Properties of FT →

* (1.) Linearity → $a_1 x_1(t) + a_2 x_2(t) \rightleftharpoons a_1 X_1(\omega) + a_2 X_2(\omega)$

* (2.) Time reversal → $x(-t) \rightleftharpoons X(-\omega)$

* (3.) Conjugation → $x^*(t) \rightleftharpoons X^*(-\omega)$

* (4.) Time shifting → $x(t-t_0) \rightleftharpoons X(\omega) e^{-j\omega t_0}$

* (5.) Time scaling → $x(at) \rightleftharpoons \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$, $a \neq 0$

* (6.) Freq. shifting → $e^{-j\omega_0 t} x(t) \rightleftharpoons X(\omega + \omega_0)$

* (7.) Diff. in time → $\frac{d^n x(t)}{dt^n} \rightleftharpoons (j\omega)^n X(\omega)$

* (8.) Integration in time → $\int_{-\infty}^{t} x(t)\,dt \rightleftharpoons \frac{X(\omega)}{j\omega} + \pi X(0) \cdot \delta(\omega)$

  where; $X(0) = X(\omega)\Big|_{\omega=0}$

* (9.) Convolution in time → $x(t) * x_2(t) \rightleftharpoons [X_1(\omega) \cdot X_2(\omega)]$

* (10.) multiplication in time → $x_1(t) \cdot x_2(t) \rightleftharpoons \frac{1}{2\pi}[X_1(\omega) * X_2(\omega)]$

  $x_1(t) \cdot x_2(t) \rightleftharpoons X_1(f) * X_2(f)$

* (11.) Diff. in freq. → $t^n x(t) \rightleftharpoons (j)^n \frac{d^n X(\omega)}{d\omega^n}$

* (12.) Parseval's energy theorem → $E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega.$

* (13.) Modulation → $x_1(t) \cdot \cos\omega_0 t \rightleftharpoons \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$

  $x(t) \cdot \sin\omega_0 t \rightleftharpoons \frac{j}{2}[X(\omega+\omega_0) - X(\omega-\omega_0)]$

* (14.) Area of time-domain → $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$

  $\Big\downarrow \omega = 0$

  $X(0) = \int_{-\infty}^{\infty} x(t) \cdot dt$

Eg:- $x(t) = e^{-at}u(t),\ a>0 \rightleftharpoons X(\omega) = \frac{1}{a+j\omega}$

$$\text{area of } x(t) = X(\omega)\Big|_{\omega=0} = \frac{1}{a}$$

* (15.) Area under Freq. domain →

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, d\omega$$

$$\int_{-\infty}^{\infty} X(\omega)\, d\omega = 2\pi x(0)$$

$$\boxed{\text{Area under } X(\omega) = 2\pi\, x(t)\Big|_{t=0}}$$

DATE-20/10/14

Que → $x(t) = e^{at}u(-t) \rightleftharpoons X(\omega) = ?\quad a>0$

Sol^n →

$$e^{-at}u(-t) \rightleftharpoons X(\omega)$$

$\downarrow (t = -t) \qquad \downarrow (\omega = -\omega)$ → time reversal.

$$e^{at}u(-t) \rightleftharpoons \frac{1}{a-j\omega}$$

Que → $y(t) = e^{-a|t|},\ a>0 \rightleftharpoons Y(\omega) = ?$

Sol^n →

$$y(t) = e^{-a|t|}$$

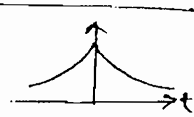
$$= \begin{cases} e^{at}, & t<0 \\ e^{-at}, & t>0 \end{cases}$$

$$= e^{at}u(-t) + e^{-at}u(t)$$

$$Y(\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$Y(\omega) = \frac{2a}{a^2+\omega^2}$$

$e^{-a|t|},\ a>0 \rightleftharpoons \frac{2a}{a^2+\omega^2}$



* Property of duality →

(1.)

$$x(t) \rightleftharpoons X(\omega)$$
$$(\omega = t) \qquad (t = -\omega)$$
$$X(t) \rightleftharpoons 2\pi\, x(-\omega)$$

$$x(t) \rightleftharpoons X(f)$$
$$(f = t) \qquad (t = -f)$$
$$X(t) \rightleftharpoons x(-f)$$

Q.→ $x(t) = \dfrac{1}{a+jt} \rightleftharpoons X(\omega) = ?$

Sol^n →

$$x(t) = \frac{1}{a+jt}$$
$$(t = \omega)$$
$$e^{-at}u(t), a>0 \rightleftharpoons \frac{1}{a+j\omega}$$
$$(\omega = t) \qquad (t = -\omega)$$
$$\boxed{\frac{1}{a+jt} \rightleftharpoons 2\pi e^{a\omega}u(-\omega),\ a>0}$$

Q.→ $x(t) = \dfrac{2a}{a^2+t^2} \rightleftharpoons X(\omega) = ?$

Sol^n →

$$x(t) = \frac{2a}{a^2+t^2}$$
$$(t = \omega)$$
$$e^{-a|t|}, a>0 \rightleftharpoons \frac{2a}{a^2+\omega^2}$$
$$(\omega = t) \qquad (t = -\omega)$$
$$\frac{2a}{a^2+t^2} \rightleftharpoons 2\pi e^{-a|-\omega|},\ a>0$$
$$\boxed{\frac{2a}{a^2+t^2} \rightleftharpoons 2\pi e^{-a|\omega|},\ a>0}$$

***

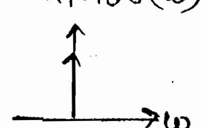
$$\boxed{A_0\,\delta(t) \rightleftharpoons A_0}$$

Q.→ $x(t) = A_0 \rightleftharpoons X(\omega) = ?$

Sol^n →

$$A_0\,\delta(t) \rightleftharpoons A_0$$
$$\omega = t \qquad t = -\omega$$
$$A_0 \rightleftharpoons \cancel{2\pi A_0[\delta(\omega)]}\ 2\pi A_0\,\delta(-\omega)$$

$$\boxed{A_0 = \text{dc signal} \rightleftharpoons 2\pi A_0\,\delta(\omega)}$$



∞

Q → Find $Y(\omega)$ in terms of $X(\omega)$

Soln →

$$x(t) = X(\omega)$$

$$y(t) = Y(\omega)$$

(i) $y(t) = e^{j2t}x(t)$

Soln → $Y(\omega) = X(\omega-2)$ } freq. shifting property

(ii) $y(t) = x(-2t)$

Soln →

Q → $Y(\omega) = \frac{1}{2}X\left(\frac{-\omega}{2}\right)$ } time scaling

(iii) $y(t) = x(2t-3)$

Soln → $y(t) = x(2t-3) = x\left[2\left(t-\frac{3}{2}\right)\right]$

$$x(t) \longrightarrow x(2t) \longrightarrow y(t) = x\left[2(t-1.5)\right] = f(t-1.5)$$

$$X(\omega) \quad F(\omega) = \frac{1}{2}X\left(\frac{\omega}{2}\right) \quad Y(\omega) = F(\omega)\,e^{-j1.5\omega}$$

$$= \frac{1}{2}X\left(\frac{\omega}{2}\right)e^{-j1.5\omega}$$

(OR)

$$y(t) = x(2t-3) = x\left[2(t-1.5)\right]$$

$$= \frac{1}{2}X\left(\frac{\omega}{2}\right)e^{-j\omega 1.5}$$

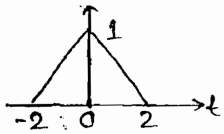
(iv) $y(t) = x(-2t-4)$

Soln → $y(t) = x\left[-2(t+2)\right]$

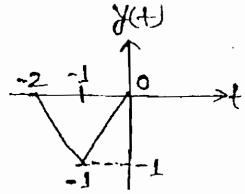
$t_0 = 2$

$$Y(\omega) = \frac{1}{2}X\left(\frac{-\omega}{2}\right)e^{j\omega \cdot 2}$$

(V.) $x(t)$



$y(t)$

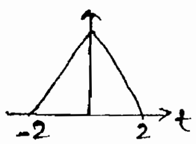


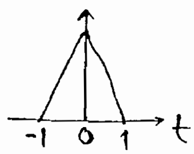
$Y(\omega) = ?$

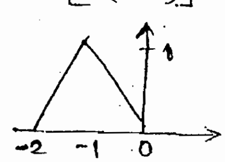
Sol^n →

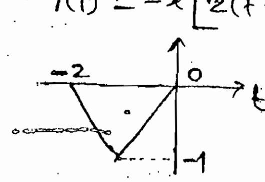
$T_{01} = 4 \quad T_{02} = 2$

$x(t) \longrightarrow x(2t) \longrightarrow x[2(t+1)] \longrightarrow y(t) = -x[2(t+1)]$









$y(t) = -x[2(t+1)]$

$Y(\omega) = -\frac{1}{2} X\left(\frac{\omega}{2}\right) e^{j\omega} \quad (t_0 = 1)$

Q.→ $y(t) = x(t) * h(t)$ ------ (i)

$g(t) = x(3t) * h(3t)$ ------ (ii)

If $g(t) = Ay(Bt)$ then calculate values of A & B.

Sol^n → from eq^n (i)

$Y(\omega) = X(\omega) H(\omega)$ ------ (iii)

From eq^n(ii)

$G(\omega) = \left[\frac{1}{3} X\left(\frac{\omega}{3}\right)\right]\left[\frac{1}{3} H\left(\frac{\omega}{3}\right)\right]$

$G(\omega) = \frac{1}{9}\left[X\left(\frac{\omega}{3}\right) H\left(\frac{\omega}{3}\right)\right]$

$= \frac{1}{9}\left[Y\left(\frac{\omega}{3}\right)\right]$ from eq^n (iii)

$= \frac{1}{3}\left[\frac{1}{3} Y\left(\frac{\omega}{3}\right)\right]$

$g(t) = \frac{1}{3} y(3t)$

$g(t) = Ay(Bt) = \frac{1}{3} y(3t)$

$\boxed{A = \frac{1}{3},\ B = 3}$

2nd method →

$x(t) * h(t) = y(t)$

$x(at) * h(at) = \frac{1}{|a|} y(at)$

$\downarrow (a=3)$

$x(3t) * h(3t) = \frac{1}{3} y(3t)$

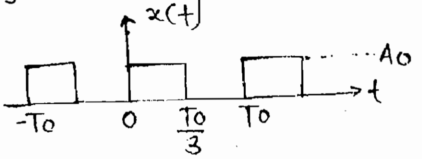
$g(t) = Ay(Bt)$

By comparision;

$\boxed{A = \frac{1}{3} \text{ (or) } B = 3}$

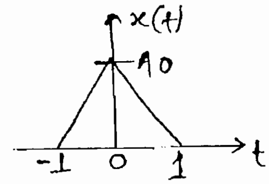
Avg. Value →

(1.)



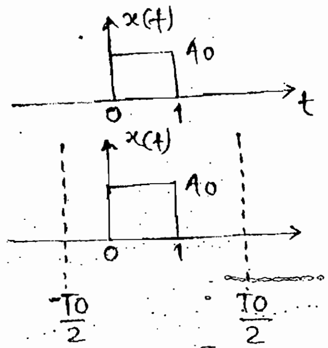
$$\boxed{avg. = \frac{Ao}{3}}$$

(3.)



$$\boxed{avg = Ao}$$

(2.)

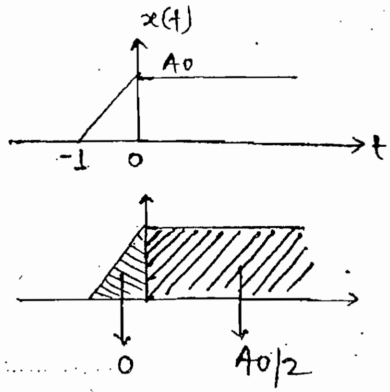


$$Avg. = \lim_{To\to\infty} \frac{1}{To}\int_{-To/2}^{To/2} x(t)\,dt$$

$$= \lim_{To\to\infty} \frac{1}{To}\int_{0}^{1} Ao\,dt = \frac{Ao}{To\to\infty} = 0$$

$$\boxed{Avg. = 0}$$

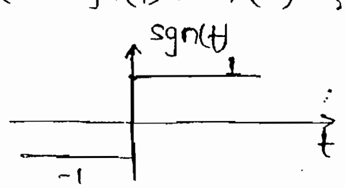
(4.)



$$avg = \frac{Ao}{2}$$

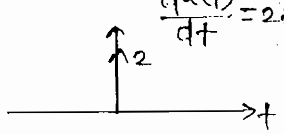
★★★ For any finite duration pulse avg. value will be = 0

Q.→ $x(t) = sgn(t) \rightleftharpoons X(\omega) = ?$



Sol^n →

$$\frac{dx(t)}{dt} = 2\delta(t)$$



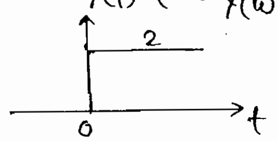
$$\frac{dx(t)}{dt} = 2\delta(t)$$

↓ FT.

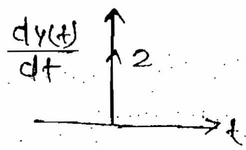
$$j\omega X(\omega) = 2$$

$$\boxed{x(t) = sgn(t) \rightleftharpoons X(\omega) = \frac{2}{j\omega}}$$

Que.→ $y(t) \rightleftharpoons Y(\omega) = ?$



Soln→



$\longrightarrow avg = \frac{2}{2}$

$= 1$

$= 2\pi\delta(\omega)$

$\frac{dy(t)}{dt} = 2\delta(t)$

$[j\omega]\, Y(\omega) = 2$

$\boxed{Y(\omega) = \frac{2}{j\omega}}$ X

2nd method →

$y(t) = 1 + x(t)$

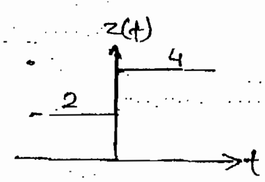
$\big\downarrow$ (FT)

$Y(\omega) = 2\pi\delta(\omega) + X(\omega)$

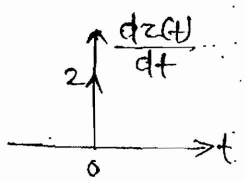
$= \frac{2}{j\omega} + 2\pi\delta(\omega)$

$\boxed{Y(\omega) = \frac{2}{j\omega} + 2\pi\delta(\omega)}$

Que.→



Soln→



$\frac{dz(t)}{dt} = 2\delta(t)$

$j\omega Z(\omega) = 2$

$\boxed{Z(\omega) = \frac{2}{j\omega}}$ X

$Avg. = \frac{4+2}{2} = 3$

$\downarrow$

$3 \times 2\pi\delta(\omega)$

$6\pi\delta(\omega)$

$\boxed{Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)}$

2nd method →

$z(t) = 3 + x(t)$

$\big\downarrow$ FT

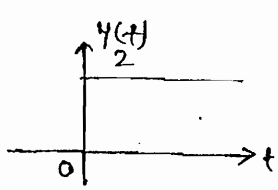
$Z(\omega) = 6\pi\delta(\omega) + \cancel{\frac{2}{j\omega}}\, X(\omega)$

$= X(\omega) + 6\pi\delta(\omega)$

$Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)$

$\boxed{Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)}$

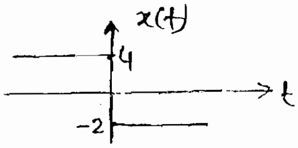
**



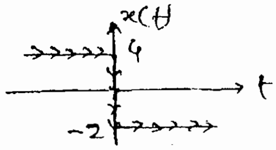
$2u(t) = \left[\frac{2}{j\omega} + 2\pi\delta(\omega)\right]$

$\frac{2u(t)}{2} = \frac{1}{2}\left[\frac{2}{j\omega} + 2\pi\delta(\omega)\right]$

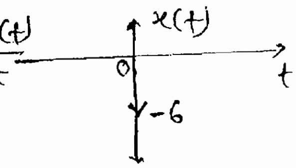
Q.→



Sol^n→



$\frac{dx(t)}{dt}$



$$\frac{dx(t)}{dt} = -6\delta(t)$$

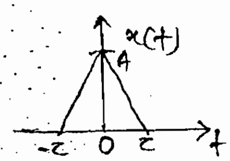
$$j\omega X(\omega) = -6$$

$$X(\omega) = \frac{-6}{j\omega}$$

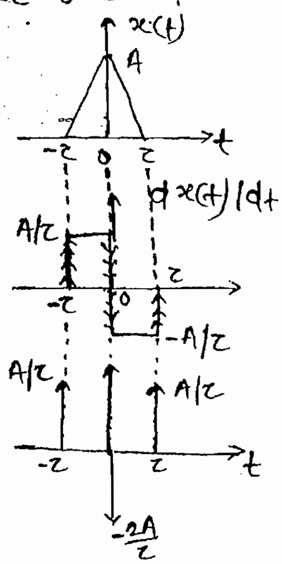
$$Avg. = \frac{4-2}{2} = 1$$

$$\boxed{X(\omega) = \frac{-6}{j\omega} + 2\pi\delta(\omega)}$$

Q.→



Sol^n→



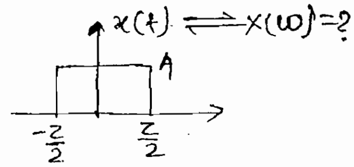
$\frac{d^2x(t)}{dt}$

$$\frac{d^2x(t)}{dx} = \frac{A}{\tau}\delta(t+\tau) + \frac{A}{\tau}\delta(t-\tau) - \frac{2A}{\tau}\delta(t)$$

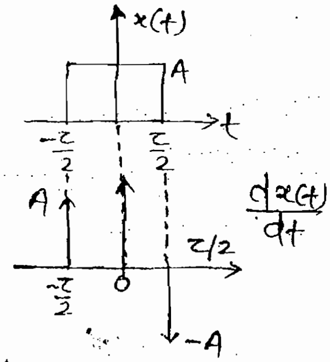
$$\Big\downarrow FT$$

$$(j\omega)^2 X(\omega) = A\, e^{j\omega\tau} \; A\, e^{-j\omega\tau}$$

Que.→



Sol^n→



$$\frac{dx(t)}{dt} = A\delta\left(t+\frac{\tau}{2}\right) - A\delta\left(t-\frac{\tau}{2}\right)$$

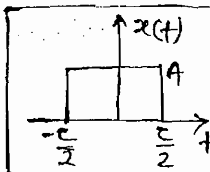
$$j\omega X(\omega) = Ae^{j\omega\tau/2} - Ae^{-j\omega\tau/2}$$

$$X(\omega) = \frac{A}{j\omega} \times \left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$$

$$= \frac{A}{j\omega} \times 2j \times \sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{2A}{\omega}\left[\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}\right] \times \left(\frac{\omega\tau}{2}\right)$$

$$= A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$



$$X(\omega) \rightleftharpoons A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$$X(\omega) = \frac{A}{\tau(-\omega^2)}\left[\left(e^{j\omega\tau} + e^{-j\omega\tau}\right) - 2\right]$$

$$X(\omega) = \frac{A}{-\tau\omega^2}\left[2\cos\omega\tau - 2\right]$$

$$X(\omega) = \frac{2A}{+\tau\omega^2}(1 - \cos\omega\tau)$$

$$= \frac{2A}{\tau\omega^2}\, 2\left[\sin^2\left(\frac{\omega\tau}{2}\right)\right]$$

$$= \frac{2A}{\tau\omega^2} \times 2\, \frac{\sin^2\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)^2} \times \left(\frac{\omega\tau}{2}\right)^2$$

$$\boxed{X(\omega) = A\tau\, Sa^2\left(\frac{\omega\tau}{2}\right)}$$

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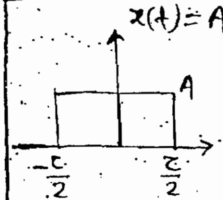
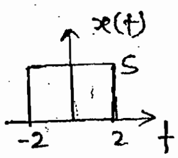
$x(t) = A\,rect\left(\frac{A}{\tau}\right)$



$$\Longleftrightarrow X(\omega) = A\tau\, Sa\left(\frac{\omega\tau}{2}\right) = Area\ Sa\left(\frac{\omega\, duration}{2}\right)$$

Q.→

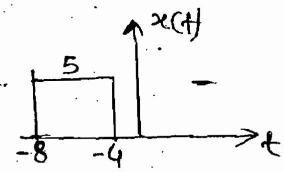


Sol^n→ $X(\omega) = 20\, Sa\left(\omega \frac{4}{2}\right)$

$$= 20\, Sa(2\omega)$$

$$\boxed{X(\omega) = 20\, Sa(2\omega)}$$

Q.→



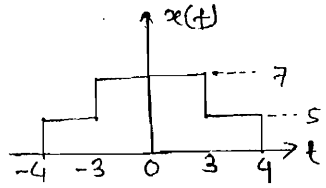
Sol^n→ $y(t) = x(t+6)$

$$Y(\omega) = X(\omega)e^{j\omega 6}$$

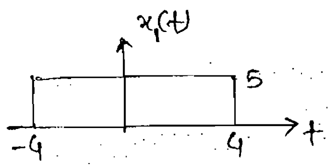
$$= 20\, Sa(2\omega)\, e^{j\omega 6}$$

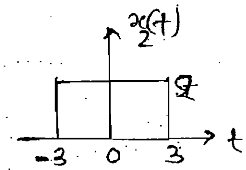
$$\boxed{Y(\omega) = 20\, Sa(2\omega)\, e^{j\omega 6}}$$

Q.→



Soln→





$X_1(\omega) = 40 Sa(4\omega)$ $\qquad$ $X_2(\omega) = 12 Sa(3\omega)$

$$x(t) = x_1(t) + x_2(t)$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$\therefore X(\omega) = 40 Sa(4\omega) + 12 Sa(3\omega)$$

Q.→ $x(t) = rect\left(t - \frac{1}{2}\right)$

$y(t) = x(t) + x(-t) \rightleftharpoons Y(\omega)$ ... Find $Y(\omega) = ?$

(a.) $sinc\left(\frac{\omega}{2\pi}\right)$ $\qquad$ (c.) $2 sinc\left(\frac{\omega}{2\pi}\right) cos\left(\frac{\omega}{2}\right)$

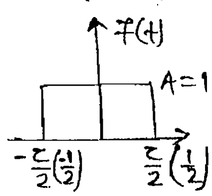
(b.) $2 sinc\left(\frac{\omega}{2\pi}\right)$

Soln→ $\quad x(t) = rect\left(t - \frac{1}{2}\right)$

$$x(-t) = rect\left(-t - \frac{1}{2}\right)$$

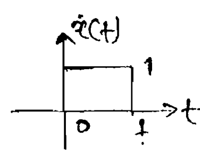
$$y(t) = x(t) + x(-t) = rect\left(t - \frac{1}{2}\right) + rect\left(-t - \frac{1}{2}\right)$$

$$f(t) = rect(t) = A\, rect\left(\frac{t}{\tau}\right)$$



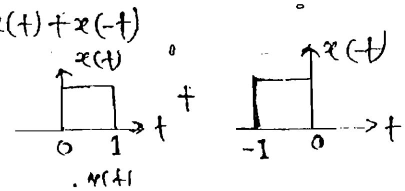
$\because Sa(k) = sinc\left(\frac{k}{\pi}\right)$

$$x(t) = f\left(t - \frac{1}{2}\right)$$



$$y(t) = x(t) + x(-t)$$

$=$



$$Y(\omega) = 2Sa(\omega) = 2\,\text{sinc}\left(\frac{\omega}{\pi}\right)$$

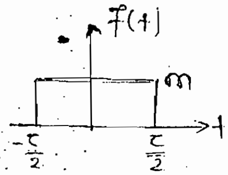
$$= \frac{2\sin\omega}{\omega} = 2\cancel{x}\left(\frac{\sin\frac{\omega}{2}}{\omega/2}\right) \times \cos\frac{\omega}{2} = 2Sa\left(\frac{\omega}{2}\right)\cos\left(\frac{\omega}{2}\right)$$

$$= \cancel{\frac{2\times \sin\frac{\omega}{2}\cos\frac{\omega}{2}}{\omega}} \qquad = 2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)\cdot\cos\left(\frac{\omega}{2}\right)$$

$$\boxed{Y(\omega) = 2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)\cdot\cos\left(\frac{\omega}{2}\right)}$$

Q.→ $x(t) = A_0 Sa(t) \rightleftharpoons$ Draw FT $X(\omega)$

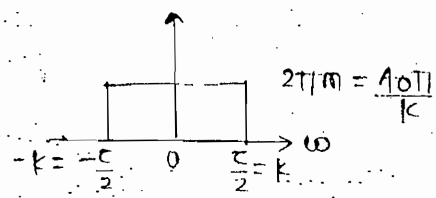
Sol^n→



$$\rightleftharpoons m\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$(\omega = t)$ $\qquad$ $(t = -\omega)$

$$m\tau\, Sa\left(\frac{t\tau}{2}\right) \rightleftharpoons 2\pi f(-\omega)$$



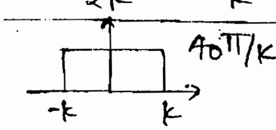
$$m\tau\, Sa\left(\frac{t\tau}{2}\right) = A_0 Sa(kt)$$

$$m\tau = A_0, \quad k = \frac{\tau}{2}$$

$$2\pi m = 2\pi\cdot\frac{A_0}{\tau} = \frac{2\pi \times A_0}{2k} = \frac{A_0\pi}{k}$$

★★★

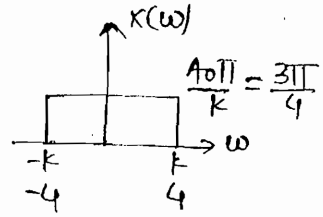
$A_0 Sa(kt) \rightleftharpoons$



Q.→ $x(t) = 3Sa(4t) \rightleftharpoons X(\omega)$

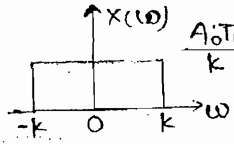
Sol^n→ $\quad A_0 = 3 \qquad A_0 Sa(kt)$

$k = 4$



Q→ Calculate area ~~of~~ energy of $x(t) = Sa(t)$

Soln→ $x(t) = Sa(t)$



$\frac{A_0 \pi}{k} = \pi \Rightarrow \cancel{A_0 Sa(t)}\ A_0 Sa(kt)$

$x(t) = A_0 Sa(kt) = Sa(t)$

$A_0 = 1,\ k = 1$

area under time-domain →

area of $x(t) = X(\omega)|_{\omega=0}$

$\boxed{A = \pi}$

Parseval's energy theorem →

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} \pi^2 d\omega$$

$\boxed{E = \pi}$

Q.→ $h(t) \rightleftharpoons H(\omega) = \frac{2\cos\omega \cdot \sin 2\omega}{\omega}$

Find $h(0) = ?$

(a) 1/4 (b) 1/2 (c) 1 (d) 2

Soln→ $H(\omega) = \frac{2\cos\omega \cdot \sin 2\omega}{\omega}$

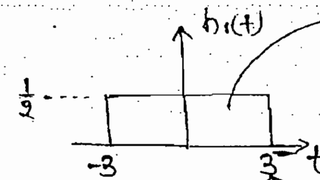
$= \frac{\sin(3\omega) + \sin\omega}{\omega}$

$= \frac{\sin(3\omega)}{\omega} + \frac{\sin\omega}{\omega}$

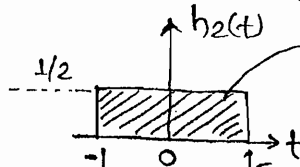
$= \frac{3\sin(3\omega)}{3\omega} + \frac{\sin\omega}{\omega}$

$H(\omega) = 3Sa(3\omega) + Sa(\omega)$

$= H_1(\omega) + H_2(\omega)$



$\rightleftharpoons H_1(\omega) = 3Sa(3\omega)$



$\rightleftharpoons H_2(\omega) = Sa(\omega)$

$h(t) = h_1(t) + h_2(t)$

$\downarrow t=0$

$h(0) = h_1(0) + h_2(0)$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

Area

$x(t) \longrightarrow A$

$x(kt) \longrightarrow \frac{A}{k}$

$k\,x(t) \longrightarrow kA$

$Sa(\omega) \longrightarrow \pi$

$Sa(3\omega) \longrightarrow \pi/3$

$3\,Sa(3\omega) \longrightarrow 3\times\frac{\pi}{3} = \pi$

2nd method →

Area under freq. domain

$2\pi h(0) = \text{Area of } H(\omega)$

$h(0) = \dfrac{\text{Area of } H(\omega)}{2\pi}$

$h(0) = \dfrac{2\pi + \pi}{2\pi} = 1$

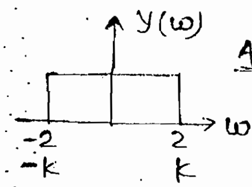
$\boxed{h(0) = 1}$

Q.→ $y(t) = x(t)\cos t \rightleftharpoons Y(\omega) = \begin{cases} 2, & |\omega| \le 2 \\ 0 & \text{otherwise.} \end{cases}$

Find $x(t)$

(a) $\frac{4}{\pi}\frac{\sin t}{t}$ (b) $\frac{2\sin t}{t}$ (c) $4\frac{\sin t}{t}$ (d) $2\pi\cdot\frac{\sin t}{t}$

Soln → **1st method →**



$\frac{A_0\pi}{k} = 2$

$y(t) = A_0 Sa(kt) \Longrightarrow$

$= \frac{4}{\pi} Sa(2t)$

$k = 2, A_0 = 4/\pi$

$= \frac{4}{\pi}\cdot\frac{\sin 2t}{2t} = \left[\frac{4}{\pi}\cdot\frac{\cancel{2}\sin t}{\cancel{2}t}\right]\cdot\cos t$

$= \left[\frac{4}{\pi}\cdot\frac{\sin t}{t}\right]\cdot\cos t$

$= x(t)\cdot\cos t$

2nd method →

area under freq. domain

$2\pi y(0) = \text{area of } Y(\omega)$

$2\pi y(0) = 8$

$y(0) = \frac{8}{2\pi} = \frac{4}{\pi} = x(0)$

$y(0) = x(0)$ at ...

Q.→ Find FT of $\cos\omega_0 t$

$$x(t) = \cos\omega_0 t \Longleftrightarrow X(\omega) = ?$$

Sol^n→

$$x(t) = \cos\omega_0 t$$

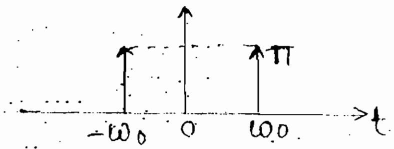
$$x(t) = \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$$

$$X(\omega) = \frac{1}{2}\left[2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right]$$

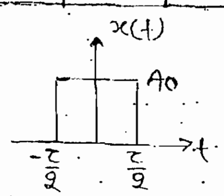
$$X(\omega) = \pi\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\pi$$

$$A_0 = 2\pi A_0\delta(\omega)$$
$$\downarrow A_0 = 1$$
$$1 = 2\pi\delta(\omega)$$
$$1 \cdot e^{j\omega_0 t} = 2\pi\delta(\omega-\omega_0)$$
$$e^{-j\omega_0 t} = 2\pi\delta(\omega+\omega_0)$$

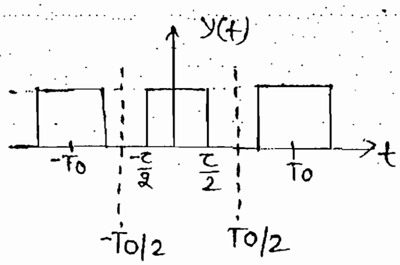
$$\cos\omega_0 t = \pi\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\pi$$



✱ Calculation of C_n by using FT→



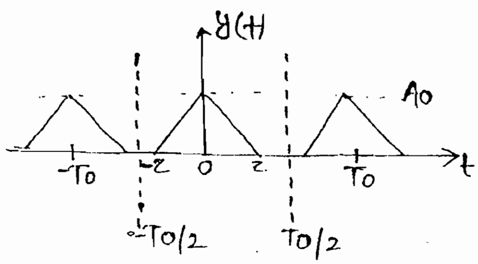
$$X(\omega) = A_0\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$



$$\boxed{C_n = \frac{X(n\omega_0)}{T_0}}$$

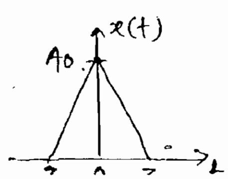
$$X(\omega) = \frac{A_0\tau}{T_0} Sa\left(\frac{n\omega_0\tau}{2}\right)$$

Q.→



$$C_n = ?$$

Sol^n→



$$X(\omega) = A_0\tau\, Sa^2\left(\frac{\omega\tau}{2}\right)$$

$$C_n = \frac{X(n\omega_0)}{T_0} = \frac{A_0\tau}{T_0} Sa^2\left(\frac{n\omega_0\tau}{2}\right)$$

* FT for periodic signal →

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \qquad X(\omega) = ?$$

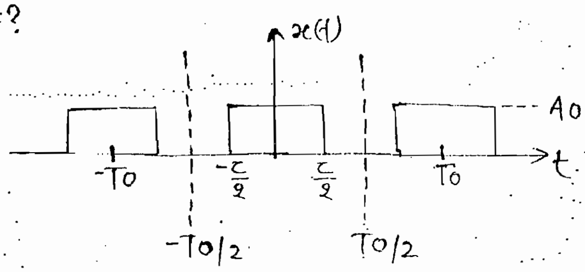
$$1 = 2\pi\,\delta(\omega)$$

$$C_n = 2\pi C_n\,\delta(\omega)$$

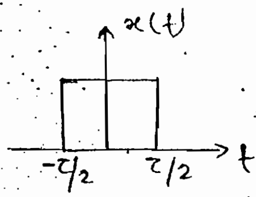
$$C_n e^{jn\omega_0 t} = 2\pi C_n\,\delta(\omega - n\omega_0) \quad \text{(Freq. shifting)}$$

$$\boxed{\sum C_n e^{jn\omega_0 t} = 2\pi \sum_{n=-\infty}^{\infty} C_n\,\delta(\omega - n\omega_0)}$$

Que.→ $X(\omega) = ?$



Soln→



$$x(\omega) \longrightarrow A_0 \tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

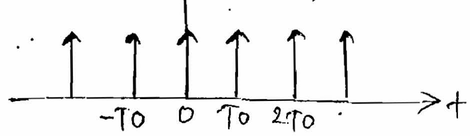
$$C_n = \frac{A_0\tau}{T_0}\, Sa\left(\frac{n\omega_0\tau}{2}\right)$$

$$X(\omega) = 2\pi \sum C_n\,\delta(\omega - n\omega_0)$$

$$= 2\pi \sum_{n=-\infty}^{\infty} \left[\frac{A_0\tau}{T_0}\, Sa\left(\frac{n\omega_0\tau}{2}\right)\right] \delta(\omega - n\omega_0)$$

Que.→ $X(\omega) = ?$

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

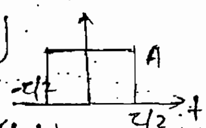
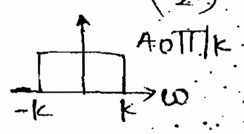
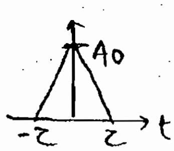


Soln→

$$X(\omega) = 2\pi \sum C_n\,\delta(\omega - n\omega_0)$$

$$= 2\pi \sum \frac{1}{T_0}\,\delta(\omega - n\omega_0)$$

*Important signal →

	$x(t)$	$X(\omega)$
(1.)	$\delta(t)$	1
(2.)	$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
(3.)	$sgn(t)$	$\frac{2}{j\omega}$
(4.)	A_0	$2\pi A_0\delta(\omega)$
(5.)	$e^{-at}u(t),\ a>0$	$\frac{1}{a+j\omega}$
(6.)	$e^{-a\lvert t\rvert}u(t),\ a>0$	$\frac{2a}{a^2+\omega^2}$
(7.)	$\cos\omega_0 t$	$\pi\left[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right]$
(8.)	$\sin\omega_0 t$	$\pi j\left[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right]$
(9.)	$A\,rect\left(\frac{t}{\tau}\right)$ 	$A\tau\, sa\left(\frac{\omega\tau}{2}\right)$
(10.)	$A_0\, sa(kt)$	
(11.)	Periodic sig.	$2\pi\sum_{n=-\infty}^{\infty} C_n\,\delta(\omega-n\omega_0)$
(12.)	$\sum\delta(t-nT_0)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$
(13.)		$A\tau\, sa^2\left(\frac{\omega\tau}{2}\right)$
~~(14.)~~	$\cancel{e^{j\omega_0 t}}$	$2\pi\delta(\omega-\omega_0)$
(15.)	$e^{-j\omega_0 t}$	$2\pi\delta(\omega+\omega_0)$

DATE- 21/10/14

* $x(t)$ - $X(\omega)$ pairs →

$x(t)$	$X(\omega)$
Real	CS
CS	Real
Img.	CAS
CAS	Img.
[illegible]	[illegible]

R+O	I+O
I+O	R+O

$x(t)$	$X(\omega)$
Continues	Non-periodic
Non-periodic	contineous
Discrete	periodic
periodic	discrete

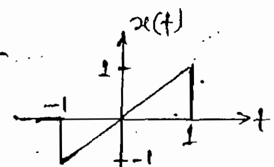
C+P → D+NP

C+NP → C+NP

D+P → D+P

D+NP → C+P

Q.)



$X(\omega) = ?$

(a.) $4\pi j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega}\right]$

(b.) $4\pi j\left[\frac{\cos\omega}{\omega^2} - \frac{\sin\omega}{\omega}\right]$

(c.) $2j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega^2}\right]$

(d.) $2j\left[\frac{\cos\omega}{\omega^2} - \frac{\sin\omega}{\omega}\right]$

Solⁿ → For solⁿ go through the option.

ans.(c)

I+O

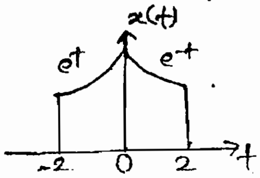
$x(t) \rightarrow R+O$

$X(\omega) \rightarrow I+O$

$\frac{\sin\omega}{\omega} = E$

$$X(\omega) = 2j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega^2}\right]$$

Q.→



$\Longrightarrow X(\omega) = ?$

(a) $2 - 2e^{-2}\sin 2\omega + 2\omega e^{-2}\sin 2\omega$

(b) $2 + 2e^{-2}\cos 2\omega - 2\omega e^{-2}\cos 2\omega$

(c.) $2 + 2e^{-2}\cos 2\omega - 2\omega e^{-2}\sin 2\omega$

(d.) $2 - 2e^{-2}\cos 2\omega + 2\omega e^{-2}\sin 2\omega$

Sol^n→ $x(t) = R+E$; $X(\omega) = R+E$

so; option (a) & (b) $\neq R+E$

Area under time domain;

$$X(0) = \int_{-\infty}^{\infty} x(t)dt$$

$$= \int_{-2}^{2} x(t)dt = 2\int_{0}^{2} x(t)dt$$

$$= 2\int_{0}^{2} e^{-t}dt$$

$$= \frac{2[e^{-t}]_0^2}{-1} = 2(1-e^{-2})$$

$$= 2-2e^{-2}$$

Now; put $x(0)$ in the option (c) & (D)

$\boxed{\text{Ans. (d)}}$

Q.→ $f(t) \rightleftharpoons F(\omega)$

$$g(t) = \int_{-\infty}^{\infty} F(u)e^{-jut}du$$

what is the relationship bet^n $f(t)$ & $g(t)$?

(a.) $g(t)$ would always be proportional to $f(t)$.

(b.) $g(t)$ would always be proportional to $f(t)$ is $f(t)$ is an even sig.

(c.) $g(t)$ would proportional to $f(t)$ only if $f(t)$ is sinusoidal f^n

(d) $g(t)$ would ~~area~~ never to be proportional to $f(t)$.

Sol^n→ IFT (inverse FT)

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

$$2\pi f(t) = \int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

$\downarrow (\omega = u)$

$$2\pi f(t) = \int_{-\infty}^{\infty} F(u)e^{jut}du$$

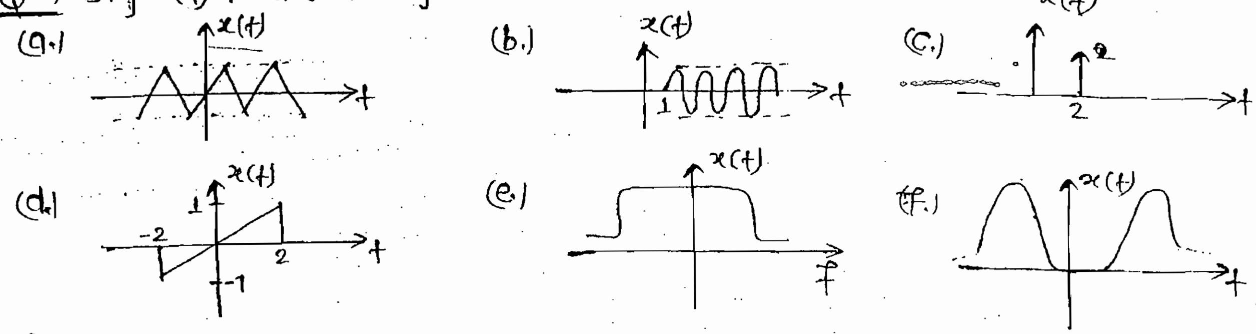
$\downarrow (t = -t)$

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(u)e^{-jut}du$$

$$\boxed{g(t) = 2\pi f(t)}$$

If $f(t)$ is even
$f(-t) = f(t)$

Q.→ sig. $x(t)$ is a real sig.



(i) $Re[X(\omega)] = 0$

(a) a,d (b) e,f (c) b,c (d) b,d.

Sol^n→

$x(t) \longrightarrow$ Real

$X(\omega) \longrightarrow CS_{0}$ (given)

$= \text{Real}[X(\omega)] + i\,\text{Img}[X(\omega)]$

$\downarrow$ even $\qquad \downarrow$ odd

$X(\omega) \rightarrow CS$ & nature is imag. odd.

so $x(t) \rightarrow R + O$.

$\boxed{\text{ans (a.)}}$

(ii) $\int_{-\infty}^{\infty} X(\omega)\, d\omega = 0$

(a) e (b) a, b, c, d, f (c) b, c (d) a, d, e, f.

Sol^n→ Area under freq. domain

$$2\pi x(0) = \int_{-\infty}^{\infty} X(\omega)\, d\omega$$

$$2\pi x(0) = 0$$

$$\boxed{x(0) = 0}$$

$\boxed{\text{ans (b)}}$

(iii) $\int_{-\infty}^{\infty} \omega X(\omega) d\omega = 0$

(a.) a, b, c, d, f (b) b, c, e, f (c) e (d) b, c

Sol^n→

$$x(t) \rightleftharpoons X(\omega)$$

$$\frac{dx(t)}{dt} = (j\omega) X(\omega)$$

$$\frac{1}{j}\frac{dx(t)}{dt} = \omega X(\omega)$$

$$y(t) = Y(\omega)$$

area under Freq. domain

$$2\pi y(0) = \int_{-\infty}^{\infty} Y(\omega) d\omega$$

$$= \int_{-\infty}^{\infty} \omega X(\omega) d\omega$$

$$2\pi y(0) = 0$$

$$y(0) = (0)$$

$$y(t)\Big|_{t=0} = 0$$

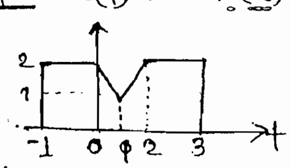
$$\frac{1}{j}\frac{dx(t)}{dt}\Big|_{t=0} = 0$$

$$\boxed{\frac{dx(t)}{dt} = 0}$$ (slope is zero at the origin)

$\boxed{\text{ans. (b)}}$

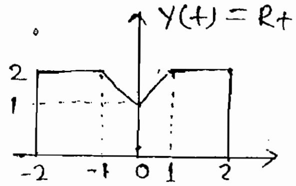
Q.→ $x(t) \rightleftharpoons X(\omega)$

Find $\angle X(\omega) = ?$



Sol^n→

$$Y(t) = R + E \rightleftharpoons Y(\omega) = R + E$$



$$\angle X(\omega) = \angle Y(\omega) + (-\omega)$$

$$x(t) = y(t-1)$$

$\because$ $Y(\omega) = R + E$ & $\angle Y(\omega) = 0$

where; $f(0^-) = |f(t)|_{t=0^-}$

$$f'(0^-) = \frac{df(t)}{dt}\Big|_{t=0^-} \qquad f''(0^-) = \frac{d^2 f(t)}{dt^2}\Big|_{t=0^-} \quad \cdots$$

(8.) Integration in time →

$$\int_{-\infty}^{t} f(t)\,dt = \begin{cases} \dfrac{F(s)}{s}, & \text{Bilateral FT.} \\ \dfrac{F(s)}{s} + \dfrac{\int_{-\infty}^{0^-} f(t)\,dt}{s}, & \text{unilateral LT.} \end{cases}$$

(9.) Convolution in time →

$$f_1(t) * f_2(t) = F_1(s) \cdot F_2(s)$$

(10.) multiplication in time →

$$f_1(t) \cdot f_2(t) = \frac{1}{2\pi j}\left[F_1(s) * F_2(s)\right]$$

(11.) Differentiation in freq →

$$t^n f(t) = (-1)^n \frac{d^n F(s)}{ds^n}$$

(12.) Integration in freq →

$$\frac{f(t)}{t} = \int_{s}^{\infty} F(s)\,ds$$

(13.) Initial value theorem →

$$x(0) = \lim_{s\to\infty}\left[sX(s)\right]$$

Condition:- It is applicable only for causal type signals.

i.e. $x(t) = 0;\ t < 0$

(14.) Final value theorem →

$$x(\infty) = \lim_{s\to 0}\left[sX(s)\right]$$

Condition →

(i.) It is applicable only for causal type signals.

i.e.; $x(t) = 0, t<0$

*** (ii) "$sX(s)$" should have only LHS poles in s-plance

Que → $F(s) = \frac{1}{s^2+1} \rightleftharpoons f(t)$. Calculate $f(\infty) = ?$

(a) -1 (b) 0 (c) 1 (d) $-1 \le f(\infty) \le 1$.

Sol^n → $sF(s) = \frac{s}{s^2+1}$

Poles:- $s = \pm j \neq$ LHS poles

* FVT is not applicable because $s=\pm j$ poles)

$f(t) = \sin t\, u(t)$

$f(\infty) = \sin\infty\, u(\infty)$

$(-1, 1)$

$f(\infty) = -1 \le f(\infty) \le 1$

Que → $F(t) = F(s) = \frac{1}{s(s-1)}$; $f(\infty) = ?$

(a) 0, (b) ∞ (c) -1 (d) 1

Sol^n → (1.) signal is causal because depand on past $(s-1)$

(2.) $sF(s) = \frac{s}{s(s-1)} = \frac{1}{s-1}$

Pole: $s=1 \neq$ LHS plane

FVT is not applicable.

$F(s) = \frac{1}{s(s-1)} = -\frac{1}{s} + \frac{1}{(s-1)}$

~~$f(t) = e^t u(t)$~~ $\quad f(t) = e^t u(t) - u(t)$

$f(\infty) = \infty - 1 + \infty$

$\boxed{F(\infty) \simeq \infty}$

Q→ $f(t) = -e^{-at}u(-t) \rightleftharpoons F(S) = ?,\ ROC = ?$

Sol^n →

$$f(t) = e^{-at}u(-t)$$

$$e^{-at}u(t) \rightleftharpoons \frac{1}{s+a} ;\ (\sigma > -a)$$

$$-e^{-at}u(t) \rightleftharpoons \frac{-1}{s+a} ;\ (\sigma > -a)$$

$\downarrow (t = -t)$ $\downarrow (s = -s)$ $\downarrow$ (By time Reversal) $(\sigma = -\sigma)$

$\because s = \sigma + j\omega$

$-s = -\sigma - j\omega$

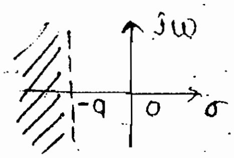
$$-e^{at}u(-t) \rightleftharpoons \frac{-1}{-s+a} ;\ (-\sigma > -a)$$

$\downarrow (a = -a)$ $\downarrow (a = -a)$

$$-e^{-at}u(-t) \rightleftharpoons \frac{-1}{-s-a} ;\ (-\sigma > a)$$

$$-e^{-at}u(-t) \rightleftharpoons \frac{1}{s+a} ;\ (\sigma < -a)$$

$$-e^{-at}u(-t) \rightleftharpoons \frac{1}{s+a} ;\ (\sigma < -a)$$



$(a=0)$

& also;

$$e^{-at}u(t) \rightleftharpoons \frac{1}{s+a} ;\ (\sigma > -a)$$

$(a=0)$

$$u(-t) \longrightarrow \frac{1}{s} ;\ \sigma < 0$$

$$u(t) \longrightarrow \frac{1}{s} ;\ \sigma > 0$$

$$\boxed{\begin{array}{l} e^{-at}u(t) \rightarrow \frac{1}{s+a} ;\ (\sigma > -a) \\ -e^{-at}u(-t) \longrightarrow \frac{1}{s+a} ;\ (\sigma < -a) \end{array}}$$

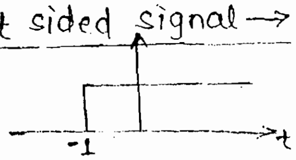
$$\boxed{\begin{array}{l} u(-t) \longrightarrow \frac{1}{s} ;\ \sigma < 0 \\ u(t) \longrightarrow \frac{1}{s} ;\ \sigma > 0 \end{array}}$$

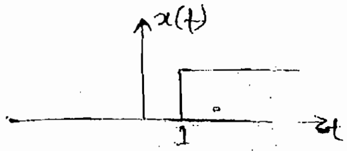
Region of convergance (ROC) → It is defined as the range of complex variable as in s-planes for which LT of signal is convergant (or) finite.

Properties →

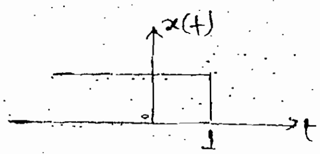
(i) ROC doesn't include any pole.

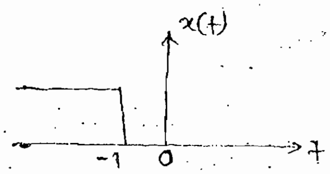
Right sided signal →



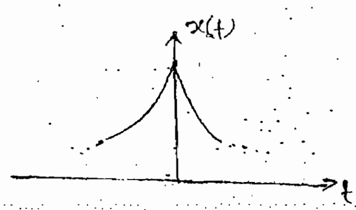


Left sided signal →





Both sided signal →



(ii) For right side signal ROC will be right side to the right most pole. For left sided signal ROC will be left side to the left most pole.

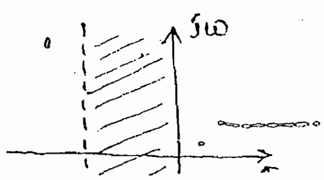
(iii) For stability ROC includes imaginary axis in s-plane.

(iv) For both sided sig. ROC is a strip in s-plane.

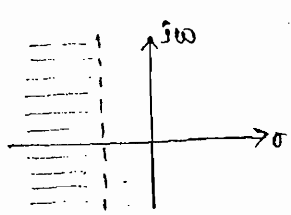
(v) for ∞ duration signal ROC is entire s-plane excluding possibly $s=0$ or $\pm\infty$.

Q.→ Check stability of LTI sys. of & comment about extention of $h(t)$.

(i) ROC: $\sigma > -1$

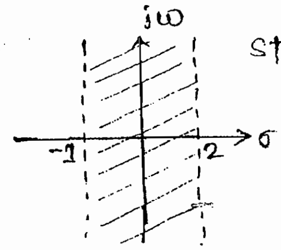


(ii) ROC: $\sigma < -2$



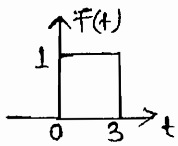
$h(t)$ = Unstable + LS

(3) ROC: $-1<\sigma<2$



strip type $\quad h(t) =$ stable + Both side.

Que.→



$\rightleftharpoons F(s) = ?\ ;\ ROC = ?$

Sol^n→ $F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st} dt$

$= \int_0^3 e^{-st} dt = \int_0^3 e^{-st} dt = \frac{(e^{-st})_0^3}{-s} = \frac{1-e^{-3s}}{s}$

At $(s=0)$ $\quad F(0) = \frac{1-e^{-3s}}{s} = \frac{0}{0}$ then use L-Hospital Rule

diff w.r.t. $s = 3e^{-3s}\Big|_{s=0} = 3$

at $(s=\infty)$

$F(\infty) = 0$

at $(s=-\infty)$ $\quad F(-\infty) = 3e^{-3s}\Big|_{(s=-\infty)} = 3e^{\infty} = \infty$

ROC : Entire s-plane excluding $s=-\infty$

$e^{2t}u(-t+3)$ $\qquad\qquad$ $e^{2t}u(-t-4)$

ROC: $\sigma<2$ $\qquad\qquad$ ROC: $\sigma<2$

Que.→ $f(t) = e^{-2t}u(-t) + e^{3t}u(t)$

Sol^n→ $f(t) = \underbrace{e^{-2t}u(-t)}_{\text{ROC: } \sigma<-2} + \underbrace{e^{3t}u(t)}_{\sigma>3}$

LT of $f(t)$ will not exist because of no common ROC

Que.→ $f(t) = -e^{-2t}u(-t) + e^{-3t}u(t)$

Sol^n→ $f(t) = -e^{-2t}u(-t) + e^{-3t}u(t)$

common ROC: $-3<\sigma<-2$ → strip

Q.→ $f(t) = e^{-3|t|}$; ROC = ?

Sol^n→ $f(t) = \begin{cases} e^{3t} & ; t<0 \\ e^{-3t} & ; t>0 \end{cases}$

ROC:- $-3<\sigma<3$ $\qquad f(t) = \underbrace{e^{3t}u(-t)}_{\sigma<3} + \underbrace{e^{-3t}u(t)}_{\sigma>-3}$

Que.→ $f(t) = e^{r(t)} = e^{t}u(t)$; ROC = ?

Sol^n→ $f(t) = \begin{cases} 1 & ; t<0 \\ e^t & ; t>0 \end{cases}$ (No common ROC)

$= \underbrace{u(-t)}_{\sigma<0} + \underbrace{e^t u(t)}_{\sigma>1}$

Que.→ $f(t) = e^{-at}u(t)$

Sol^n→ where $a = b+jc$

$f(t) = e^{-(b+jc)t}u(t)$

For existance of LT

$= \int_{-\infty}^{\infty} |f(t)\, e^{-\sigma t}|\, dt < \infty = \int_{-\infty}^{\infty} |e^{-(b+jc)t} e^{-\sigma t}|\, dt < \infty = \int_{-\infty}^{\infty} |e^{-(b+\sigma)t}|\, |\overbrace{e^{-jct}}^{1}|\, dt < \infty$

$= \int_{0}^{\infty} e^{-(\sigma+b)t}\, dt < \infty = (\sigma+b)>0 = \sigma>-b = \sigma>-Re(a)$

Que.→ $f(t) = \cos\omega_0 t\, u(t) \rightleftharpoons F(s) = ?$, ROC = ?

Sol^n→ $\cos\omega_0 t = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$f(t) = \frac{1}{2}\left[e^{j\omega_0 t}u(t) + e^{-j\omega_0 t}u(t)\right]$

$e^{-at}u(t) \rightleftharpoons \dfrac{1}{s+a}$; $\sigma>-Re(a)$

$a = 0+j\omega_0$

$e^{-j\omega_0 t}u(t) \rightleftharpoons \dfrac{1}{s+j\omega_0}$; $\sigma>0$

$a = 0-j\omega_0$

$e^{j\omega_0 t}u(t) \rightleftharpoons \dfrac{1}{s-j\omega_0}$; $\sigma>0$

Important signals →

$F(t)$	$F(S)$	ROC
$\delta(t)$	1	entire s-plane
$u(t)$	$1/s$	$\sigma > 0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\sigma > -a$
$e^{-at}u(-t)$	$\frac{1}{s+a}$	$\sigma < -a$
$t u(t)$	$\frac{1}{s^2}$	
$e^{-at} t \cdot u(t)$	$\frac{1}{(s+a)^2}$	$\sigma > -a$
$\cos\omega_0 t\, u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\sigma > 0$
$-t e^{-at} u(-t)$	$\frac{1}{(s+a)^2}$	$\sigma < -a$
$\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\sigma > 0$
$t^n u(t), n \geq 0$	$\frac{\lfloor n}{s^{n+1}}$	$\sigma > 0$

Que.→ $r(t) = t u(t) \rightleftharpoons R(s) = 1/s^2$

Que.→ $f(t) = t u(t-1) \rightleftharpoons [(t-1)+1]\cdot u(t-1) = (t-1)u(t-1) + u(t-1)$

$$= \frac{1}{s^2}e^{-s} + \frac{e^{-s}}{s}$$

$$y(t) = r(t-1) \rightleftharpoons Y(s) = R(s)e^{-s} = \frac{1}{s^2}e^{-s}$$

$$= (t-1)u(t-1)$$

Que.→ $f(t) = (t^2+5t-2)u(t-1)$

Sol^n→ $[(t-1)^2 + 7t - 3]\, u(t-1)$

$$= [(t-1)^2 + 7(t-1) + 4]\, u(t-1)$$

$$= (t-1)^2 u(t-1) + 7(t-1)u(t-1) + 4u(t-1)$$

$$F(S) = \frac{2}{s^3}e^{-s} + \frac{7}{s^2}e^{-s} + \frac{4}{s}e^{-s}$$

Que.→ $f(t) = (t^3+5t^2+3t+1)\, u(t-1)$

Sol^n→ $(t = t+1)$

$$f(t+1) = [(t+1)^3 + 5(t+1)^2 + 3(t+1) + 1]\, u(t)$$

$$f(t+1) = [t^3 + 8t^2 + 16t + 10]\, u(t)$$

$\downarrow$ LT.

$$F(S) = \left(\frac{6}{} + \frac{16}{} + 16 + 10\right)e^{-s}$$

DATE-28/10/14

Q.→ $f(t) = F(s) = \log\left(\frac{s+5}{s+6}\right)$

Find $f(t)$:- (a) $\frac{1}{t}(e^{-6t} - e^{-5t})u(t)$ (c.) $t(e^{-6t} - e^{-5t})u(t)$

(b.) $\frac{1}{t}(e^{-5t} - e^{-6t})u(t)$ (d.) $t(e^{-5t} - e^{-6t})u(t)$

Sol^n→ Diff. in freq. domain;

$$f(t) \rightleftharpoons F(s)$$

$$t f(t) \rightleftharpoons -\frac{dF(s)}{ds} = -\left[\frac{1}{s+5} - \frac{1}{s+6}\right]$$

$$t f(t) = \frac{1}{s+6} - \frac{1}{s+5}$$

$$t f(t) = e^{-6t}u(t) - e^{-5t}u(t)$$

$$f(t) = \frac{1}{t}(e^{-6t} - e^{-5t})u(t)$$

Q.→ $f(t) = \frac{(1-e^{t})u(t)}{t}$; $F(s) = ?$

(a.) $\log\left(\frac{s}{s-1}\right)$ (c.) $\log\left(\frac{s-1}{s+1}\right)$

(b.) $\log\left(\frac{s-1}{s}\right)$ (d.) $\log\left(\frac{s+1}{s-1}\right)$

Sol^n→ Integration in freq. domain:-

$$y(t) = (1-e^{t})u(t) \rightleftharpoons Y(s)$$

$$f(t) = \frac{y(t)}{t} \rightleftharpoons F(s) = \int_s^\infty Y(s)\cdot ds$$

$$F(s) = \int_s^\infty Y(s)\cdot ds = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s-1}\right) ds$$

$$F(s) = \left[\log(s) - \log(s-1)\right]_s^\infty = \left[\log\left(\frac{s}{s-1}\right)\right]_s^\infty$$

$$= \log\left[\lim_{s\to\infty}\left(\frac{s}{s-1}\right)\right] - \log\left(\frac{s}{s-1}\right)$$

$$= \log(1) - \log\left(\frac{s}{s-1}\right)$$

$$= 0 - \log\left(\frac{s}{s-1}\right)$$

Que.→ Find inverse LT of $F(s) = \dfrac{s^2+2s+5}{(s+3)(s+5)^2}$

For (i) $\sigma > -3$ (ii) $\sigma < -5$ (iii) $-5 < \sigma < -3$

Soln→

$$F(s) = \frac{A}{(s+3)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

$$A = 2,\ B = -1,\ C = -10$$

$$F(s) = \frac{2}{s+3} - \frac{1}{s+5} - \frac{10}{(s+5)^2}$$

Poles:- $s = -3, -5$

(i) $\sigma > -3$; $f(t)$ will be Right sided.

$$f(t) = 2e^{-3t}u(t) - e^{-5t}u(t) - 10te^{-5t}u(t)$$

(ii) $\sigma < -5$; $f(t)$ will be left sided.

$$f(t) = -2e^{-3t}u(-t) - [-e^{-5t}u(-t)] - 10[-te^{-5t}u(-t)]$$

(iii) $-5 < \sigma < -3$; $f(t)$ will be both sided.
↳ strip

ROC; $\sigma > -5$ → Right sided   ROC; $\sigma < -3$ → left sided

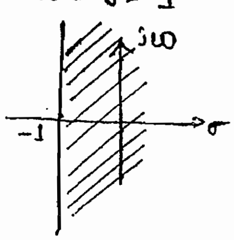
$$f(t) = -2e^{-3t}u(-t) - e^{-5t}u(t) - 10te^{-5t}u(t).$$

***Causal system→**

(1.) $h(t) = 0;\ t < 0$

For causal sys. ROC will be right side to the right most pole.

Eg:- ROC: $\sigma > -1$



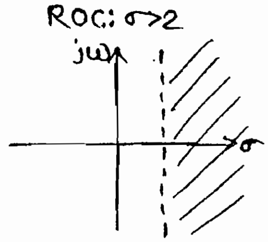
(Stable)

$H(s) = \dfrac{1}{s+1},\ \sigma > -1$

$h(t) = e^{-t}u(t)$ (energy)

(stable)

ROC: $\sigma > 2$



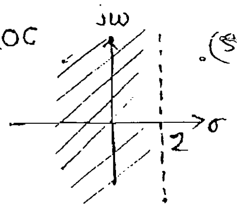
(Unstable)

$H(s) = \dfrac{1}{s-2};\ \sigma > 2$

$h(t) = e^{2t}u(t)$ (NENP)

Note → For stability of causal sys., poles of TF should lie in the LHS of s-plane.

Eg:- ROC



~~(stable)~~

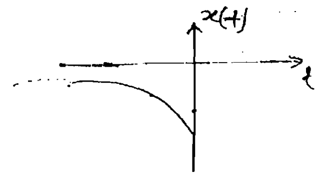
$$H(s) = \frac{1}{(s-2)} \; ; \; \sigma < 2$$

$$h(t) = -e^{2t} u(-t)$$

(Energy)

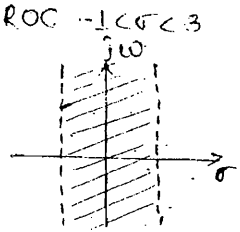
↓

stable.



* For anticausal sys. ROC will be left side to the left most pole.
* For stability of anticausal sys., poles of TF should lie in the RHS of s-plane.

Eg:-(4) ROC $-1 < \sigma < 3$



(stable) + NC

Que → Consider a contineous time LTI sys. whose i/p $x(t)$ & o/p $y(t)$ are related by the differential eqn.-

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

determine $h(t)$ of sys.

(a) when the sys. is causal.

(b) when the sys. is ~~NC~~ stable.

(c) Neithe stable nor causal.

Soln →

$$s^2 Y(s) - sY(s) - 2Y(s) = X(s)$$

$$Y(s)[s^2 - s - 2] = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}$$

$$= \quad 1$$

$$H(s) = \frac{\left(\frac{1}{3}\right)}{(s-2)} + \frac{\left(\frac{1}{-3}\right)}{(s+1)}$$

Poles:- $-1, 2$

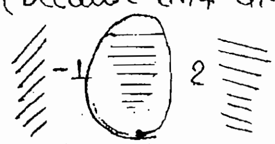
(i) when the sys is causal $h(t)$ will be Right sided.

$$h(t) = -\frac{1}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$$

(ii) When sys is stable. $h(t)$ will be both sided.

ROC:- $-1<\sigma<2$ (because img axis is included)

Poles:-



$s+1 \longrightarrow$ Right sided.

$s-2 \longrightarrow$ Left sided.

$$h(t) = -\frac{1}{3}e^{-t}u(t) - \frac{1}{3}e^{2t}u(-t)$$

(iii) ROC: $\sigma<-1$; $h(t)$ will be left sided.

$$h(t) = \left(\frac{1}{3}\right)e^{-t}u(-t) - \frac{1}{3}e^{2t}u(-t)$$

Que.→ For diff. eqn

$$\frac{d^2y(t)}{dt^2} + \frac{6dy(t)}{dt} + 8y(t) = 0$$

with initial condn $y(0^-)=1$, $y'(0^-)=0$; the soln of $y(t)$ is;

(a) $2e^{-2t}-e^{-4t}$ (b) $2e^{-6t}-e^{-2t}$ (c) $-e^{-6t}+2e^{-4t}$ (d) $e^{-2t}+2e^{-4t}$

Soln→

$$\cancel{s^2Y(s)+6Y(s)s+8Y(s)=0}$$

$$\cancel{Y(s)(s^2+6s+8)=0}$$

By applying LT on given diff eqn

$$\left[s^2Y(s) - sy(0^-) - y'(0^-)\right] + 6\left[sY(s) - y(0^-)\right] + Y(s) = 0$$

$$s^2Y(s) - s + 6[sY(s) \underset{-1}{]} + Y(s) = 0$$

$$Y(s)\left[s^2+6s+1\right] = s+6$$

$$Y(s) = \frac{s+6}{s^2+6s+1} = \frac{s+6}{(s+2)(s+4)}$$

$$Y(s) = \frac{2}{(s+2)} - \frac{1}{(s+4)}$$

$$\boxed{y(t) = 2e^{-2t} - e^{-4t}}$$

Que.→ For DE $\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + y(t) = \delta(t)$

with initial condn; $y(0^-) = -2,\ y'(0^-) = 0$

the value of $\left.\frac{dy(t)}{dt}\right|_{t=0^+}$ is

(a) -1 (b) 1 (c) 0 (d) 2

Soln→

$$s^2Y(s) - sy(0^-) - y'(0^-) + 2[sY(s) - y(0^-)] + Y(s) = 1$$

$$Y(s)[s^2+2s+1] - (-2s + 2\times(2)) = 1$$

$$Y(s) = \frac{-2s-3}{s^2+2s+1}$$

$$Y(s) = \frac{-2s-3}{(s+1)^2} = \frac{-2(s+1)-1}{(s+1)^2}$$

$$Y(s) = \frac{-2}{(s+1)} - \frac{1}{(s+1)^2}$$

$$y(t) = -2e^{-t} - te^{-t}$$

$$\frac{dy(t)}{dt} = 2e^{-t} - (e^{-t} - te^{-t})$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0^+} = 2(1) - (1-0)$$

$$\boxed{\text{Ans.} = 1}$$

Que.→ $y(t) = \sum_{n=0}^{\infty} f(t-nT_0)$ Find $Y(s)$ in terms of $F(s)$

Soln→ $y(t) = \sum_{n=0}^{\infty} f(t-nT_0)$

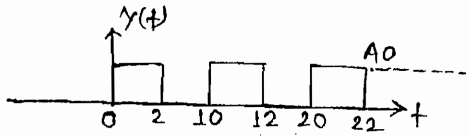
$$y(t) = f(t) + f(t-T_0) + f(t-2T_0) + \cdots$$

| LT

$$y(s) = F(s)\left[1 + e^{-sT_0} + (e^{-sT_0})^2 + \cdots\right]$$

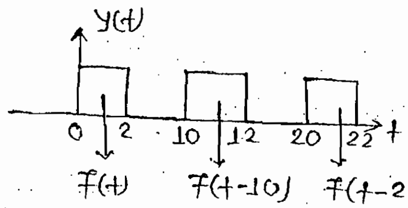
$$\boxed{y(s) = \frac{F(s)}{1-e^{-sT_0}}}$$

Que.→



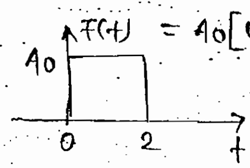
Find y(s)

Sol^n→



$$y(t) = f(t) + f(t-10) + f(t-2\times10) + \cdots$$

$$= \sum_{n=0}^{\infty} f(t-nT_0) \; ; \; T_0 = 10$$



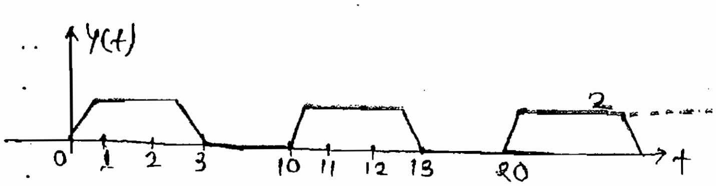
$$= A_0[u(t) - u(t-2)]$$

$$F(s) = \frac{A_0}{s}(1-e^{-2s})$$

$$y(s) = \frac{F(s)}{(1-e^{-sT_0})}$$

$$\boxed{y(s) = \frac{A_0(1-e^{-2s})}{s(1-e^{-10s})}}$$

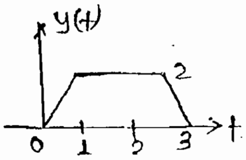
Que→



Find LT of the signal.

Sol^n→

$$y(t) = \sum_{n=0}^{\infty} f(t-nT_0)$$



$$f(t) = 2r(t) - 2r(t-1) - 2r(t-2) + 2r(t-3)$$

$$F(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} - \frac{2e^{-2s}}{s^2} + \frac{2e^{-3s}}{s^2}$$

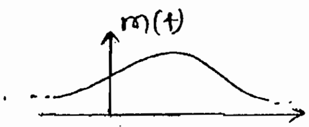
$$F(s) = \frac{2}{s^2}\left[1 - e^{-s} + e^{-2s} + e^{-3s}\right]$$

$$Y(s) = \frac{F(s)}{1 - e^{-sT_0}}$$

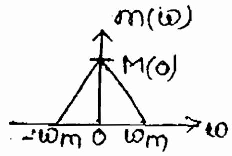
$$Y(s) = \frac{\frac{2}{s^2}\left(1 - e^{-s} + e^{-2s} + e^{-3s}\right)}{1 - e^{-10s}}$$

Chapter-07
Sampling Theorem

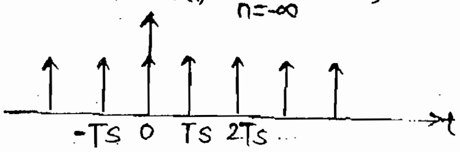
Sampling theorem →



$\rightleftharpoons$

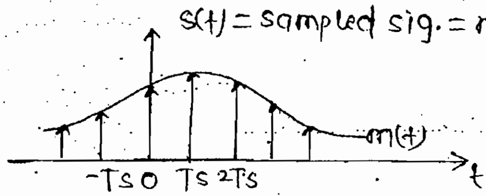


$C(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

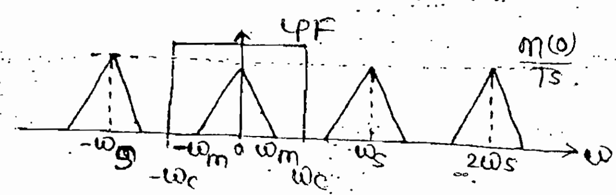


$\rightleftharpoons \quad C(\omega) = \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$

$s(t) = \text{sampled sig.} = m(t) \cdot c(t)$



$\rightleftharpoons$



$$s(t) = m(t) \cdot c(t)$$

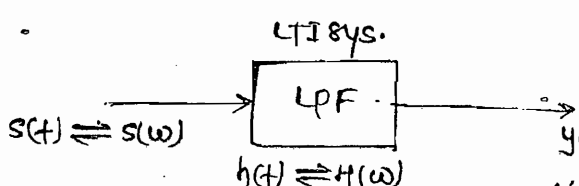
$$S(\omega) = \frac{1}{2\pi}\left[M(\omega) * C(\omega)\right]$$

$$= \frac{1}{2\pi}\left[M(\omega) * \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)\right]$$

$$= \frac{1}{T_s}\left[\sum_{n=-\infty}^{\infty} M(\omega - n\omega_s)\right]$$

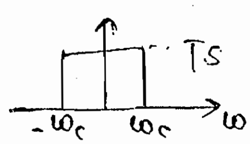
$$= \frac{1}{T_s}\left[- - - - - - - + M(\omega+\omega_s) + M(\omega) + M(\omega-\omega_s) + M(\omega - 2\omega_s) + \cdots\right]$$

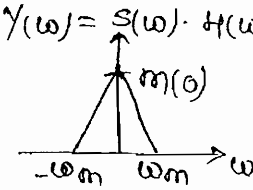
$$\boxed{T_s = \text{sampling interval} = \frac{2\pi}{\omega_s}}$$



$y(t) = s(t) * h(t)$

$Y(\omega) = S(\omega) \cdot H(\omega)$





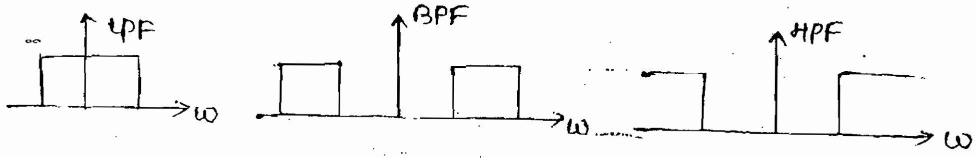
$$\boxed{\omega_m \le \omega_c \le \omega_s - \omega_m}$$

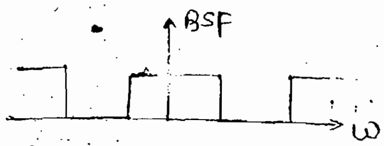
* To avoid overlapping in sampled sig. spectrum:-

$$\omega_m \le \omega_s - \omega_m$$

$$\boxed{\omega_s \ge 2\omega_m}$$

Note:-





Statement → A sig. can be represented by its samples (or) recovered back from its samples if sampling freq. is greater then (or) equal to twice of max^m freq. componant present in signal.

Nquist Rate →

$$F_{ny} = 2F_m$$

Nquist-interval →

$$T_{ny} = \frac{1}{F_{ny}} = \frac{1}{2F_m}$$

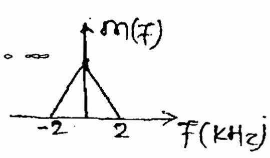
Over sampling →

* $F_s > 2F_m$
* Allowable case

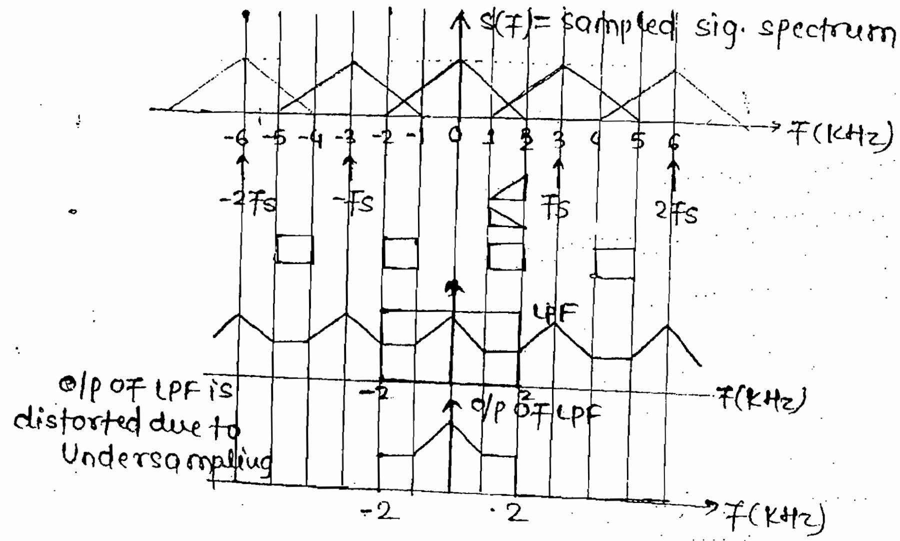
Under-sampling →

* $F_s < 2F_m$
* not allowable.

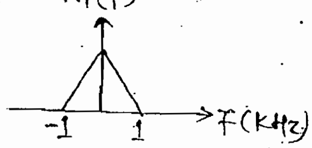
Eg:- $m(t) \rightleftharpoons$



* $F_m = 2KHz$
* $F_s = 3KHz < 2F_m$
* Undersampling



Que.→ $m(t) \rightleftharpoons m(f)$

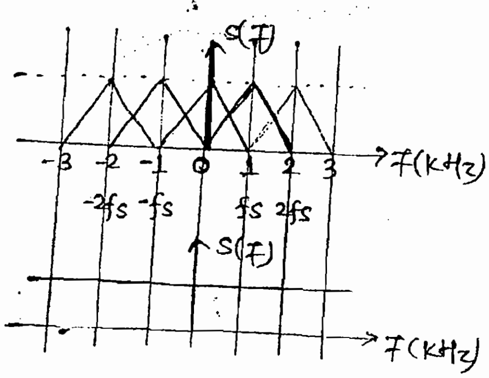


$T_s = 1ms$.

draw sampled sig. spectrum.

Soln→ $f_m = 1kHz$

$f_s = \frac{1}{T_s} = 1kHz < 2f_m$ (Undersampling)



Que.→ Calculate Nquist Rate in (rad/sec)

(i) $m(t) = 2\sin 4\pi t \cdot \cos 2\pi t$.

Soln→ $m(t) = 2\sin 4\pi t \cdot \cos 2\pi t$

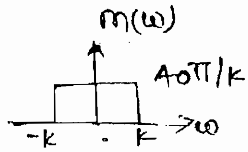
$= \sin 6\pi t + \sin 2\pi t$

(ii) $m(t) = Sa(4\pi t)$

Soln →

$m(t) = Sa(4\pi t)$

$= A_0 \, Sa(kt)$



$\omega_m = k = 4\pi$

$\omega_{ny} = 2\omega_m = 8\pi$

(iii) $m(t) = Sa^3(5\pi t)$

Soln → $\omega_m = 3 \times 5\pi$

$\omega_{ny} = 2\omega_m$

$= 30\pi$

$$\boxed{\begin{array}{l} y(t) = [x(t)]^n \\ \qquad\qquad \downarrow \\ \qquad\qquad \omega_m \\ \omega_m' = n\omega_m \end{array}}$$

(iv) $m(t) = Sa^2(4\pi t) \cdot Sa^4(3\pi t)$

Soln → $\omega_{m1} = 2 \times 4\pi$

$\omega_{m2} = 4 \times 3\pi$

$\omega_m = \omega_{m1} * \omega_{m2} = 20\pi$

$\omega_{ny} = 2\omega_m = 40\pi$

$$\boxed{\begin{array}{l} m(t) = m_1(t) \cdot m_2(t) \\ \qquad\quad \downarrow \qquad\quad \downarrow \\ \qquad\quad \omega_{m1} \quad\; \omega_{m2} \\ \omega_m = \omega_{m1} + \omega_{m2} \end{array}}$$

(v) $m(t) = m_1(t) * m_2(t)$

$\qquad\quad \downarrow \qquad\qquad \downarrow$

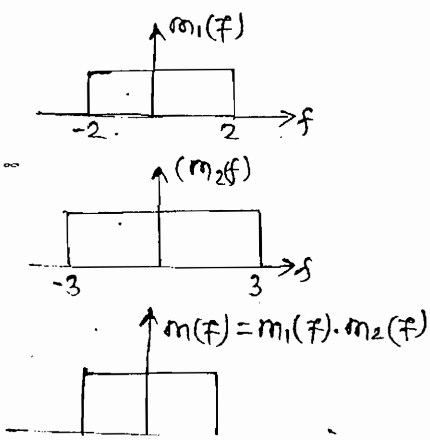
$f_{m_1} = 2\,kHz \quad f_{m_2} = 3\,kHz$

$f_{ny} = ?$

(a) 4 kHz (b) 6 kHz (c) 10 kHz (d) 12 kHz

Soln →

$m(f) = m_1(f) \cdot m_2(f)$

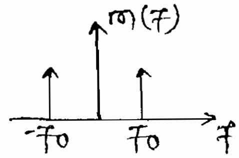


$f_m = 2\,kHz$

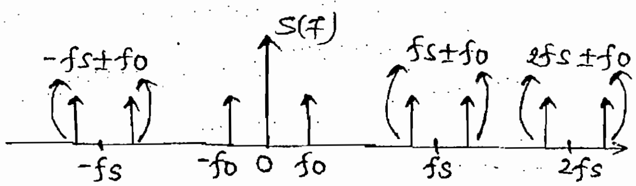
$f_{ny} = 2f_m = 4\,kHz$

Important points →

(1.) $m(t) = \cos 2\pi f_0 t \rightleftharpoons$



$$C(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$



Freq. componants present in $S(f)$

: $\pm f_0$, $f_s \pm f_0$, $2f_s \pm f_0$, ...

$\boxed{: n f_s \pm f_0}$

where; n = an integer

$= \pm 1, \pm 2, \pm 3, \ldots$

(2.) $m(t) = \cos 2\pi f_1 t + \cos 2\pi f_2 (t)$

Freq. componant present in $S(f)$

$\boxed{: n f_s \pm f_1, \; n f_s \pm f_2}$

Que. → A sig. $m(t) = 100\cos(24\pi \times 10^3 t)$ is idealy sampled at $T_s = 50\mu s$ & passed through an LPF with $f_c = 15kHz$. Which of the following freq. is/are present at the o/p of the LPF?

(a.) 8kHz (c) 8 & 10 kHz

(b.) 12kHz (d) 8 & 12 kHz

Soln →

$f_0 = 12kHz$

$f_s = \frac{1}{T_s} = 20kHz$

LPF: $f_c = 15kHz$

Freq. compo. present in $S(f)$

: f_0, $f_s \pm f_0$, $2f_s \pm f_0$, ...

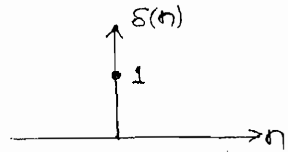
: ± 12, 20 ± 12, 40 ± 12, ...

: 12, 8, 32, 28, 52, ...

Chapter-08
Discrete time signal

1) Unit-impulse signal $\delta(n) \rightarrow$

$$\delta(n) = \begin{cases} 1 ; & n=0 \\ 0 ; & n \neq 0 \end{cases}$$



properties →

(1.) $\delta(n)$ is an even signal.

(2.) $\delta(n)$ is an energy signal.

$\delta(t)$ is NENP sig.

(3.) $\delta(an) = \delta(n)$

$\delta(at) = \frac{1}{|a|}\delta(t)$

(4.) $x(n)\cdot\delta(n-n_1) = x(n_1)\,\delta(n-n_1)$

(5.) $x(n) * \delta(n-n_1) = x(n-n_1)$

(6.) $\int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$

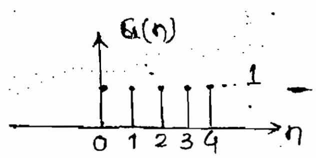
$\downarrow$

$\int = \Sigma$, $t = n$, $\tau = k$

$$\sum_{k=-\infty}^{n} \delta(k) = u(n)$$

(2.) Unit-step sig. →

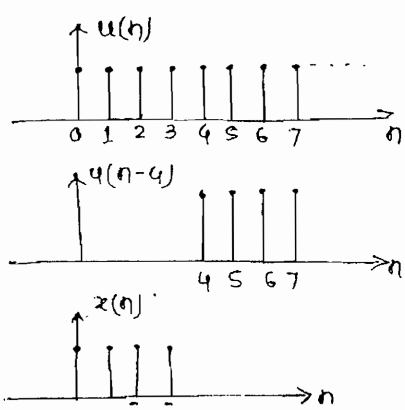
$$u(n) = \begin{cases} 1 , & n \geq 0 \\ 0 , & n < 0 \end{cases}$$



Q.→ Draw sig. $x(n)$

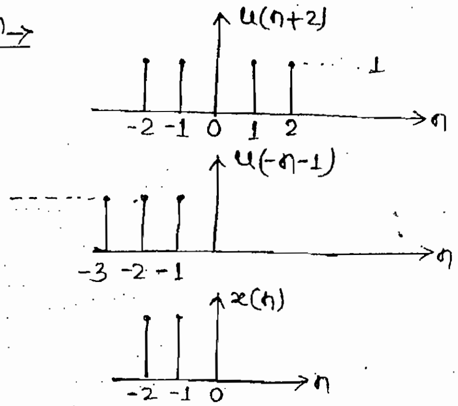
$$x(n) = u(n) - u(n-4)$$

Sol^n →



Que→ Draw the signal $x(n) = u(n+2) \cdot u(-n-1)$

Soln→



$-n-1 = 0$

$n = -1 \rightarrow$ starting point

Because LS.

Operations on signal→

(1) Time-shifting→

$x(n) = \{\overset{n=-2}{5}\ 3\ \underset{\uparrow}{7}\ 4\ 8\ \overset{n=3}{9}\}$

axis not shifts only signal shifts

$x(n-1) = \{5\ \underset{\uparrow}{3}\ 7\ 4\ 8\ 9\}$
(RS)

$x(n+2) = \{5\ 3\ 7\ 4\ \underset{\uparrow}{8}\ 9\}$
(LS)

(2) Time Compression→(Decimation)→

$x(n) = \{\overset{n=-3}{5}, 3, 7, \underset{\uparrow}{8}, -2, 4, \overset{n=3}{9}\}$

$f(n) = x(2n) = \{3, \underset{\uparrow}{8}, 4\}$

$f(n) = x(3n) = \{5, \underset{\uparrow}{8}, 9\}$

$f(-2) = x(-4) = 0$
$f(-1) = x(-2) = 3$
$f(0) = x(0) = 8$
$f(1) = x(2) = 4$
$f(2) = x(4) = 0$

(3) Time expansion→(Interpolation)

$x(n) = \{\overset{n=-1}{4}, \underset{\uparrow}{3}, \overset{n=1}{5}\}$

$f(n) = x\left(\frac{n}{2}\right) = \{4, 0, 3, 0, 5\}$

$1 = 2-1$

$x\left(\frac{n}{3}\right) = \{4, 0, 0, 3, 0, 0, 5\}$

$2 = 3-1$

$f(-3) = \cancel{f}(-3/2) = 0$
$f(-2) = \cancel{f}(-1) = 4$
$f(-1) = \cancel{f}(-1/2) = 0$
$f(0) = 0 \quad = 3$
$f(1) = x(1/2) = 0$
$f(2) = x(1) = 5$
$f(3) = x(3/2) = 0$

Que.→ $x(n)=\{1,2,\underset{\uparrow}{3},4,5\}$ Find $y(n)$

(i) $y(n)=x\left(\frac{2n}{3}\right)$

Sol^n→ $x(n) \xrightarrow{\text{Dec}} x(2n) \xrightarrow{\text{Int}} x\left(\frac{2n}{3}\right)$

$\{1,2,\underset{\uparrow}{3},4,5\}$ $\{1,\underset{\uparrow}{3},5\}$ $\{1,0,0,\underset{\uparrow}{3},0,0,5\}$

(ii) $y(n)=x(-2n)$

Sol^n→ $x(n) \longrightarrow x(2n) \longrightarrow x(-2n)$ (-ve-folding about arrow)

$\{1,2,\underset{\uparrow}{3},4,5\}$ $\{1,\underset{\uparrow}{3},5\}$ $(5,\underset{\uparrow}{3},1)$

(iii) $y(n)=x(-n-1)$

Sol^n→ $x(n) \longrightarrow x(n-1) \longrightarrow x(-n-1)$

$(1,2,\underset{\uparrow}{3},4,5)$ $(1,\underset{\uparrow}{2},3,4,5)$ $(5,4,3,\underset{\uparrow}{2},1)$

(iv) $y(n)=x(2n-1)$

Sol^n→ $x(n) \longrightarrow x(n-1) \longrightarrow x(2n-1)$

$(1,2,\underset{\uparrow}{3},4,5)$ $(1,\underset{\uparrow}{2},3,4,5)$ $(\underset{\uparrow}{2},4)$

(4) Convolution→

$$y(n)=x_1(n) * x_2(n)$$

$$=\sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

signal	extension	Length
$x_1(n)$	$n_1 \le n \le n_2$	L_1
$x_2(n)$	$n_3 \le n \le n_4$	L_2
$y(n)$	$n_1+n_3 \le n \le n_2+n_4$	L_1+L_2-1

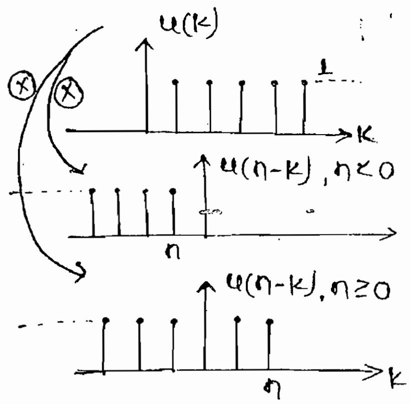
length = no. of samples

Que.→ $y(n)=u(n) * u(n)$

Sol^n→

$$y(n)=u(n) * u(n)$$

$$=\sum_{k=-\infty}^{\infty} u(k)\cdot u(n-k)$$



$n-k=0 \rightarrow -ve(LS)$
$n=k$

$$y(n) = u(n) * u(n)$$
$$= \sum_{-\infty}^{\infty} u(k) \cdot u(n-k)$$
$$= \begin{cases} 0 & , n<0 \\ \sum_{k=0}^{n} 1 & ; n \geq 0 \end{cases}$$
$$= \begin{cases} 0 & , n<0 \\ n+1 & , n \geq 0 \end{cases}$$
$$\boxed{y(n) = (n+1)\, u(n)}$$

Que.→ $x_1(n) = (1, \underset{\uparrow}{2}, -2)$; $x_2(n) = (2, 0, \underset{\uparrow}{1})$

$y(n) = x_1(n) * x_2(n) = ?$

Solⁿ→ Tabular method →

$x_2(n)$ \ $x_1(n)$	1	2	−2
2	2	4	−4
0	0	0	0
→ 1	1	2	−2

$x_1(n) = (1, \underset{\uparrow}{2}, -2)$ = 2nd element
$x_2(n) = (2, 0, \underset{\uparrow}{1})$ = 3rd element

$2+3=5$
$5-1=4$
(arrow point)

$y(n) = (2, 4, -3, \underset{\uparrow}{2}, -2)$

Que.→ $x_1(n) = (-1, \underset{\uparrow}{2}, 0, 1)$ $x_2(n) = (3, 1, \underset{\uparrow}{0}, -1)$

$y(n) = x_1(n) * x_2(n)$

Solⁿ→

$x_2(n)$ \ $x_1(n)$	−1	2	0	1
3	−3	6	0	3
1	−1	2	0	1
→ 0	0	0	0	0
−1	1	−2	0	−1

$y(n) = (-3, 5, 2, \underset{\uparrow}{4}, -1, 0, -1)$

Que→

$x(n) = (\underset{\uparrow}{1}, -1) \rightarrow \boxed{\text{LTI sys.}}\ h(n) \rightarrow y(n) = (\underset{\uparrow}{1}, 0, 0, 0, -1)$

Find $h(n) = ?$

(a.) $(\underset{\uparrow}{1}, 0, 0, 1)$ ~~(b) $(\underset{\uparrow}{1}, 0, 1)$~~

(c.) $(\underset{\uparrow}{1}, 1, 1, 1)$ (d) $(\underset{\uparrow}{1}, 1, 1)$

Sol^n→

$y(n) = x(n) * h(n)$

$x(n) = 2$

$y(n) = 5$

$5 = 2 + L_2 - 1$

$L_2 = 4$ {ans (C) or (a)}

Tabular method →

$h(n)$ \ $x(n)$	1	−1
1	1	−1
1	1	−1
1	1	−1
1	1	−1

Que→ $y(n) = h(n) * g(n)$, $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$g(n)$ is causal sequence.

If $y(0) = 1$, $y(1) = 1/2$ then $g(1)$ is equal to

(a) 0 (b) 1/2 (c) 1 (d) 3/2

Sol^n→

$g(n)$ \ $h(n)$	1	1/2	1/4	1/8 --------
1	1	1/2	1/4	1/8 -------
0	0	0	0	0 -------

ans-(a)

* Energy & power signal →

* Energy signal →

* E = Finite ; P = 0
* This are absolutely summable signal i.e.

$$\boxed{\sum_{n=-\infty}^{\infty} |x(n)| < \infty} \qquad \boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$$

Que → Calculate energy of signal :–

(i) $x(n) = \delta(n)$

Sol^n → $x(n) = \delta(n)$

$= (\cdots 0, \underset{\uparrow}{1}, 0 \cdots)$

$E = \sum |x(n)|^2 = 1$

(ii) $x(n) = \left(\frac{1}{3}\right)^n u(n)$

Sol^n → $x(n) = \left(\frac{1}{3}\right)^n u(n)$

$E = \sum_{-\infty}^{\infty} |x(n)|^2 = \sum_{0}^{\infty} \left(\frac{1}{9}\right)^n$

$= 1 + \frac{1}{9} + \left(\frac{1}{9}\right)^n + \cdots$

$= \frac{1}{1 - 1/9} = \frac{9}{8}$

(iii) $x(n) = (\underset{\uparrow}{1+j}, 1-j, -2, 2)$

Sol^n → $E = \sum |x(n)|^2$

$= (\sqrt{2})^2 + (\sqrt{2})^2 + (2)^2 + (2)^2$

$= 12$

Que → calculate energy of $y(n)$

$x(n) = (\underset{\uparrow}{1}, 2, 3, 4, 5)$

(i) $y(n) = x(-n)$ (ii) $y(n) = x(n-1)$ (iii) $y(n) = x\left(\frac{n}{2}\right)$ (iv) $y(n) = -x(n)$ (v) $y(n) = x(3n)$

Sol^n → (i) $y(n) = x(-n)$

$x(n) = E[x(n)] = 55$

$x(-n) = 55$

(ii) $y(n) = x(n-1)$

$E_{x(n-1)} = 55$

(iii) $y(n) = x\left(\frac{n}{2}\right)$

$= (\underset{\uparrow}{1}, 0, 2, 0, 3, 0, 4, 0, 5)$

$E_{x(n/2)} = 55.$

(iv) $y(n) = -x(n)$

$E_{y(n)} = 55$

(v) $y(n) = x(3n)$

$= (1, 4)$

$E_{y(n)} = 1^2 + 4^2 = 17$

Note:- Energy calculation is independant of time shifting, time reversal, amp. reversal & interpolation.

* Power signal →

(1) P = finite, $E=\infty$

(2) $P = \begin{cases} \frac{1}{N}\sum_{n=N} |x(n)|^2 \text{ ; For periodic signal.} \\ \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \text{ ; For Non-periodic signal.} \end{cases}$

Que.→ Calculate power of signal.

(i) $x(n) = A_0 u(n)$

Sol^n→ $x(n) = A_0 u(n)$

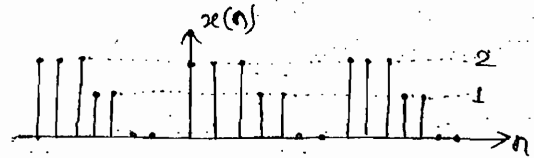
$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} A_0^2 (N+1)$$

$$= \lim_{N\to\infty} \frac{A_0^2 (N+1)}{2N+1}$$

$$\boxed{P = \frac{A_0^2}{2}}$$

(ii)



Sol^n→

$$P = \frac{1}{N}\sum |x(n)|^2$$

$$= \frac{2^2+2^2+2^2+1^2+1^2+0^2+0^2}{7}$$

$$= \frac{14}{7}$$

$$\boxed{P=2}$$

Que→ Find even, odd, CS & CAS for sig. x(n)

$$x(n) = (-4-5j,\ \underset{\uparrow}{1+2j},\ 4)$$

Sol^n→ * Even part → $\frac{x(n)+x(-n)}{2}$

$$x(-n) = (4\ \ \underset{\uparrow}{1+2j}\ \ -4-5j)$$

$$= \left(\frac{-5j}{2}\ \ \underset{\uparrow}{1+2j}\ \ \frac{-5j}{2}\right)$$

* Odd part → $\frac{x(n)-x(-n)}{2}$

$$= \left(-4-\frac{5j}{2},\ 0,\ 4+\frac{5j}{2}\right)$$

* CAS part → $\frac{x(n)-x^*(-n)}{2}$

$$x^*(-n) = (4,\ 1-2j,\ -4+5j)$$

* Periodic Signal →

$$\boxed{x(n) = x(n \pm KN)}$$

where; K = an-integer

N = FTP = integer

$$x(n) = \underbrace{x_1(n)}_{N_1} + \underbrace{x_2(n)}_{N_2}$$

$$\longrightarrow \frac{N_1}{N_2} = \frac{\text{integer}}{\text{integer}} = \text{Rational no. (always)}$$

Note:- The sum of 2 (or) more 2 periodic signals in case of discrete time system will be always periodic.

Complex-exponential → Complex exponential & sinusoidal signals are always periodic in case of contineous time signal.

Eg:- $x(n) = A_0 e^{j\omega_0 n}$

let N be the FTP of $x(n)$ i.e.

$$x(n) = x(n+N)$$

$$A_0 e^{j\omega_0 n} = A_0 e^{j\omega_0 (n+N)}$$

$$e^{j\omega_0 N} = 1 = e^{j2\pi K} \quad (K = \text{an integer})$$

$$\omega_0 N = 2\pi K$$

$$\boxed{\frac{2\pi}{\omega_0} = \frac{N}{K} = \text{Rational no.}}$$

In case of discrete time sys; complex exponantials & sinusoidal sig. will be periodic only if ratio $2\pi/\omega_0$ is rational no.

$$\boxed{N = \frac{2\pi}{\omega_0} K}$$

K is a least int. for which N is an integer.

Que.→ Calculate FTP of sig. if it is periodic

(i) $x(n) = e^{j2n}$.

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi = \text{irrational no.}$$

(ii) $x(n) = \cos \frac{3\pi}{4} n$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi/4} = \frac{8}{3} \text{ (R. no.)}$$

iii) $x(n) = \sin\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{5\pi}{4}n\right)$

Soln →

$$x(n) = \underbrace{\sin\left(\frac{3\pi}{4}n\right)}_{N_1 = 8} + \underbrace{\cos\left(\frac{5\pi}{4}n\right)}_{N_2}$$

$$N_2 = \frac{2\pi}{\omega_2} k_2 = \frac{2\pi}{5\pi/7} k_2 = \frac{14}{5} k_2 = 14$$

$$N = \text{LCM}(N_1, N_2)$$

$$= \text{LCM}(8, 14)$$

$$= 56.$$

$e^{j2t} \longrightarrow$ Periodic

$e^{j2n} \longrightarrow$ NP

$\sin 4t \longrightarrow$ P

$\sin 4n \longrightarrow$ NP

$\underset{(P)}{\sin 2t} + \underset{(P)}{\cos 4t} \longrightarrow$ P

$\underset{(NP)}{\sin 3n} + \underset{(NP)}{\cos 4n} \longrightarrow$ NP

Chapter-09
Z-transform

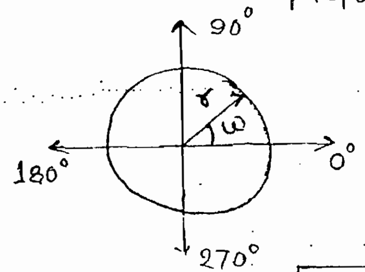
* Discrete time fourier transform (DTFT) exists for E & P signals whereas Z-TF also exist for NENP sig. (upto certain only).
* The replacement $z = e^{j\omega}$ is used for z-transform to DTFT conversion only for absolutely summable signal.

$$x(n) \rightleftharpoons X(z)$$

where; z = complex variable

$$= r e^{j\omega}$$

damping factor ← r; $e^{j\omega}$ → oscillation factor.



$$z \longrightarrow 2\pi$$
$$X(z) \longrightarrow 2\pi$$

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}}$$

<u>Que.</u> → $x(n) = a^n u(n) \rightleftharpoons X(z) = ?$

<u>Soln</u> →

$$X(z) = \sum_{n=-\infty}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

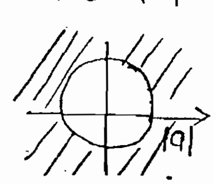
$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= (az^{-1})^0 + (az^{-1}) + (az^{-1})^2 + \dots\dots$$

$$X(z) = \frac{1}{1 - az^{-1}} \quad ; \quad |az^{-1}| < 1$$

$$\boxed{a^n u(n) \rightleftharpoons \frac{1}{1 - az^{-1}} \quad ; \quad ROC: |z| > a}$$

z-plane



Que.→ $x(n) = -a^n u(-n-1)$ Cal. ZTF & ROC.

Sol^n→

$$X(z) = \sum_{-\infty}^{\infty} x(n)\, z^{-n}$$

$$= \sum_{-\infty}^{\infty} -a^n u(-n-1)\, z^{-n}$$

$$= \sum_{-\infty}^{-1} (-a^n)\, z^{-n}$$

$$= -\sum_{-\infty}^{-1} (a z^{-1})^n$$

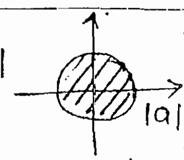
$$X(z) = -\left[(a z^{-1})^{-1} + (a z^{-1})^{-2} + \cdots\cdots\right]$$

$$= -\left[a^{-1}z + (a^{-1}z)^2 + \cdots\cdots\right]$$

$$= \frac{-a^{-1}z}{1-a^{-1}z} \; ; \quad |a^{-1}z| < 1$$

$$X(z) = \frac{1}{1-az^{-1}} \; ; \quad |z| < |a|$$

$$\boxed{-a^n u(-n-1) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \quad |z|<|a|}$$



$$-a^n u(-n-1) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \quad |z|<|a|$$

$$a^n u(n) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \quad |z|>|a|$$

(a=1) (a=1)

$$u(n) \rightleftharpoons \frac{1}{1-z^{-1}} \; ; \quad |z|>1$$

$$-u(n-1) \rightleftharpoons \frac{1}{1-z^{-1}} \; ; \quad |z|<1$$

Properties of z-transform →

1.) linearity :- $a_1 x_1(n) + a_2 x_2(n) \rightleftharpoons a_1 X_1(z) + a_2 X_2(z)$

2.) Time-reversal :- $x(-n) \rightleftharpoons X(z^{-1})$

3.) Conjugation :- $x^*(n) \rightleftharpoons X^*(z^*)$

4.) Time-shifting :- $x(n-n_0) \rightleftharpoons X(z) . z^{-n_0}$

$$\downarrow (n_0 = 1)$$

$$x(n-1) \rightleftharpoons z^{-1} X(z)$$

5.) Scaling of z :- $a^n x(n) \rightleftharpoons X(a^{-1} z)$

6.) Convolution in time :-

$$x_1(n) * x_2(n) \rightleftharpoons X_1(z) . X_2(z)$$

7.) Multiplication in time :-

$$x_1(n) . x_2(n) \rightleftharpoons \frac{1}{2\pi j}\left[X_1(z) * X_2(z)\right]$$

8.) Successive diff./difference in time →

$$\frac{dx(n)}{dn} = \frac{x(n) - x(n-1)}{n-(n-1)}$$

$$= x(n) - x(n-1) \rightleftharpoons X(z) - z^{-1} X(z)$$

$$x(n) - x(n-1) \rightleftharpoons (1 - z^{-1}) X(z)$$

9.) Accumulation / Integration in time →

$$\sum_{k=-\infty}^{n} x(k) \rightleftharpoons \frac{X(z)}{1 - z^{-1}}$$

10.) Differentiation in Freq. →

$$n . x(n) \rightleftharpoons -z \frac{dX(z)}{dz}$$

(11) Initial value theorem →

$$x(t)\Big|_{t=0} = \lim_{s \to \infty} sX(s)$$

Condn.:- Applicable only for causal type signal i.e

$x(n) = 0, \; n < 0$

(12) Final value theorem →

$$x(t)\Big|_{t=\infty} = \lim_{s\to 0}\left[sX(s)\right]$$

$$x(n)\Big|_{n=\infty} = \lim_{z\to 1}\left[(1-z^{-1})X(z)\right]$$

Cond^n :- (i) Applicable only for causal signals. i.e.

$$x(n)=0,\ n<0$$

(ii) poles of term $\left[(1-z^{-1})X(z)\right]$ should lie inside unit circle in z-plane.

DATE-30/10/14

Region of Convergance (ROC) → It is defined as the range of complex variable z in z-plane for which z-transform of signal is convergant (or) Finite.

Properties of ROC →

(1.) ROC does not include any pole.

(2.) For right sided signal ROC will be outside circule in z-plane.

(3.) For left sided signal ROC will be inside circule in z-plane.

RS LS
ROC I
out side inside

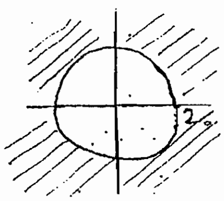
(4.) For both sided signal ROC is a ring in z-plane.

(5.) For stability, ROC includes unit circle in z-plane.

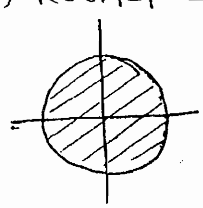
(6.) For finite duration sig. ROC is intire z-plane excluding possibly z=0 &/or $\pm\infty$.

Que → Check stability of sys. & comment about extension of h(n).

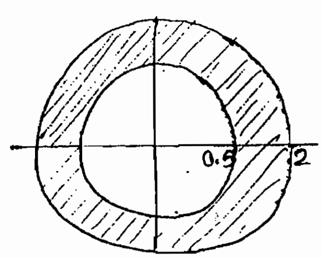
(i) ROC: $|z|>2$



(ii) ROC: $|z|<2$



(iii) ROC: $0.5<|z|<2$



Sol^n → h(n) = RS + US

h(n) = LS + S

h(n) = BS + S

Que.→ $x(n) = (2, \underset{\uparrow}{5}, 3, 7, 8)$ $X(z) = ?$, $ROC = ?$

Sol^n→

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{-1}^{3} x(n) z^{-n}$$

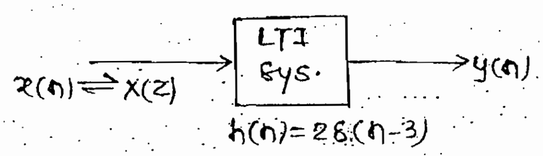
$$= x(-1)z^{1} + x(0)z^{0} + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3}$$

$$= 2z + 5 + 3z^{-1} + 7z^{-2} + 8z^{-3}$$

ROC→ Entire z-plane excluding

$z = 0, +\infty, -\infty$ (excluded) [Because they give ∞ sol^n in $X(z)$]

Que.→



$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$. Find $y(4)$

(a) -6 (b) 0 (c) 2 (d) -4

Sol^n→

$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$

$x(n) = (1, 0, 1, -2, 2, 0, 0, 0, -3)$

$\delta(n) \rightleftharpoons 1$

$2\delta(n) \rightleftharpoons 2$

$2\delta(n-3) \rightleftharpoons 2z^{-3}$

$y(n) = x(n) * h(n)$

$Y(z) = X(z) \cdot H(z)$

$= 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4})$

$Y(z) = 2z + 2z^{-1} - 4z^{-2} + 4z^{-3} - 6z^{-7}$

$y(n) = (2, \underset{\uparrow}{0}, 2, -4, 4, +0, 0, 0, -6)$

$y(4) = 0$

$$\boxed{y(4) = 0}$$

Que.→ $X(z) = 1 - 3z^{-1} \rightleftharpoons x(n) \neq i/p$

$Y(z) = 1 + 2z^{-2} \rightleftharpoons y(n) \neq o/p$

An LTI sys. has impulse responce $h(n)$ defined as $h(n) = x(n-1) * y(n)$.
The o/p of sys. for i/p $\delta(n-1)$ ~~has ZT~~.

soln→ $\quad i/p \rightarrow \delta(n-1) = z^{-1}$

$$p(n) = P(z) \longrightarrow \boxed{\text{LTI system}} \longrightarrow q(n) = Q(z)$$

$P(z) = z^{-1}$ $\qquad h(n) \rightleftharpoons H(z)$

$$h(n) = x(n-1) * y(n)$$

$$H(z) = z^{-1}X(z) \cdot Y(z)$$

$$q(n) = p(n) * h(n)$$

$$Q(z) = P(z) \cdot H(z)$$

$$= (z^{-1})\left[z^{-1}X(z) \cdot Y(z)\right]$$

$$Q(z) = z^{-2}X(z) \cdot Y(z)$$

$$= z^{-2}(1-3z^{-1})(1+2z^{-2})$$

$$= z^{-2}(1-3z^{-1}+2z^{-2}-6z^{-3})$$

$$Q(z) = z^{-2} - 3z^{-3} + 2z^{-4} - 6z^{-5}$$

$$q(n) = \delta(n-2) - 3\delta(n-3) + 2\delta(n-4) - 6\delta(n-5)$$

ans (b).

2nd method →

$$q(n) = p(n) * h(n)$$

$$= \delta(n-1) * h(n)$$

$$= h(n-1)$$

$$h(n) = x(n-1) * y(n)$$

$$X(z) = 1-3z^{-1} \rightleftharpoons x(n) = (\underset{\uparrow}{1}, -3)$$

$$x(n-1) = (\underset{\uparrow}{0}, 1, -3)$$

$$Y(z) = 1+2z^{-2} \rightleftharpoons y(n) = (\underset{\uparrow}{1}, 0, 2)$$

$$h(n) = (\underset{\uparrow}{0}, 1, 2) * (\underset{\uparrow}{1}, 0, 2)$$

Que.→ $x(n) = \left(-\frac{1}{2}\right)^{-n} u(-n+1) + 3^n u(n)$

$X(z) = ?$

Soln→

$\cancel{|z| < \left|\frac{-1}{2}\right|}$ $\quad$ $\cancel{|z| > |3|}$

$\cancel{|z| < (1/2)}$ $\quad$ $\cancel{|z| > 3}$

$\cancel{\left(\frac{1}{2}\right) < |z| < 3}$

$(-2)^n u(-n-3)$ LS

$|z| < |-2|$

$|z| < 2$

$x(n) = \left(-\frac{1}{2}\right)^{-n} u(-n+1) + 3^n u(n)$

$= \underbrace{(-2)^n u(-n+1)}_{|z|<2} + \underbrace{3^n u(n)}_{|z|>3}$

ZT of the x(n) will not exist because no common ROC

Que.→ $x(n) = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u(n)$ $\quad$ ROC = ?

Soln→ $x(n) = \left(\frac{1}{3}\right)^{n} - \left(\frac{1}{2}\right)^n u(n)$

$$\left(\frac{1}{3}\right)^{n} = \begin{cases} \left(\frac{1}{3}\right)^{-n} & ; n<0 \\ \left(\frac{1}{3}\right)^{n} & ; n\geq 0 \end{cases}$$

$$= \begin{cases} 3^n & ; n<0 \\ (1/3)^n & ; n\geq 0 \end{cases}$$

$= 3^n u(-n-1) + \left(\frac{1}{3}\right)^n u(n)$

$= |z|<3 \qquad |z| > 1/3$

$x(n) = \left(\frac{1}{3}\right)^{n} - \left(\frac{1}{2}\right)^n u(n)$

$= \underbrace{3^n u(-n-1)}_{|z|<3} + \underbrace{\left(\frac{1}{3}\right)^n u(n)}_{|z|>1/3} - \underbrace{\left(\frac{1}{2}\right)^n u(n)}_{|z|>1/2}$

$|z|>1/3$ and $|z|>1/2$ → $|z|>1/3$

$|z|<3$ and $|z|>1/3$ → $\frac{1}{3}<|z|<3$

$$\boxed{ROC = \frac{1}{3} < |z| < 3}$$

Que.→ $x(n) = 2^{r(n)}$ ZT=?

Soln→ $x(n) = 2^{r(n)}$

$$\cancel{X(z) = \sum_{-\infty}^{\infty} 2^{r(n)} z^{-n}}$$

$$x(n) = 2^{r(n)}$$
$$= 2^{nu(n)}$$
$$= \begin{cases} 1 & ; n<0 \\ 2^n & ; n \geq 0 \end{cases}$$
$$= u(-n-1) + 2^n u(n)$$
$$|z|<1 \qquad |z|>2$$

ZT will not exists because no common ROC.

Que.→ $x(n) = \cos\omega_0 n \cdot u(n)$ $X(z)=?$, $ROC=?$

Soln→ $x(n) = \cos\omega_0 n \, u(n)$

$$a^n u(n) \rightleftharpoons \frac{z}{z-a}, |z|>|a|$$

$a = e^{j\omega_0}$

$$e^{j\omega_0 n} u(n) \longrightarrow \frac{z}{z-e^{j\omega_0}} \quad |z|>1$$

$a = e^{-j\omega_0}$

$$e^{-j\omega_0 n} u(n) \longrightarrow \frac{z}{z-e^{-j\omega_0}} \quad |z|>1$$

$$\cos\omega_0 n = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}$$

$$x(n) = \frac{1}{2} e^{j\omega_0 n} u(n) + \frac{1}{2} e^{-j\omega_0 n} u(n)$$

$$X(z) = \frac{1}{2}\left[\frac{z}{z-e^{j\omega_0}} + \frac{z}{z-e^{-j\omega_0}}\right] ; |z|>1$$

$$= \frac{1}{2}\left[\frac{z(z-e^{-j\omega_0}) + z(z-e^{j\omega_0})}{z^2 - z(e^{j\omega_0} + e^{-j\omega_0}) + 1}\right], |z|>1$$

$$= \frac{1}{2}\left[\frac{2z^2 - z(e^{j\omega_0} + e^{-j\omega_0})}{z^2 - 2z\cos\omega_0 + 1}\right], |z|>1$$

$$= \frac{1}{2}\left[\frac{2z^2 - 2z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}\right], |z|>1$$

$$= \frac{z^2 - z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1} \quad |z|>1$$

Important signals →

$x(n)$	$X(z)$	ROC
$\delta(n)$	1	entire z-plane
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\|>\|a\|$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\|<\|a\|$
$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\|>1$
$n \cdot a^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|>\|a\|$
$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|<\|a\|$
$\cos\omega_0 n$	$\frac{z^2 z\cos\omega_0}{z^2-2z\cos\omega_0+1}$	$\|z\|>1$
$\sin\omega_0 n$	$\frac{z\sin\omega_0}{z^2-2z\cos\omega_0+1}$	$\|z\|>1$

Que.→ $x(n) \rightleftharpoons X(z) = \frac{0.5}{1-2z^{-1}}$

It is given that ROC of $X(z)$ includes unit circle. is
The value of $x(0)$ is :-
(a) 0.5 (b) 0 (c) 0.25 (d) 0.5

Sol^n →

$$X(z) = \frac{0.5}{1-2z^{-1}}$$

Pole:- $1-2z^{-1}=0$

$z=2$

ROC:- $|z|<2$ (Given)

so inverse will be left sided, $x(n)$

$$x(n) = -0.5(2)^n u(-n-1)$$

$u(-n-1) \rightarrow (-\infty \text{ to } -1)$

Que.→ Find inverse ZT of

$$X(z) = \frac{z}{(z-1)(z-2)^2} \quad \text{if}$$

(i) $|z|>2$ (ii) $|z|<1$ (iii) $1<|z|<2$

Soln→

$$X(z) = \frac{z}{(z-1)(z-2)^2}$$

$$\boxed{\begin{aligned} &f(n) = a^n u(n) \rightarrow F(z) = \frac{z}{z-a} \\ &f(n-1) = a^{n-1}u(n-1) \\ &z^{-1}F(z) = \frac{1}{(z-a)} \end{aligned}}$$

$$X(z) = \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-2)^2}$$

$$\frac{X(z)}{z} = \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-2)^2}$$

$$X(z) = \frac{Az}{(z-1)} + \frac{Bz}{(z-2)} + \frac{Cz}{(z-2)} \times \frac{z^{-2}}{z^{-2}}$$

$$X(z) = \frac{A}{1-z^{-1}} + \frac{B}{1-2z^{-1}} + \frac{C}{2}\left[\frac{2z^{-1}}{(1-2z^{-1})^2}\right]$$

$A=1,\ B=-1,\ C=1$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{z}{1-2z^{-1}} + \frac{1}{2}\left[\frac{2z^{-1}}{(1-2z^{-1})^2}\right]$$

Poles→ $z = 1, 2$

(i) ROC: $|z|>2$

* $x(n)$ will be RS.
* $x(n) = u(n) - 2^n u(n) + \frac{1}{2} n 2^n u(n)$

(ii) ROC: $|z|<1$

* $x(n)$ will be LS.
* $x(n) = -u(-n-1) - [-2^n u(-n-1)] + \frac{1}{2}[-n 2^n u(-n-1)]$

(iii) ROC: $1<|z|<2$

* $x(n)$ will be both sided.
* $x(n) = u(n) - [-2^n u(-u-1)] + \frac{1}{2}[-n 2^n u(-n-1)]$

Que→ $x(n) = 4^n u(n) \rightleftharpoons X(z)$

$y(n) \rightleftharpoons Y(z) = X^2(z)$ Find $y(n)$

(a) $4^n u(n)$ (b) $(n+1)4^n u(n+1)$ (c) $(n+1)4^n u(n)$ (d) $n \cdot 4^n u(n+1)$

Solⁿ→

$$X(z) = \frac{1}{1-4z^{-1}}$$

$$X^2(z) = \left(\frac{1}{1-4z^{-1}}\right)^2 = \frac{1}{(1-4z^{-1})^2} = \frac{1}{1+16z^{-2}-8z^{-1}}$$

$$Y(z) = X^2(z) = \frac{1}{(1-4z^{-1})^2}$$

$$Y(z) = \frac{z^2}{(z-4)^2} = \frac{z(z-4)+4z}{(z-4)^2}$$

$$= \frac{z}{(z-4)} + \frac{4z}{(z-4)^2} \times \frac{z^{-2}}{z^{-2}}$$

$$Y(z) = \frac{1}{1-4z^{-1}} + \frac{4z^{-1}}{(1-4z^{-1})^2}$$

$$y(n) = 4^n u(n) + n 4^n u(n)$$

$$\boxed{y(n) = (n+1)\, 4^n u(n)}$$

ans: (b) & (c)

$$(n+1)4^n u(n+1) = (0, \underset{\uparrow}{1}, 8 \ldots\ldots)$$

$$(n+1)4^n u(n) = (\underset{\uparrow}{1}, 8 \ldots\ldots)$$

Que→ $X(z) = \log(1+az^{-1})$; $|z|>|a|$ Find $x(n) = ?$

Solⁿ→ diff in Freq:

$$x(n) \rightleftharpoons X(z)$$

$$n x(n) \rightleftharpoons -z\frac{dX(z)}{dz} = -z\left[\frac{1}{1+az^{-1}}(-az^{-2})\right]$$

$$n \cdot x(n) = \frac{az^{-1}}{1+az^{-1}} \; ; \; |z|>|a|$$

$$n x(n) = 1 - \frac{1}{1+az^{-1}} \quad |z|>|a|$$

$$n x(n) = \delta(n) - (-a)^n u(n)$$

Que→ $X(z) = \dfrac{1+z^{-1}}{1+\frac{1}{3}z^{-1}} \rightleftharpoons x(n)$

(a) Assuming ROC to be $|z| < 1/3$. determine $x(0), x(-1), x(-2)$

(b) Assuming ROC to be $|z| > 1/3$ determine $x(0), x(1)$ & $x(2)$

Solⁿ→

$$X(z) = \frac{1+z^{-1}}{1+(\frac{1}{3})z^{-1}}$$

(a) $|z| < \frac{1}{3}$; $x(n)$ will be LS sig.

Arrange numerator & denominator polynomials in asending powers of 'z'.

$$X(z) = \frac{z^{-1}+z^{0}}{(\frac{1}{3})z^{-1}+z^{0}}$$

LED (LA)

$$\tfrac{1}{3}z^{-1}+1 \,\Big)\, z^{-1}+1 \,\Big(\, 3-6z+18z^2+\cdots$$

$$\begin{array}{r} z^{-1}+1 \\ \underline{-\,z^{-1} \mp 3} \\ -2 \\ \underline{\mp 2 \mp 6z} \\ 6z \\ \underline{-\,6z - 18z^2} \\ -18z^2 \\ \vdots \end{array}$$

$$X(z) = 3-6z+18z^2+\cdots\cdots$$
$$= x(0)+x(-1)z+x(-2)z^2+\cdots\cdots$$

$x(0)=3$, $x(-1)=-6$, $x(-2)=18$

(b) $|z| > \frac{1}{3}$; $x(n)$ will be RS sig.

Arrange nume. & deno. poly. in decensing powers of 'z'.

$$X(z) = \frac{1+z^{-1}}{1+(\frac{1}{3})z^{-1}}$$

$$1+\tfrac{1}{3}z^{-1} \,\Big)\, 1+z^{-1} \,\Big(\, 1+\tfrac{2}{3}z^{-1}-\tfrac{2}{9}z^{-2}+\cdots$$

$$\begin{array}{r} 1+z^{-1} \\ \underline{-\,1 - \frac{1}{3}z^{-1}} \\ \frac{2}{3}z^{-1} \\ \underline{-\,\frac{2}{3}z^{-1} - \frac{2}{9}z^{-2}} \\ -\frac{2}{9}z^{-2} \end{array}$$

$$X(z) = 1+\frac{2}{3}z^{-1}-\frac{2}{9}z^{-2}+\cdots\cdots$$

$$= x(0) + x(1)z^{-1} + x(2)z^{-2} + \cdots\cdots$$

$$x(0)=1,\ x(1)=\frac{2}{3},\ x(2)=-\frac{2}{9}$$

Que.→ $x(n) \rightleftharpoons X(z) = \frac{z}{2z^2-3z+1}$, $|z|<\frac{1}{2}$. Find $x(-2)$

(a) 0 (b) 1 (c) 2 (d) 3

Soln→

$$X(z) = \frac{z}{1-3z+2z^2}$$

$$1-3z+2z^2 \,)\, \cancel{z} \,(\, z+3z^2+7z^3+\cdots\cdots$$

$$\cancel{z}-3z^2+2z^3$$

$$3\cancel{z^2}-2z^3$$

$$3\cancel{z^2}-9z^3$$

$$7\cancel{z^3}$$

$$7\cancel{z^3}-21z^4+14z^5$$

$$\boxed{x(-2)=3}$$

Que.→ $x(n) \rightleftharpoons X(z) = \frac{z+z^{-3}}{z+z^{-1}}$; $x(n)$ series has

(a) Alternate -1s (c) Alternate 1s

(b) Alternate 0s (d) Alternate 2s.

Soln→

$$X(z) = \frac{z+z^{-3}}{z+z^{-1}}$$

Here ROC is not given, so devide in the given form.

$$z+z^{-1} \,)\, \cancel{z}+z^{-3} \,(\, 1-z^{-2}+2z^{-4}-2z^{-6}+2z^{-8}-2z^{-10}$$

$$\cancel{z}+z^{-1}$$

$$-\cancel{z^{-1}}+z^{-3}$$

$$-\cancel{z^{-1}}-z^{-3}$$

$$2\cancel{z^{-3}}$$

$$2\cancel{z^{-3}}+2z^{-5}$$

$$-2z^{-5}$$

$$-2z^{-5}-2z^{-7}$$

$$\vdots$$

$$X(z) = 1 - z^{-2} + 2z^{-4} + 2z^{-6} + 2z^{-8} + \cdots$$

$$x(n) = (1, 0, -1, 0, 2, 0, -2, 0, 2 \cdots)$$

ans. (b).

* Causal system →

(1.) $h(n) = 0,\ n < 0$

(2.) $H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = \sum_{n=0}^{\infty} h(n) z^{-n}$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + \cdots = \frac{N(z)}{D(z)}$$

Note → For causal sys, expansion of TF does not include +ve powers of z.

(3.) $\lim_{z \to \infty} H(z) = h(0) = 0$ (or) finite.

Note → *For causal sys.; order of numerator can't exceed order of denominator

* For causal sys. ROC will be outside circle in z-plane.
* For stability of ~~d~~ discrete time causal sys., poles of TF should lie inside unit circle in z-plane.

* Anticausal system →

* For this sys. Roc will be inside circle in z-plane.
* For stability of this sys, poles of TF should lie outside unit circle in z-plane.

Que. → A causal LTI sys. is described by the difference eqn

$$2y(n) = a y(n-2) - 2x(n) + \beta x(n-1)$$

The sys. is stable only if

(a.) $|a| = 2,\ |\beta| < 2$

(b.) $|a| > 2,\ |\beta| > 2$

(c.) $|a| < 2$, for any value of 'β'

(d.) $|\beta| < 2$, for any value of 'a'

Sol^n→ $$2Y(z) = aY(z)\cdot z^{-2} - 2X(z) + \beta X(z) z^{-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-2+\beta z^{-1}}{2-az^{-2}}$$

Poles:- $$2-az^{-2}=0$$
$$z=\sqrt{\frac{a}{2}}$$

For stability of causal sys.
$$|pole|<1$$
$$\sqrt{\frac{a}{2}}<1$$
$$\boxed{|a|<2}$$

Que.→ $x(n) \rightleftharpoons X(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$

Find $x(\infty) = ?$ (a) 1/4 (b) 0 (c) 1 (d) ∞

Sol^n→
$$x(n) \rightleftharpoons X(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$$

(i) $\lim_{z\to\infty} X(z) = 0$

Cond^n for causality is satisfied.

(ii) $$(1-z^{-1})X(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})}$$

$$= \frac{z^{-1}(1+z^{-2})(1-z^{-2})}{4(1-z^{-1})}$$

$$= \frac{z^{-1}(1+z^{-2})(1+z^{-1})\cancel{(1-z^{-1})}}{4\cancel{(1-z^{-1})}}$$

$$= \frac{z^{-1}(1+z^{-2})(1+z^{-1})}{4}$$

Pole:- $z=0<1$

Both the cond^n are satisfied. So we can use final value theorem.

Final value theorem →

$$x(\infty) = \lim_{z\to 1}\left[(1-z^{-1})X(z)\right]$$

$$= \lim_{z\to 1}\left[\frac{z^{-1}(1+z^{-2})(1+z^{-1})}{4}\right]$$

$$= \frac{2\times 2}{4}$$

$$= 1$$

Que → A stable & causal sys. is described by the diff. eqn

$$y(n)+\frac{1}{4}y(n-1)+\frac{1}{8}y(n-2) = -2x(n)+\frac{5}{4}x(n-1)$$

h(n) of the sys. is

(a.) $\left(\frac{1}{4}\right)^n u(n) + 3\left(\frac{-1}{2}\right)^n u(n)$ (b.) $\left(\frac{1}{4}\right)^n u(n) - 3\left(\frac{-1}{2}\right)^n u(n)$

(c.) $\left(\frac{1}{4}\right)^n u(n) - 3u(n)$ (d.) $\left(\frac{1}{4}\right)^{n-1} u(n-1) - 3u(n-1)$

Soln →

$$Y(z)+\frac{1}{4}Y(z)z^{-1}-\frac{1}{8}Y(z)z^{-2} = -2X(z)+\frac{5}{4}X(z)z^{-1}$$

$$Y(z)\left[1+\frac{1}{4}z^{-1}-\frac{z^{-2}}{8}\right] = X(z)\left[-2+\frac{5}{4}z^{-1}\right]$$

$$H(z) = \frac{-2+\frac{5}{4}z^{-1}}{1+\frac{1}{4}z^{-1}-\frac{z^{-2}}{8}}$$

from option

(a.) $|z|>\frac{1}{4}$ $|z|>\frac{1}{2}$ common ROC $|z|>\frac{1}{2}$ (stable)

(b.) common ROC $|z|>\frac{1}{2}$ (stable)

(c.) common ROC $|z|>1$ (US)

(d.) common ROC $|z|>1$ (US)

Here sys. is causal so follow the Initial value theorem

$$h(0) = \lim_{z\to\infty} H(z) = -2$$

(a.) $h(0) = 1+3 = 4$ (b.) $h(0) = 1-3 = -2$

* Jury Test →

(1.) To check stability of contineous time causal LTI sys. Routh-hurwitz criteria is used.

(2.) Jury test is used to check stability of discrete time causal LTI sys.

$$H(z) = \frac{k_m z^m + k_{m-1} z^{m-1} + \cdots + k_0}{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0} = \frac{N(z)}{D(z)}$$

For causlity :- $n \geq m$

c/s eqn :- $D(z) = 0$

necessary condn for stability →

(1.) $D(1) > 0$

(2.) $D(-1) \begin{cases} > 0 \; ; n = \text{even} \\ < 0 \; ; n = \text{odd} \end{cases}$

Jury table → no. of rows $= 2n-3$

	z^0	z^1	z^2 -----	z^n
1.	a_0	a_1	a_2 -----	a_n
2.	a_n	a_{n-1}	a_{n-2} -----	a_0
3.	b_0	b_1	b_2 ------	b_{n-1}
4.	b_{n-1}	b_{n-2}	b_{n-3} ----	b_0
5.	c_0	c_1	c_2 ------	c_{n-2}
	⋮			

$$b_i = \begin{vmatrix} a_0 & a_{n-i} \\ a_n & a_i \end{vmatrix} = a_0 a_i - a_n a_{n-i}$$

$$b_0 = a_0 a_0 - a_n a_{n-0} = a_0^2 - a_n^2$$

$$b_1 = a_0 a_1 - a_n \cdot a_{n-1}$$

$$c_0 = b_0^2 - b_{n-1}^2$$

$$c_1 = b_0 b_1 - b_{n-1} \cdot b_{n-2}$$

⋮

sufficient condⁿ →

(1.) $|a_n| > |a_0|$

(2.) $|b_{n-1}| < |b_0|$

(3.) $|c_{n-2}| < |c_0|$

(4.) $|d_{n-3}| < |d_0|$

Que. → $H(z) = \dfrac{2z^3 + 2z^2 + 3z + 1}{2z^4 + 3z^3 + z^2 - 1}$. check stability of sys.

Solⁿ → $D(z) = 2z^4 + 3z^3 + z^2 - 1$

Order → $n = 4$, (even)

necessity condⁿ :-

(i) $D(1) > 0$ ✓

(ii) $D(-1) > 0$, n = even ✗

Que. → check stability of sys. having

$D(z) = 5z^3 + 2z^2 + 4z + 1$

Solⁿ → $D(z) = 5z^3 + 2z^2 + 4z + 1$

Order – odd ($n = 3$)

Condⁿ → (i) $D(1) > 0$

(ii) $D(-1) < 0$; n = odd

$\downarrow$

-6

Jury table → No. of rows $2n - 3 = 2 \times 3 - 3 = 3$

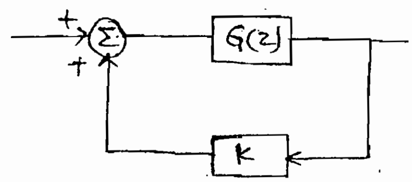
		z^0	z^1	z^2	z^3	
a_0	1.	1	4	2	5	a_n
	2.	5	2	4	1	
b_0	3.	-24	-6	-18	0	b_{n-1}
	4	$\cancel{0}$	$\cancel{-18}$	$\cancel{-6}$	$\cancel{-24}$	

Sufficient condⁿ :-

(1.) $|a_n| > |a_0|$ ✓ (2.) $|b_{n-1}| < |b_0|$ ✓

sys. is stable

Que.→



$g(n) \rightleftharpoons G(z)$

$g(n) = (\underset{\uparrow}{0}, 1, 1)$

The sys. is stable for range of. ~~value~~

value of 'k'

(a) $(-1, 1/2)$ (b) $(-1, 1)$ (c) $(-1/2, 1)$ (d) $(-\frac{1}{2}, 2)$

Sol^n→

$$G(z) = z^{-1} + z^{-2}$$

$$H(z) = \frac{G(z)}{1 - KG(z)}$$

$$= \frac{z^{-1} + z^{-2}}{1 - K(z^{-1} + z^{-2})} \times \frac{z^2}{z^2}$$

$$= \frac{z+1}{z^2 - K(z+1)}$$

$$D(z) = z^{2} - Kz + 1 \xrightarrow{} \text{even}$$

* $D(1) > 0$, $1 - 2K > 0 \Rightarrow K < \frac{1}{2}$ ---- (i)

* $D(-1) > 0$; $1 > 0$

* $|a_n| > |a_0|$

$1 > |K|$

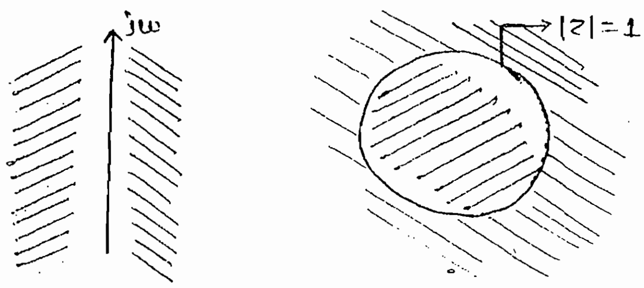
$(-1 < K < 1)$ ----- (ii)

from (i) & (ii)

$$\boxed{-1 < K < \frac{1}{2}}$$

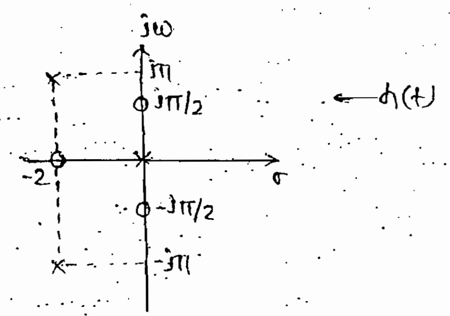
DATE-31/10/14

Mapping between s-plane & z-plane →



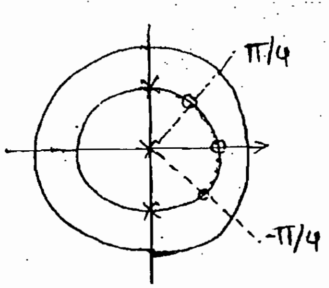
Pole mapping - $\boxed{z=e^{sT}}$ T - sampling time interval

Que.→

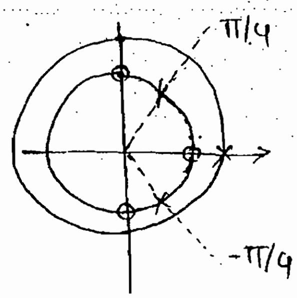


The impulse response of h(t) is sampled at 2kHz to get h(n). Which one of the following represents equivalent pole-zero plot of H(z) in z-plane? (The concentric circles are $|z|=1$, $|z|=\frac{1}{e}$)

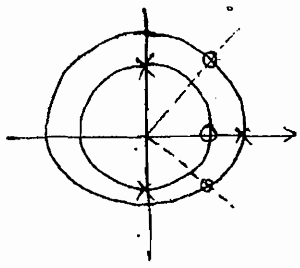
(a.)



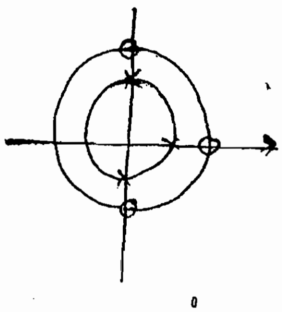
(b.)



(c.)



(d.)



Sol^n → **S-plane**

$$s = 0,\ -2+j\pi,\ -2-j\pi$$

Z-plane

$$z = e^{sT} = e^{s/f} = e^{s/2}$$

$$= 1,\ e^{-1+j\pi/2},\ e^{-1-j\pi/2}$$

$$= 1e^{0j},\ \frac{1}{e}e^{j\pi/2},\ \frac{1}{e}e^{-j\pi/2}$$

*** DTFT →**

$$x(n) = X(e^{j\omega})$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\Big\downarrow z = e^{j\omega}$$

$$\boxed{X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}}$$

$$\Big\downarrow (\omega = 0)$$

***

$$\boxed{X(e^{j0}) = \sum_{n=-\infty}^{\infty} x(n)}$$

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega.$$

$$\Big\downarrow (n = 0)$$

$$x(0) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) d\omega$$

***

$$\boxed{2\pi x(0) = \int_{2\pi} X(e^{j\omega}) d\omega}$$

16/18

$$Y(e^{j0}) = \sum_{-\infty}^{\infty} y(n)$$

$$= \sum_{0}^{\infty} \left(\frac{1}{4}\right)^{n}$$

$$= 1+\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^{2}+\cdots$$

$$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

31/20

$$h(n) = \frac{1}{2}\left[\delta(n)+\delta(n-2)\right] \qquad |H(e^{j\omega})| = ?, \quad \omega=\Omega$$

$$H(z) = \frac{1}{2}(1+z^{-2})$$

$$z=e^{j\omega}$$

$$H(e^{j\omega}) = \frac{1}{2}(1+e^{-j2\omega})$$

$$H(e^{j\omega}) = e^{-j\omega}\left[\frac{e^{j\omega}+e^{-j\omega}}{2}\right]$$

$$|H(e^{j\omega})| = |\overset{1}{e^{-j\omega}}|\,|\cos\omega|$$

$$|H(e^{j\omega})| = |\cos\omega|$$

45/31

$$x(n) = \sin(\omega_0 n+\phi) \longrightarrow \boxed{\text{LTI sys.}} \longrightarrow y(n) = Ax(n-n_0)$$

$$H(e^{j\omega})$$

$$\angle H(e^{j\omega_0}) = ?$$

$$y(n) = Ax(n-n_0)$$

$$Y(z) = AX(z)\cdot z^{-n_0}$$

$$H(z) = Az^{-n_0}$$

$$\downarrow (z=e^{j\omega})$$

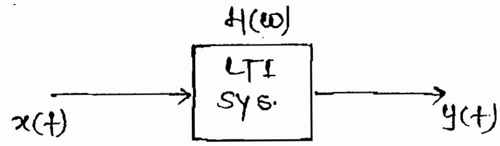
$$H(e^{j\omega}) = A\cdot e^{-j\omega n_0} \qquad \downarrow \omega=\omega_0$$

$$H(e^{j\omega_0}) = Ae^{-j\omega_0 n_0}\cdot e^{j2\pi k} \qquad \to k = \text{an int.}$$

$$= Ae^{j(2\pi k - n_0\omega_0)}$$

$$\angle H(e^{j\omega_0}) = 2\pi k - n_0\omega_0$$

**

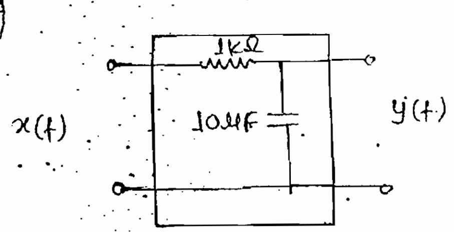


$$x(t) = A_0 \sin(\omega_0 t + \phi)$$
$$y(t) = A_0 \times |H(\omega_0)| \times \sin\left\{[(\omega_0 t + \phi)] + \angle H(\omega_0)\right\}$$

(21/19)

$$h(t) = e^{-2t}u(t)$$
$$x(t) = 2\cos(2t) \; ; \omega_0 = 2$$
$$y(t) = ?$$
$$H(\omega) = \frac{1}{j\omega + 2}$$
$$H(\omega_0) = \frac{1}{2+2j} \qquad |H(\omega_0)| = \frac{1}{2\sqrt{2}} \; , \angle H(\omega_0) = -\pi/4$$
$$y(t) = 2 \times \frac{1}{2\sqrt{2}} \times \cos(2t - \pi/4)$$
$$= e^{-0.5}\cos(2t - 0.25\pi)$$

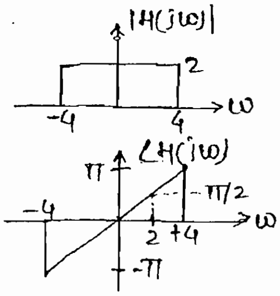
(11/22)



$$x(t) = 3 + 4\sin 100t$$
$$\omega_0 = 100$$
$$H(s) = \frac{1}{1 + sCR}$$
$$(s = j\omega)\downarrow$$
$$H(\omega) = \frac{1}{1 + j\omega RC}$$
$$\downarrow(\omega = \omega_0)$$
$$H(\omega_0) = \frac{1}{1 + j\omega_0 RC}$$
$$\left\{ \begin{aligned} \omega_0 RC &= 100 \times 10^3 \times 10 \times 10^{-6} \\ &= 1 \end{aligned} \right.$$
$$= \frac{1}{1+j}$$
$$|H(\omega_0)| = \frac{1}{\sqrt{2}} \; , \angle H(\omega_0) = -\frac{\pi}{4}$$

$$y(t) = 3 + 4 \times \frac{1}{\sqrt{2}} \times \sin[100t + (-\pi/4)]$$
$$= 3 + \frac{4}{\sqrt{2}} \sin(100t - \pi/4)$$

Que.→



$x(t) = 2\sin 2t$

$y(t) = ?$

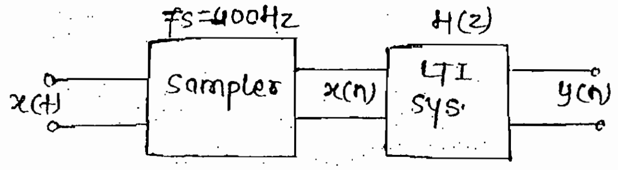
Sol^n→

$\omega_0 = 2 \qquad x(t) = 2\sin 2t$

$y(t) = 2\times 2\times \sin(2t + \pi/2)$

$y(t) = 4\cos 2t$

Que.→



$x(t) = 2 + 5\sin(100\pi t)$

$H(z) = \frac{1}{N}\left[\frac{1-z^{-N}}{1-z^{-1}}\right]$ where N = no. of samples per cycle.

The o/p. y(n) of sys. under steady state is

(a) 0 (b) 1 (c) 2 (d) 5

Sol^n→ $y(\infty) = ?$; N = no. of samples per cycle = sampling freq.

$= 400$

$x(t) = 2 + 5\sin(100\pi t)$

$\Big\downarrow \; t = nT_s = \frac{n}{f_s} = \frac{n}{400}$

$x(n) = 2 + 5\sin\left(100\pi\times\frac{n}{400}\right)$

$= 2 + 5\sin\left(\frac{n\pi}{4}\right) \quad ; \; \omega_0 = \frac{\pi}{4}$

$(z = e^{j\omega}) \qquad H(\omega) = \frac{1}{N}\left[\frac{1-e^{-j\omega N}}{1-e^{-j\omega}}\right] \Big\downarrow \left(\omega = \omega_0 = \frac{\pi}{4}\right)$

$H(\omega_0) = \frac{1}{N}\left[\frac{1-e^{-j\frac{\pi}{4}N}}{1-e^{-j\pi/4}}\right]^{N=400 \, 100} \neq 1 \qquad (\because e^{-j100\pi} = 1)$

$H(\omega_0) = 0$

$y(n) = 2 +$ o/p due to $\cancel{\text{sin}}$usoidal part $\to 0$

$= 2$

$\boxed{y(\infty) = 2}$

Que.→ $x(n) \rightleftharpoons X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$

$x(n)$ is orthogonal to the signal.

(a.) $y_1(n) = Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$ (c.) $y_3(n) = Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$

(b.) $y_2(n) = Y_2(z) = \sum_{n=0}^{\infty} (5^n - n) z^{-(2n+1)}$ (d.) $y_4(n) = Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

Sol→

$$\sum_{n=-\infty}^{\infty} x_1(n) \cdot x_2(n) = 0$$

(orthogonal)

$y_2(n)$ is available for odd instants of 'n'

ans.(b)

$x_1(n)\, y_2(n) = 0$ so $\sum_{-\infty}^{\infty} x(n)\, y_2(n) = 0$

so; they are orthogonal.

3/4 $3y^2(t) + 2y^2(t) + y(t) = x^2(t) + x(t)$

all values are present (static)

NL (sq. term)

10/4 anticipator − anticausal sys.

11/4 $h(n) = \delta(n+2) - \delta(n-2)$ (OR) $H(z) = z^2 - z^{-2}$

$y(n) = x(n+2) - x(n-2)$ (a.) $H(z) =$

$(n = n-2)$

$y(n-2) = x(n) - x(n-4)$

14/5 $y(n) = x(n) * h(n)$

$= \sum x(k) \cdot h(n-k)$

$= \sum a^k u(k) \cdot b^{n-k} u(n-k)$

16.
5

$$x(n) = \begin{cases} 0 & , n < -2 \text{ or } n > 4 \\ 1 & \text{otherwise.} \end{cases}$$

$(n-2)$

$x(n-2) = 0 \quad n-2 < -2 \quad (-n-2) > 4$

$n > 0 \qquad n < -6$

22
6

even part of $u(t) = \dfrac{u(t)+u(-t)}{2} = \dfrac{1}{2}$

Odd part of $u(t) = \dfrac{u(t)-u(-t)}{2}$ $\qquad \because u(t)+u(-t) = 1$

$u(-t) = 1 - u(t)$

$= \dfrac{u(t)-1+u(t)}{2}$

$= \dfrac{2u(t)-1}{2}$ $\qquad \because u(t) = \dfrac{1+\text{sgn}(t)}{2}$

$= \dfrac{\text{sgn}(t)}{2}$ $\qquad \text{sgn}(t) = 2u(t)-1$

$= \dfrac{x(t)}{2}$

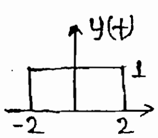
Ans. $\left(\dfrac{1}{2}, \dfrac{x(t)}{2}\right)$

27
7

$x(t) = \delta(t+2) - \delta(t-2)$

$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$= u(t+2) - u(t-2)$



$E_y = \int_{-\infty}^{\infty} |y(t)|^2 dt$

$= \int_{-2}^{2} 1^2 dt = 4$

28/7 scaling property of convo^n

$(t^2+t) \quad (t)$

$x_1(t) * x_2(t) = y(t)$

$x_1(at) * x_2(at) = \frac{1}{|a|} y(at)$

$(t = t + 3)$

$\downarrow a=3$

$x_1(3t) * x_2(3t) = \frac{1}{3} y(3t)$

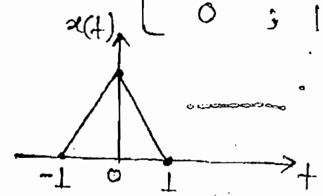
$(9t^2+3t) \quad (3t)$

32/8

$(-1 \le t \le 1)$

$$f(t) = \begin{cases} 1-|t| & |t| \le 1 \\ 0 & |t| \ge 1 \end{cases}$$

$$= \begin{cases} 1+t & ; -1 \le t \le 0 \\ 1-t & ; 0 \le t \le 1 \\ 0 & ; |t| > 1 \end{cases}$$



Chapter-02

1/9

$y(n+2) - 5y(n+1) + 6y(n) = x(n)$

$H(z) = \frac{1}{z^2 - 5z + 6}$

$H(z) = \frac{1}{(z-3)(z-2)}$

Poles:- 3, 2

9/9

$y(t) + \int_{0^-}^{\infty} y(\tau) x(t-\tau) d\tau = \delta(t) + x(t)$

$y(t) + y(t) * x(t) = \delta(t) + x(t)$

$\downarrow (LT)$

$Y(s) + Y(s) \cdot X(s) = 1 + X(s)$

$Y(s)[1 + X(s)] = 1 + X(s)$

1/1

$h(n) = (1, \frac{1}{2}, \frac{1}{4})$

$x(n) = (1, 0, 1)$

$y(n) = x(n) * h(n)$

Tabular method

15/10

$x(t) = u(t) \longrightarrow y(t) = 0.5(1 - e^{-2t}) u(t)$

$X(s) = \frac{1}{s} \qquad Y(s) = 0.5\left(\frac{1}{s} - \frac{1}{s+2}\right)$

$= \frac{1}{s(s+2)}$

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$

$h(t) = e^{-2t} u(t)$

17/10

(i) $y(t) = t x(t) \longrightarrow L$

(ii) $y(t) = t x^2(t) \longrightarrow NL$

(iii) $y(t) = x(2t) \longrightarrow L$

18/11

$y(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau) d\tau$

$\downarrow y(t) = h(t) \qquad \downarrow x(t) = \delta(t)$

$h(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} \delta(\tau) d\tau$

$h(t) = \frac{1}{T} [u(\tau)]_{t-T/2}^{t+T/2}$

$= \frac{1}{T} [u(t+T/2) - u(t-T/2)]$

$h(t)$

(19/11) $h(t) = \delta(t) + \delta(t-1) \rightleftharpoons H(s) = 1 + e^{-s}$

$y(t) = u(t) + u(t-1) \rightleftharpoons Y(s) = \frac{1}{s} + \frac{1}{s}e^{-s}$

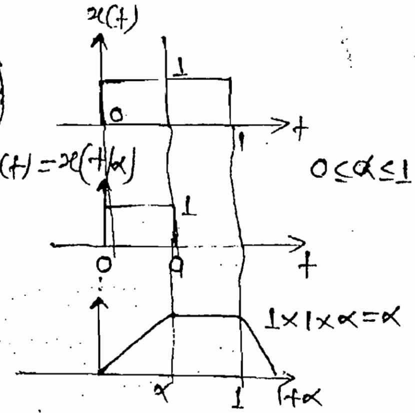
$= \frac{1}{s}(1 + e^{-s})$

$H(s) = \frac{Y(s)}{X(s)}$

$X(s) = \frac{1}{s}$

$x(t) = u(t)$

(20/11)



$0 \le \alpha \le 1$

(21/12) LPF; $f_c = 100Hz$

$v(t) = 30\sqrt{2}\sin(1256t)$

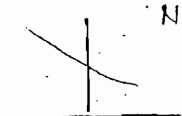
$f_i = \frac{1256}{2\pi} \approx 200Hz$

Chapter-03

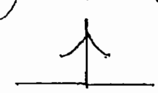
(9/14) $x_1(t) \rightarrow e^{j20t}$ (P)

$x_2(t) \rightarrow e^{(-2+j)t} = e^{-2t} e^{jt}$

NP N P



(13/14) $e^{-|t|}$ $\sin 25t$



NP P

(13/15) $v(t) = (10\sin 2\pi t)\ 100t$

NP P NP

Chapter-04

(4/16) C+P → D+NP

CTFS

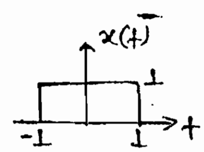
CTFT C+NP

D+P DTFS

DTFT D+NP

Dirchelt cond NP

(20/19)



$X(\omega) = 2Sa(\omega)$

$X(\omega) = 0,\ \omega = ?$

$2Sa(\omega) = 0$

$Sa(\omega) = 0$

$\frac{\sin\omega}{\omega} = 0$

$\sin\omega = 0$

$\omega = n\pi;\ n \ne 0$

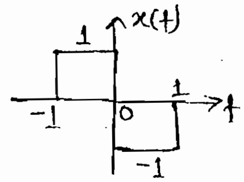
(8/17) $y(n) - \frac{1}{2}y(n-1) = x(n) = k\delta(n)$

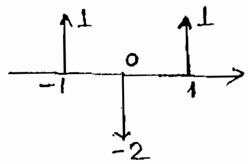
$Y(z) - \frac{1}{2}Y(z)z^{-1} = X(z) = k$

$Y(z) = \frac{k}{1 - \frac{1}{2}z^{-1}}$

$y(n) = k\left(\frac{1}{2}\right)^n u(n)$

24/19



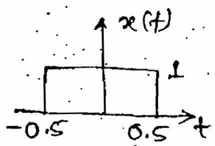


$$\frac{dx(t)}{dt} = \delta(t+1) + \delta(t-1) - 2\delta(t)$$

$$j\omega X(\omega) = (e^{j\omega} + e^{-j\omega}) - 2$$

$$X(\omega) = \frac{2\cos\omega - 2}{j\omega}$$

25/19



$$\rightleftharpoons X(\omega) = Sa\left(\frac{\omega}{2}\right)$$

$$h(t) = e^{j\omega_0 t} \rightleftharpoons H(\omega) = 2\pi\delta(\omega - \omega_0)$$

$$y(t) = 0, \ \omega_0 = ?$$

$$Y(\omega) = 0$$

$$X(\omega) \cdot H(\omega) = 0$$

$$Sa\left(\frac{\omega}{2}\right) \cdot 2\pi\delta(\omega - \omega_0) = 0$$

$$Sa\left(\frac{\omega_0}{2}\right) 2\pi\delta(\omega - \omega_0) = 0$$

$$Sa\left(\frac{\omega_0}{2}\right) = 0$$

$$\frac{\sin\left(\frac{\omega_0}{2}\right)}{\left(\frac{\omega_0}{2}\right)} = 0$$

$$\sin\left(\frac{\omega_0}{2}\right) = 0$$

$$\frac{\omega_0}{2} = n\pi, \ n \neq 0$$

$$\omega_0 = 2n\pi, \ n \neq 0$$

Chapter-05

2/21

$$x(t) = u(t) \rightleftharpoons y(t) = t^2 e^{-2t} u(t)$$

$$t^2 u(t) \rightleftharpoons \frac{2}{s^3}$$

$$e^{-2t} t^2 u(t) \rightleftharpoons \frac{2}{(s+2)^3}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{2}{(s+2)^3}}{1/s} = \frac{2s}{(s+2)^3}$$

9/21

$$\frac{d^2y(t)}{dt^2} = x(t-2)u(t-2) + \frac{d^2x(t)}{dt^2}$$

$$H(s) \longrightarrow LTI$$

12/22

$$f(t) \rightleftharpoons F(s) = \frac{s+2}{s^2+1}$$

$$g(t) \rightleftharpoons G(s) = \frac{s^2+1}{(s+3)(s+2)}$$

$$h(t) = \int_0^t f(\tau) \cdot g(t-\tau)d\tau \rightleftharpoons H(s) = ?$$

$$= \int_{-\infty}^{\infty} f(\tau) \cdot g(t-\tau)d\tau$$

$$h(t) = f(t) * g(t)$$

Let assump $\rightarrow$ $f(t)$ & $g(t) \rightarrow$ causal

$$H(s) = F(s)\, G(s) = \frac{1}{s+3}$$

23/24

$$x(t) = \sin\omega t$$

$$y(\infty) = 0, \ \omega = ?$$

$$y(t) = 0, \ \omega = ?$$

$$Y(s) = 0$$

$$H(s) \cdot X(s) = 0$$

$$H(s) = 0$$

$(s = j\omega)$

$$H(\omega) = 0, \ \omega = ?$$

(26/24)

$f(t) \rightleftharpoons F_1(S)$

$f(t-\tau) \rightleftharpoons F_2(S) = F_1(S)\, e^{-S\tau}$

$g(t) \rightleftharpoons G(S) = \dfrac{F_2(S)\cdot F_1(S)}{|F_1(S)|^2}$

?

$G(S) = \dfrac{F_2(S)\cdot \cancel{F_1^*(S)}}{F_1(S)\cdot \cancel{F_1^*(S)}} = e^{-S\tau}$

$g(t) = \delta(t-\tau)$

(29/24)

$y(t)=0,\ \omega = ?$

$Y(S)=0$

$\dfrac{1}{S} - \dfrac{3}{S+1} + \dfrac{3}{S+2} = 0$

$(S+1)(S+2) - 3S(S+2) + 3S(S+1) = 0$

$S^2 + \cancel{3S} + 2 - \cancel{3S^2} - \cancel{6S} + \cancel{3S^2} + \cancel{3S} = 0$

$S^2 + 2 = 0$

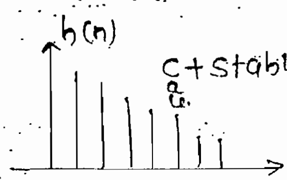
$(j\omega)^2 + 2 = 0$

$-\omega^2 + 2 = 0$

$\omega = \sqrt{2}$ rad/s.

$\boxed{\text{Chapter-06}}$

(1/25)



C + Stable

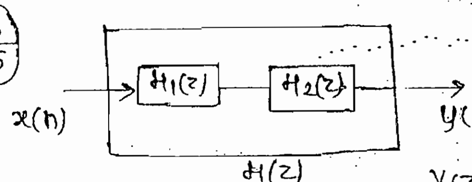
|Poles| < 1

(7/26)

$a^n u(n) \rightleftharpoons \dfrac{z}{z-a},\ |z|>|a|$

$+a^n u(-n-1) \rightleftharpoons \dfrac{-z}{z-a}\ \ |z|<|a|$

(9/25)



$y(n) = x(n-2)$

$Y(z) = X(z) z^{-2}$

$H(z) = z^{-2}$

$H(z) = H_1(z)\cdot H_2(z)$

$H_2(z) = \dfrac{H(z)}{H_1(z)}$

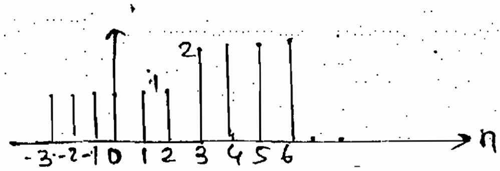
$H_2(z) = \dfrac{z^{-2}(z-0.8)}{z-0.5}$

$= \dfrac{z^{-1} - 0.8z^{-2}}{z-0.5} \times \dfrac{z^{-1}}{z^{-1}}$

$= \dfrac{z^{-2} - 0.8z^{-3}}{1-0.5z^{-1}}$

(15/27)

$h(n) = u(n+3) + u(n-3) - 2u(n-7)$



(35/30)

$x(n)$ $y(n)$ LTI Sys.

$u(n) \longrightarrow s(n) =$ step responce

$\downarrow d/dn$

$\delta(n) \longrightarrow h(n) = \dfrac{ds(n)}{dn}$

$= s(n) - s(n-1)$

$h(n) = s(n) - s(n-1)$

$s(n) = (\underset{\uparrow}{1}, 1/2, 1/4, 1/8 \cdots)$

$s(n-1) = (\underset{\uparrow}{0}, 1, 1/2, 1/4 \cdots)$

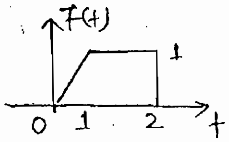
$h(n) = (1, -1/2, -1/4, -1/8 \ldots)$

(54/32)

$G(S) \rightarrow$ Poles $\Rightarrow S = 0, -5$

$z = e^{ST} = 1, e^{-5T}$

1.1)



$$f(t) = r(t) - r(t-1) - u(t-2)$$

$$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}$$

$$= \frac{1}{s^2}\left(1 - e^{-s} - s e^{-2s}\right)$$

1.1)

$$\frac{d^2y}{dt^2} = x(t-2) + \frac{d^2x}{dt^2}$$

$$s^2 Y(s) = e^{-2s} X(s) + s^2 X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{s^2 + e^{-2s}}{s^2} = 1 + \frac{e^{-2s}}{s^2}$$

)

$$F(s) = \frac{27s+97}{s(s+33)} \Longrightarrow f(t)$$

$$\frac{27s+97}{s(s+33)} = \frac{A}{s} + \frac{B}{s+33} \qquad s=0,\ A = \frac{97}{33}$$

$$s=-33,\ B = \frac{-27\times 33+97}{-33\times 33} = \frac{794}{1089}$$

$$f(t) = \frac{97}{33}u(t) + \frac{794}{1089}e^{-33t}u(t)$$

$$f(0^+) = \frac{97}{33}u(0^+) + \frac{794}{1089}e^{-33(0^+)} = \frac{97}{33}$$

2)

$$x_1(t) = e^{k_1 t}u(t) \quad x_2(t) = e^{-k_2 t}u(t)$$