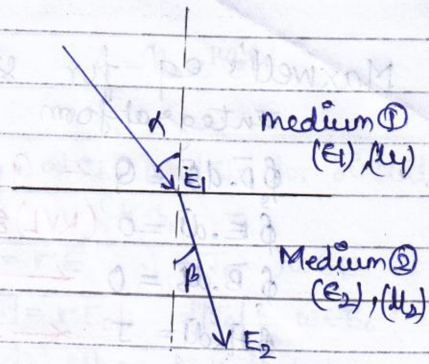
$\boxed{E = 0} \Rightarrow V_{AB} = -\int E\cdot dl = 0$

Perfect conductor is an equipotenⁿ surface.

Type of configuration	Potential	Electric field
(i) Q • (monopole)	$V \propto \frac{1}{r}$	$E \propto \frac{1}{r^2}$
(ii) $+Q$ • - - - • $-Q$ (dipole)	$V \propto 1/r^2$	$E \propto 1/r^3$
(iii) $+Q$, $-Q$, $-Q$, $+Q$ (Quaterpole)	$V \propto 1/r^3$	$E \propto 1/r^4$
(iv) $+Q$, $-Q$, $-Q$, $+Q$, $-Q$, $+Q$, $+Q$, $-Q$ (Octapole)	$V \propto 1/r^4$	$E \propto 1/r^5$

Boundry Condition

If field exist in two diffn media seperated by a boundry, then field in one medium canbe calculated by field in another medium.



$$\boxed{\frac{\tan\alpha}{\tan\beta} = \frac{\epsilon_1}{\epsilon_2}}$$

Case (i) Field is normal to the boundry

(i) For Electric field

(a) $\boxed{D_{n_1} - D_{n_2} = 0}$

→ If Boundry has no charge

(b) If bound has surface charge density $\boxed{D_{n_1} - D_{n_2} = \rho_s}$

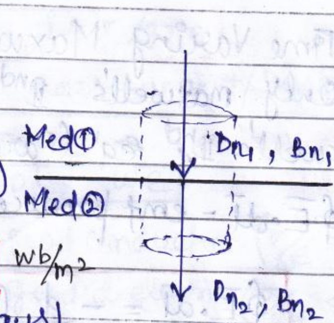
$\rho_s \rightarrow$ surface charge density (C/m^2)

(ii) For Magn Field

$\oint B \cdot ds = 0$ OR $\nabla \cdot \bar{B} = 0$ (always)

$\therefore \boxed{B_{n_1} - B_{n_2} = 0}$ Wb/m^2

(always)



Case (ii) Field is tangential to the Boundry

(i) for Electric field

(a) $\oint E \cdot d\bar{l} = 0$

$\boxed{E_{t_1} - E_{t_2} = 0}$ (always)

→ Tangential compn of e field is continous on either side

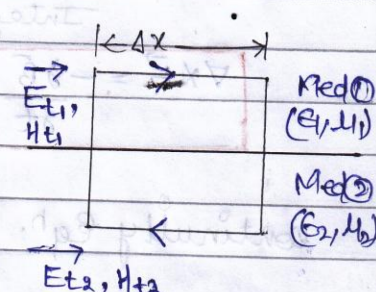
(ii) For Magn field

(a) $\oint \bar{H} \cdot d\bar{l} = I$

$\boxed{H_{t_1} - H_{t_2} = 0}$

→ for charge free Region

(b) If boundry has surface current density $\boxed{H_{t_1} - H_{t_2} = \bar{K}}$



For perfect conductor ($\sigma = \infty$)

$J = \sigma E \Rightarrow \boxed{E = 0}$

for perfect conductor electric field cannot exist because charge accumulation ~~exist~~ doesnot exist only flow exist

$\boxed{\begin{matrix} E_{\text{along the surface}} = 0 \\ E_{tan} = 0 \end{matrix}}$ but $D_{normal} \neq 0$

$\boxed{D_{n_2} - D_{n_1} = \rho_s}$

Continuity Eqn and Maxwell's Eqns

Maxwell's eqn for static electric and Magn field

Integral form — Point form

$\oint_s \bar{D}\cdot d\bar{S} = Q$ ← Gauss law → $\nabla\cdot\bar{D} = \rho_v$

$\oint \bar{E}\cdot d\bar{l} = 0$ (KVL) Eqn — $\nabla\times E = 0$ → electric field is irrotational

$\oint_s \bar{B}\cdot d\bar{S} = 0$ ← (always) → $\nabla\cdot\bar{B} = 0$ → Magn field is always solenoidal.

$\oint_l \bar{H}\cdot d\bar{l} = I$ ← Ampere's law → $\nabla\times\bar{H} = \bar{J}_c$

J_c → conduction current density

Time Varing Maxwell' Eqn:

Only maxwell's IInd and IVth Eqns are modified.

Maxwell's IInd eqn (Faraday's law)

$\oint \bar{E}\cdot d\bar{l}$ = emf produced $= -\frac{d\phi_m}{dt}$

$$\boxed{\oint_l \bar{E}\cdot d\bar{l} = -\frac{d}{dt}\int \bar{B}\cdot d\bar{S}}$$

Integral form

$$\boxed{\nabla\times\vec{E} = -\frac{\partial\bar{B}}{\partial t}}$$ Point form.

Maxwell's IVth eqn:

(Maxwell's introduced concept of displacement current)

$$\boxed{\oint_l \bar{H}\cdot d\bar{l} = I_{total} = I_c + I_d}$$

$$\boxed{\begin{aligned}\nabla\times\bar{H} &= J_c + J_D\\ \nabla\times\bar{H} &= \sigma E + \frac{\partial\bar{D}}{\partial t}\end{aligned}}$$

continuity Eqn. Continuity Eqn follows when region is bounded by a closed surface.

$$I = \oint_s \bar{J}\cdot d\bar{S} = \int_v (\nabla\cdot J)\,dv = \int\left(\frac{\partial\rho_v}{\partial t}\right)\rho_v \Rightarrow \boxed{\nabla\cdot J = \frac{\partial\rho_v}{\partial t}}$$ → continuity Eqn.

Note:- (i) $\oint \bar{J}\cdot d\bar{S} = 0$ → law of conservation of charge — KCL

(ii) $\nabla\cdot J = -\frac{\partial\rho_v}{\partial t}$ — (1), $J = \sigma E$ and $\nabla\cdot D = \rho_v$

outward current

after solving eqn (1) $\boxed{\rho_v = K e^{-t/(\epsilon/\sigma)}}$

$\boxed{t_r = \frac{\epsilon}{\sigma}}$ → Relaxation time

(i) For Good conductor

$t_r \downarrow$ as $\sigma\uparrow$

(ii) For Good dielectric

$t_r (\uparrow)$ as $\sigma\downarrow$

Maxwell eq^n's for sinusoidal case

$\bar{E} = E_0 e^{j\omega_0 t}$ $\bar{D} = D_0 e^{j\omega_0 t}$ $\bar{H} = H_0 e^{j\omega_0 t}$ $\bar{B} = B_0 e^{j\omega_0 t}$

I^st eq^n $\boxed{\nabla \cdot D = \rho_v}$

II^nd eq^n $\nabla \times \bar{E} = -\frac{\partial B}{\partial t} = -j\omega_0 B = -j\mu\omega_0 H$

$$\boxed{\nabla \times \bar{E} = -j\mu\omega_0 H}$$

III^rd eq^n $\nabla \cdot \bar{B} = 0$ (always)

IV^th eq^n $\nabla \times \vec{H} = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t}$

$$\nabla \times \bar{H} = \sigma \bar{E} + j\omega_0 \bar{D}$$

$$\nabla \times \bar{H} = \sigma \bar{E} + j\omega\epsilon \bar{E}$$

$$\boxed{\nabla \times \bar{H} = (\sigma + j\omega\epsilon)\vec{E}}$$

Value of J_C and J_D for sinusoidal case

$J_C = \sigma \vec{E}$ $J_D = j\omega\epsilon \vec{E}$

$\boxed{|J_C| = \sigma E_0}$ $\boxed{|J_D| = \omega\epsilon E_0}$

case (i) when $|J_C| = |J_D|$

$|J_C| = |J_D|$

$\boxed{f = \sigma / 2\pi\epsilon}$

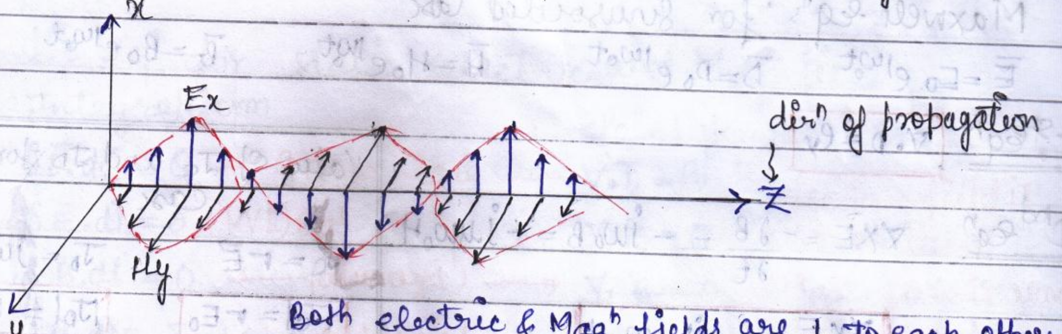
case (ii) Loss tangent / Dissipation factor

$\boxed{\left|\frac{J_C}{J_D}\right| = \frac{\sigma}{\omega\epsilon}}$ ← loss tangent (unitless)

a) for good conductor $\left(\frac{\sigma}{\omega\epsilon} >> 1\right)$

b) for good dielectric $\left(\frac{\sigma}{\omega\epsilon} << 1\right)$

Uniform plane wave / Plane Wave / Transverse Electromagnetic Wave



Both electric & Magⁿ fields are ⊥ to each other and dirⁿ of propagation. $(E \perp H) \perp$ dirⁿ of propⁿ.

therefore dirⁿ of propagation is given by $(\bar{E} \times \bar{H})$.

(i) If plane wave is propagating in +z dirⁿ then,

a) $\boxed{E_Z = H_Z = 0}$

b) there is no variation in x-y plane OR Z=0 plane. It means

$$\frac{\partial}{\partial x}(E \text{ OR } H) = 0 \quad \text{and} \quad \frac{\partial}{\partial y}(E \text{ OR } H) = 0$$

Eqⁿ of plane wave for free space ($\sigma = 0$, $\epsilon = \epsilon_0$, $\mu = \mu_0$)

$$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}} \qquad \boxed{\nabla^2 H = \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2}} \rightarrow \text{Harmonic Solution}$$

where $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ → speed of light (3×10^8 m/s)

Solution of plane wave

(a)
$$\boxed{\begin{aligned} E_x(z,t) &= E_0 e^{j\omega t} e^{-\gamma z} \\ H_y(z,t) &= H_0 e^{j\omega t} e^{-\gamma z} \end{aligned}} \rightarrow \text{dirⁿ of propⁿ (+z dirⁿ)}$$

(If wave is travelling in +z dirⁿ)

$$\boxed{\begin{aligned} E_x(z,t) &= E_0 e^{j\omega t} e^{\gamma z} \\ H_y(z,t) &= H_0 e^{j\omega t} e^{\gamma z} \end{aligned}}$$

→ dirⁿ of propⁿ (−z dirⁿ)

If wave is travelling in −z dirⁿ.

Here γ is called propagation constⁿ.

$$\gamma = \sqrt{j\mu\omega(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

α → attenuation constⁿ (np/m) [1 np = 8.68 dB]

β → phase constant (rad/m)

(b) Consider plane wave is travelling +z dir^n It means $\bar{E}$ field is in +x dir^n and Mag^n field is in +y dir^n.

$$\boxed{\begin{aligned} E_x(z,t) &= E_0 e^{-\alpha z} e^{j(\omega t-\beta z)} \text{ V/m} \\ \text{OR } E_x(z,t) &= E_0 e^{-\alpha z} \cos(\omega t - \beta z) \text{ V/m}\end{aligned}} \qquad \boxed{\begin{aligned} H_y(z,t) &= H_0 e^{-\alpha z} e^{j(\omega t-\beta z)} \text{ A/m} \\ H_y(z,t) &= H_0 e^{-\alpha z} \cos(\omega t - \beta z) \text{ A/m}\end{aligned}}$$

Shows dir^n of prop^n (+z dir^n) — Shows dir^n of prop^n (+z dir^n)

(c) Amplitude is exponentially decaying by rate of $e^{-\alpha}$

(d) For UPW, $\bar{E}$ and $\bar{H}$ component should be func^n of space and time

(e) For UPW, there should not be any non linear variation in space and time.

eg (i) $(x+3t^2)$ X Nonlinear var^n (iv) $\sin^2(3t+9)$ X Non linear var^n

(ii) $\cos(\omega t+\beta z)$ ✓ UPW (v) $\cos(9t^2+\beta z)$ X "

(iii) $\cos(\omega z+\beta y)$ X Not a f^n of time (vi) $3+4\sin(\omega t-\beta z)$ X "

(iv) $e^{(3t-9y)}$ X Non linear var^n (v) $e^{j(\omega t-9z)}$ ✓ UPW

Propagation constant (γ)

$$\gamma = \sqrt{j\mu\omega(\sigma+j\omega\epsilon)} = \alpha + j\beta$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]} \qquad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]}$$

attenuation const^n (Np/m OR dB/m) — phase const^n (rad/m)

Intrinsic Impedence (η) $\boxed{\eta = |\eta|\angle\theta_n}$

$$\eta = \frac{E}{H} \Rightarrow \eta = \frac{|E_x|}{|H_y|}\angle\theta_n \quad \text{OR} \quad \eta = \frac{|E_y|}{|E_x|}\angle\theta_n$$

$\theta_n \rightarrow$ phase shift b/w E and H components

It is always assumed that E field is leading H-field and Hence

$\boxed{\eta = R + jX}$ ← Inductive (always)

$$\boxed{\eta = \frac{j\mu\omega}{\gamma} = \sqrt{\frac{j\mu\omega}{(\sigma+j\omega\epsilon)}}}$$

Relationship b/w loss tangent and θ_n

$$\boxed{\tan(2\theta_n) = \frac{\sigma}{\omega\epsilon}}$$

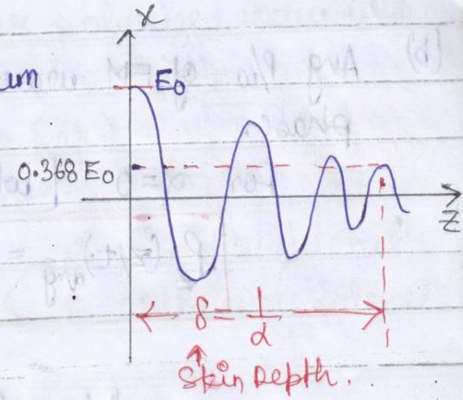
Phase Velocity (v_p)

$$\boxed{v_p = \frac{\omega}{\beta}} \quad m/s$$

Free Space	Loss Less Dielectric	Case (ii) Good Dielectric	Good Conductor
($\sigma=0$, $\mu=\mu_0$, $\epsilon=\epsilon_0$)	(i) Perfect dielectric ($\sigma=0$, $\mu=\mu_0\mu_r$, $\epsilon=\epsilon_0\epsilon_r$)	Dielectric ($\sigma\neq0$) ($\frac{\sigma}{\omega\epsilon}<<1$)	($\sigma/\omega\epsilon>>1$)
(i) $\gamma=\sqrt{j\mu\omega(\sigma+j\omega\epsilon)}$ $\sigma=0$ $\gamma=0+j\omega\sqrt{\mu_0\epsilon_0}$ $\boxed{\alpha=0}$, $\boxed{\beta=\frac{\omega}{c}}$ $\frac{rad}{m}$ No attenuⁿ, only propⁿ	(i) $\gamma=0+j\omega\sqrt{\mu\epsilon}$ $\boxed{\alpha=0}$, $\boxed{\beta=\omega\sqrt{\mu\epsilon}}$ For Non Magⁿ Medⁿ ($\mu_r=1$) $\boxed{\beta=\frac{\omega}{c}\sqrt{\epsilon_r}}$	(i) $\boxed{\alpha=\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}}$ $\boxed{\beta=\omega\sqrt{\mu\epsilon}}$	(a) α and β $\alpha=\sqrt{\frac{\sigma\mu\omega}{2}}$ $\beta=\sqrt{\frac{\sigma\mu\omega}{2}}$
(ii) $\eta=\sqrt{\frac{j\mu\omega}{\sigma+j\omega\epsilon}}$ $\eta=\sqrt{\frac{\mu_0}{\epsilon_0}}$ $\boxed{\eta=120\pi\,\Omega \text{ OR } \eta=377\,\Omega}$	(ii) Intrinsic Impⁿ $\eta=120\pi\sqrt{\frac{\mu_r}{\epsilon_r}}\angle0^\circ$ For Non Magⁿ Medᵐ. $\boxed{\eta=\frac{120\pi}{\sqrt{\epsilon_r}}}$	(ii) Phase velocity $\boxed{v_p=\frac{c}{\sqrt{\mu_r\epsilon_r}}}$	(b) Intrinsic Impⁿ (η) $\eta=\sqrt{\frac{j\mu\omega}{\sigma}}$ OR $\eta=(1+j)\sqrt{\frac{\mu\omega}{2\sigma}}$ $\eta=\sqrt{\frac{\mu\omega}{\sigma}}\angle45^\circ$
(iii) Phase velocity $v_p=\frac{\omega}{\beta}$ $v_p=\omega/\omega\sqrt{\mu_0\epsilon_0}$ $\boxed{v_p=c=3\times10^8\, m/s}$ Incase of free space plane wave travels with speed of light	(iii) Phase Velocity $v_p=\frac{c}{\sqrt{\mu_r\epsilon_r}}$ For Non magⁿ Medᵐ. $\boxed{v_p=\frac{c}{\sqrt{\epsilon_r}}}$	LOSSY Dielectric OR Conducting Medᵐ ($\sigma\neq0$) $\alpha=\omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]}$ $\beta=\omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]}$	(c) Phase velocity $v_p=\omega/\beta$ (d) $E_x(z,t)=E_0e^{-\alpha z}\cos(\omega t-\beta z)$
(iv) $\bar{E}=E_0e^{j(\omega t-\beta z)}\hat{a}_x$ $\bar{H}=H_0e^{j(\omega t-\beta z)}\hat{a}_y$ where $H_0=\frac{E_0}{\eta_0}$.	(iv) $E_x(z,t)=E_0e^{j(\omega t-\beta z)}$ $H_y(z,t)=\frac{E_0}{\eta}e^{j(\omega t-\beta z)}$ where $\eta=\frac{120\pi}{\sqrt{\epsilon_r}}$	Note Loss tangent: (i) $\sigma/\omega\epsilon>>1$ Good Conductor (b) $\sigma/\omega\epsilon<<1$ Good dielectric (c) $\frac{\sigma}{\omega\epsilon}=1$ Quasi conductor	$H_y(z,t)=\frac{E_0}{\lvert\eta\rvert}e^{-\alpha z}\cos(\omega t-\beta z-45^\circ)$

Skin Depth / Depth of Penetration

When plane wave travels in conducting medium (Good conductor / lossy Med^n), its amplitude is attenuated by factor α. Therefore after travelling $1/\alpha$ distance its major strength is removed. This distance is called **Skin Depth.**



$$\boxed{\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\sigma\mu\omega}} = \frac{1}{\sigma \cdot R}}$$ where $R = \sqrt{\frac{\mu\omega}{2\sigma}} = X$ for Good conductor.

Note :-

a) $\delta \propto \frac{1}{\sqrt{f}}$ $f(\uparrow) - \delta(\downarrow)$, therefore
- (a) at low freq^n, skin depth is high, hence we use thick conductor.
- (b) at high freq^n, skin depth is low, hence we use thin conductor.

$f(\uparrow) \to R(\uparrow) \to A(\downarrow)$ ∵ $R = \frac{\rho l}{A}$ thin conductor

b) Phase Velocity $v_p = \frac{c}{\sqrt{\epsilon_r}}$ (For Non mag^n Med^m)

$v_p = 3\times10^8 \longrightarrow v_p = \frac{c}{\sqrt{\epsilon_r}} \longrightarrow v_p = \frac{\omega}{\beta} \longrightarrow v_p = $ few m/s

(free space)   (Good Dielectric)   (lossy Med^n)   (Good conductor)

⟶ decreasing

Poynting theorm and Avg P/w of EM wave.

$$\oint_s (\bar{E}\times\bar{H})\cdot d\bar{S} = -\int_V \sigma|E|^2 dV - \frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\epsilon|E|^2 + \frac{1}{2}\mu|H|^2\right] dV$$

Total P/w leaving the closed surface.   Ohmic P/w Dissipation   Rate of ↓ in energy stored in E and magn field.

dir^n of Poy^n vector = dir^n of $(\bar{E}\times\bar{H})$

Average P/w of EM wave $\boxed{P_z(z,t)\Big|_{avg} = \frac{1}{2}Re(\bar{E}\times\bar{H}^*)}$ $\frac{Watt}{m^2}$

(b) Avg P/w of EM wave depends on Mag^n only & never on the phase.

For $\alpha = 0$ [where $\sigma = 0$] No attenuation

$$\boxed{P_z(z,t)_{avg} = \frac{1}{2}\frac{E_0^2}{\eta} = \frac{1}{2}\eta H_0^2} \quad \text{watt/m}^2$$

Wave Polarization

Time varing behaviour of $\vec{E}$ field at a fixed point in space along dir^n of prop^n.

(i) Linear Polarization ($\theta_p = \pm n\pi$)

(i) If plane wave has single $\vec{E}$ field component either E_x OR E_y, then it is linearly polarized

eg $E_x = 3\cos(\omega t - \beta z)$ V/m

(ii) If plane has two components

$E = E_{x_0}\cos(\omega t - \beta z)\hat{a}_x + E_{y_0}\cos(\omega t - \beta z + \theta_p)\hat{a}_y$

then $\theta_p = \pm n\pi$

Circular polarization

For Circular Polar^s

(i) $\boxed{E_{x_0} = E_{y_0}}$

(ii) $\boxed{\theta_p = \pm\frac{\pi}{2}(2n+1)}$

(a) Left Circular Pol^r wave

$E = E_0\cos(\omega t - \beta z)\hat{a}_x + E_0\cos(\omega t - \beta z + 90^\circ)\hat{a}_y$

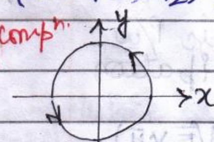
Here $\hat{a}_y$ component is leading (left)

LCP wave

(b) Right Circular Polariz^n

$E = E_0\cos(\omega t - \beta z)\hat{a}_x + E_0\cos(\omega t - \beta z - \pi/2)\hat{a}_y$

Here $\hat{a}_y$ comp^n is lagg^n (Right)



Elliplical polarizat^n

for elliptical polar^n

(a) $\boxed{E_{x_0} \neq E_{y_0}}$

(b) θ_p maybe any value

eg :-

(i) $E = 2\cos(\omega t - \beta z)\hat{a}_x + 3\cos(\omega t - \beta z + 30^\circ)\hat{a}_y$

Here $E_{x_0} \neq E_{y_0}$, $\theta_p = 30^\circ$

$\therefore$ elliptical pol^n.

Here $\hat{a}_y$ leads therefore it is left elliptical Polarized wave.

Note :- (a) If both component have diff^t freq^n component then, plane wave is unpolarized. eg : $E = 2\cos(2t - 3z)\hat{a}_x + 4\cos(3t - 4z)\hat{a}_y$

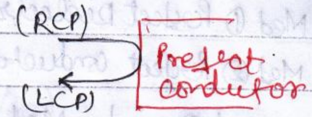
diff^n freq^n hence unpolarized wave.

(b) If dir^n of prop^n changes

left hand $\rightleftarrows$ Right hand

(C) Sum of two opposite rotating circular polarized wave with same mag^n will be Linearly polarized.

(d) When right circularly polar^3 wave incident on Perfect conductor, the reflected wave is LCP OR vica-versa.



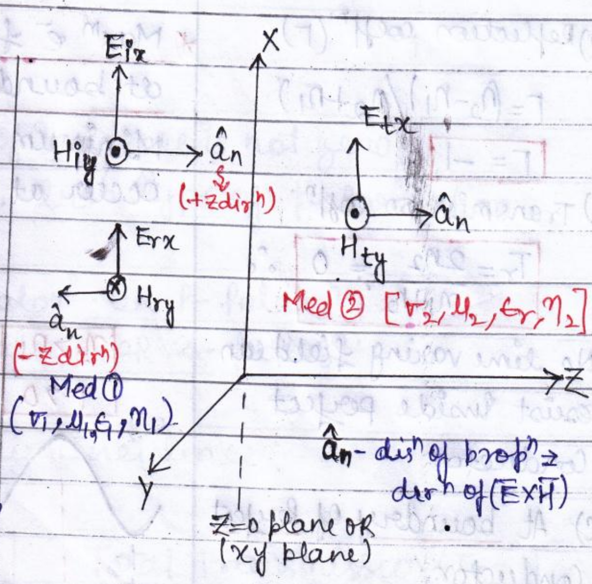
Wave Reflaction and Refraction

(i) Normal Incidence
(ii) Oblique Incidence.

(i) Normal Incidence

$\hat{a}_n = dr^n \text{ of } (\bar{E} \times \bar{H})$

$\uparrow$ dir^n of $prop^n$.



(a) Incident wave eq^n

$E_{ix}(z,t) = E_{i0}\, e^{j\omega t}\, e^{-\gamma_1 z}$ V/m
$H_{iy}(z,t) = H_{i0}\, e^{j\omega t}\, e^{-\gamma_1 z}$ A/m
} $Prop^n$ in $+z\ dir^n$

$\gamma_1 \rightarrow prop^n\ const^n$ in med^n ①

where $H_{i0} = E_{i0}/\eta_1$

(b) Reflected wave eq^n. due to reflection $prop^n\ dir^n$ becomes $-z\ dir^n$.

$E_{rx}(z,t) = E_{r0}\, e^{j\omega t}\, e^{\gamma_1 z}$ V/m. ($\bar{E}$ field in $+x\ dir^n$)

$H_{ry}(z,t) = -H_{r0}\, e^{j\omega t}\, e^{\gamma_1 z}$ A/m. (Mag^n field in $-y\ dir^n$)

(C) Transmitted wave eq^n.

$E_{tx}(z,t) = E_{t0}\, e^{j\omega t}\, e^{-\gamma_2 z}$ V/m
$H_{ty}(z,t) = H_{t0}\, e^{j\omega t}\, e^{-\gamma_2 z}$ A/m

where $\gamma_2 \rightarrow Prop^n\ const^n$ in med 2

$H_{t0} = E_{t0}/\eta_2$

Reflection $coff^n$.

$$\Gamma = \frac{E_{r0}}{E_{i0}} = -\frac{H_{r0}}{H_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Transmission $coeff^n$ (T_r)

$$T_r = \frac{E_{t0}}{E_{i0}} = 1 + \underset{\text{Reflection coff}^n}{\Gamma} = \frac{2\eta_2}{(\eta_1 + \eta_2)}$$

(a) $Ref^n\ coff^n$ and transmission $coff^n$, both are complex quantity.

(b) For lossless medium ($\sigma = 0$), η is real quantity hence $Ref^n\ coff^n$ and transmission $coff^n$ are real.

Case (i)

Med ① Perfect Dielectric ($\sigma=0$)

Med ② Perfect conductor ($\sigma=\infty$)

Med ① ($\sigma=0$)	Med ② ($\sigma=\infty$)
$\eta_1 = 120\pi\sqrt{\frac{\mu_r}{\epsilon_r}}$	$\eta_2 = 0$

(a) Reflection coeffⁿ (Γ)

$$\Gamma = (\eta_2-\eta_1)/(\eta_2+\eta_1)$$

$$\boxed{\Gamma = -1}$$

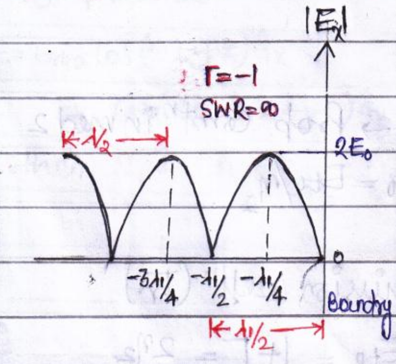
(b) Transmission coeffⁿ:

$$\boxed{T_r = \frac{2\eta_2}{\eta_1+\eta_2} = 0} \quad \therefore$$

No time varing field can exist inside perfect conductor.

(C) At boundry of Perfect Conductor,

* $\bar{e}$ field is minimum (zero)
* Magⁿ field is maxᵐ ($2H_{i0}$)



Case (ii)

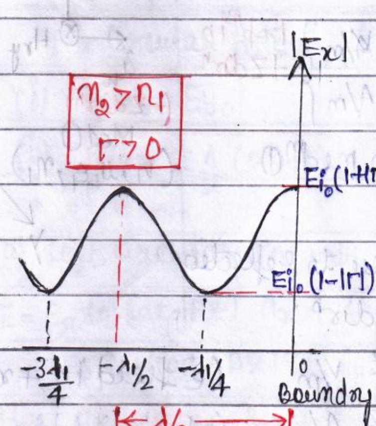
($\eta_2 > \eta_1$) ($\Gamma > 0$)

Med ①	Med ②
($\sigma_1, \mu_1, \epsilon_1, \eta_1$)	($\sigma_2, \mu_2, \epsilon_2, \eta_2$)

($\eta_2 > \eta_1$)

In this case,

* Maxᵐ $\bar{e}$ field occur at boundry OR Minimum magⁿ field occur at boundry



Case (iii)

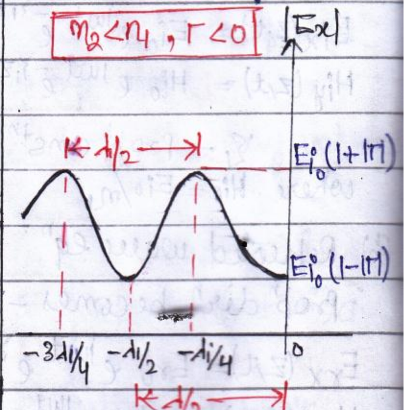
($\eta_2 < \eta_1$) ($\Gamma < 0$)

Med ①	Med ②
($\sigma_1, \mu_1, \epsilon_1, \eta_1$)	($\sigma_2, \mu_2, \epsilon_2, \eta_2$)

($\eta_2 < \eta_1$)

In this case,

Minimum $\bar{e}$ field Occur at the boundry OR Maxᵐ magⁿ field occur at boundry



Standing wave Ratio

Ratio of Maxᵐ to Minᵐ field

$$SWR = \frac{|E_x|\,max}{|E_x|\,min} \text{ OR } \frac{|H_y|\,max}{|H_y|\,min}$$

$$\boxed{SWR = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

$S \geq 1$ → SWR is always greater than or equal to 1.

where $\bar{e}$ field is maxᵐ, magⁿ field is minimum OR vica versa.

Between two successive maxima OR minima distance is $\lambda/2$.

Reflected and Transmitted Avg P/w

Reflected P/w $\quad P_{r_z}(avg) = |\Gamma|^2 \, P_{i_z}(avg) \quad (W/m^2)$

Transmitted Avg P/w $\quad P_{t_z}(avg) = (1-|\Gamma|^2) \, P_{i_z}(avg) \quad W/m^2$

(ii) Oblique Incidence [angle of incidence is not zero]

In case of oblique incidence, two type of polarization occurs

(a) Parallel Polarization OR TM Polar^n OR P-Polarz^n.

(b) Perpendicular Polar^n OR TE Polar^n OR S-polarz^n.

Two special cases in oblique Incidence.

Total internal Reflection ($\Gamma=-1$) OR Zero Transmission ($T_r=0$)

It exists in both polariz^n (s and p polariz^n)

Total Reflection, ($\theta_t = 90°$) and $\theta_i = \theta_c \rightarrow$ critical angle.

$$\boxed{\sin\theta_i \geq \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}}$$

$n_1 \rightarrow$ refractive Index of med ①

$n_2 \rightarrow$ refractive Index of med ②

$n \propto \sqrt{\epsilon_r}$ — refractive Index $\left(\frac{c}{v}\right)$

$\eta \propto \frac{1}{\sqrt{\epsilon_r}}$ — intrinsic Imp^n.

Total Transmission ($T_r=1$) Zero Reflection ($\Gamma=0$)

It exists only for p-polariz^n.

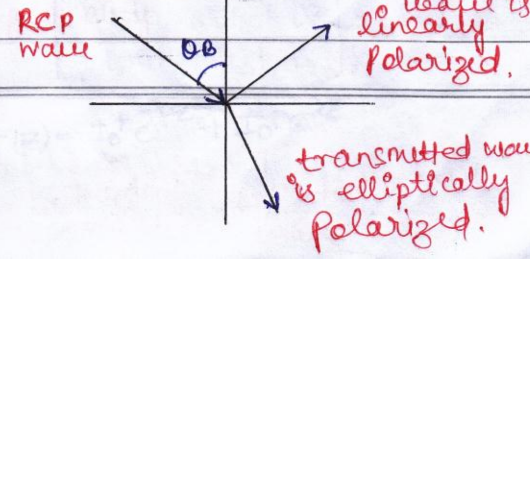
($T_r=1$) and ($\Gamma=0$), $\eta_1=\eta_2$

It is possible only when.

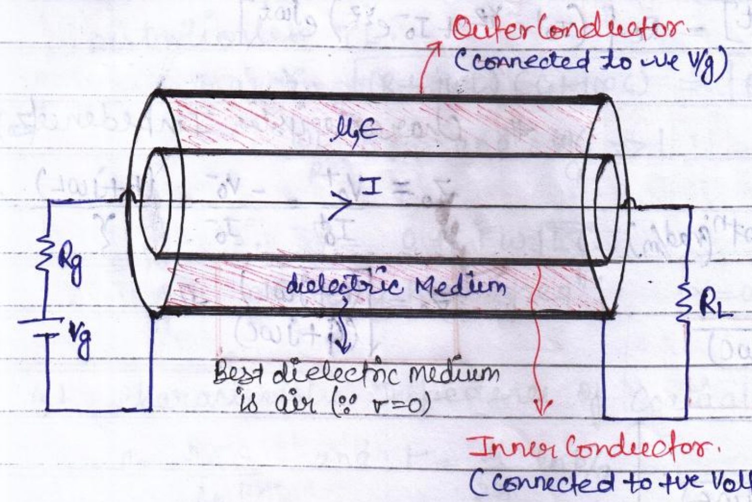
$\boxed{\theta_i = \theta_B}$ Brewster Angle.

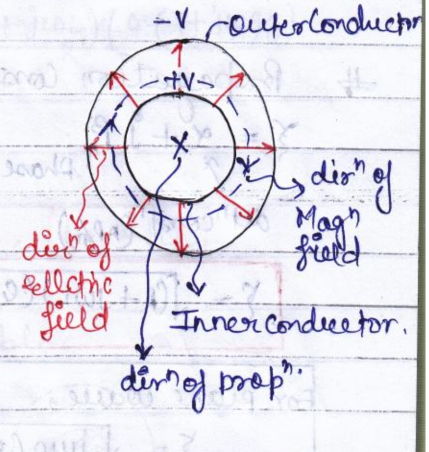
$$\boxed{\tan\theta_B = \sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}}$$

at Brewster Angle (θ_B)



Transmission Line





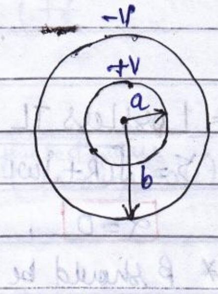
(a) For TL, primary constants are R, L, G, C which are distributed along the length of TL.

(b) Inner and outer conductors are equipotential surface.

(c) For TL, $R \neq \frac{1}{G}$, because Resistance per unit length of conductor represents lossy nature of conductor whereas conductance per unit length of dielectric represents lossy nature of dielectric. Therefore, for Lossless TL, $\boxed{R = G = 0}$.

(d) In TL, two conductors are seperated by dielectric medium, Hence capacitance perunit length is given by

$$\boxed{C = \frac{2\pi\epsilon}{\ln(b/a)} \text{ per length}} \ (F/m)$$



(e) Due to current on a line, there is a sourrounded mag^n field hence inductance per unit length is given by

$$\boxed{L = \frac{\mu}{2\pi} \ln(b/a) \text{ /length.}} \ (H/m)$$

(f) For Transmission line of any geometry

$$\boxed{LC = \mu\epsilon}$$

(g) V-I Eq^n of TL

$$V(z) = \underset{\substack{\uparrow \\ \text{Incident wave}}}{V_0^+ e^{-\gamma z}} + \underset{\substack{\uparrow \\ \text{Reflected wave}}}{V_0^- e^{\gamma z}} \qquad I(z) = \underset{\substack{\uparrow \\ \text{Incident wave}}}{I_0^+ e^{-\gamma z}} + \underset{\substack{\uparrow \\ \text{Reflected wave.}}}{I_0^- e^{\gamma z}}$$

(h) Time harmonic func^n:

$$V(z,t) = Re\left[V(z)e^{j\omega t}\right] = Re\left[\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right)e^{j\omega t}\right]$$

$$I(z,t) = Re\left[I(z)e^{j\omega t}\right] = Re\left[\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right)e^{j\omega t}\right]$$

Propagation Const^n:

$$\gamma = \underset{\substack{\uparrow \\ \text{att}^n\text{ const}^n\,(np/m)}}{\alpha} + j\underset{\substack{\uparrow \\ \text{Phase Const}^n\,(rad/m)}}{\beta}$$

$$\boxed{\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}}$$

For plane wave,

$$\boxed{\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

Characteristic Impedence (Z_0)

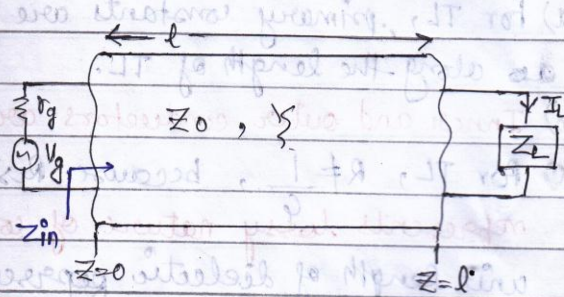
$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \frac{(R+j\omega L)}{\gamma}$$

$$\boxed{Z_0 = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}}}\ \Omega$$

Input Impedence

$$\boxed{Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}\right]}$$

$\rightarrow$ for lossy T.L



$l \rightarrow$ physical length of TL

$\beta l \rightarrow$ electrical length of TL.

Lossless TL $[R=G=0]$

(a) $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = 0 + j\omega\sqrt{LC}$

$\boxed{\alpha = 0}$, $\boxed{\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon} = \frac{2\pi}{\lambda}}$

* β should be linear f^n of 'ω'.

(b) Characteristic Imp^n:

$\boxed{Z_0 = \sqrt{L/C}}$

(c) Input Impedence Z_{in}

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

Distortionless TL $[R/L = G/C]$

* α should be const^n. If it depends on freq^n, then there will be freq^n distortion.

* β should be linear f^n of 'ω' Otherwise, there will be phase distortion.

$\boxed{\alpha = \sqrt{RG}}$

$\boxed{\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon}}$ $\boxed{\beta = \frac{2\pi}{\lambda}}$

$\boxed{Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}}$

Note:- (a) Every Lossless TL $[R=G=0]$ is distortionless but vica-versa is not true.

(b) Transmission line at very high freq^n: behaves as Lossless / Distortionless TL.

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{R\left(1+\frac{j\omega L}{R}\right)\left(G\left(1+\frac{j\omega C}{G}\right)\right)}$$

$\because \frac{\omega L}{R} >> R$ and $\frac{\omega C}{G} >> 1$

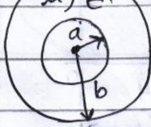
$\therefore \gamma = 0 + j\omega\sqrt{LC}$

$\therefore$ at very high freq^n; $\boxed{\alpha = 0}$, $\boxed{\beta = \omega\sqrt{LC}}$

(c) Characteristic Impedence of Coaxial cable

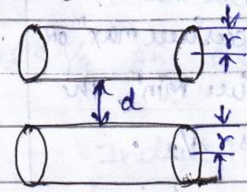
$C = \frac{2\pi\epsilon}{\ln b/a}$ and $L = \frac{\mu}{2\pi}\ln b/a$

$$\therefore \boxed{Z = \sqrt{\frac{L}{C}} = \frac{60}{\sqrt{\epsilon_r}}\ln\left(\frac{b}{a}\right)}$$

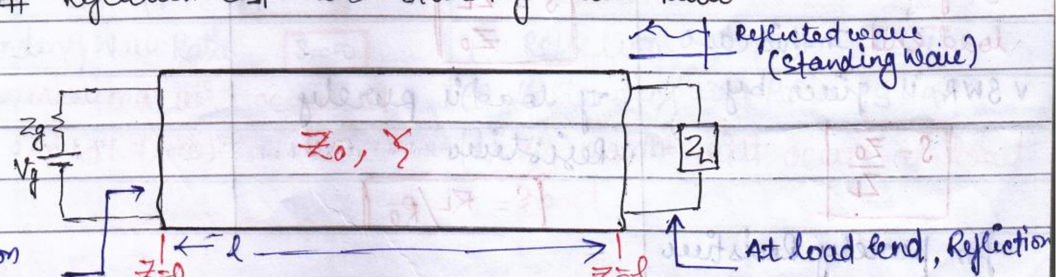


(d) Characteristic Impedence of parallel wire line

$$\boxed{Z_0 = \frac{120}{\sqrt{\epsilon_r}}\ln\left(\frac{d}{r}\right)}$$



Reflection coff^n and Standing wave Ratio



I/p Reflection coff^n. $\boxed{\Gamma_i = \Gamma_L e^{-j2\beta l}}$ (If Z_g is not mentioned)

At load end, Reflection coff^n is $\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$

$\boxed{0 \leq |\Gamma_L| \leq 1}$

case (i) $Z_L = Z_0$ (Matched TL) $\boxed{\Gamma_L = 0}$ No standing wave will exist hence total P/w is absorbed by load and nothing reflected back. The cond^n is also called Max^m P/w Transfer cond^n.

Voltage Standing WAVE Ratio

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

(a) $\because 0 \leq |\Gamma| \leq 1$ therefore VSWR is $\boxed{1 \leq VSWR \leq \infty}$

(b) For better impedence matching VSWR should be minimum.

(C) For Matched TL ($Z_L = Z_0$), no standing wave / Reflected wave exists and hence $\boxed{|\Gamma| = 0}$ and $\boxed{VSWR = 1}$

case (i) $Z_L < Z_0$, ($\Gamma_L < 0$)

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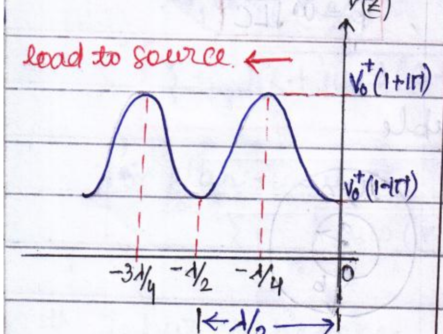

* where the voltage is max^m; current will be min^m.
* B/w two successive max^m OR two successive Min^m, the distance is $\lambda/2$.
* when $Z_L < Z_0$ ($\Gamma_L < 0$) Voltage min^m will occur at load end In this case VSWR is given by

$$\boxed{S = \frac{Z_0}{Z_L}}$$

for purely Resistive load $\boxed{S = R_0/R_L}$

Case (ii) $Z_L > Z_0$, ($\Gamma_L > 0$)

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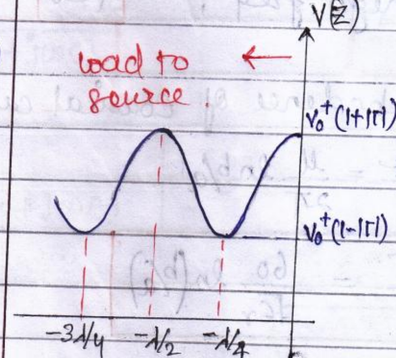


when $Z_L > Z_0$, ($\Gamma > 0$) Voltage Maxima will occur at load end

In this case VSWR is given by

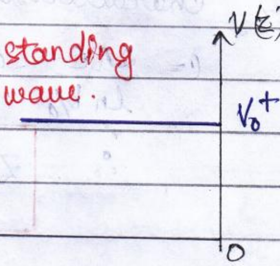
$$\boxed{S = \frac{Z_L}{Z_0}}$$

If load is purely Resistive

$\boxed{S = R_L/R_0}$

Case (iii) Matched TL ($Z_L = Z_0$) and ($\Gamma_L = 0$)

No standing wave.



* when $Z_L = Z_0$, Max^m P/w is delivered to load.

For Matched TL, VSWR is given by

$\boxed{VSWR = 1}$

Reflected Power

$$P_r(avg) = |\Gamma|^2 P_{i\,avg} \quad \text{where } P_i(avg) = (V_0^+)^2/2Z_0$$

Transmitted avg P/w

$$P_t(avg) = (1 - |\Gamma|^2) P_i(avg)$$

Shorted TL ($Z_L=0$) / Opened Line ($Z_L=\infty$) / Matched line ($Z_L=Z_0$)

For lossless TL, Input Impⁿ is given by

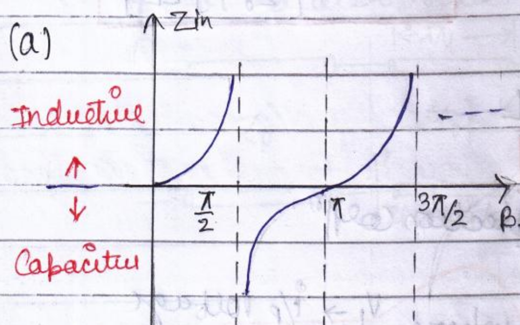
$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

l – physical length.
βl – electrical length.
where $\beta = \frac{2\pi}{\lambda} = \omega\sqrt{LC}$

Shorted TL

$$\boxed{Z_{in(SC)} = jZ_0 \tan\beta l}$$

(a)



$Z_{in(SC)} \rightarrow$ Inductive when

$\boxed{0 < \beta l < \pi/2 \quad 0 < l < \lambda/4}$ Inductive

$Z_{in(SC)} \rightarrow$ Capacitive when:

$\boxed{\pi/2 < \beta l < \pi \quad \lambda/4 < l < \lambda/2}$ Capacitive

(b) Reflection coeffⁿ (Γ) $\boxed{\Gamma = -1}$

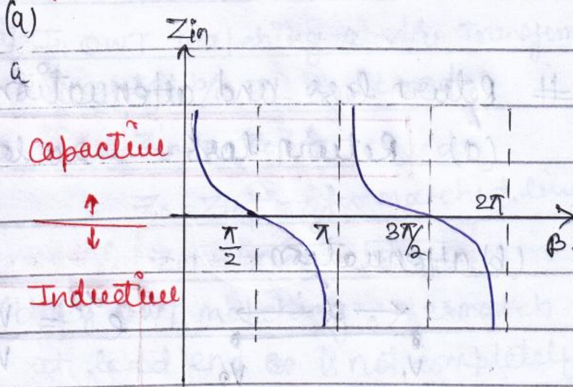
(c) V/g Standing Wave Ratio $\boxed{S = \infty}$

(d) Voltage minima will occur at load end at ($l=0$).

Open circuit TL

$$\boxed{Z_{in(OC)} = \frac{Z_0}{j\tan\beta l}}$$

(a)



$Z_{in(OC)} \rightarrow$ Capacitive

$\boxed{0 < \beta l < \pi/2 \quad \text{OR} \quad 0 < l < \lambda/4}$

$Z_{in(OC)} \rightarrow$ Inductive

$\boxed{\pi/2 < \beta l < \pi \quad \text{OR} \quad \frac{\lambda}{4} < l < \lambda/2}$

(b) Reflection coeffⁿ (Γ) $\boxed{\Gamma = 1}$

(c) V/g standing wave Ratio $\boxed{S = \infty}$

(d) V/g maxima will occur at load end.

For Matched TL ($Z_L = Z_0$) $\boxed{Z_{in} = Z_0}$

For Loss Less TL

$$\boxed{Z_0 = \sqrt{Z_{in(SC)} Z_{in(OC)}}}$$

Different type of TL (lossless)

$l = \lambda$ — (ii) $l = \lambda/2$

$\boxed{Z_{in} = Z_L}$ ($\because \beta l = 2\pi$)

$\boxed{Z_{in} = Z_L}$ ($\because \beta l = \pi$)

$l = \lambda/4$ (Quater wavelength TL)

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

↑ used for Imp^n (matching)

$l = \lambda/8$

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + j Z_0}{Z_0 + j Z_L}\right]}$$

Return loss and attenuation in TL

(a) $\boxed{\text{Return loss} = -20 \log (\Gamma)}$

↑ reflection coeff^n.

(b) attenuation

$\leftarrow l \rightarrow$

V_1 ——— V_2

$(V_1 > V_2)$

$$\boxed{e^{\alpha l} = \frac{V_1}{V_2}}$$

where $V_1 \rightarrow$ i/p voltage
$V_2 \rightarrow$ o/p voltage (Received V/g)
$l \rightarrow$ length of TL

$$\alpha l = \ln\left(\frac{V_1}{V_2}\right)$$

$$\boxed{\alpha = \frac{1}{l} \ln\left(\frac{V_1}{V_2}\right)}$$

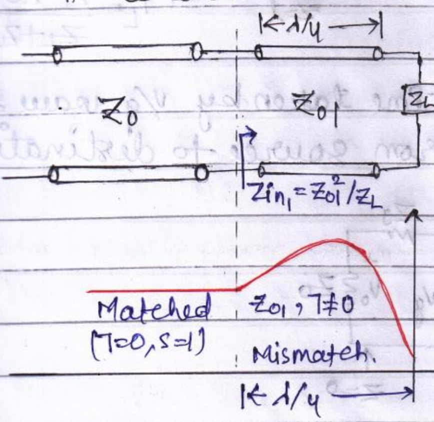
where $\alpha \rightarrow$ attenuation in (dB/m)
(1 neper = 8.686 dB)

Transmission line Matching

Quater Wave Transfomer Matching
- This techeique is preferered for Resistive load and lossless line.

Stub Matching
- Shunt
  - Opencircuit
  - Short circuit
- Series
  - Open ckt
  - Short ckt

Quater Wave Transformer Matching



(a) In QWT matching a λ/4 Transfomer is inserted b/w TL and Load.

$\boxed{Zin_1 = Z_{01}^2/Z_L}$ and

$Z_0 = Zin_1$ → for matched line

∴ $\boxed{Z_{01} = \sqrt{Zin\,Z_L} = \sqrt{Z_0 Z_L}}$

(b) By QWT matching, Mismatch at load end is not completely removed.

The length being λ/4, it restricts the freq^n range to narrowband.

Stub Mathing

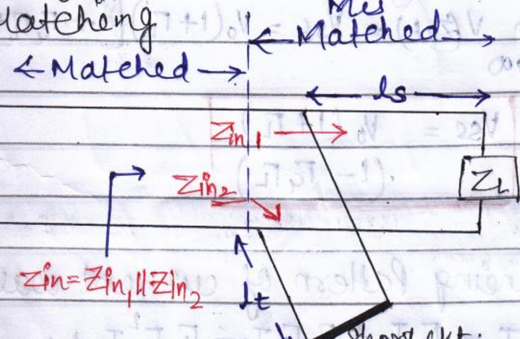
Short circuit shunt stub Matching

l_s → stub position

$$\boxed{l_s = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Z_0}{Z_L}}}$$

l_t → length of stub

$$\boxed{l_t = \frac{1}{\beta}\tan^{-1}\left(\frac{Z_L - Z_0}{\sqrt{Z_L Z_0}}\right)}$$

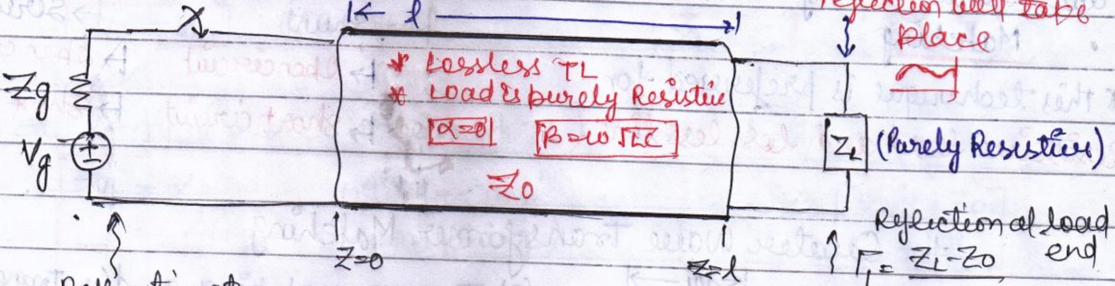


Stub position and stub length, both are freq^n dependent therefore in single stub matching it is dyficult to move stub but stub length can be adjusted.

For wide range of freq^n Double stub matching tech^n is used where l_{s1}, l_{s2} → fixed but l_{t1}, l_{t2} → variable

Transient on Transmission line

line is mismatched ($Z_L \neq Z_0$) reflection will take place



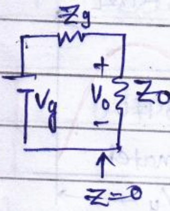
Reflection at load end $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

Reflection at generator end $\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$

$T = \frac{l}{v_p}$ → Time taken by v/g wave to reach from source to destination (load)

(i) at $t = 0^+$ and $Z = 0$

$$V(Z,t) = V(0,0^+) = V_0 = \frac{Z_0}{(Z_0 + Z_g)} V_g$$



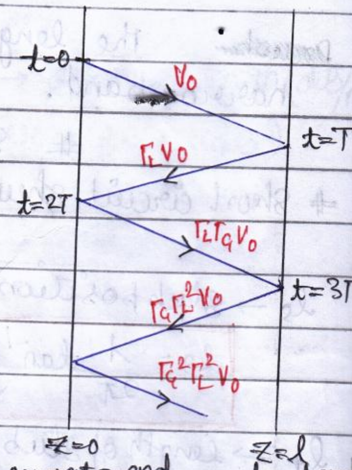
(ii) Bouncing Patter of voltage waveform

$$V(Z,t) = V_0 + \Gamma_L V_0 + \Gamma_G \Gamma_L V_0 + \Gamma_G \Gamma_L^2 V_0 + ---$$

steady state value

$$\lim_{t \to \infty} V(Z,t) = V_{SS} = V_0(1+\Gamma_L)\left[1 + \Gamma_G\Gamma_L + \Gamma_G^2\Gamma_L^2 + ---\right]$$

$$\boxed{V_{SS} = \frac{V_0(1+\Gamma_L)}{(1 - \Gamma_G\Gamma_L)}}$$



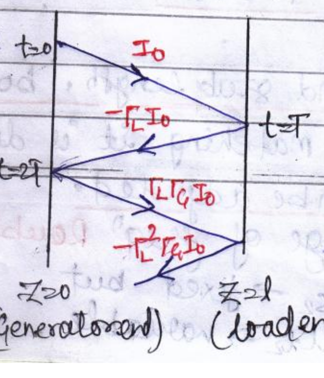
$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$ $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

(iii) Bouncing Pattern of current waveform.

$$I(Z,t) = \underset{\text{Incident wave}}{I_0} - \underbrace{\Gamma_L I_0}_{\text{Ist reflected wave at load.}} + \Gamma_G \Gamma_L I_0 - \Gamma_G \Gamma_L^2 I_0 + ----$$

Steady state value

$$\lim_{t \to \infty} V(Z,t) = V_{SS} = \frac{I_0(1-\Gamma_L)}{(1 + \Gamma_L\Gamma_G)}$$

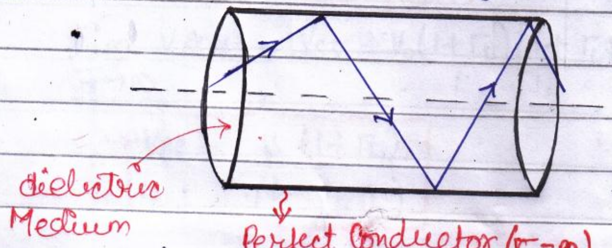


Waveguides

(i) Transmission Line	Waveguides
(a) TL can support only TEM wave.	(a) Waveguide can support Many modes.
(b) TL behaves as All pass filter. Therefore, it can pass all freq^n.	(b) Waveguides are High Pass Filter. It can pass freq^n which are greater than cutoff freq^n (f_c).
(c) TL can pass DC signal as well as AC signal	(c) DC s/g cannot pass through waveguides.
(d) In TL, analysis is done in terms of voltage and current.	(d) In waveguides, analysis is done in terms of E and H.
(e) TLs are preffered at low freq^n because at high freq^n, TL have problem of radiation loss	If $f(\downarrow) \rightarrow \lambda(\uparrow) \rightarrow$ size of w/g $(\uparrow)$ hence w/gs are preffered at high freq^n due to size problem.

Common w/gs are Rectangular ~~and~~ waveguide and Cylindrical w/g.

Working of waveguide w/g is a hollow mettalic tube of uniform Crosssection for transmitting EM waves.



[Best dielectric Medium is air]

\# Conductivity of walls should be infinite so that there will be complete reflection of wave.

\# To improve conductivity and min^m losses, inner walls are either Coated with Gold, Silver OR Platinum.

Different type of Modes :-

(i) TEM mode :- $E \perp H \perp$ dir^n of Propagation. Hence $E_z = 0$, $H_z = 0$

\# Rectangular waveguide can't support TEM mode.

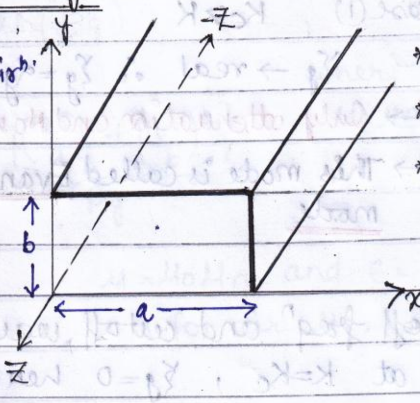
(ii) TE mode :- Only e field is $\perp$ dir^n of prop^n. Hence $E_z = 0$ and $H_z \neq 0$.

(iii) TM mode :- Only H field is $\perp$ to dir^n of prop^n. Hence $H_z = 0$ and $E_z \neq 0$.

(iv) HE mode / Hybrid Parameter :- Neither e field nor H field $\perp$ to dir^n of prop^n. Hence $E_z \neq 0$, $H_z \neq 0$

Rectangular w/g supports both TE and TM modes.

Wave propagation Rectangular w/g



* $(a > b)$
* $\frac{a}{b} \rightarrow$ aspect Ratio
* assume w/g is filled with dielectric med^m ($\sigma = 0$) and walls are Perfectly conducting.

Let EM wave is propagating in +z dir^n.

$\frac{\partial}{\partial z} = \gamma_g \rightarrow$ Propag^n const^n of wave in waveguide

$$\boxed{\begin{aligned} E_z(x,y,z) &= E_0 \sin(K_x x) \sin(K_y y) e^{-\gamma_g z} \\ H_z(x,y,z) &= H_0 \cos(K_x x) \cos(K_y y) e^{-\gamma_g z} \end{aligned}}$$

where

$K_x = \frac{m\pi}{a}$ and $K_y = \frac{n\pi}{b}$

Here m, n are called half wave variation in x and y dir^n respectively

Propagation Const^n of wave in w/g

$$\boxed{\gamma_g = \sqrt{(K_x^2 + K_y^2) - K^2}}$$

where $K^2 = \omega^2 \mu \epsilon$ and $\sqrt{K_x^2 + K_y^2} = K_c \leftarrow$ cutoff wave no.

Note :- For TEM mode $E \perp H \perp$ dir^n of prop^n. and

$\frac{\partial}{\partial x}(E \text{ OR } H) = \frac{\partial}{\partial y}(E \text{ or } H) = 0 \quad \therefore \quad K_x = \frac{\partial}{\partial x} = 0$, and $K_y = \frac{\partial}{\partial y} = 0$

Hence $\gamma_g = \sqrt{0 - K^2} = jK \Rightarrow \alpha_g = 0, \ \beta_g = \omega\sqrt{\mu\epsilon}$

$\therefore$ TEM has no cutoff freq^n and so not supported by Rectangular w/g. Rectangular w/g supports only TE_{mn} and TM_{mn}

$TE_{m \times n}$ Mode ($E_z = 0$)

Case (i) $m = n = 0$ only $H_z \neq 0$

($E_x = E_y = E_z = H_x = H_y = 0$) In this case no wave will propagate. Hence

TE_{00} is invalid mode

Case (ii) $m \neq 0, n = 0$

In TE_{m0}, only E_y will exist ($H_x \neq 0, H_z \neq 0$)

case (iii) $m = 0, n \neq 0$

In TE_{0n}, only E_x will exists ($H_y \neq 0, H_z \neq 0$)

$TM_{m \times n}$ mode ($H_z = 0$)

Case (i) $m = 0, n \neq 0$ OR $n = 0, m \neq 0$

($E_x = E_y = E_z = H_x = H_y = H_z = 0$)

No wave exists Hence

* TM_{m0} OR TM_{0n} are invalid mode
* In Rectangular w/g, TM_{mn} mode exists only when $m \neq 0, n \neq 0$. Hence Lowest order mode TM_{11}.

PART (I) Propagation of wave in rectangular W/g with TE_{mn} and TM_{mn} mode

(i) Propagation constⁿ.
$$\gamma_g = \sqrt{\underbrace{(K_x^2+K_y^2)}_{K_c^2 \leftarrow \text{cutoff wave number}} - K^2} \quad \text{where } K = \omega\sqrt{\mu\epsilon}$$

Case (i) $K_c > K$

$\gamma_g \rightarrow$ real $\therefore \gamma_g = \alpha_g + j0$

↦ Only attenuation and No wave propⁿ.

↦ This mode is called Evanescent mode

Case (ii) $K_c < K$

$\gamma_g \longrightarrow$ Imaginary

$\gamma_g = 0 + j\beta_g$ and $\alpha_g = 0$

↦ Only propagation

↦ No attenuation

(ii) cutoff freqⁿ and cutoff wavelength. ($K_c = K$)

at $K = K_c$, $\gamma_g = 0$ hence $\alpha_g = \beta_g = 0$

$K = K_c$

(a) Cutoff freqⁿ. (f_c)

$\omega_c\sqrt{\mu\epsilon} = \sqrt{K_x^2+K_y^2}$.

$$\boxed{f_c = \frac{v}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

where $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$

for non magnetic Medⁿ.

$$\boxed{v = \frac{c}{\sqrt{\epsilon_r}}}$$

(b) Cutoff wavelength (λ_c) $\lambda_c = \frac{v}{f_c}$

$$\boxed{\lambda_c = \frac{2\pi}{\sqrt{(m\pi/a)^2 + (n\pi/b)^2}}}$$

Note:- If there are various mode with cutoff freqⁿ $f_{c_1}, f_{c_2}, f_{c_3}$ and freqⁿ of operation is f then for propagation of these mode

$\boxed{f > f_{c_1}}$ $\boxed{f > f_{c_2}}$ $\boxed{f > f_{c_3}}$

* Dominant Mode:- Mode that has lowest cutoff freqⁿ (highest wavelength) and minᵐ distortion

a) For TE_{mn} mode

* If $a > b$, TE_{10} is dominant mode for which cutoff freqⁿ is $\boxed{f_c = v/2a}$

* If $a < b$, TE_{01} is dominant mode for which cutoff freqⁿ $\boxed{f_c = v/2b}$

b) For TM_{mn} mode

* TM_{0n} OR TM_{m0} → Evanescent Mode.

* TM_{11} is dominant mode.

* For rectangular w/g, Overall dominant mode is TE_{10}.

* For Circular W/g dominant mode is TE_{11}.

Degenerate Mode: Two modes have same cutoff freqⁿ for diffⁿ value of 'm' and 'n'.

Different value of m and n suggest different field distortion.

Note:- If mode is not mentioned in problem then always take dominant mode.

(iii) Phase constant of waveguide (β_g)

$$\gamma_g = \sqrt{K_x^2 + K_y^2 - K^2}$$

where $\omega_c^2 \mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

when $K > K_c$, $\gamma_g = \underset{(\alpha_g = 0)}{0} + j \underbrace{\sqrt{K^2 - \left\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right\}}}_{\beta_g}$

$$\boxed{\beta_g = \omega\sqrt{\mu\epsilon}\sqrt{1 + \left(\frac{f_c}{f}\right)^2}}$$

$\mu = \mu_0\mu_r$ and $\epsilon = \epsilon_0\epsilon_r$
(for air $\mu_r = 1$, $\epsilon_r = 1$)

(iv) phase Velocity of Guide (v_p) $v_p = \omega/\beta_g$

$$\boxed{v_p = \frac{v}{\sqrt{1-(f_c/f)^2}}}$$

where $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$

for non magn medm.

$$\boxed{v = c/\sqrt{\epsilon_r}} \text{ m/s}$$

Note:-(a) For air filled w/g ($\epsilon_r = 1$) $v = c$ therefore $v_p = \frac{c}{\sqrt{1-(f_c/f)^2}}$

Phase velocity is always greater than speed of light

(b) at $f = f_c$, phase velocity becomes infinite.

(v) Group velocity (v_g) $v_g = \frac{\partial\omega}{\partial\beta_g}$

$$\boxed{v_g = v\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

where $v = c/\sqrt{\mu_r\epsilon_r}$ m/s.

Note:- (a) $\boxed{v_p \cdot v_g = v^2}$

(b) For Non dispersive Medm
Phase velocity = Group velocity
∵ velocity does not depends upon freq^n.

(vi) Guide Wavelength. ($\lambda_g = v_p/f$)

$$\boxed{1/\lambda^2 = 1/\lambda_c^2 + 1/\lambda_g^2}$$

$$\boxed{\lambda_g = v/f\sqrt{1-(f_c/f)^2}}$$ where ($\lambda = v/f$)

(vii) Guide Impedence $\eta_g = \frac{E_x}{H_y} = -E_y/H_x$

For TE mode

$$\boxed{\eta_g = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

For TM mode

$$\boxed{\eta_g = \eta\sqrt{1-(f_c/f)^2}}$$

where $\eta = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}}$

η_0: intrinsic Imp^n of air.