ANUPAM SHUKLA

NOTE BOOK

ELECTRICAL-SIGNAL&SYSTEM

Signal & system

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DATE-09/10/14

Syllabus →

(1.) Signal definations & its classifications.

(2.) Different operation on signal.

(a.) Shifting (d.) Differentiation

(b.) Scaling (e.) Integration

(c.) Reversal (f.) Convolution.

(3.) Basic system operations.

(a.) Static/dynamic

(b.) linear/non-linear

(c.) Causal/Non-causal

(d.) Time invarient/time-varient

(e.) Stable/Unstable.

(4.) Contineous time fourier series
(5.) Contineous time fourier xform
(6.) Laplace xform.

→ contineous time sig & sys.

(7.) Sampling theorem
(8.) Discrete time sys.
(9.) Z-transform

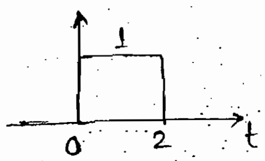
→ discrete time sig & sys.

* Different operations on signal →

* Amplitude shifting
* Time shifting
* Time scaling
* Time reversal
* Amplitude Reversal
*

(1.) Time shifting →

$$x(t) \longrightarrow y(t) = x(t+k)$$

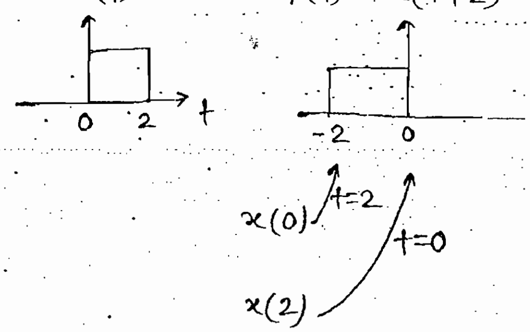


Case(1)

when $k>0$

Eg:- $k=2$

$$x(t) \longrightarrow y(t) = x(t+2)$$



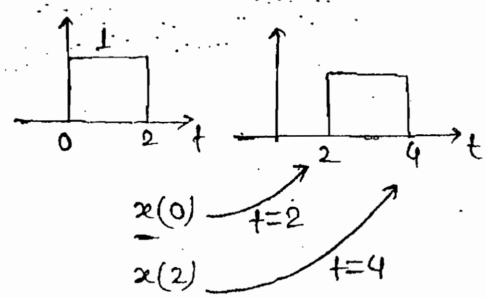
* It is a case of left shifting.

Case(2)

when $k<0$

Eg:- $k=-2$

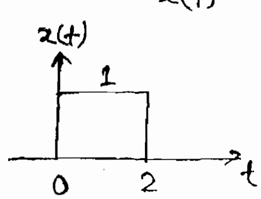
$$x(t) \longrightarrow y(t) = x(t-2)$$



* It is a case of right shifting

(2.) Amplitude Shifting →

$$x(t) \longrightarrow y(t) = k + x(t)$$



$$x(t) = \begin{cases} 0 & , t<0 \\ 1 & \ldots \end{cases}$$

Case(1) → when $k<0$

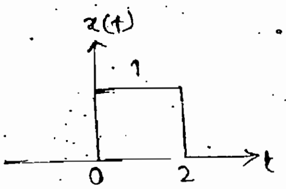
Eg:- $k=-1$

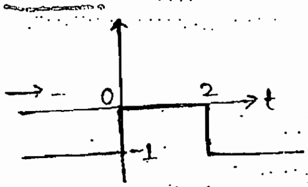
$x(t) \longrightarrow y(t) = -1+x(t)$

$$y(t) = -1+x(t)$$

$$= \begin{cases} -1+0 & , t<0 \\ -1+1 & ; 0\le t\le 2 \\ -1+0 & ; t>2 \end{cases}$$

$$= \begin{cases} -1 & , t<0 \\ 0 & ; 0\le t\le 2 \\ -1 & ; t>2 \end{cases}$$





* It is a case of <u>downward</u> shifting

Case(2) → when $k>0$

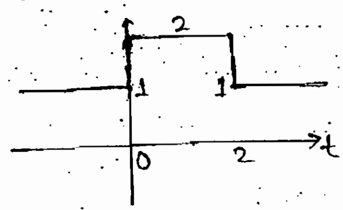
Eg:- $k=+1$

$x(t) \longrightarrow y(t) = 1+x(t)$

$$y(t) = 1+x(t)$$

$$= \begin{cases} 1+0 & , t<0 \\ 1+1 & ; 0\le t\le 2 \\ 1+0 & ; t>2 \end{cases}$$

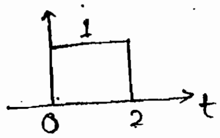
$$= \begin{cases} 1 & ; t<0 \\ 2 & ; 0\le t\le 2 \\ 1 & ; t>2 \end{cases}$$



* It is a case of <u>upward</u> shifting

(3.) Time Scaling →

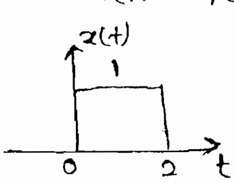
$x(t) \longrightarrow y(t) = x(at)$

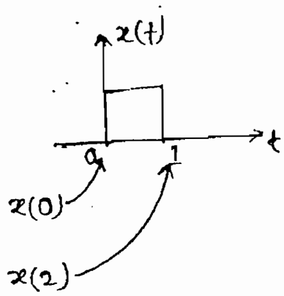


Case(1) → when $a>1$

Ep:- $a=2$

$x(t) = y(t) = x(2t)$



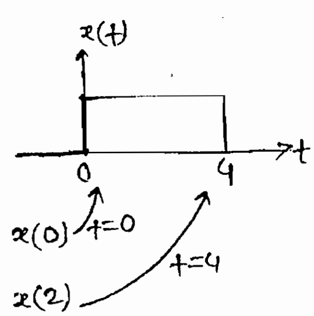


Time Compression

Case(2) → when $a<1$

Eg:- $a=0.5$

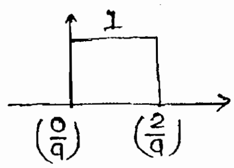
$x(t) = y(t) = x(0.5t)$



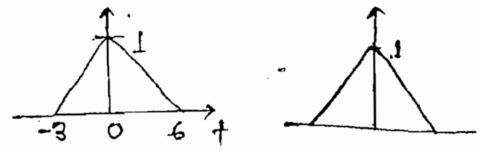
Time expansion

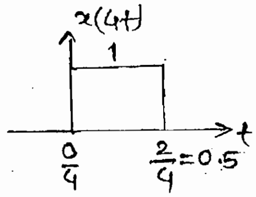
Rule General →

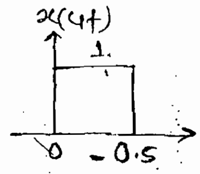
$x(at) \longrightarrow t$



Ep:- $x(t) \longrightarrow x(-3t)$





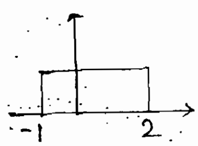


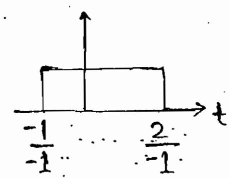
(4.) Time-reversal →

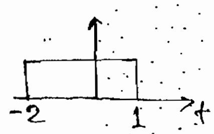
$$x(t) = y(t) = x(-t)$$

* Time reversal is a special case of time scaling in which signal folding will take palce around y-axis.

$x(-t) = a(=-1)$



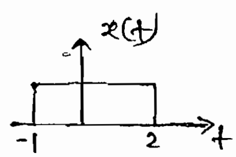


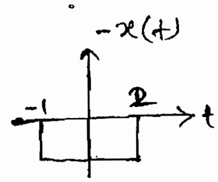


(5.) Amplitude Reversal →

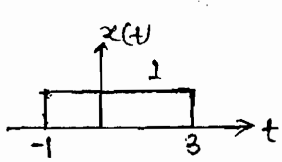
$$x(t) \longrightarrow y(t) = -x(t)$$

* In this case, signal folding will take place about x-axis





Q →



Draw signal $y(t)$ if $y(t) = 2x(2t+1)$

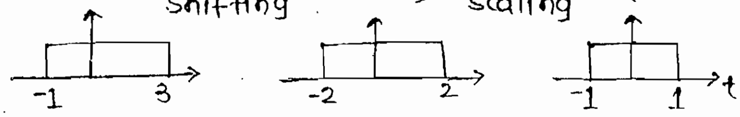
Sol^n → 1st method →

$$x(t) \xrightarrow[\text{scaling}]{\text{time}} x(2t) \xrightarrow[\text{shifting}]{\text{time}} x[2(t+0.5)]$$

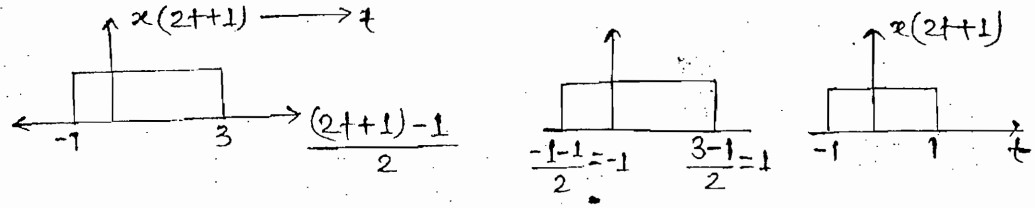
$\uparrow 1 \qquad \uparrow \qquad \uparrow y(t)$

2nd method →

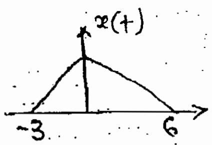
$$x(t) \xrightarrow[\text{shifting}]{\text{Time}} x(t+1) \xrightarrow[\text{scaling}]{\text{time}} x(2t+1)$$



3rd method → (Trick)



Q.→

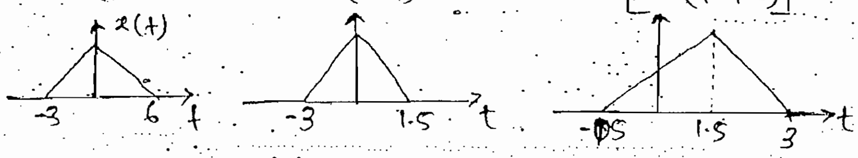


draw sig y(t) if $y(t) = x(-2t+3)$

Soln → 1st method →

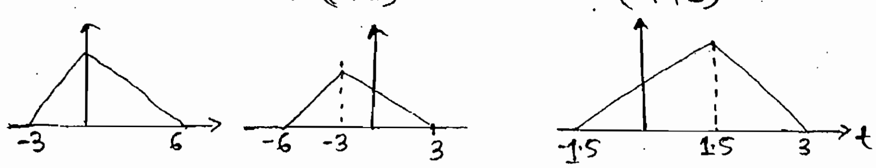
$$y(t) = x[-2(t-1.5)]$$

$$x(t) \longrightarrow x(-2t) \longrightarrow x[-2(t-1.5)]$$

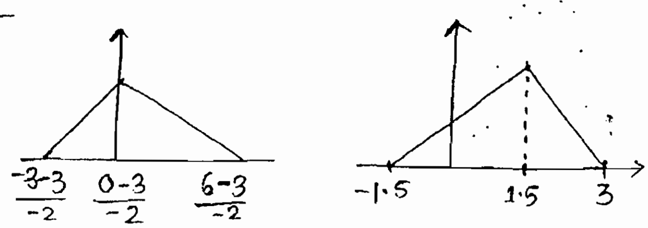


2nd method →

$$x(t) \longrightarrow x(t+3) \longrightarrow x(-2t+3)$$



3rd method →



Chapter-01
Signal defination & Classifications

Signal → A signal is a f^n which contains some information.

System → A sys. is interconnection of devices (or) componants which converts signal from one form to another form.

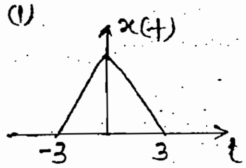
Classification of signals →

1) Even & odd signals →

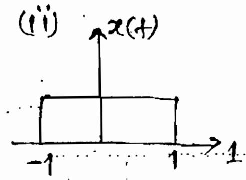
* **Even** → This are symmetrical (or) mirror image about y-axis.

i.e. → $\boxed{x(t) = x(-t)}$ → time reversal

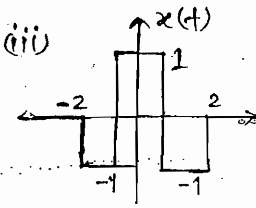
Eg:- (i)



(ii)



(iii)



(iv) $x(t) = \cos\omega_0 t$ (even)

$t = -t$

$x(-t) = \cos\omega_0(-t)$

$= \cos\omega_0 t$

$x(-t) = x(t)$

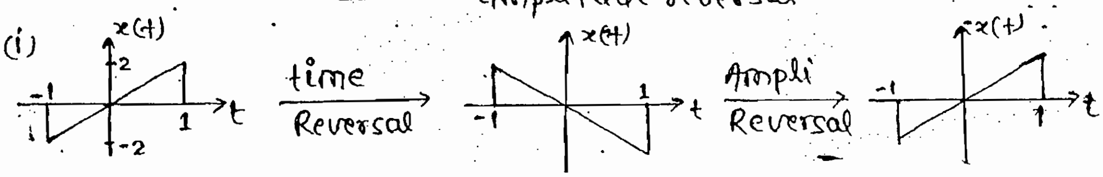
* **Odd** → This are antisymmetrical about y-axis.

i.e. $\boxed{\begin{matrix} x(-t) = -x(t) \\ (\text{or}) \\ x(t) = -x(-t) \end{matrix}}$

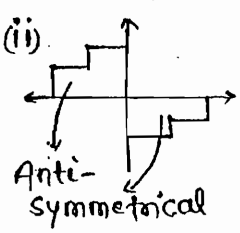
time reversal

amplitude reversal

Eg:- (i)



(ii)



(iii) $x(t) = \sin\omega_0 t$ → odd signal.

$(t = -t)$

$x(-t) = \sin\omega_0(-t)$

$x(-t) = -\sin\omega_0 t$

$\boxed{x(-t) = -x(t)}$

* The avg. value of an odd signal is 0; but converse of this statement is not true.

Important points →

Important points →

(1.) Even × Even = Even ; $t^2 \times t^4 = t^6$

(2.) Even × odd = odd ; $t^2 \times t^3 = t^5$

(3.) Odd $\underset{\div}{\times}$ odd = Even ; $t^3 \times t^5 = t^8$

(4.) Even ± Even = Even

$x(t) = t^2 + \cos t$

$x(-t) = t^2 + \cos t = x(t)$

(5.) Odd + Odd = Odd

$x(t) = \sin t + t^3$

$x(-t) = -\sin t - t^3$

$\boxed{x(-t) = -x(t)}$

(6.) Even + odd = Neither even nor odd.

$x(t) = t^2 + \sin t$

$x(-t) = t^2 - \sin t$

$\boxed{x(-t) \neq x(t)}$

* Any signal can be devided into 2 part in which one part will be even & the other part will be odd.

i.e. $\boxed{x(t) = x_E(t) + x_O(t)}$

where;

$x_E(t)$ = even part of $x(t)$ = $\dfrac{x(t) + x(-t)}{2}$

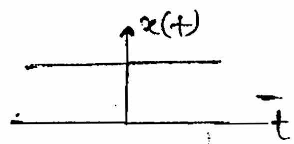
$x_O(t)$ = Odd part of $x(t)$ = $\dfrac{x(t) - x(-t)}{2}$

Eg → $x(t) = 2$ = dc signal

$\downarrow t = -t$

$x(-t) = 2 = x(t)$ [Even signal]

dc signal is a Even signal.



(2.) $f(k) = \sin(k^2)$

$\downarrow k = -k$

$f(-k) = \sin(k^2) = f(k)$ [Even signal]

(3.) $f(r) = \sin \pi/2$

$= 1$

$f(r) = f(-r)$ [Even signal]

(4.) Find $x_E(t)$ & $x_O(t)$ of the signal.

$$x(t) = 3 - \frac{t^2}{\sin t} + \frac{\cos t}{t} - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t$$

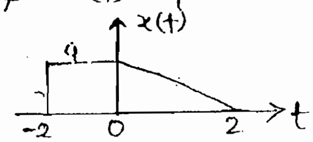
$$E - \frac{E}{O} + \frac{E}{O} - \frac{E}{E} + O \times O$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

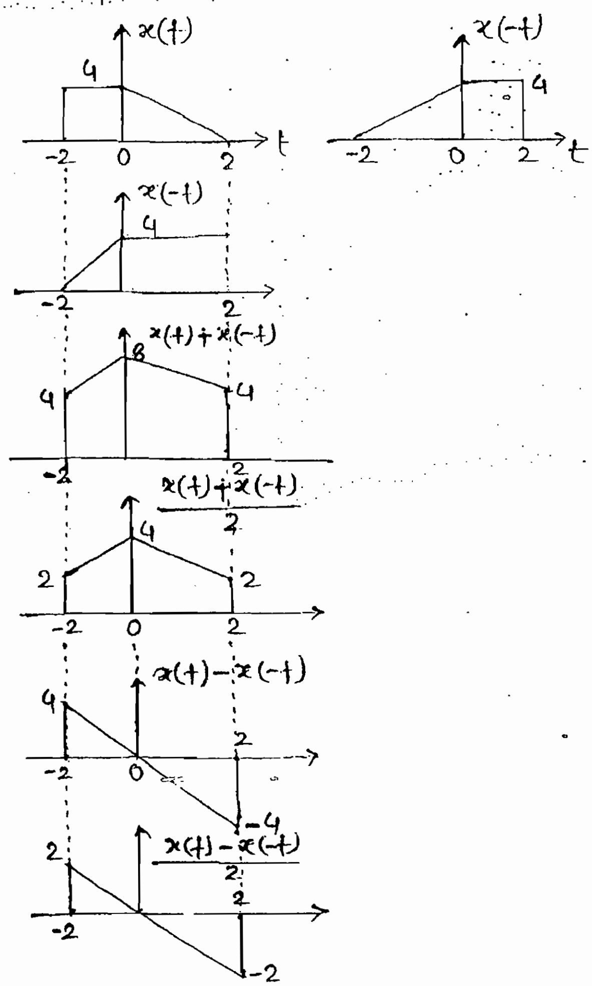
$$E \quad O \quad O \quad E \quad E$$

$$x_E(t) = 3 - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t \quad , \quad x_O(t) = \frac{-t^2}{\sin t} + \frac{\cos t}{t}$$

Que.→ Draw $x_E(t)$ & $x_O(t)$ of



Sol^n→ for even part of $x(t)$



(2) Conjugate Symmetric (CS) & Conjugate antisymmetric (CAS) signal →

* Conjugate symmetric (CS)

$$\boxed{x(t) = x^*(-t)}$$

$$x(t) = a(t) + jb(t) \text{ ——— (i)}$$

$$(t = -t)$$

$$x(-t) = a(-t) + jb(-t)$$

$$x^*(-t) = a(-t) - jb(-t) \text{ ——— (ii)}$$

From eq^n (i) & (ii)

$$a(t) = a(-t) \longrightarrow \text{Even}$$

$$b(t) = -b(-t) \longrightarrow \text{Odd}$$

Eg:- $x(t) = \underbrace{t^2}_{E} + \underbrace{\sin t}_{O}$

* Conjugate antisymmetric (CAS)

$$\boxed{x(t) = -x^*(-t)}$$

$$x(t) = a(t) + j\,b(t)$$

$$a(t) = -a(-t) \longrightarrow \text{Odd}$$

$$b(t) = b(-t) \longrightarrow \text{Even}$$

Eg:- $x(t) = \underbrace{\sin t}_{O} + j\underbrace{t^2}_{E}$

(3) Halfwave symmetric signal (HWS) →

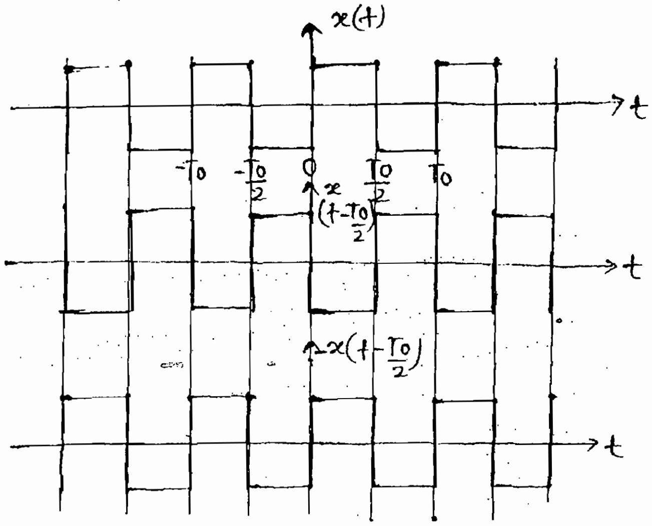
For Half-wave symmetry (HWS)

$$\boxed{x(t) = -x\left(t \pm \frac{T_0}{2}\right)}$$

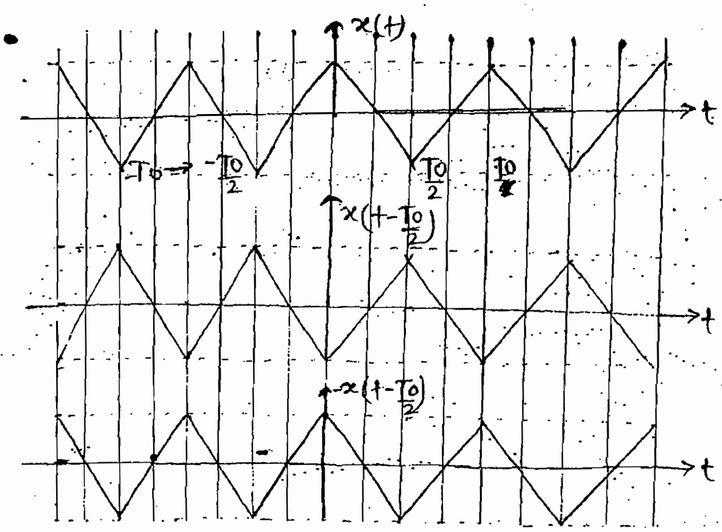
time shifting

amp. reversal

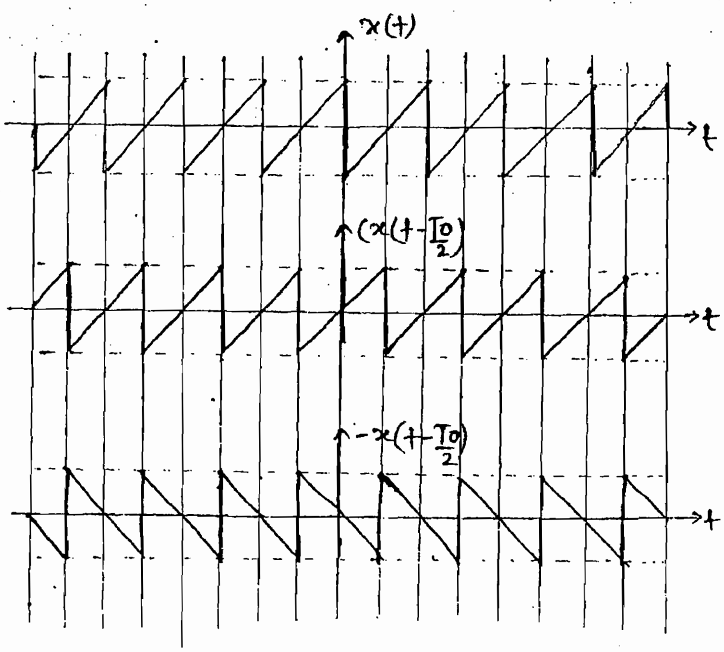
Eg → (1.)



(2.)



(3.)



so; sawtooth wave doesn't follow the HWS.

* The avg. value of a HWS is 0. but converse of this statement is not true.

DATE-10/10/14

(4.) Periodic & non-periodic signal →

Periodic → A signal repeats itself after some time period, the signal is said to be periodic.

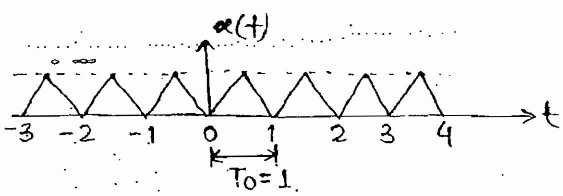
i.e. $\boxed{x(t) = x(t \pm nT_0)}$

where, n = an integer

T_0 = Fundamental time period. $\left\{ \begin{matrix} T_0 \neq 0 \\ T_0 \neq \infty \end{matrix} \right\}$

FTP → It is the smallest, +ve & fixed value of the time for which signal is periodic.

Eg →



FTP=1

Q → Find FTP of signal $x(t)$

$$x(t) = A_0 e^{j\omega_0 t}$$

Sol^n → Let 'T_0' be the FTP of the signal

i.e.

$$x(t) = x(t+T_0)$$

$$x(t+T_0) = A_0 e^{j\omega_0(t+T_0)}$$

$$A_0 e^{j\omega_0 t} = A_0 e^{j\omega_0(t+T_0)}$$

$$\cancel{A_0 e^{j\omega_0 t}} = \cancel{A_0 e^{j\omega_0 t}} e^{j\omega_0 T_0}$$

$$e^{j\omega_0 T_0} = 1 = e^{j2\pi K} \quad \text{(where } K = \text{an integer)}$$

$$j\omega_0 T_0 = j2\pi K$$

$$\underset{\text{(smallest)}}{T_0} = \frac{2\pi \cancel{K}^{\,1 \text{(least integer)}}}{\omega_0}$$

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

Q.→ Find FTP of following signal →

(i.) $x_1(t) = A_0 \sin(2\pi t)$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{2\pi} = 1$

(ii) $x_2(t) = A_0 \sin(2\pi t + 30^\circ)$

$\omega_0 = 2\pi$

$T_0 = 1$

(iii) $x_3(t) = -x_1(t)$

$= -A_0 \sin(2\pi t)$

$\omega_0 = 2\pi, \; T_0 = 1$

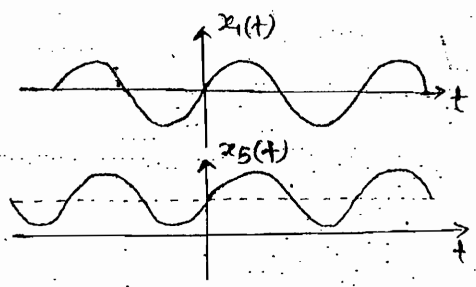
(iv) $x_4(t) = x_1(-t)$

$= -A_0 \sin 2\pi t$

$\omega_0 = 2\pi \;, T_0 = 2\pi$

(v.) $x_5(t) = A_0 + x_1(t)$

$= A_0 + A_0 \sin(2\pi t)$



(vi) $x_6(t) = x_1(t - t_0)$

$= A_0 \sin[2\pi(t - t_0)]$

$\omega_0 = 2\pi$

$T_0 = 1$

* Time period of signal is unaffected by time shifting, time reversal, amp. reversal, amp. shifting & change in phase of signal.

(vii) $f(t) = \sin^2(4\pi t)$

$= \frac{1 - \cos 8\pi t}{2}$

$\omega_0 = 8\pi$

$T_0 = \frac{2\pi}{8\pi} = \frac{1}{4}$

* The sum of 2 (or) more than 2 periodic signal will be periodic if ratios of there fundamental time period (or) freq. are rational.

i.e. $x(t) = x_1(t) + x_2(t)$

$x_1(t) \to T_1, f_1, \omega_1$

$x_2(t) \to T_2, f_2, \omega_2$

→ $\frac{T_1}{T_2}$ (or) $\frac{\omega_1}{\omega_2}$ (or) $\frac{f_1}{f_2}$ (Rational no.)

→ $T_0 = \text{LCM}[T_1, T_2]$

→ $f_0 = \text{HCF}[f_1, f_2]$

Q.→ Find FTP of signal if it is periodic :-

(i) $x(t) = \sin 2t + \cos 3\pi t$

Sol^n→ $\omega_1 = 2$ $\quad \frac{\omega_1}{\omega_2} = \frac{2}{3\pi}$ (Irrational no)

$\omega_2 = 3\pi$

Hence it is non-periodic

(ii) $x(t) = \sin 2\pi t + \cos \sqrt{2}\pi t$

Sol^n→ $\omega_1 = 2\pi, \quad \omega_2 = \sqrt{2}\pi$

$$\frac{\omega_1}{\omega_2} = \frac{2\pi}{\sqrt{2}\pi} = \sqrt{2} \quad \text{(Irrational no.)}$$

Hence it is Non-periodic

(iii) $x(t) = \sin 4\pi t + \sin 7\pi t$

Sol^n→ $\omega_1 = 4\pi, \quad \omega_2 = 7\pi$

$$\frac{\omega_1}{\omega_2} = \frac{4\pi}{7\pi} = \frac{4}{7} \quad \text{(Rational no.)}$$

Hence it is periodic. Then calculate To.

1st method :-

$$\omega_0 = \cancel{2\pi}\, HCF[\omega_1, \omega_2] = HCF[4\pi, 7\pi]$$

$$\omega_0 = \pi$$

$$T_0 = \frac{2\pi}{\omega_0} = 2$$

*** $$HCF\left[\frac{P_1}{q_1}, \frac{P_2}{q_2}\right] = \frac{HCF[P_1, P_2]}{LCM[q_1, q_2]} \qquad LCM\left[\frac{P_1}{q_1}, \frac{P_2}{q_2}\right] = \frac{LCM[P_1, P_2]}{HCF[q_1, q_2]}$$

2nd method →

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4\pi} = \frac{1}{2} \qquad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{7\pi} = \frac{2}{7}$$

$$T_0 = LCM[T_1, T_2] = LCM\left[\frac{1}{2}, \frac{2}{7}\right]$$

$$= \frac{LCM[1,2]}{HCF[2,7]} = \frac{2}{1} = 2$$

* Area & avg. value of signal →

Area of $x(t)$:-

$$\text{Area} = \int_{-\infty}^{\infty} x(\tau)\,d\tau$$

Area of $x(t)$ over Range (t_1, t_2)

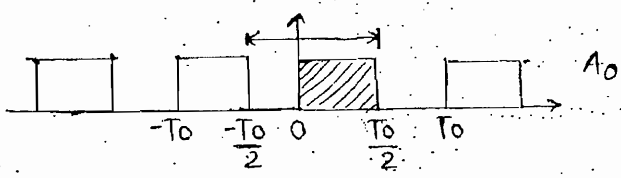
$$\text{Area} = \int_{t_1}^{t_2} x(\tau)\,d\tau$$

Avg. value of $x(t)$:

$$\text{Avg.} = \begin{cases} \dfrac{1}{T_0}\displaystyle\int_{-T_0/2}^{T_0/2} x(\tau)\,d\tau, & \text{For periodic sig.} \\ \displaystyle\lim_{T\to\infty} \dfrac{1}{T}\int_{-T/2}^{T/2} x(\tau)\,d\tau, & \text{For Non-periodic sig.} \end{cases}$$

Que.→ Find the avg. value of sig.

(i)



Sol^n →

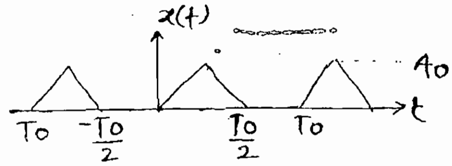
$$\text{avg.} = \frac{\displaystyle\int_{-T_0/2}^{T_0/2} x(\tau)\,d\tau}{T_0}$$

$$= \frac{\text{Area of } x(t) \text{ over } `T_0'}{T_0}$$

$$= \frac{A_0 \times \frac{T_0}{2}}{T_0}$$

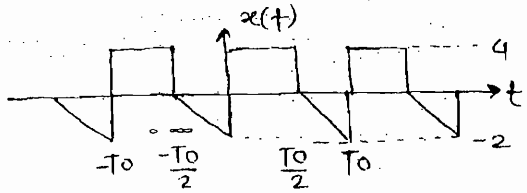
$$= \frac{A_0}{2}$$

(2.)



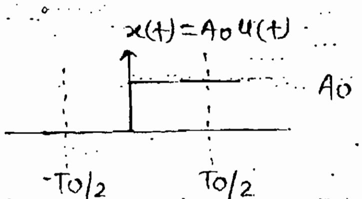
Sol^n → $\text{avg.} = \frac{\text{Area over } T_0}{T_0} = \frac{1/2 \times A_0 \times \cancel{T_0}/2}{\cancel{T_0}} = \frac{A_0}{4}$

(3.)



Sol^n → $\text{Avg.} = \frac{\text{Area over } T_0}{T_0} = \frac{-1/2 \times \frac{T_0}{2} \times 2 + 4 \times \frac{T_0}{2}}{T_0} = \frac{3}{2}$

(iv)



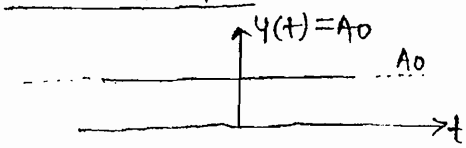
Sol^n → $\text{Avg.} = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(z)\,dz$

$= \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{0}^{T_0/2} A_0\,dz$

$= \lim_{T_0 \to \infty} \frac{A_0 \times T_0/2}{T_0}$

$= \frac{A_0}{2}$

2nd method →



$\text{avg } y(t) = A_0$

$\text{avg } x(t) = \frac{\text{avg } y(t)}{2}$

$= \frac{A_0}{2}$

(5.) Energy & power signal →

* Energy of $x(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$\boxed{E = \int_{-\infty}^{\infty} |x(t)|^2 dt}$$

* Power of $x(t)$

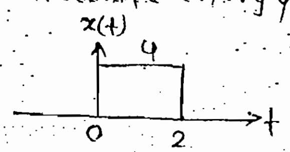
$$P = \begin{cases} \dfrac{1}{T_0} \displaystyle\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \ ; & \text{For periodic sig.} \\ \lim\limits_{T_0 \to \infty} \displaystyle\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \ ; & \text{Non periodic sig.} \end{cases}$$

* For an energy sig., energy should be finite & power should be zero.
* Energy signals are absoulately integrable signal.

i.e. $\boxed{\int_{-\infty}^{\infty} |x(t)|\, dt < \infty}$

Q.→ Calculate energy of sig.

(i)



Sol^n→ $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

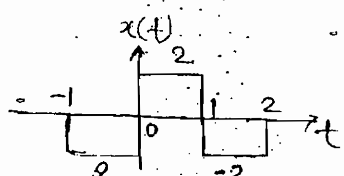
$$= \int_0^2 4^2 dt = 32$$

2nd method →

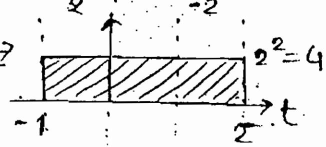
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= 16 \times 2 = 32$$

(ii)



Sol^n→

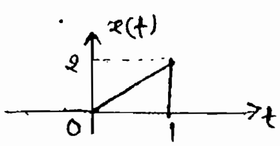


$E_{x(t)}$ = area of $|x(t)|^2$

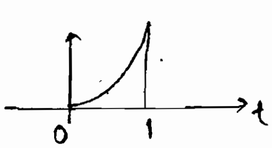
$= 4 \times 3$

$= 12$

(iii)



Sol^n→

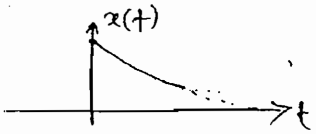


$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_0^1 (2t)^2 dt = \frac{4}{3}$$

Q.→ Cal. area & energy of signal:-

(i) $x(t) = e^{-at}u(t)$, $a>0$



Soln→

$$\text{area} = \int_{-\infty}^{\infty} x(t)\,dt$$

$$= \int_{0}^{\infty} e^{-at}\,dt$$

$$= \left(\frac{e^{-at}}{-a}\right)_0^{\infty} = \frac{e^{-a\infty} - e^{0}}{-a}$$

$\because e^{-a\infty} = 0,\ a>0$ $(a=2)$

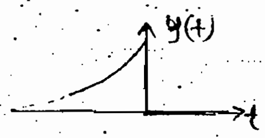
$e^{-2\infty} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$$= \frac{0-1}{-a} = \frac{1}{a}$$

$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

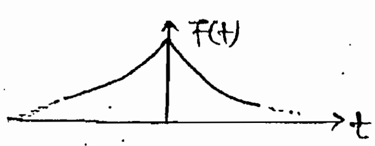
$$= \int_{0}^{\infty} e^{-2at}\,dt = \left(\frac{e^{-2at}}{-2a}\right)_0^{\infty} = \frac{e^{-2a\infty} - e^{0}}{-2a} = \frac{1}{2a}$$

(ii) $y(t) = x(-t) = e^{at}u(-t)$, $a>0$



Soln→ Area $= \frac{1}{a}$, Energy $= \frac{1}{2a}$

(iii) $f(t) = x(t) + y(t) = e^{-a|t|}$, $a>0$



Soln→

$f(t) = e^{-a|t|}$, $a>0$

$$= \begin{cases} e^{at}, & t<0 \\ e^{-at}, & t>0 \end{cases}$$

$*\ |t| = \begin{cases} -t, & t<0 \\ t, & t>0 \end{cases}$

$$\text{area} = \frac{1}{a} + \frac{1}{a} = \frac{2}{a}$$

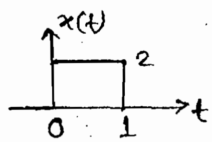
Energy = 1 1 1

2.→ $x(t) \longrightarrow E$

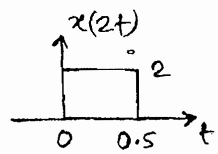
$x(2t) \longrightarrow ?$

(a.) $\frac{E}{4}$ (b.) $\frac{E}{2}$ (c) $2E$ (d) E

Soln→



$E \to 4$



$E \to 2 = \frac{E}{2}$

$x(t) \longrightarrow E$

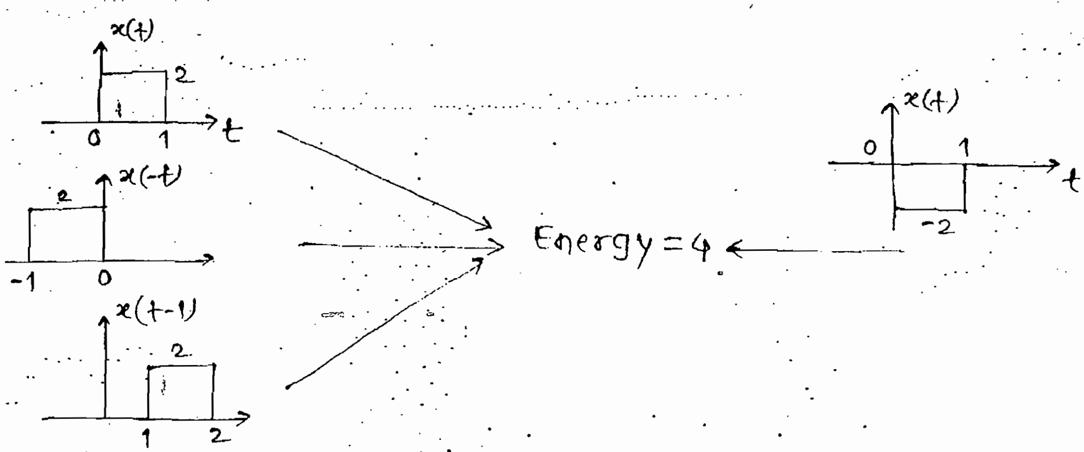
$x(2t) \longrightarrow \frac{E}{2}$

$x(-2t) \longrightarrow \frac{E}{2}$

***

$$\boxed{x(at)_{a \neq 0} \longrightarrow \frac{E}{|a|}}$$

*



* Energy of signal is independent of amp. reversal, time reversal, time shifting.

*** Power Signal →** * For this signal power should be finite & energy should be ∞.

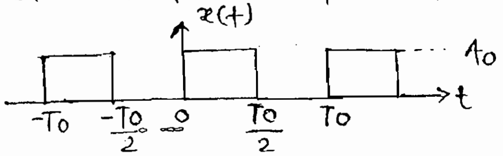
* Periodic power signals are absolutely integrable over there time period.

i.e. $\boxed{\int_{T_0} |x(t)|\,dt < \infty}$ periodic power sig.

$$p = \begin{cases} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & ; \text{ for periodic signal.} \\ \lim_{T_0 \to \infty} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & ; \text{ for Non-periodic} \end{cases}$$

Q→ Calculate power of signal:-

(i.)



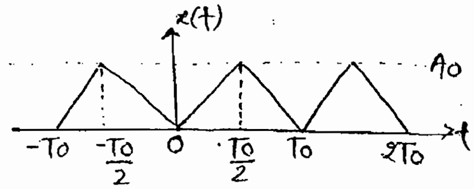
Sol^n→

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{0}^{T_0/2} A_0^2 dt$$

$$\boxed{P = \frac{A_0^2}{2}}$$

(ii)



Sol^n→

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} |x(t)|^2 dt$$

$$x(t) = mt = \left(\frac{2A_0}{T_0}\right)t \quad \left(\because m = \frac{A_0}{T_0/2}\right)$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} \left(\frac{2A_0}{T_0}\right)^2 t^2 dt$$

$$= \frac{8\times A_0^2}{T_0^3}\int_{0}^{T_0/2} t^2 dt$$

$$= \frac{8A_0^2}{T_0^3} \times \frac{T_0^3}{8\times 3}$$

$$\boxed{P = \frac{A_0^2}{3}}$$

(iii) $x(t) = A_0 \sin\omega_0 t$

Sol^n→

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (A_0 \sin\omega_0 t)^2 dt$$

$$P = \frac{A_0^2}{T_0}\int_{-T_0/2}^{T_0/2} \left(\frac{1-\cos 2\omega_0 t}{2}\right) dt$$

$$P = \frac{2A_0^2}{2T_0}\int_{0}^{T_0/2} (1-\cos 2\omega_0 t)\, dt$$

$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \left(\frac{\sin 2\omega_0 t}{2\omega_0}\right)_0^{T_0/2}\right]$$

$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \frac{\sin \cancel{2}\omega_0 \frac{T_0}{\cancel{2}}}{2\omega_0}\right]$$

$(\because \omega_0 T_0 = 2\pi)$

$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \frac{\cancel{\sin\omega_0 T_0}^{\,0}}{2\omega_0}\right]$$

$$= \frac{A_0^2}{T_0} \times \frac{T_0}{2}$$

$$\boxed{P = \frac{A_0^2}{2}}$$

∵ Rms of the Given signal is $\frac{A_0}{\sqrt{2}}$

*** $\boxed{Rms^2 = \frac{A_0^2}{2} = P}$

* Power is also known as mean square value of signal.

Q.→ Calculate power of signal

(i.) $x_1(t) = A_0 \sin \omega_0 t$

(ii) $x_2(t) = x_1(t-t_0) = A_0 \sin[\omega_0(t-t_0)]$

(iii) $x_3(t) = x_1(2t) = A_0 \sin 2\omega_0 t$

(iv) $x_4(t) = A_0 \sin(\omega_0 t + \phi)$

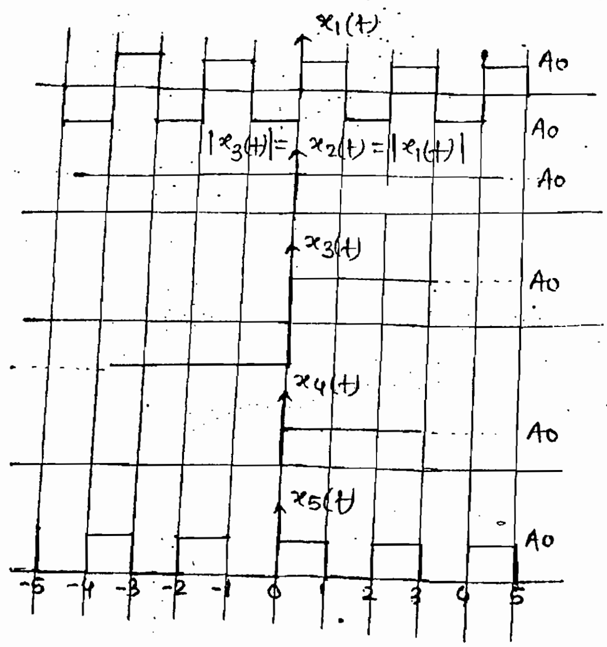
Sol^n→ For above all signals

$$Rms = \frac{A_0}{\sqrt{2}}$$

$$power = \frac{A_0^2}{2}$$

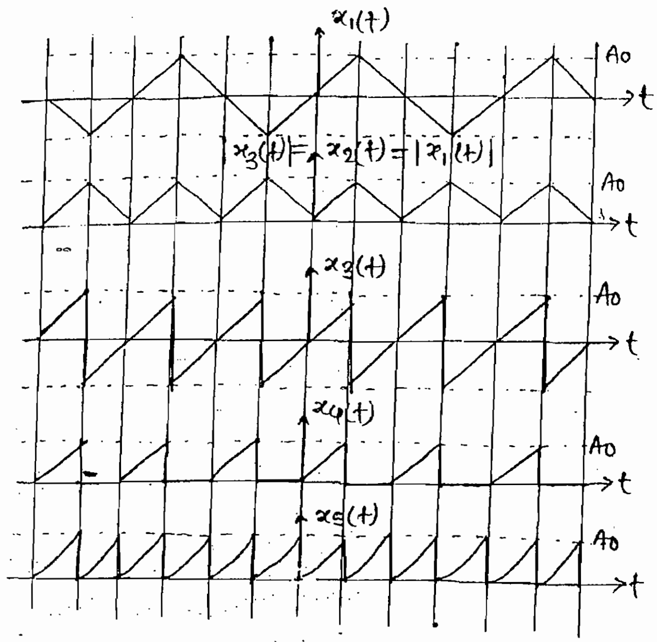
* Power calculation is independant of time shifting, time scaling, change in freq. (or) time period & change in phase of signals.

Q.→



Power	Avg.
$P_1 = A_0^2$	0
$P_2 = A_0^2$	A_0
$P_3 = A_0^2$	0
$P_4 = \frac{P_2}{2} = \frac{A_0^2}{2}$	$\frac{A_0}{2}$
$P_5 = \frac{P_2}{2} = \frac{A_0^2}{2}$	$\frac{A_0}{2}$

Q.→



	Power	Avg.
	$P_1 = \frac{A_0^2}{3}$	0
	$P_2 = \frac{A_0^2}{3}$	A_0
	$P_3 = \frac{A_0^2}{3}$	0
	$P_4 = \frac{A_0^2}{6}$	$\frac{A_0}{4}$
	$P_5 = 2P_4 = \frac{A_0^2}{3}$	$\frac{A_0}{2}$

DATE-13/10/14

Concept of Orthogonality → 2 signals $x_1(t)$ & $x_2(t)$ are said to be orthogonal if

* $\int_{-\infty}^{\infty} x_1(t)\cdot x_2(t)\,dt = 0$, For non-periodic sig.

* $\int_{T_0} x_1(t)\cdot x_2(t)\,dt = 0$; For periodic sig.

Use of orthogonality for energy & power calculation →

If $x_1(t)$ & $x_2(t)$ are orthogonal & $z(t) = x_1(t) \pm x_2(t)$ then;

$$P_z = P_{x_1} + P_{x_2} \quad \{\text{If } x_1 \,\&\, x_2 \text{ are power signal}\}$$

(or)

$$E_z = E_{x_1} + E_{x_2} \quad (\text{If } x_1 \,\&\, x_2 \text{ are energy signals})$$

Important trignomatrical results →

(1) $\int_{T_0} \sin(m\omega_0 t + \phi)\,dt = 0$, (m = an integer, $T_0 = \frac{2\pi}{\omega_0}$)

(2) $\int_{T_0} \cos(m\omega_0 t + \phi)\,dt = 0$

(3) $\int_{T_0} \sin^2(m\omega_0 t + \phi)\,dt = \frac{T_0}{2}$

★ (4.) $\int_{T_0} \cos^2(m\omega_0 t + \phi)\, dt = \frac{T_0}{2}$

★ (5.) $\int_{T_0} \sin(m\omega_0 t + \phi_1) \cdot \sin(n\omega_0 t + \phi_2)\, dt = 0$; ($m \neq n$ & both are integer)

Q→ $z(t) = 2\sin(3\pi t + 30^\circ) - 4\sin(7\pi t + 40^\circ)$

Sol^n→ In the above sig. the freq. of the signals are diff. ($m \neq n$). So that they are orthognal.

$$P_z = P_{x_1} + P_{x_2}$$

$$P_{x_1} = \frac{2^2}{2} = 2 \qquad P_{x_2} = \frac{4^2}{2} = 8$$

$$P_z = 10$$

Q.→ $z(t) = 2\sin 3\pi t + 4\sin(7\pi t + 30^\circ) + 5\sin(10\pi t + 45^\circ)$

Sol^n→

$$P_z = P_1 + P_2 + P_3$$

$$= \frac{2^2}{2} + \frac{4^2}{2} + \frac{5^2}{2}$$

★ (6.) $\int_{T_0} \cos(m\omega_0 t + \phi_1) \cdot \cos(n\omega_0 t + \phi_2) = 0$ $\{m \neq n\}$

Q.→ $z(t) = 3\cos(3\pi t + 70^\circ) + 4\cos(7\pi t + 85^\circ)$

Sol^n→

$$P_z = \frac{3^2}{2} + \frac{4^2}{2}$$

★ (7.) $\int_{T_0} \cos(m\omega_0 t + \phi_1) \cdot \sin(n\omega_0 t + \phi_2)\, dt = 0$ → ($m \neq n$), → ($m = n$, $\phi_1 = \phi_2$).

Q→ $z(t) = 2\sin(3\pi t + 40^\circ) + 3\cos(7\pi t)$

Sol^n→

$$P = \frac{2^2}{2} + \frac{3^2}{2}$$

Q→ $z(t) = 2\sin(2\pi t + 45^\circ) + 3\cos(2\pi t + 45^\circ)$

Sol^n→

$$P = \frac{2^2}{2} + \frac{3^2}{2}$$

★ (8.) $\int_{T_0} A_0 \sin(m\omega_0 t + \phi) = 0$

$\int_{T_0} \downarrow$ AC

$\sin(m\omega_0 t + \phi) \downarrow$ sinusoidal (sin, cos)

Q→ $z(t) = 2 + 4\sin(3\pi t + 45^\circ)$

Sol^n→ $P_z = P_1 + P_2$

$$= 2^2 + \frac{4^2}{2}$$

$\boxed{\begin{array}{l} \text{* Harmonics of diff. freq. are orthogonal.} \\ \text{* Sine \& cosine } f^n \text{ of same freq. \& same phase are also orthogonal.} \\ \text{* DC \& sinusoidal } f^n \text{ are also orthogonal.} \end{array}}$

Q.→ $z(t) = A_1\sin(\omega_0 t + \phi_1) + A_2\sin(\omega_0 t + \phi_2)$ where $\phi_1 - \phi_2 \neq \frac{n\pi}{2}$; (n = integer)

Sol^n→ $P = \frac{1}{T_0}\int_{T_0} z^2(t)\,dt$

$$P = \frac{1}{T_0}\int_{T_0}\left[A_1\sin(\omega_0 t + \phi_1) + A_2\sin(\omega_0 t + \phi_2)\right]^2 dt$$

$$= \frac{1}{T_0}\int_{T_0}\left[A_1^2\sin^2(\omega_0 t + \phi_1) + \frac{A_2^2}{2}\sin^2(\omega_0 t + \phi_2) + 2A_1A_2\sin(\omega_0 t + \phi_1)\cdot\sin(\omega_0 t + \phi_2)\right]$$

$$= \frac{1}{T_0}\int_{T_0}\left[A_1'\left(1 - \cos(2\omega_0 t\right.\right.$$

$$\boxed{\begin{array}{l} P = \frac{A_0^2}{2}, \quad Rms = \frac{A_0}{\sqrt{2}} \\ A_0 = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2)} \end{array}}$$

Q.→ $z(t) = 2\sin 3\pi t + 3\cos(3\pi t + \frac{\pi}{3})$

Sol^n→ $A_0 = \sqrt{2^2 + 3^2 + 2\times 2\times 3\cos(0 - \pi/3)}$

$$= \sqrt{13 + 12\times\frac{1}{2}}$$

$$= \frac{\sqrt{19}}{\sqrt{2}}$$

Above calculation is wrong because sin & cos is present.

$$z(t) = 2\sin 3\pi t + 3\sin\left(3\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

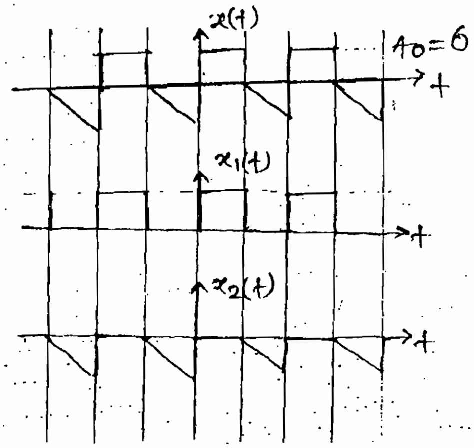
$= 2\sin 3\pi t + 3\sin(3\pi t \ldots$

$$\phi_1 - \phi_2 = 150^\circ$$

$$A_0 = \sqrt{2^2 + 3^2 + 2 \times 2 \times 3 \cos(150^\circ)}$$

$$\text{Rms} = \frac{A_0}{\sqrt{2}} = 1.14$$

Q.→ Rms = ?

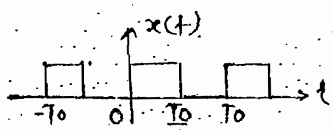


Soln→ For this 1st check that are they orthogonal (or) not.

$$P_z = P_1 + P_2 = \frac{A_0^2}{2} + \frac{A_0^2}{6} = \frac{6^2}{2} + \frac{6^2}{6} = 24$$

$$\text{RMS} = \sqrt{24} = 2\sqrt{6}$$

Q.→



Soln→ $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

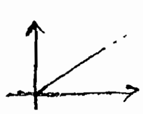
$= \underbrace{\text{no. of pulses}}_{\infty} \times \underbrace{\int_{T_0} |x(t)|^2 dt}_{\text{finite}} = \infty$

Note→

* Periodic signals are not energy signals because there energy content is ∞.

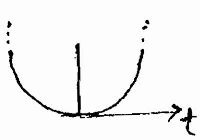
* (1.) If magnitude of sig. is ∞ at any instant of time then signal will be <u>neither energy nor power.</u>

eg.→ (a.)



$x(t) = r(t)$ [Ramp sig]

(b.)



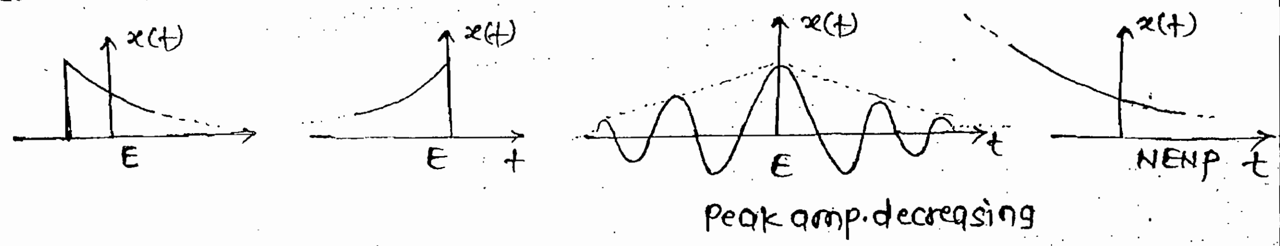
(c.) $x(t) = \tan t$

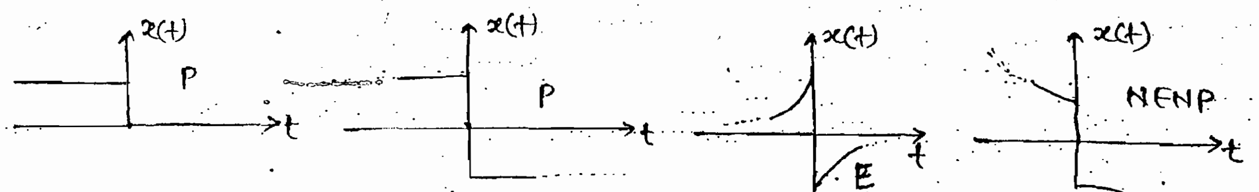
(d.) $x(t) = \frac{1}{t}$ (because $t = 0$, $x(t) = \infty$)

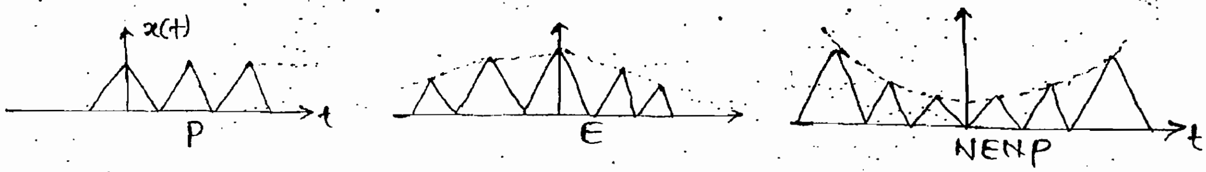
*(2.) Energy signals are:-

(i) Finite duration signals having finite amp. for each & every instant of time.

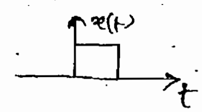
(ii) ∞ extension signals with amp. (or) peak amp decreasing in nature.

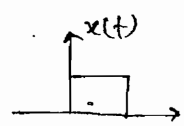
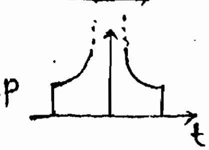






* Periodic Signals
  - P → sin t
  - NENP → tan t

* Non-Periodic
  - E → 
  - P → u(t)
  - NENP → r(t) = t u(t)

* Finite duration
  - E 
  - NENP 

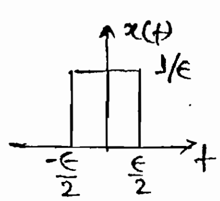
* ∞ extension
  - E 
  - P → u(t)
  - NENP → r(t)

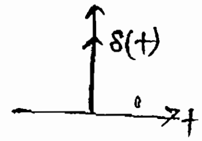
*Basic Signals →

(1.) Unit-impulse :- $\delta(t)$

$$\delta(t) = \lim_{\epsilon \to 0} x(t)$$

$$= \begin{cases} \infty, & t=0 \\ 0, & t \neq 0 \end{cases}$$





Properties →

* (1.) $\delta(t)$ is an even signal.

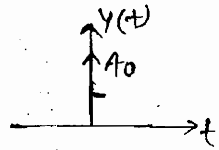
* (2.) It is a NENP signal.

* (3.) Area under impulse :-

$$= \int_{-\infty}^{\infty} \delta(t)\,dt = \int_{-\infty}^{\infty} \left[\lim_{\epsilon \to 0} x(t)\right] dt = \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} x(t)\cdot dt = 1$$

* (4.) Weight/ strength of impulse :-

$$y(t) = A_0\,\delta_0(t)$$



Area of weighted impulse $y(t)$

$$= \int_{-\infty}^{\infty} y(t)\,dt = A_0 \int_{-\infty}^{\infty} \delta(t)\cdot dt = A_0 = \text{weight of impulse.}$$

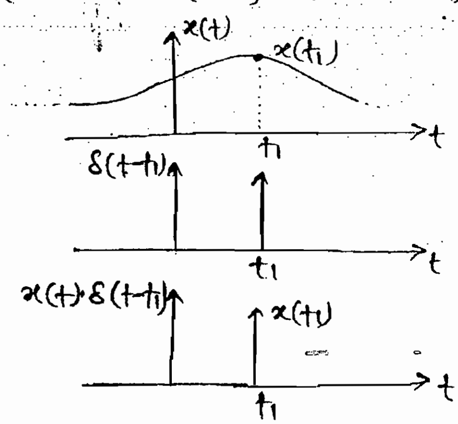
* (5.) Scaling property of impulse :-

$$\delta[a(t-t_1)]_{a \neq 0} = \frac{1}{|a|}\,\delta(t)$$

Eg. → (1.) $\delta(-2t) = \frac{1}{2}\,\delta(t)$

(2.) $\delta(2t-3) = \delta[2(t-3/2)] = \frac{1}{2}\,\delta(t-3/2)$

* (6.) $x(t) \times \delta(t-t_1) = ? = x(t_1)\cdot\delta(t-t_1)$



Eg :- (1.) $y(t) = 2\sin t\cdot\delta(t-\frac{\pi}{2})$

$= 2\sin(\frac{\pi}{2})\,\delta(t-\frac{\pi}{2})$

$= 2\delta(t-\pi/2)$

(2.) $y(t) = e^{-2t^2}\cdot\delta(2t-1)$

$= e^{-2t^2}\,\delta[2(t-\frac{1}{2})]$

$= e^{-2t^2}\cdot\frac{1}{2}\,\delta(t-\frac{1}{2})$

$= \frac{1}{2}\cdot e^{-2\times\frac{1}{4}}\,\delta(t-\frac{1}{2})$

$= \frac{1}{2}\,e^{-1/2}\,\delta(t-\frac{1}{2})$

*(7.) $\int_{-\infty}^{\infty} x(t)\cdot\delta(t-t_1)\,dt = ?$

$= \int_{-\infty}^{\infty} x(t_1)\,\delta(t-t_1)\,dt$

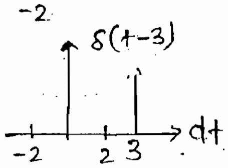
$= x(t_1)\int_{-\infty}^{\infty} \cancel{\delta(t-t_1)\,dt}^{\,1}$

$= x(t_1)$

Q.→ Calculate the value of

(i) $I = \int_{-2}^{2} \delta(t-3)\,dt$

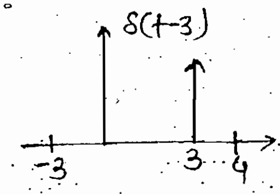
Soln→



$\boxed{I=0}$

(ii) $I = \int_{-3}^{4} \delta(t-3)\,dt$

Soln→



$\boxed{I=0}$

(iii) $I = \int_{-\infty}^{\infty}\left[2\cos\left(\frac{t}{2}\right)+t^2\right]\delta(t-\pi)\,dt$

Soln→ $I = \int_{-\infty}^{\infty}\underbrace{\left[2\cos\left(\frac{t}{2}\right)+t^2\right]}_{x(t)}\ \underbrace{\delta(t-\pi)}_{\delta(t-t_1)}\,dt$

$= x(t_1)$

$= \left[2\cos\frac{\pi}{2}+\pi^2\right]$

$= \pi^2$

*(8.) $\int_{-\infty}^{\infty} x(t)\cdot\frac{d^n\delta(t-t_1)}{dt^n}\cdot dt = (-1)^n \left.\frac{d^n x(t)}{dt^n}\right|_{t=t_1}$

Soln→

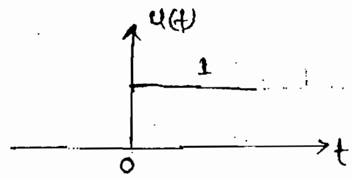
Eg.→ $\int_{-\infty}^{\infty}(t^2+3t)\,\delta'(t-2)\,dt$

$= (-1)^1 \left.\frac{d}{dt}(t^2+3t)\right|_{t=2}$

$= -(2t+3)\big|_{t=2}$

$= -(4+3) = -7$

(2) Unit-step signal → $u(t)$



* $u(t)$ is discontinuous at $t=0$.

Gibb's phenomenon → At the point of discontinuity signal value is given by the avg. of signal value taking just before & after the point of discontinuity.

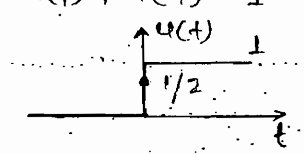
$$u(0) = \frac{u(0^-) + u(0^+)}{2}$$

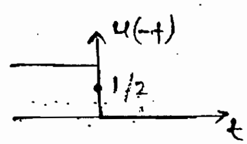
$$= \frac{0+1}{2}$$

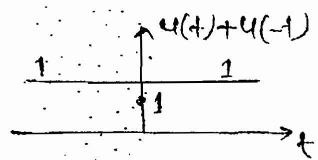
$$\boxed{u(0) = \frac{1}{2}}$$

* Properties →

(1) $u(t) + u(-t) = 1$





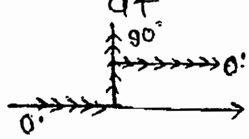


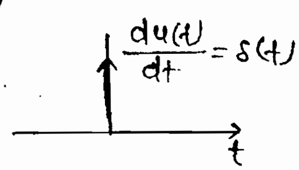
(2) $u(t)$ is a power signal.

Power $= \frac{1}{2}$, Rms $= \frac{1}{\sqrt{2}}$; avg. $= \frac{1}{2}$

(3) Derivative of $u(t)$

$$\frac{du(t)}{dt} = \delta(t)$$



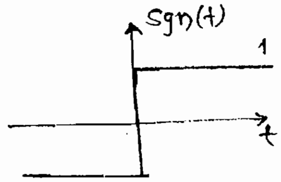


$$\left\{ \frac{dx(t)}{dt} = \text{slope of } x(t) \text{ wrt } 't' \right.$$

* And $\int_{-\infty}^{t} \delta(t)\,dt = u(t)$

$$\begin{tikzcd} \delta(t) \arrow[r, "I"] & u(t) \arrow[l, bend right, "D"'] \arrow[r, "I"] & r(t) \arrow[l, bend right, "D"'] \arrow[r, "I"] & p(t) \arrow[l, bend right, "D"'] \end{tikzcd}$$

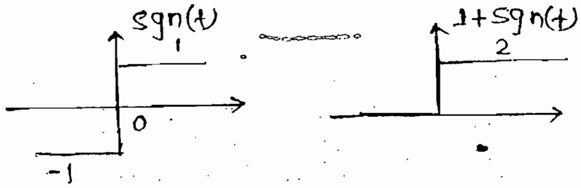
(3.) Signum function →



* This is a power signal.

$P = 1$, $RMS = 1$, $Avg = 0$

*

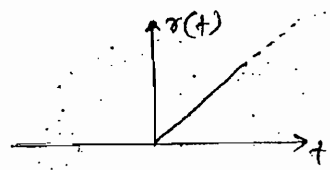


$1 + sgn(t) = 2u(t)$

*** $\boxed{u(t) = \dfrac{1 + sgn(t)}{2}}$

(4.) Ramp Signal → $r(t)$

$$r(t) = \int_{-\infty}^{t} u(t)\,dt = t\,u(t)$$



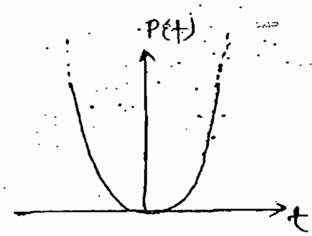
* This is NENP sig.

(5.) Parabolic signal →

$$P(t) = \int_{-\infty}^{t} r(t)\,dt$$

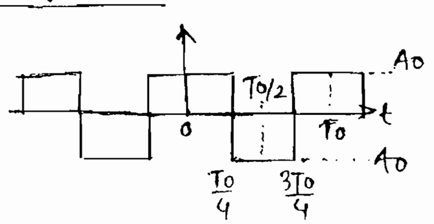
$$= \int_{-\infty}^{t} t\,u(t)\,dt$$

$$= \frac{t^2}{2} u(t)$$



* This is NENP signal.

(6.) Square signal →

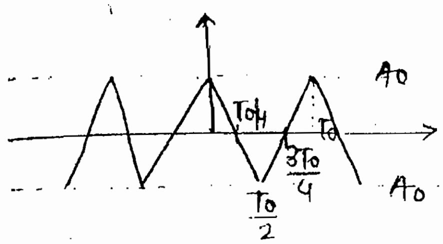


$P = A_0^2$

$RMS = A_0$

$Avg = 0$

(7.) Triangular wave→



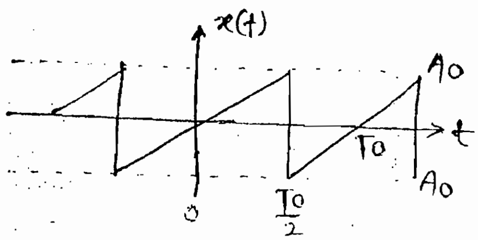
$P = A_0^2/3$

$Rms = A_0/\sqrt{3}$

avg. = 0

HWS = Yes

(8.) Sawtooth wave→



$P = A_0^2/3$

$Rms = A_0/\sqrt{3}$

Avg. = 0

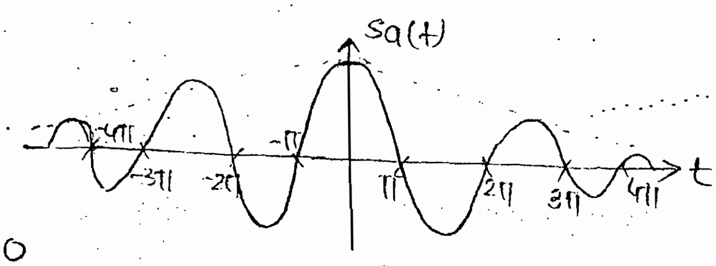
HWS = No.

(9.) Sampling signal→

$$Sa(t) = \frac{\sin t}{t}$$

* $Sa(0) = \lim_{t \to 0} \frac{\sin t}{t} = 1$

* $Sa(\infty) = \frac{\sin \infty}{\infty} = \frac{(-1,1)}{\infty} = 0$



* If $Sa(t) = 0$, then $\frac{\sin t}{t} = 0$

  so $\sin t = 0$, $\boxed{t = n\pi, n \neq 0}$

* This is a energy signal.

$$E = x(t) = \int_{-\infty}^{\infty} \frac{\sin^2 t}{t^2} dt$$

$$= \int_{-\infty}^{\infty} \left(\frac{1 - \cos 2t}{2t^2} \right) dt$$

$$= \frac{1}{2} \left[\left(\frac{1}{-2t^2} \right)_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{\cos 2t}{t^2} dt \right]$$

$$\boxed{E = \pi}$$

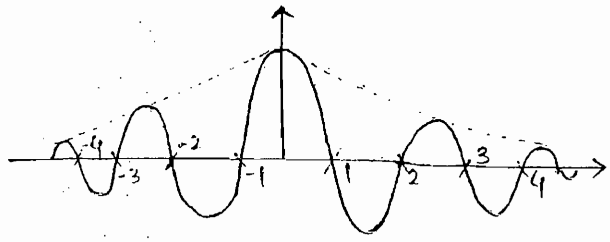
(10.) Sinc function →

$$\text{sinc}\,t = \frac{\sin \pi t}{\pi t} = sa(\pi t)$$

* $\text{sinc}(0) = \frac{\sin \pi t}{\pi t} = 1$

* $\text{sinc}(\infty) = 0$

* If $\text{sinc}(t) = 0$, $\frac{\sin(\pi t)}{\pi t} = 0$



$\sin \pi t = 0$

$\cancel{\pi} t = n\cancel{\pi}$, $n \neq 0$

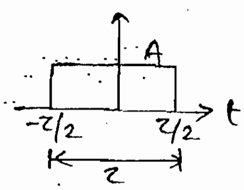
$\boxed{t = n,\ n \neq 0}$

* Energy = 1

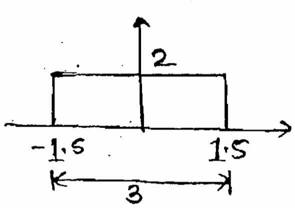
$x(t) \longrightarrow E$	$sa(t) = \pi$
$x(at) \longrightarrow \frac{E}{\lvert a \rvert}$	$\text{sinc}(t) = sa(\pi t) = 1$

(11.) Rect function →

$$x(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)$$

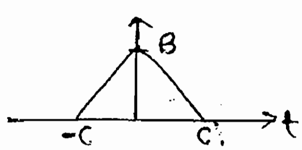


$$x(t) = 2\,\text{rect}\left(\frac{t}{3}\right)$$

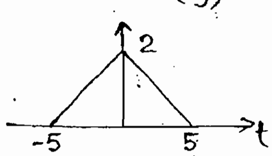


(12) Tri-function →

$$x(t) = B\,\text{tri}\left(\frac{t}{c}\right)$$



$$x(t) = 2\,\text{tri}\left(\frac{t}{5}\right)$$



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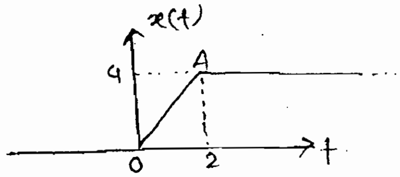
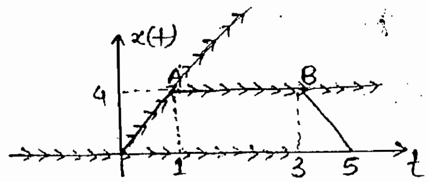
Mathematical representation of waveform →

(1.)



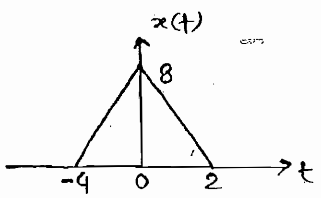
$$\ast\ x(t) = 0 + 2r(t-0) - 2r(t-2)$$

(2.)



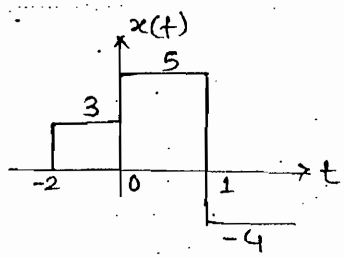
$$\ast\ x(t) = 0 + 4r(t-0) - 4r(t-1) - 2r(t-3) + 2r(t-5)$$

(3.)



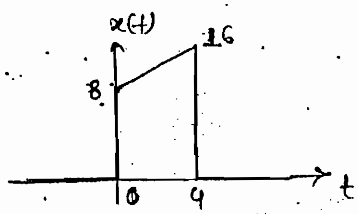
$$\ast\ x(t) = 0 + (+2)r(t+4) \cancel{-4r(t-0)} - 2r(t-0) + 4r(t-2)$$

(4.)



$$\ast\ x(t) = 0 + 3u(t+2) + 2u(t-0) - 9u(t-1)$$

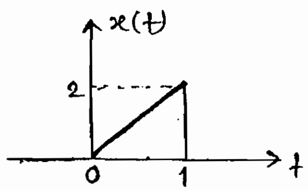
(5.)



$$\ast\ x(t) = 0 + 8u(t-0) + 2r(t-0) - 2r(t-4) - 16u(t-4)$$

$$= 8u(t) + 2r(t) - 2r(t-4) - 16u(t-4)$$

(6.)



$$\ast\ x(t) = 2r(t) - 2r(t-1) - 2u(t-1)$$

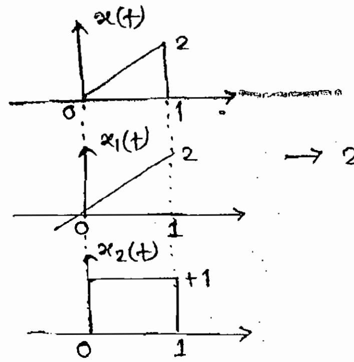
$$= 2tu(t) - 2(t-1)u(t-1) - 2\cancel{(t-1)}u(t-1)$$

$$= 2tu(t) - 2tu(t-1) + 2\cancel{u(t-1)} - 2\cancel{u(t-1)}$$

$$= 2t[u(t) - u(t-1)]$$

$$= 2t[u(t) - u(t-1)]$$

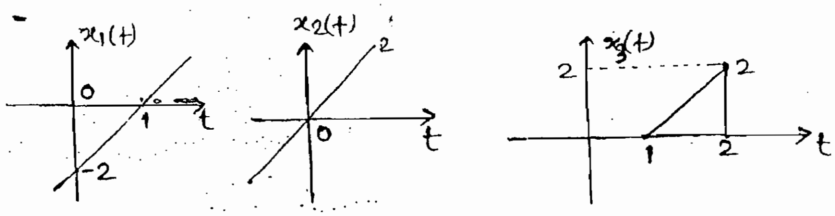
2nd method →



$\rightarrow 2t[u(t) - u(t-1)]$

$$\because x(t) = x_1(t) \cdot x_2(t)$$
$$= 2t[u(t) - u(t-1)]$$

Q →



Ans →
$x_2(t) = 2t$

$x_1(t) = 2(t-1)$

$x_3(t) = 2(t-1)[u(t-1) - u(t-2)]$

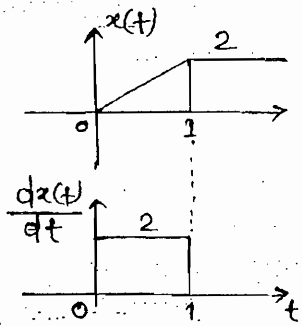
Chapter-02
Different operations of signal

(1.) Differentiation →

$$x(t) = \frac{dx(t)}{dt} = \text{slope of } x(t) \text{ wrt 't'}$$

* Graphical diff is applicable for triangular & rectangular type signal.

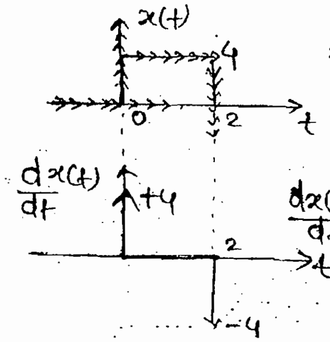
(1.)



$x(t) = 2r(t) - 2r(t-1)$

$$\frac{dx(t)}{dt} = 2u(t) - 2u(t-1)$$

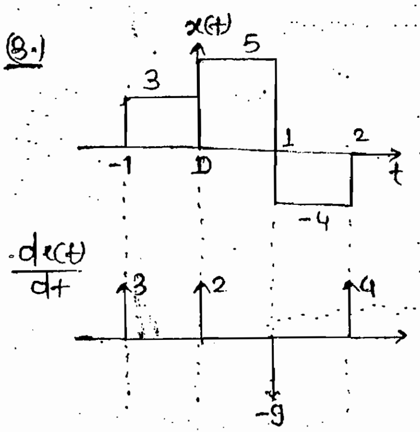
(2.)



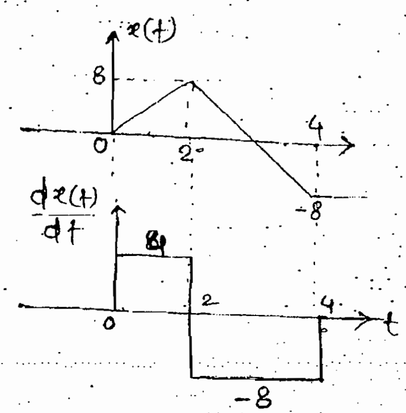
$x(t) = 4[u(t) - u(t-2)]$

$\frac{dx(t)}{dt} = 4[\delta(t) - \delta(t-2)]$

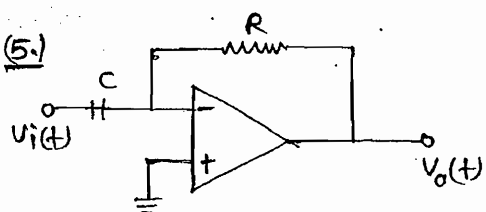
(3.)



(4.)

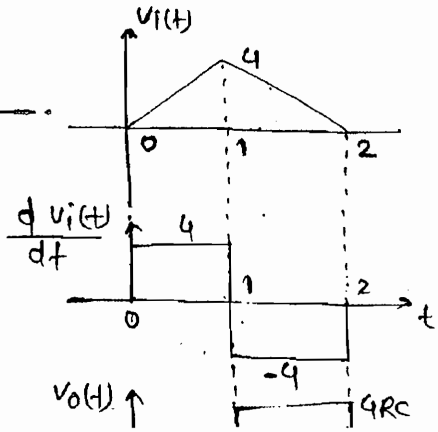


(5.)



$$V_o(t) = -RC\frac{dv_i(t)}{dt}$$

amp reversal



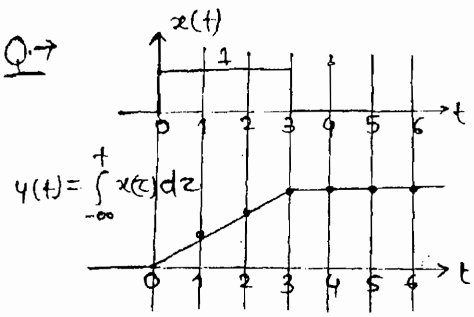
(2.) Integration →

$$x(t) = y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

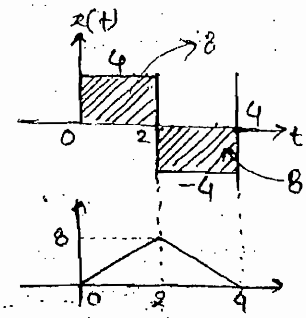
$$= \text{area of signal } x(t) \text{ wrt } 't'$$

* Graphical integration is applicable only for rectangular type waveform.

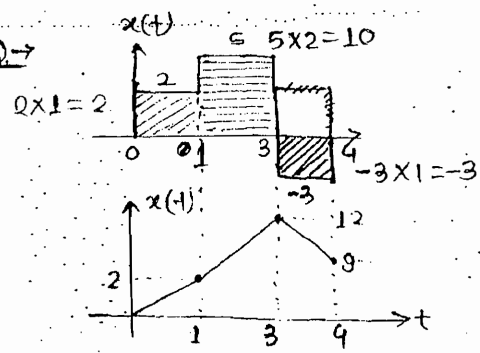
Q.→



Q.→

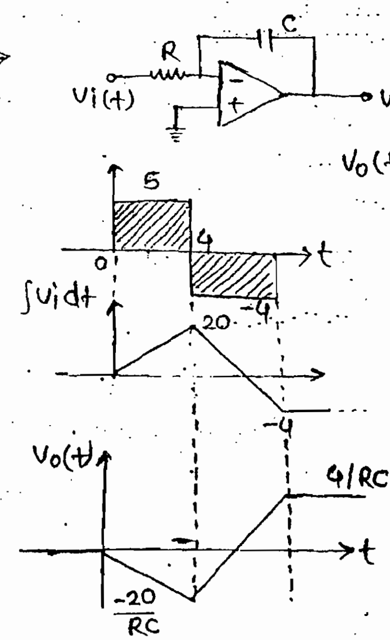


Q.→



Total area = 2+10−3 = 9

Q.→



$$V_o(t) = \frac{-1}{RC}\int v_i\,dt$$

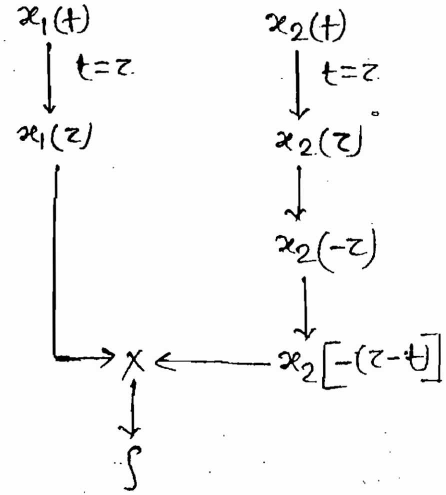
(3.) Convolution →

It is a mathematical operator & it is used for calculation of responce of LTI system.

$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau).x_2(t-\tau)\,d\tau$$

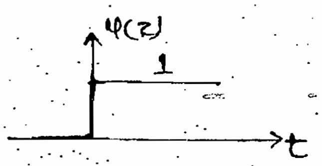
$$\boxed{x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\cdot x_2(t-\tau)\,d\tau}$$

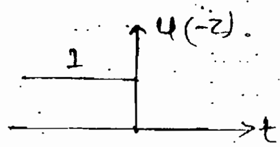


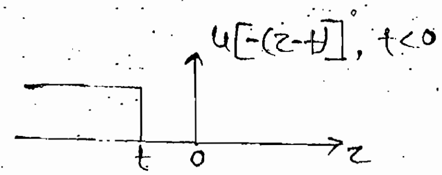
Steps →
(1.) Folding
(2.) shifting
(3.) multiplication
(4.) Integration.

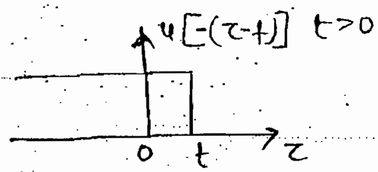
Q → $y(t) = u(t) * u(t)$

sol^n → $= \int_{-\infty}^{\infty} u(\tau) \cdot u(t-\tau)\, d\tau$









$$y(t) = \int_{-\infty}^{\infty} u(\tau) \cdot u(t-\tau)\, d\tau$$

$$= \begin{cases} 0 & ; t<0 \\ \int_0^t dt & ; t>0 \end{cases}$$

$$= \begin{cases} 0 & , t<0 \\ t & , t>0 \end{cases}$$

$$= r(t)$$

2nd method →

$$y(t) = x_1(t) * x_2(t)$$

$$Y(s) = X_1(s) \cdot X_2(s)$$

$$Y(s) = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2}$$

$$\boxed{y(t) = r(t)}$$

* Properties of Convolution →

(1.) Commutative →

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

$$\int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau$$

(2.) Associative →

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$

(3.) Distributive →

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

(4.) Impulse Responce →

$$x(t) * \delta(t-t_1) = x(t-t_1)$$

$$\downarrow t_1 = 0$$

$$x(t) * \delta(t) = x(t)$$

eg:- (1.) $u(t-1) * \delta(t+2) = u[(t+2)-1] = u(t+1)$

(5.) Derivative →

$$y(t) = x_1(t) * x_2(t)$$

$$\frac{d\,y(t)}{dt} = \frac{d\,x_1(t)}{dt} * x_2(t) = x_1(t) * \frac{d\,x_2(t)}{dt}$$

eg:- Find $y(t) = ?$

$$y(t) = \frac{d}{dt}[r(t) * u(t)]$$

Sol^n →

$$y(t) = \frac{d}{dt} r(t) * u(t) \qquad \text{(OR)} \quad y(t) = r(t) * \frac{d}{dt} u(t)$$

$$= u(t) * u(t) \qquad\qquad = r(t) * \delta(t)$$

$$= r(t) \qquad\qquad = r(t)$$

(6.) Step Responce →

$$y(t) = x(t) * u(t) = ?$$

$$y(t) = \int_{-\infty}^{t} \frac{d\,y(t)}{dt}\, dt = \int_{-\infty}^{t} \left[x(t) * \frac{d\,u(t)}{dt}\right] dt$$

Eg → (1.) $u(t) * u(t) = \int_{-\infty}^{t} u(t)\,dt = r(t)$

(2.) $r(t) * u(t) = \int_{-\infty}^{t} r(t)\,dt = p(t)$

(7.) Time scaling →

If $x_1(t) * x_2(t) = y(t)$ then;

$$x_1(at) * x_2(at) = \frac{1}{|a|}\, y(at) \quad (a \neq 0)$$

(8.) Area →

If $x_1(t) * x_2(t) = y(t)$ then;

$$\text{Area}_{y(t)} = \text{Area}_{x_1(t)} \times \text{Area}_{x_2(t)}$$

(9.) Time delay →

$$x_1(t) * x_2(t) = y(t)$$

$$x_1[(t-t_1)] * x_2(t-t_2) = y[t-(t_1+t_2)]$$

Eg:- (1.) $u(t-1) * u(t-2) = r(t-3)$

$= (t-3)\,u(t-3)$

(2.) $r(t-1) * u(t+3) = p(t+2)$

$= \frac{(t+2)^2}{2}\, u(t+2)$

*** (10) Duration →

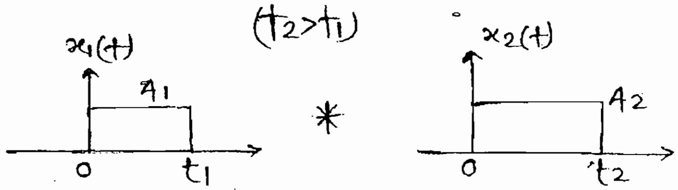
$$y(t) = x_1(t) * x_2(t)$$

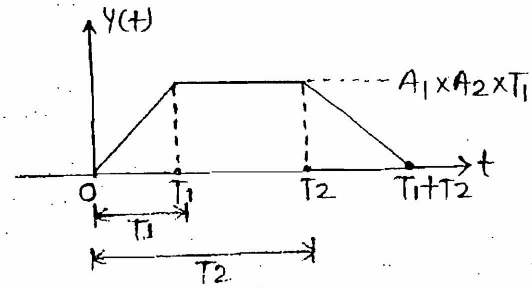
Signal	Extension
$x_1(t)$	$t_1 \leq t \leq t_2$
$x_2(t)$	$t_3 \leq t \leq t_4$
$y(t)$	$t_1+t_3 \leq t \leq t_2+t_4$

* Convolution of 2 rectangular pulses of equal duration will be a triangle.

* Convolution of 2 rectangular pulses of unequal duration will be a Trapezoid.

Trapezoid → $(t_2 > t_1)$

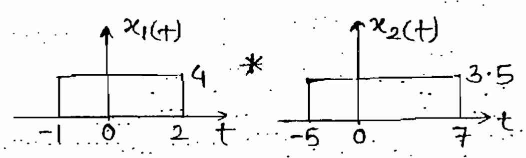




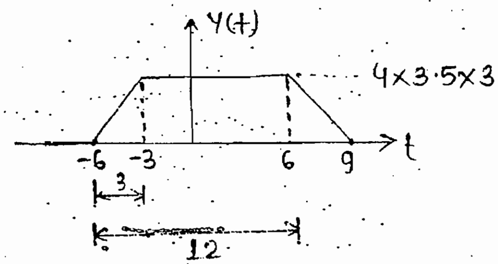
T_1 = smaller duration

T_2 = larger duration

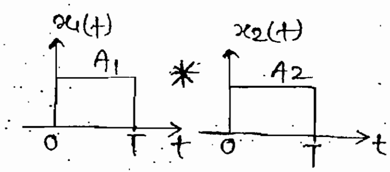
Que →



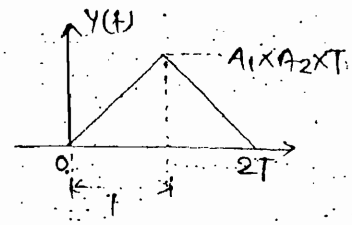
Sol^n →



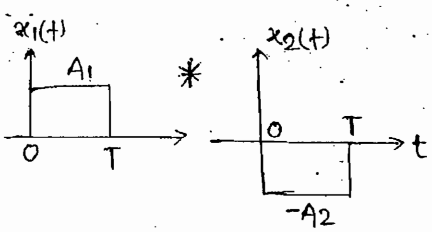
Que →



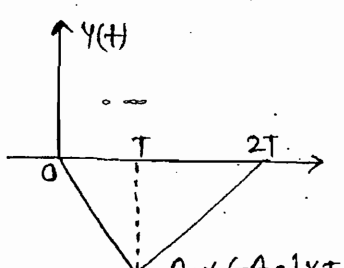
Sol^n →



Que →

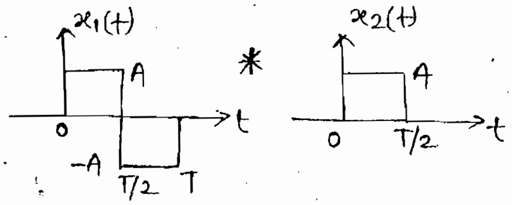


Sol^n →

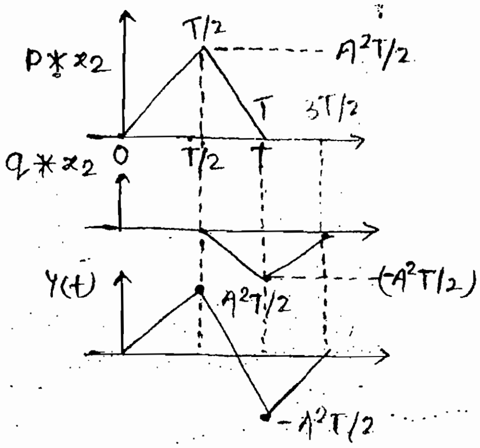


Because $(-A_2)$ Δ will be –ve.

Q.→

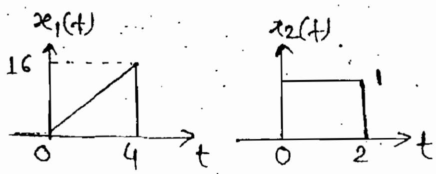


Soln→



$$y(t) = x_1(t) * x_2(t)$$
$$= [p(t) + q(t)] * x_2(t)$$
$$= [p(t) * x_2(t)] + [q(t) * x_2(t)]$$

Q.→



$y(t) = x_1(t) * x_2(t)$

Find value of $y(2)$

(a) 4 (b) 8 (c) 16 (d) 32

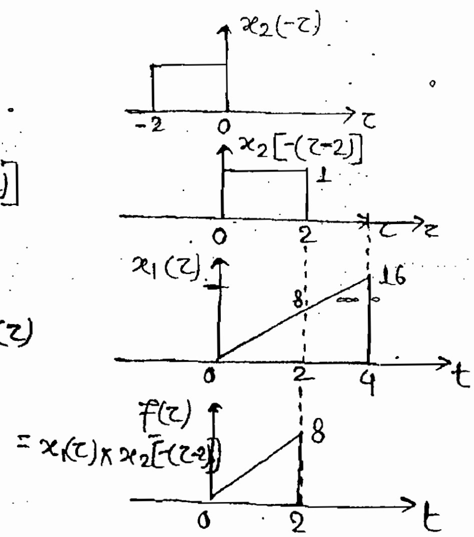
Soln→

$$y(t) = x_1(t) * x_2(t)$$
$$= \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

$$y(2) = \int_{-\infty}^{\infty} x_1(\tau)\, \underbrace{x_2(2-\tau)}_{\to\, x_2[-(\tau-2)]}\, d\tau$$

$$= \int_{-\infty}^{\infty} f(\tau)\, d\tau = \text{area of } f(\tau)$$

where $f(\tau) = x_1(\tau) \cdot x_2[-(\tau-2)]$



$$\text{Area} = \frac{1}{2} \times 2 \times 8 = 8$$

$$\boxed{\text{Area} = 8}$$

DATE-15/10/14

Chapter-03
Basic system properties

$$\text{i/p} \rightarrow x(t) \longrightarrow \boxed{\text{sys.}} \longrightarrow y(t) \longrightarrow \text{o/p}$$

$y(1)$ $y(t)$
- $\rightarrow x(t-1) = x(0) \longrightarrow$ Past
- $\rightarrow x(t) = x(1) \longrightarrow$ Present
- $\rightarrow x(t+1) = x(2) \longrightarrow$ Future

(1) Static & dynamic sys. →

Static → If o/p of sys. depands only on present values of i/p at each & every instant of time then sys. will be static.

* These sys. are also known as memoryless system.

Dynamic → * If o/p of sys. depands on past (or) Future values of i/p at any instant of time then sys. will be dynamic.

* This sys. are also known as sys. with memory.

Q.→ Check static/ dynamic sys.

(1.) $y(t) = x(t) + x(t-1)$

(2.) $y(t) = x(-t)$

(3.) $y(t) = x(\sin t)$

(4.) $y(t) = x(t-1)$

(5.) $y(t) = \text{Even}[x(t)]$

(6.) $y(t) = \text{Real}[x(t)]$

(7.) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(8.) $y(t) = e^{-(t+1)}x(t)$

Ans.→ (1.) Dynamic.

(2.) Dynamic.

(3.) $y(t) = x(\sin t)$

$y(-\pi) = x(0)$

$-3.14\,\text{sec} = x(0)$ Future — system is dynamic.

(4.) Dynamic

(5.) $y(t) = \dfrac{x(t) + x(-t)}{2}$

$(t=1)$

$y(1) = \dfrac{x(1) + x(-1)}{2}$ past — system is dynamic.

(6.) $y(t) = \dfrac{x(t) + x^*(t)}{2}$ system is static

(7.) ... system is dynamic

Note →

(1.) Integral & derivative sys. are dynamic sys.

(2.) In case of time scaling (or) time shifting system will be dynamic.

(2.) Causal & Non-Causal System →

* causal → * If o/p of sys. is independant of future values of i/p at each & every instant of time then sys. will be causal.

* This sys. are practical (or) physicaly relisable sys.

Eg:- (1.) $y(t) = x(t)$

(2.) $y(t) = x(t-1)$

(3.) $y(t) = x(t) + x(t-1)$

* Non-Causal system → * If o/p of sys. depands on future values of i/p at any instant of time then sys. will be non-causal.

Eg:- (1.) $y(t) = \underline{x(t+1)}$

(2.) $y(t) = x(t) + \underline{x(t+1)}$

(3.) $y(t) = x(t-1) + \underline{x(t+1)}$

(4.) $y(t) = x(t) + x(t-1) + \underline{x(t+1)}$

* Anti Causal System → * If o/p of sys. depands only on future values of i/p then sys. will be anticausal.

Eg:- $y(t) = x(t+1)$

* All anti-causal systems are non-causal but converse of this statement is not true.

Que. → Check Causal & Non-Causal system.

(1.) $y(t) = x(2t)$

(2.) $y(t) = x(-t)$

(3.) $y(t) = x(\sin t)$

(4.) $y(t) = \begin{cases} x(2t) \; ; \; t<0 \\ x(t-1) \; ; \; t \geq 0 \end{cases}$

(5.) $y(t) = \text{Odd}[x(t)]$

(6.) $y(t) = \sin(t+2) \cdot x(t-1)$

(7.) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

(8.) $y(t) = \int_{-\infty}^{t+1} x(\tau)\, d\tau$

(9.) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$

Sol^n → (i.) $y(t) = x(2t)$

$(t=1)\downarrow$

$y(t) = x(2)$ (system is non-causal)

(ii) $y(t) = x(-t)$

$(t=-1)$

$y(-1) = x(t)$ (System is Non-causal)

(iii) $y(t) = x(\sin t)$

$(t=-\pi)$

$y(-\pi) = x(0)$

$-3.14 = x(0)$ (system is non-causal)

(iv) $y(t) = \begin{cases} x(2t), & t<0 \longrightarrow \text{past} \\ x(t-1), & t\geq 0 \longrightarrow \text{past} \end{cases}$

(system is causal)

(v.) $y(t) = \text{odd } x(t)$

$= \dfrac{x(t)-x(-t)}{2}$

$(t=-1)$ → future

$y(-1) = \dfrac{x(-1)-x(1)}{2}$ (system is non-causal)

(vi) $y(t) = \sin(t+2)\cdot x(t-1)$

(Cofficient) ↓past

(system is causal.)

(vii) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau \rightarrow x(t)$

$= \int_{-\infty}^{t} x(t)$ (system is causal)

(viii) $y(t) = \int_{-\infty}^{(t+1)} x(\tau)d\tau \rightarrow x(t+1)$

(system is non-causal)

(ix) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau \rightarrow x(2t)$

(system is non-causal)

(3.) Linear & Non-linear system →

linear → * A linear sys. follows the law of superposition.

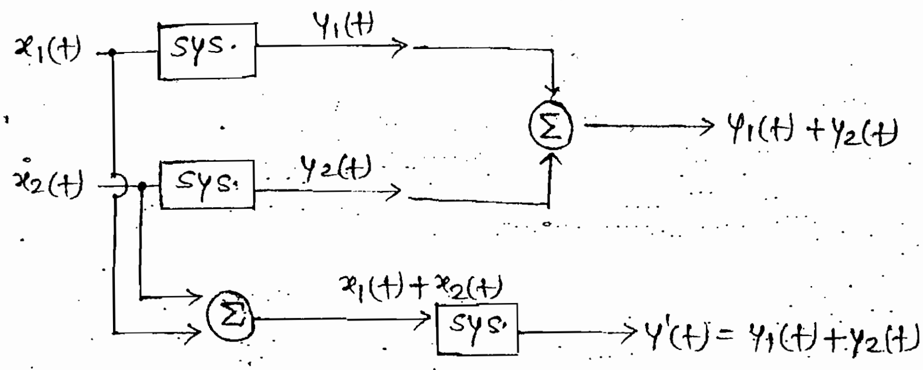
* This law is necessary & sufficient to prove linearity of system.

* It is a combination of two laws:-

(i) Law of additivity.

(ii) Law of Homogenity.

(1.) Law of additivity →



Eg:- $y(t) = x(t)+10$

o/p = i/p + 10

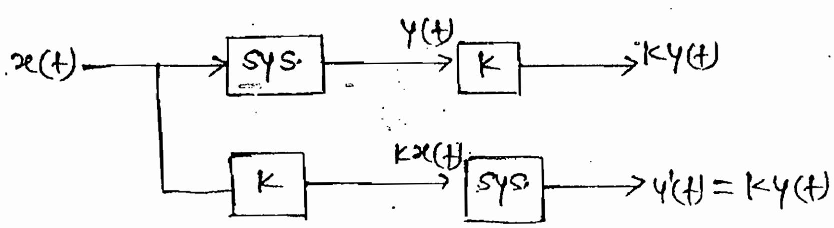
$y_1(t) = x_1(t)+10$

$y_2(t) = x_2(t)+10$ → $x_1(t)+x_2(t)+20$

$y'(t) = x_1(t)+x_2(t)+10$

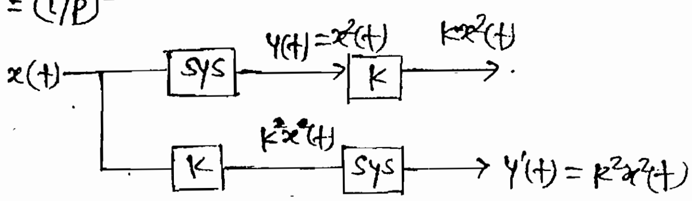
$y(t) \neq y'(t)$ [sys. is NL]

(2.) Law of Homogenity →



Eg:- $y(t) = x^2(t)$

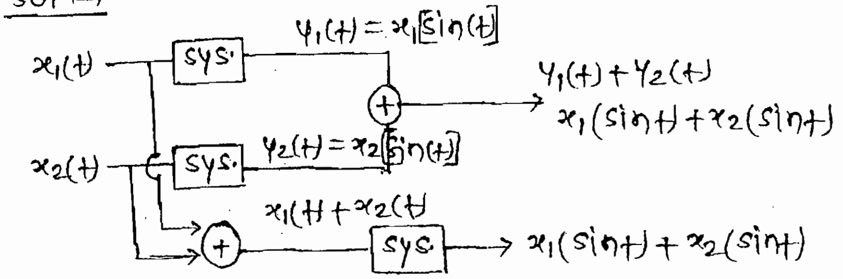
o/p = $(i/p)^2$



Que.→ Check linear/Non-linear sys.

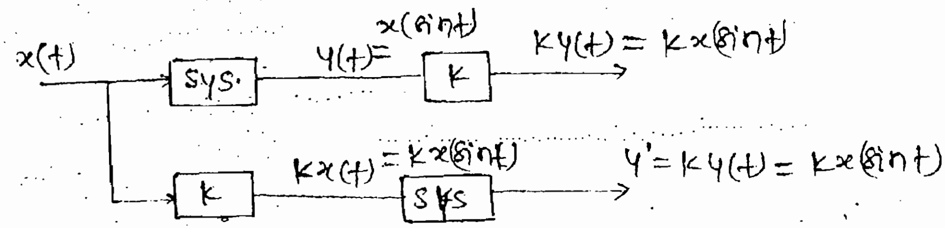
(1.) $y(t) = x(\sin t)$ (2.) $y(t) = x(\tan t)$ (3) $y(t) = x(t^2)$

Soln→



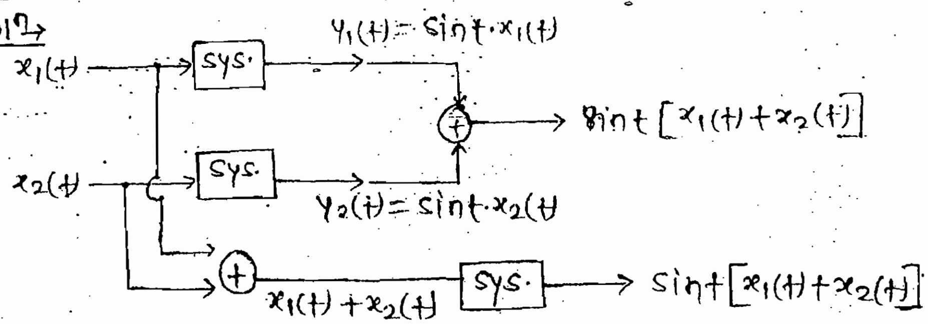
Note→

Linearity of sys. is independant of time scaling.

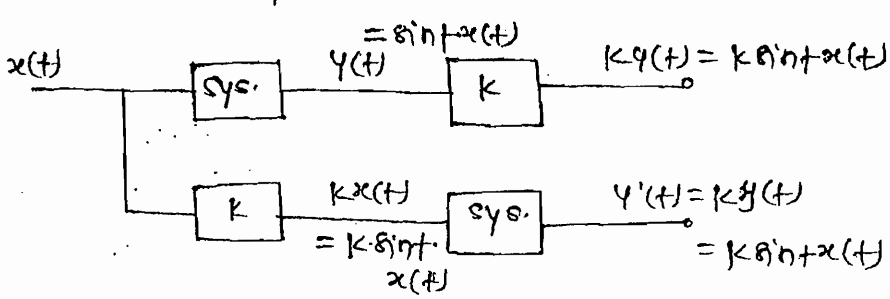


(2.) $y(t) = \sin t \cdot x(t)$ (3.) $y(t) = \log t \cdot x(t)$

Soln→



Note:- linearity of sys. is independant of cofficient used in sys. relationship.



2nd method →

for linearity :-

(i) O/p should be 0 for 0 i/p.

(ii) There should not be any 'NL' operation.

eg:- [sin, cos, tan, sec, cosec, cot, ... log, exponential, modulus, sq, cubes...... .... root, ... sq( ), sinc( ) ... sgn( ) etc either on 'x' or 'y'.]

Q.→ Check linear/NL sys.

(i) $y(t) = x(t) + 2$ ⟶ put $(t=0)$ then $y(0) \neq x(0) + 2$ ⟶ NL

(ii) $y(t) = e^{x(t)}$ ⟶ Because of $e^{x(t)}$ it is NL & also both condn not satisfying.

(iii) $y(t) = x(\tan t)$ ⟶ linear

(i). If $(t=0)$, then $y(0) = x(0)$ means No i/p no o/p

(ii) above fn is not operating on 'x', it is operating on the 't'.

(iv) $y(t) = \tan[x(t)]$

system is NL

(v.) $y(t) = x(t-1) + x(t+1)$

i/p ⟶ $\boxed{\text{sys.}}$ ⟶ o/p = $\cancel{\text{past}}^{\,0}$ i/p + $\cancel{\text{Future}}^{\,0}$ i/p

No any NL fn so this is linear

(vi) $y(t) = \text{even}[x(t)]$

$$\cancel{y(t)}^{\,0} = \frac{x(t) + \cancel{x(-t)}^{\,0}}{\cancel{2}}$$

No non-linear operator so linear

(vii) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$$\cancel{y(t)}^{\,0} = \int_{-\infty}^{t} \cancel{x(0)}^{\,0}\, d\tau$$ linear.

(viii) $y(t) = \begin{cases} x(t-1), & t<0 \\ x(t+1), & t \geq 0 \end{cases} = \begin{cases} \cancel{\text{past i/p}}^{\,0}, & t<0 \\ \cancel{\text{future i/p}}, & t \geq 0 \end{cases}$ linear

Note →

(1.) Integral & derivative operators are linear.

(2.) Even & odd operators are linear.

(ix) $y(t) = \int_{-\infty}^{t} x^2(\tau)\,d\tau$

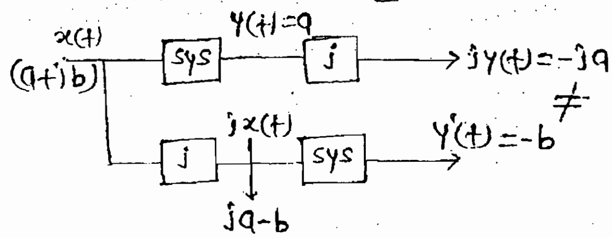
$y(t) = \int_{-\infty}^{t} x^{2}(\tau)\,d\tau$ NL

(x) $y(t) = e^{t} x(t)$

$y(t)^{0} = e^{t} x(t)^{0}$ · linear

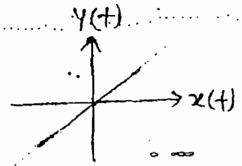
(xi) $y(t) = \text{Real}[x(t)]$

$y(t) = \frac{x(t) + x^{*}(t)}{2}$ NL



Note → Real & imaginary operators are NL.

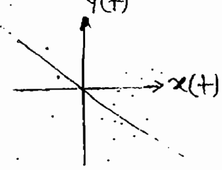
(xii)



$y(t) = m\,x(t)$

system is linear

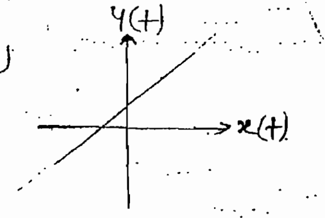
(xiii)



$y(t) = -m\,x(t)$

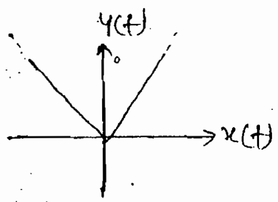
system is L

(xiv)



when i/p 0 then we got o/p NL

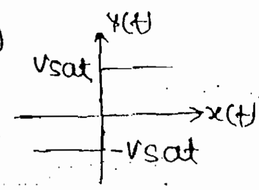
(xv)

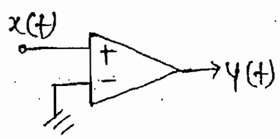


$y(t) = |x(t)|$

NL

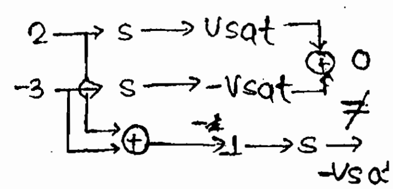
(xvi)





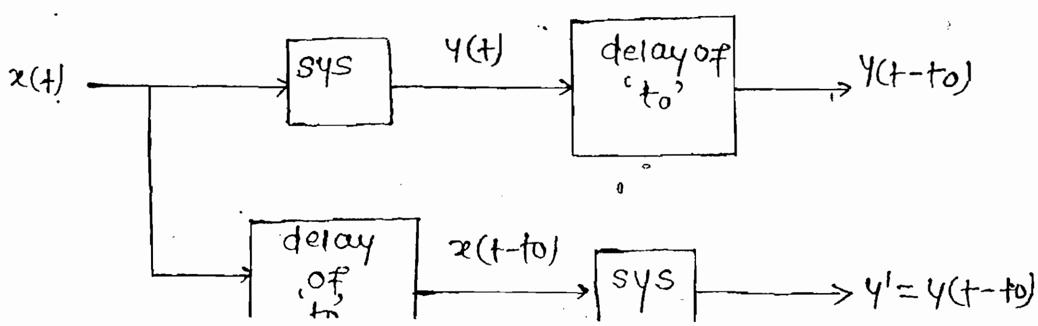
$y(t) = V_{sat}\,\text{Sgn}[x(t)]$ (or)

NL



(4.) Time invarient & time varient sys. →

Time invarient →



Note:- Any delay provided in i/p must be reflected in o/p for a time invarient system.

Que.→ Check time invarient / varient sys.

(1.) $y(t) = x(2t)$

$x(t) \rightarrow (S) \xrightarrow{x(2t)=y(t)} (D) \rightarrow y(t-t_0) = x[2(t-t_0)]$

$x(t) \rightarrow (D) \xrightarrow{x(t-t_0)} (S) \rightarrow y'(t) = f(2t) = x(2t-t_0)$

$x(t-t_0) \rightarrow f(t)$

system is (TV)

(2.) $y(t) = x(\cos t)$

Sol^n→

$x(t) \rightarrow (S) \xrightarrow{y(t)=x(\cos t)} (D) \rightarrow y(t-t_0) = x[\cos(t-t_0)]$

$x(t) \rightarrow (D) \xrightarrow{x(t-t_0)} (S) \rightarrow y'(t) = f(\cos t)$

$x(t-t_0) \rightarrow f(t)$

$= x[\cos t - t_0]$

System is (TV)

Note:- In case of time scaling sys. will be time varient.

(3.) $y(t) = \cos t \cdot x(t)$ (4.) $y(t) = \log t \cdot x(t)$

Sol^n→

$x(t) \rightarrow (S) \xrightarrow{y(t)=\cos t \cdot x(t)} (D) \rightarrow y(t-t_0) = \cos(t-t_0) \cdot x(t-t_0)$

$x(t) \rightarrow (D) \xrightarrow{x(t-t_0)} (S) \rightarrow y'(t) = \cos t \cdot f(t)$

$x(t-t_0) \rightarrow f(t)$

$\cos t \cdot x(t-t_0)$

system is (TV)

Note:- If coefficient in sys. relationship is f^n of time then sys. will be time varient.

(5.) $y(t) = odd[x(t)]$ (6.) $y(t) = cs[x(t)]$

Sol^n→ $y(t) = \frac{x(t) - x(-t)}{2}$ $y(t) = \frac{x(t) + x^*(-t)}{2}$

$x(-t)$ → Time scaling (TV) $x^*(-t)$ → Time scaling (TV)

(vii) $y(t) = x(t-1) + x(t+1)$

sol$^n \rightarrow$ $y(t) = x(t-1) + x(t+1)$

No scaling

coefficient are independant of time so TIV

(viii) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

sol$^n \rightarrow$ $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \rightarrow x(t)$

TIV

(ix) $y(t) = \int_{-\infty}^{3t} x(\tau)\,d\tau$

sol$^n \rightarrow$ $y(t) = \int_{-\infty}^{3t} x(\tau)\,d\tau \rightarrow x(3t)$ (TV)

(x) $y(t) = \int_{-\infty}^{t} \cos\tau \cdot x(\tau)\,d\tau$

sol$^n \rightarrow$ $y(t) = \int_{-\infty}^{t} \cos\tau\, x(\tau)\,d\tau \rightarrow \cos t\, x(t)$ (TV)

(xi) $y(t) = \begin{cases} x(t-1) & ; t \leq 0 \\ x(t+1) & ; t \geq 0 \end{cases}$

sol$^n \rightarrow$ $= a(t) \cdot x(t-1) + b(t) \cdot x(t+1)$

split systems are time varient system.

(5.) Stable/Unstable sys. $\rightarrow$ Finite/bounded in amplitude

Stable $\rightarrow$ Bounded i/p bounded o/p (BIBO) criteria.

BIBO $\rightarrow$ For stable system, o/p should be bounded (or) finite for finite (or) bounded i/p at each & every instant of time.

Eg:- Bounded i/p are $u(t)$, dc-signal, $\sin t$, $\cos t$, $\operatorname{sgn}(t)$

Que. $\rightarrow$ Check stable/unstable sys.

(1.) $y(t) = x(t) + 2$

$x(t)$	$y(t)$
10	12

(Stable)

(2.) $y(t) = t\,x(t)$

$x(t)$	$y(t)$
10	$10t$

$\infty = \infty$

$-\infty = \infty$

(Unstable)

(3.) $y(t) = \dfrac{x(t)}{\sin t}$

$x(t)$	$y(t)$
2	$\dfrac{2}{\sin t}$

($t = 0, \pi$)

(Unstable)

~~(4)~~ $y(t) = \sin t \cdot x(t)$

Sol^n→ $y(t) = \sin t \cdot x(t)$

$\sin t \rightarrow (-1, 1)$

$y(t) \rightarrow -x(t), x(t)$

$x(t)$	$y(t)$
2	$(-2, 2)$

(stable)

(5.) $y(t) = \sin[x(t)]$

Sol^n→ $y(t) = \sin[x(t)]$

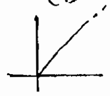
$\rightarrow (-1, 1)$

(stable)

(6.) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

Sol^n→ $x(t) = u(t) \longrightarrow$ bounded

$y(t) = r(t) \longrightarrow t\,u(t)$



(Unstable)

(7.) $y(t) = \frac{dx(t)}{dt}$

Sol^n→ $x(t) = u(t) \longrightarrow$ bounded

$y(t) = \delta(t) \longrightarrow$ Unbounded

(Unstable).

(8.) $\int_{-\infty}^{t} \cos\tau \cdot x(\tau)d\tau = y(t)$

Sol^n→ $y(t) = \int_{-\infty}^{t} \cos\tau \cdot x(\tau)d\tau$

$x(t) \longrightarrow \cos t =$ bounded sig.

$$y(t) = \int_{-\infty}^{t} \cos\tau \cdot \cos\tau \, d\tau$$

$$= \int_{-\infty}^{t} \cos^2\tau \, d\tau$$

$$= \int_{-\infty}^{t} \left(\frac{1+\cos 2\tau}{2}\right) d\tau$$

$$= \frac{1}{2}\left[\underbrace{(\tau)_{-\infty}^{t}}_{\text{(Unbounded)}} + \overbrace{\left(\frac{\sin 2\tau}{2}\right)_{-\infty}^{t}}^{\text{finite}}\right]$$

(Unstable)

Note:- so the integration & differentiation signals are unstable sys.

Static | Dynamic – D→ I, D, TS & TS

linear/NL – L→ I, D, E, O

NL→ R & I

* Linear time invariant (LTI) system →

$$x(t) \longrightarrow \boxed{\text{LTI sys.}} \longrightarrow y(t)$$

$h(t) \rightarrow$ Impulse-Responce of sys.

$H(\omega)$ (or) $H(s) \longrightarrow$ TF of sys.

* Impulse Responce & TF terms are used only for LTI system.
* Impulse Responce is used for defining LTI sys. in time domain & TF is used for defining LTI sys. in freq. domain.

Impulse Responce →

$$\delta(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(t)$$

* If i/p to LTI sys. is unit impulse then o/p of sys. is known as impulse Responce.

Transfer function →

* It is the ratio of laplace Xform of o/p to Laplace Xform of i/p when all initial condn are assumed to be 0.

$$H(s) = \left.\frac{Y(s)}{X(s)}\right|_{\text{zero intial cond}^n}$$

Total o/p = Zero i/p responce + zero state responce

$$\text{Total o/p} = \underbrace{ZIR}_{\substack{\text{due to}\\ \text{initial cond}^n/\\ \text{states}}} + \underbrace{ZSR}_{\text{due to applied i/p}}$$

* for linearity of sys., initial condn are assumed to be zero, because non-zero initial condn make the sys. non-linear.

Convolution →

$$v = x(t) \longrightarrow \boxed{\text{LTI}}_{\,h(t) = v} \longrightarrow y(t) \qquad y(t) = x(t) * h(t)$$

* Convolution is a linear time invariant operator & it is used only for LTI system.

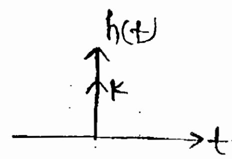
$$y(t) = x(t) * h(t)$$
$$y(t) = \int_{-\infty}^{t} h(\tau) . x(t-\tau) d\tau$$

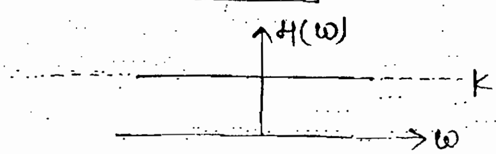
* The above relation is both linear & TIV. so it is LTI system.

Condition for LTI system to be static →

$$y(t) = K x(t)$$
$$h(t) \downarrow y(t) \qquad \downarrow x(t) = \delta(t)$$

$$\boxed{h(t) = K\delta(t)}$$
$$\boxed{H(\omega) = K}$$

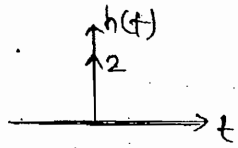




* Impulse $\delta(t)$ is the f^n whose all X_{form} is one.
* For static LTI system, impulse responce should be impulse at origin & TF should be independant of freq.

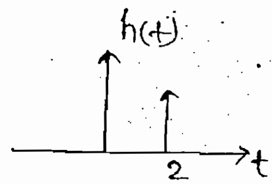
Q.→ Check S/D LTI sys.

(i)



(static)

(ii)

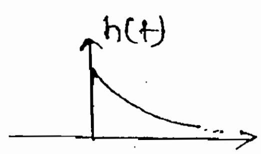


Implwe is not at origin so dynamic.

(iii) $h(t) = u(t)$

(Dynamic)

(iv)



(dynamic)

(iv) $H(S) = 2$

static (free of freq.)

(vi) $H(S) = \frac{1}{S+1}$

(dynamic)

* Filters are dynamic system because there TF depends on frq.

DATE-16/10/14

*** Cond^n For LTI system to be causal →**

$$y(t) = x(t) * h(t) \rightarrow \text{Future i/p} \neq 0$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

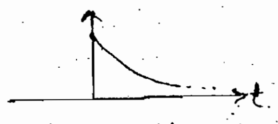
$$h(\tau) = 0 ;\ \tau < 0$$

$$\big\downarrow \tau = t$$

$$\boxed{h(t) = 0 .\ t < 0}$$

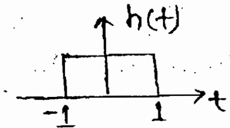
Que → Check C/NC LTI system.

(1.) $h(t) = e^{-2t} u(t)$



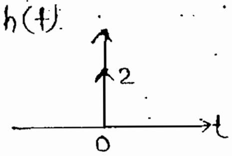
(Causal)

(2.) $h(t) = u(t+1) - u(t-1)$



(Non-causal)

(3.) $h(t)$

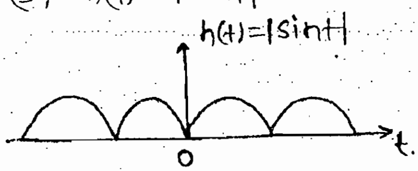


(causal)

(4.) $h(t) = e^{-(t+1)} u(t)$

(causal)

(5.) $h(t) = |\sin t|$



(Non-causal)

*** Cond^n For LTI sys. to be stable →** * If impulse Responce of LTI sys. is absolutely integrable then sys. will be stable. i.e.

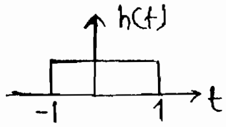
$$\boxed{\int_{-\infty}^{\infty} |h(t)|\, dt < \infty}$$

* If impulse responce of LTI sys. is represented by energy signal (or) unit impulse f^n then sys. will be stable.

i.e. $h(t) \rightarrow$ Energy/ $\delta(t) \longrightarrow$ stable

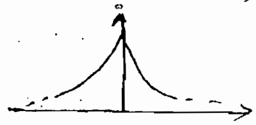
Que → Check s/us system.

(1.) $h(t) = u(t+1) - u(t-1)$



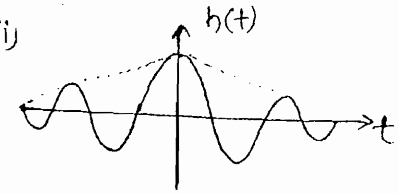
(Stable)

(2.) $h(t) = e^{-2|t|}$



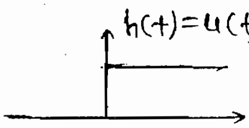
(Stable)

(iii)



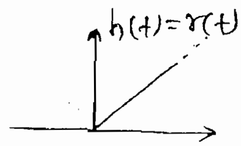
(Stable)

(iv)



(Unstable)

(v)



(Unstable)

(vi) $H(s) = \frac{1}{s^2+1}$

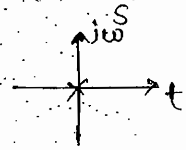
Poles $= s = \pm j$

* Because of imaginary axis lying so it is marginary stable
* $h(t) = \sin t\, u(t)$

↓

(Unstable)

(vii) $H(s) = \frac{1}{s}$



Pole → $s=0$

* marginaly stable
* $h(t) = u(t) \rightarrow$ power sig.

(Unstable)

$$H(s) = \frac{1}{s} = \frac{Y(s)}{X(s)}$$

$$Y(s) = \frac{X(s)}{s}$$

Inverse LT.

$$y(t) = \int_{-\infty}^{t} x(t)\,dt \Big] \text{ Integrator}$$

According to BIBO criteria:–

$x(t) = u(t) =$ bounded i/p

$$y(t) = \int_{-\infty}^{t} u(t)\,dt = r(t)$$

$r(t) =$ Unbounded sig.

So it is Unstable.

Note →

* All marginaly stable LTI sys. are BIBO Unstable.

* Distortions in LTI systems →

Types:- (i) magnitude/Amplitude distortion

(ii) Delay / phase distortion.

Note:-

(1.) $x(t) \longrightarrow \boxed{\text{NL sys.}} \longrightarrow y(t) = x(t) + x^2(t)$

If $x(t) = \sin\omega_0 t$ then $y(t) = \sin\omega_0 t + \sin^2\omega_0 t$

$= \sin\omega_0 t + \frac{1-\cos 2\omega_0 t}{2}$

$x(t) \downarrow \omega_0$ $\qquad$ $y(t) \downarrow \omega_0, 2\omega_0$

(2.) $x(t) \longrightarrow \boxed{\text{TV sys}} \longrightarrow y(t) = x(t) + x(2t)$

$x(t) = \sin\omega_0 t$ $\qquad$ $y(t) = \sin\omega_0 t + \sin 2\omega_0 t$

$\downarrow \omega_0$ $\qquad$ $\downarrow \omega_0, 2\omega_0$

*** For production of harmonics, nature of sys should be either NL (or) TV.

(1.) Magnitude/Amplitude distortion → If sys. provides unequal amount of amplification (or) attenuation two ~~to~~ diff freq. componants present in i/p sys, then sys. is having magnitude distortion.

$x(t) = \sin\omega_1 t + \sin\omega_2 t \longrightarrow \boxed{\text{LTI sys.}} \longrightarrow y(t) = A_1\sin\omega_1 t + A_2\sin\omega_2 t$

$\omega_1 \neq \omega_2$ $\qquad$ $A_1 \neq A_2$

(2.) Delay (or) phase distortion → If sys. provides unequal amount of time delays to diff freq. componants present in i/p signal then sys. is having delay (or) phase distortion.

$x(t) = \sin\omega_1 t + \sin\omega_2 t$ $(\omega_1 \neq \omega_2)$ $\longrightarrow$ $\boxed{\text{LTI sys.}}$ $\longrightarrow$ $y(t) = \sin[\omega_1(t-t_1)] + \sin[\omega_2(t-t_2)]$ $(t_1 \neq t_2)$

*** Cond^n for LTI sys. to be distortionless →**

$x(t) = \sin\omega_1 + \sin\omega_2$ $\longrightarrow$ $\boxed{\text{LTI sys.}}$ $\longrightarrow$ $y(t) = kx(t-t_0)$

$= k\sin[\omega_1(t-t_0)] + k\sin[\omega_2(t-t_0)]$

$$\boxed{y(t) = kx(t-t_0)}$$

So laplace transform of above eq^n

$$Y(s) = k \times X(s)\, e^{-st_0}$$

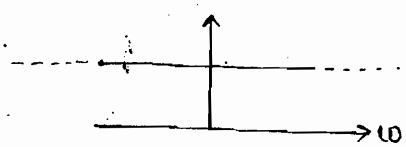
$$H(s) = \frac{Y(s)}{X(s)}$$

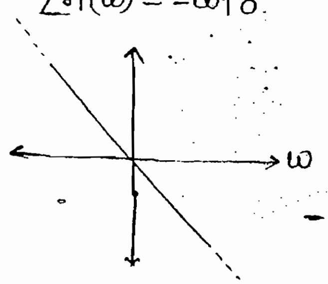
$$= k e^{-st_0}$$

$(s = j\omega)$

$$H(j\omega) = k e^{-j\omega t_0}$$

$|H(\omega)| = k$ $\qquad\qquad$ $\angle H(\omega) = -\omega t_0$





**** For distortionless LTI sys., magnitude of TF should be independant of freq. & phase of TF should be linear.

*** Differential eq^n for LTI sys. →**

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_0 y(t)$$

$$= b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + \cdots + b_0 x(t)$$

For linearity →

All initial condn should be zero.

Time-invariance →

Coefficients $a_n, a_{n-1}, \cdots a_0, b_m, b_{m-1} \cdots, b_0$ should be indepandant of time.

Que. → Check time invariance & linearity of sys. (initial condn are zero).

(1.) $\frac{2d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + y(t) = x(t)$

(2.) $\frac{2d^2y(t)}{dt^2} + 3t \cdot \frac{dy(t)}{dt} + y(t) = x(t)$

(3.) $2\left[\frac{dy(t)}{dt}\right]^2 + \frac{3dy(t)}{dt} + y(t) = x(t)$

Ans. → (1.) L, TIV

(2.) L, TV

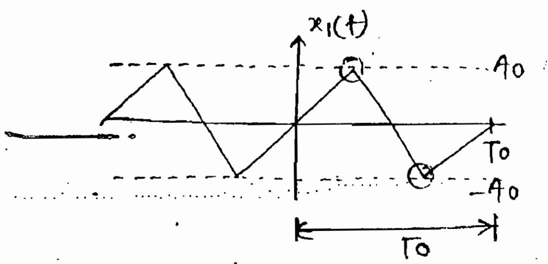
(3.) NL, TIV

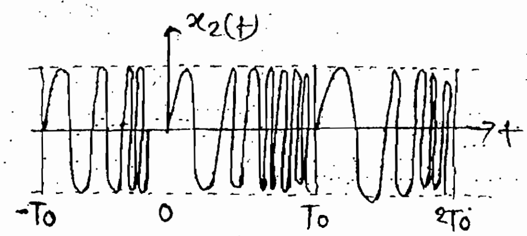
Chapter-04
Fourier Series

* FS expansion is used only for periodic signal.
* In FS sig. is expanded in terms of its harmonics which are sinusoidal & orthognal to one another.

Cond^n for existance of FS expansion → (Dirichlet cond^n)

*(1.) signal should have finite no. of maxima & minima over its time-period.

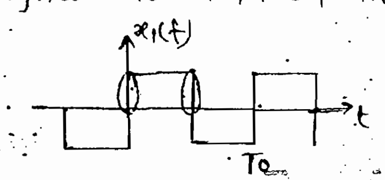




$x_1(t)$ → FS expansion is possible

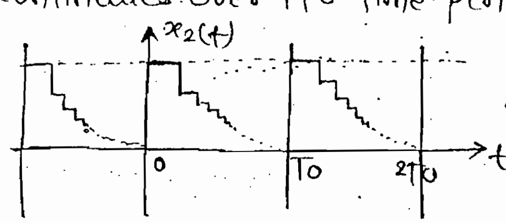
$x_2(t)$ → FS expansion is not possible.

*(2.) signal should have finite no. of discontinuties over its time-period.



$x_1(t)$ → FS expansion is possiple

No. of discontinuities = 2



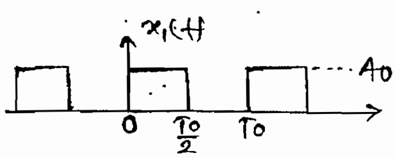
$x_2(t)$ → FS expansion is not possible

*(3.) Signal should be absolutely integrable over its time-periods.

i.e.

$$\boxed{\int_{T_0} |x(t)|\,dt < \infty}$$

Eg:- (1)



(FS is possible)

(2.) $x_2(t) = \tan(t)$

(FS is not possible)

* Types of FS expansion →

(1.) Trignometrical FS exp. →

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

where, $a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt$ = Represents avg. (or) dc value of signal.

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t\cdot dt = a_{(-n)} \text{ [even } f^n \text{ wrt 'n']}$$

$$b_n = \frac{2}{T_0}\int_{T_0} x(t)\cdot \sin n\omega_0 t\, dt = -b_{(-n)} \text{ [Odd } f^n \text{ wrt 'n']}$$

(2.) Exponential FS exp. →

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

where; C_n = Complex exponential FS coefficient

$$= \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{-jn\omega_0 t}\,dt \text{ ---------- (i)}$$

$$C_{(-n)} = \frac{1}{T_0}\int_{T_0} x(t)\, e^{jn\omega_0 t}\,dt$$

$$\Big\downarrow * \text{(conjugate)}$$

$$C^*_{(-n)} = \frac{1}{T_0}\int_{T_0} x^*(t)\, e^{-jn\omega_0 t}\,dt \text{ ------ (ii)}$$

For Conjugate symmetry (CS) C_n :-

$$C_n = C^*_{-n}$$

from eq^n (i) & (ii)

$\boxed{x(t) = x^*(t)}$ If $x(t)$ is real.

* If time domain signal is real then its exponential FS coefficient will be conjugate symmetry.

$$C_n = |C_n| e^{j\angle C_n} \text{ ------ (3.)}$$

where; $|C_n|$ = magnitude of n^{th} harmonic ($n\omega_0$)

from eq^n (3) $n = -n$

$$C_{(-n)} = |C_{(-n)}| e^{j\angle + C_{(-n)}}$$

$$\downarrow *$$

$$C^*_{(-n)} = |C_{(-n)}| e^{-j\angle C_{(-n)}} \quad \text{------ (4)}$$

for CS C_n:-

$$C_n = C^*_{-n}$$

from eq^n (3) & (4)

$$|C_n| = |C_{(-n)}| \longrightarrow \text{Even}$$

$$\angle C_n = \cancel{-\angle C_{(n)}} \; -\angle C_{(-n)} \longrightarrow \text{Odd}$$

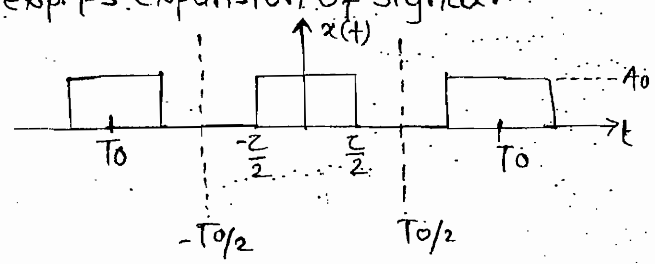
***Note:-**

For Real signal:- (RmIP)

(i) Real part of C_n will be even & imaginary part of C_n will be odd.

(ii) magnitude of C_n will be even & phase of C_n will be odd.

Que→ Find exp. FS expansion of signal.



Sol^n→

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$(x(t) = A_0) \qquad = \frac{A_0}{T_0}\int_{-\tau/2}^{\tau/2} e^{-jn\omega_0 t}\, dt$$

$$= \frac{A_0}{T_0}\left(\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right)_{-\tau/2}^{\tau/2}$$

$$= \frac{A_0}{T_0(jn\omega_0)}\left[-e^{-jn\omega_0 \frac{\tau}{2}} + e^{jn\omega_0 \tau/2}\right]$$

$$= \frac{A_0}{T_0(jn\omega_0)} \times 2j \sin\left(\frac{n\omega_0 \tau}{2}\right)$$

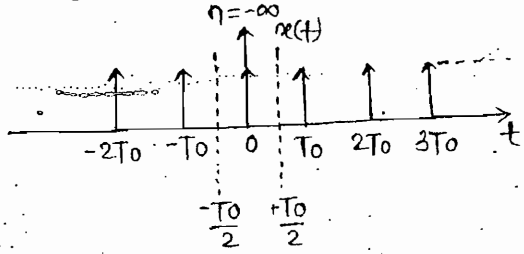
$$c_n = \frac{A_0}{T_0(n\omega_0 j)}(2j) \times \left[\frac{\sin\left(\frac{n\omega_0\tau}{2}\right)}{\left(\frac{n\omega_0\tau}{2}\right)}\right] \times \left(\frac{n\omega_0\tau}{2}\right)$$

$$\boxed{C_n = \frac{A_0\tau}{T_0}\, Sa\left(\frac{n\omega_0\tau}{2}\right)}$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty}\left[\frac{A_0\tau}{T_0}\, Sa\left(\frac{n\omega_0\tau}{2}\right)\right]\cdot e^{jn\omega_0 t}}$$

<u>Que.→</u> Find C_n for sig. $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_0)$



<u>Sol^n→</u>

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\cdot e^{-jn\omega_0 T} dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\cdot e^{-jn\omega_0 t} dt$$

$$\because f(t)\cdot\delta(t) = f(0)\cdot\delta(t)$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \underbrace{\delta(t)}_{area=1}\cdot e^{0} dt$$

$$\boxed{C_n = \frac{1}{T_0}}$$

<u>Que.→</u> The sig. $x(t)$ has '$T_0 = 2$' & following FS coefficients

$$C_K = \begin{cases} (1/2)^K, & K \geq 0 \\ 0, & K < 0 \end{cases}$$

The value of $x(0)$ will be.

Sol^n →

$$\cancel{C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (1/2)^k e^{-jk\omega_0 t}\, dt}$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

$$\Big\downarrow t=0$$

$$x(0) = \sum_{k=-\infty}^{\infty} C_k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$$

$$x(0) = 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \cdots\cdots$$

$$= \frac{1}{1-\frac{1}{2}} = \frac{1}{1/2} = 2$$

Que:→ The sig. $x(t)$ has FTP '$T_0 = 1$' & the following Fourier cofficients

$$C_k = \begin{cases} \left(\frac{-1}{3}\right)^k, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

Find $x(t) = ?$

(a.) $\dfrac{1}{1-\frac{1}{3}e^{j2\pi t}}$ (b.) $\dfrac{1}{1+\frac{1}{3}e^{+j2\pi t}}$ (c.) $\dfrac{1}{1-\frac{1}{3}e^{-j2\pi t}}$ (d.) $\dfrac{1}{-1+\frac{1}{3}e^{-j2\pi t}}$

Sol^n →

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

$$= \sum_{k=0}^{\infty} \left(\frac{-1}{3}\right)^k e^{jk\omega_0 t}$$

$$= \sum_{k=0}^{\infty} \left(\frac{-1}{3} e^{j\omega_0 t}\right)^k$$

$$= 1 + \left(\frac{-1}{3} e^{+j\omega_0 t}\right) + \left(\frac{-1}{3} e^{j\omega_0 t}\right)^2 + \cdots\cdots$$

$$= \frac{1}{1-\left(\frac{-1}{3} e^{j\omega_0 t}\right)}$$

$$= \frac{1}{1+\frac{1}{3}e^{j\omega_0 t}}$$

$$= \frac{1}{1+\frac{1}{3}e^{+j2\pi t}}$$

Note:-

$$\cancel{C_0} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \quad (*)\ C_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$$

$$\Big\downarrow n=0$$

$$C_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$$

$C_0 = a_0 =$ dc/avg. value of $x(t)$.

$$(*)\ x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$x(t) = \text{------} + C_{(-1)}e^{-j\omega_0 t} + C_0 + C_1 e^{j\omega_0 t} + \text{-----}$$

Que.→ Consider the periodic sig.

$$x(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos\left(2\omega_0 t + \frac{\pi}{4}\right)$$

Determine C_n.

Sol^n→

$$x(t) = 1 + \left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right) + \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) + \frac{1}{2}\left[e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})}\right]$$

$$x(t) = 1 + \left(1+\frac{1}{2j}\right)e^{j\omega_0 t} + \left(1-\frac{1}{2j}\right)e^{-j\omega_0 t} + \left(\frac{1}{2}\right)e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})}(1/2)$$

$$= 1 + \left(1+\frac{1}{2j}\right)e^{j\omega_0 t} + \left(1-\frac{1}{2j}\right) + \left(\frac{e^{j\pi/4}}{2}\right)\times e^{j2\omega_0 t} + \left(\frac{e^{-j\pi/4}}{2}\right)\times e^{-j2\omega_0 t}$$

$$= C_0 + C_1 e^{j\omega_0 t} + C_{(-1)}\cdot e^{-j\omega_0 t} + C_2 e^{j2\omega_0 t} + C_{(-2)}e^{-j2\omega_0 t}$$

$C_0 = 1$ $\qquad\qquad C_2 = \frac{e^{j\pi/4}}{2} = \frac{1+j}{2\sqrt{2}}$

$C_1 = \left(1+\frac{1}{2j}\right)$ $\qquad C_{(-2)} = \frac{e^{-j\pi/4}}{2} = \frac{1-j}{2\sqrt{2}}$

$C_{-1} = \left(1-\frac{1}{2j}\right)$

Que.→ Consider a periodic sig. $x(t)$ with '$T_0 = 8$' & FS coefficients.

$$c_1 = c_{-1} = 2$$
$$c_3 = 4j$$
$$c_{-3} = -4j$$

Find $x(t)$

Solⁿ→

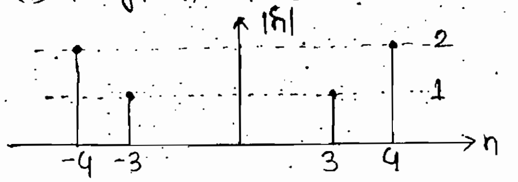
$$x(t) = c_1 e^{j\omega_0 t} + c_{-1} e^{-j\omega_0 t} + c_3 e^{j3\omega_0 t} + c_{-3} e^{-j3\omega_0 t}$$

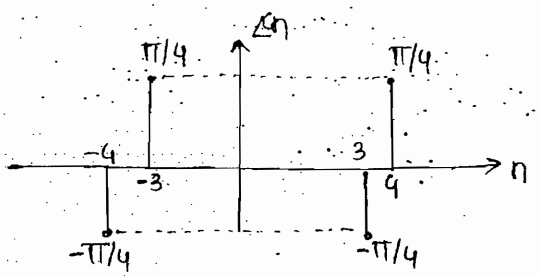
$$= 2\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] + 4j\left[e^{j3\omega_0 t} - e^{-j3\omega_0 t}\right]$$

$$= 2 \times 2\cos\omega_0 t + 4j \times 2j\sin 3\omega_0 t$$

$$= 4\cos\omega_0 t - 8\sin 3\omega_0 t \qquad \left(\omega_0 = \frac{\pi}{4}\right)$$

Que.→ C_n for sig. $x(t)$ is given below.
$(\because \omega_0 = \pi)$





Find $x(t)$ (d) $4\cos\left(4\pi t + \frac{\pi}{4}\right) + 2\cos\left(3\pi t - \frac{\pi}{4}\right)$

Solⁿ→

$$C_3 = |C_3| e^{j\angle C_3} = 1 \times e^{-j\pi/4}$$

$$C_{(-3)} = e^{j\pi/4}$$

$$C_4 = 2e^{j\pi/4} \qquad C_{(-4)} = 2e^{-j\pi/4}$$

$$x(t) = C_3 e^{j3\omega_0 t} + C_{(-3)} e^{-j3\omega_0 t} + C_4 e^{j4\omega_0 t} + C_{-4} e^{-j4\omega_0 t}$$

$$= e^{-j\pi/4} e^{3j\omega_0 t} + e^{j\pi/4} e^{(-j3\omega_0 t)} + 2e^{j\pi/4} e^{j4\omega_0 t} + 2e^{-j\pi/4} e^{-j4\omega_0 t}$$

$$= \cancel{e^{j3\pi/4t}} \cancel{+ e^{j(-\frac{3\pi}{4})t}} \cancel{+ 2e^{j5\pi/4t}} \cancel{+ 2e^{-j5\pi/4t}}$$

$$= \cancel{2\cos\left(\frac{3\pi}{4}\right)}\ 2\cos\left(\frac{3\pi t}{4} - \frac{\pi}{4}\right) + 4\cos\left(4\pi t + \frac{\pi}{4}\right)$$

$x(t) - C_n$ pairs →

	$x(t)$	C_n
(1.)		
(2.)	Real	CS
(3)	CS	Real
(4.)	Img.	CAS
(5.)	CAS	Img.
(6.)	Real + even	Real + Even
(7.)	Img. + Even	Img. + Even
(8.)	Read + Odd	Img. + odd
(9.)	Img + odd	Real + Odd

Que.→ $f(t) = \sum_{n=-\infty}^{\infty} \cos n\pi \, e^{jn\pi t}$ f(t) will be ?

Sol^n→ $C_n = \cos n\pi$

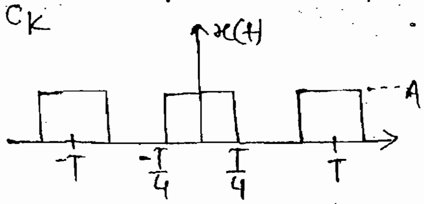
$= R + E$

Que.→ $\sum_{n=-\infty}^{\infty} j \sin \frac{n\pi}{2} e^{jn\pi t}$ f(t) will be?

Sol^n→ $C_n = I + O = j \sin \frac{n\pi}{2}$

$f(t) = R + O$.

Que.→ find C_K



(a.) $\frac{A}{j\pi k} \sin \frac{\pi}{2} k$ (b.) $\frac{A}{j\pi k} \cos\left(\frac{\pi}{2} k\right)$ (c.) $\frac{A}{\pi k} \sin\left(\frac{\pi}{2} k\right)$ (d.) $\frac{A}{\pi k} \cos\left(\frac{\pi}{2} k\right)$

Sol^n→ $x(t) = R + E$

$C_k = R + E$ (A)

$\frac{\sin\left(\frac{\pi}{2} k\right)}{(\pi k)}$ $\frac{O}{O} = E$ $\frac{\cos(\pi/2 k)}{(\pi k)} = \frac{E}{O} = O$

$f(k) = \pi k$

$f(-k) = -\pi k$

Ans.(d) (c)

<u>Que.→</u> $x(t) = c_n = \begin{cases} 2 & , n=0 \\ j\left(\frac{1}{2}\right)^{|n|} & , \text{otherwise} \end{cases}$

which of the following is true?

(a) $x(t)$ is a real sig. (c) $\frac{dx(t)}{dt}$ is an even sig.

(b) $x(t)$ is an even sig. (d) both (b) & (c)

<u>Solⁿ→</u>

$c_n \rightarrow n = -n$

so $c_n = c(-n)$ (Even signal)

& c_n = imag.

$c_n = E + I$

Ans. (b)

$\frac{dx(t)}{dt}$ is always odd. (derivative of even is odd)

<u>Relation between a_n, b_n & c_n→</u>

* $a_0 = c_0$
* $c_n = \frac{1}{2}[a_n - jb_n]$

→ valid for any type of signal $x(t)$

* $a_n = 2\,\text{Real}[c_n]$
* $b_n = 2\,\text{Img.}[c_n]$

→ valid only for real signal $x(t)$

<u>Que.→</u> FS expansion of real sig $f(t)$ is

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4+(3n\pi)^2} e^{jn\pi t}$$

Determine:-

(i) T_0

(ii) A term in that expansion is $A_0 \cos 6\pi t$, calculate the value of 'A_0'

(iii) Repeat (ii) for '$A_0 \sin 6\pi t$'.

<u>Solⁿ→</u>

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{3}{4+(3n\pi)^2} = R + E$$

$\omega_0 = \pi$

(i.) $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2$

(ii) $A_0 \cos 6\pi t = a_n \cos n\omega_0 t = a_n \cos n\pi t$

$n = 6$

$A_0 = a_6$

$a_n = 2\,\text{Real}[C_n] = 2C_n = \frac{6}{4+(3n\pi)^2}$

$A_0 = a_6 = \frac{6}{4+(3n\pi)^2}$

$a_6 = \frac{6}{4+(18\pi)^2}$

(iii) $A_0 \sin 6\pi t = b_n \sin n\omega_0 t = b_n \sin n\pi t$ $(n=6)$

$A_0 = b_6 = 0$

$b_n = 2\,\text{Img}[C_n] = 0$

*** Symmetrysity is FS →**

(1.) Even symm. →

$$x(t) = \underbrace{a_0}_{\text{Even}} + \sum_{1}^{\infty}[\underbrace{a_n \cos n\omega_0 t}_{\text{even}} + \underbrace{b_n \sin n\omega_0 t}_{\text{odd}}]$$

* FS expansion of an even signal does not contain sine terms.

(2.) Odd symm. →

* Odd symm. signal contains only any- sine terms in the FS expansion.

(3.) Half wave symm. →

$$\underbrace{x(t)}_{C_n} = -\underbrace{x\left(t + \frac{T_0}{2}\right)}_{-C_{(n)} = -C_n}$$

$\sout{C_K = C_{m}}$

$C_n = -C_n e^{\frac{jn\omega_0 T_0}{2}}$

$1 = -e^{jn\pi}$ $\left(\because \frac{\omega_0 T_0}{2} = \pi\right)$

$$\boxed{\begin{array}{l}\text{timeshifting} \\ x(t) = C_n \\ x(t-t_0) = C_n e^{-jn\omega_0 t_0}\end{array}}$$

$$1+e^{jn\pi}=0$$

$$1+(-1)^n=0$$

$$e^{jn\pi}=(e^{j\pi})^n$$
$$=(\cos\pi+i\sin\pi)^n$$
$$=(-1)^n$$

The above relation will be satiesfi only when
$n \to$ odd-integer
$\hookrightarrow n\omega_0$ } odd-integer.

∴ FS expansion of any HWS signal contains only odd harmonics.

(4.) Even + HWS →

* Contains only odd harmonics. } Because of HWS
* Avg/dc value is zero ($a_0=0$) }
* Does not contain sine terms. → Because of even.

Note → FS expansion of an even HWS signal contains odd harmonics of cos.

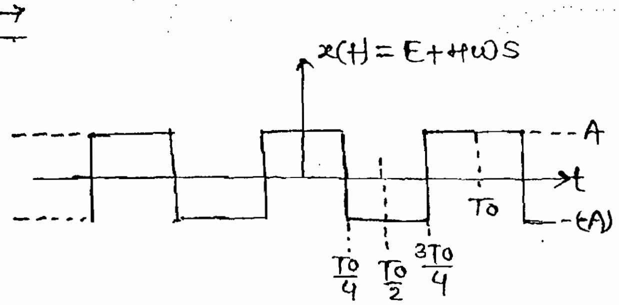
5.) Odd + HWS →

* Contains only sine terms.
* Contains only odd harmonics.

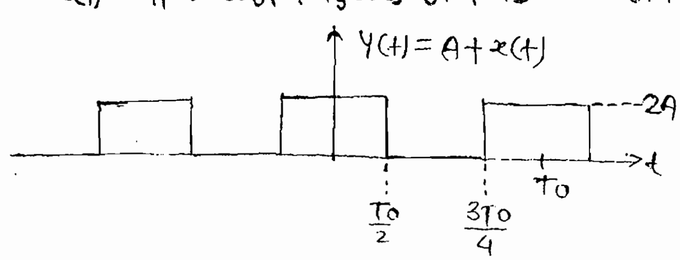
Note → FS expansion of an odd HWS signal contains sine terms without harmonics.

*

6.) Hidden Symm. →



$$x(t)=a_1\cos\omega_0 t+a_3\cos 3\omega_0 t+a_5\cos 5\omega_0 t+\cdots$$

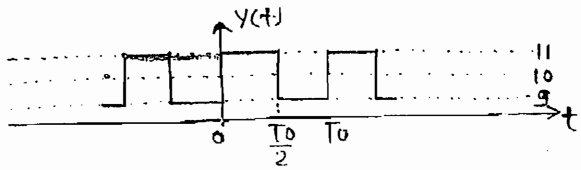


$$y(t) = A + x(t)$$

$$= A + a_1 \cos\omega_0 t + 3\cos 3\omega_0 t + \cdots\cdots$$

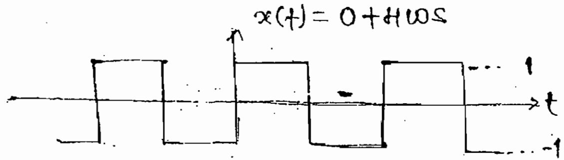
$$= dc + \text{odd harmonics of cos.}$$

Que.→



Sol^n→

$x(t) = 0 + HWS$

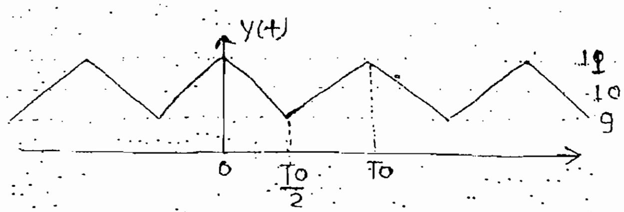


$$y(t) = 10 + x(t)$$

$$= 10 + b_1 \sin\omega_0 t + b_3 \sin 3\omega_0 t + \cdots\cdots$$

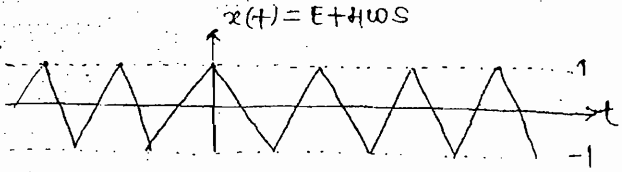
$$= dc + \text{odd harmonics of sine.}$$

Que.→



Sol^n→

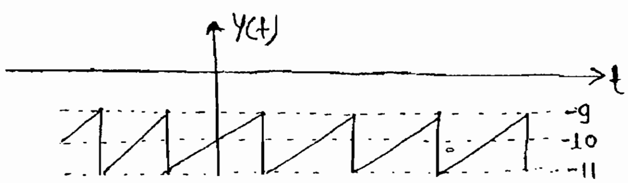
$x(t) = E + HWS$



$$y(t) = 10 + x(t)$$

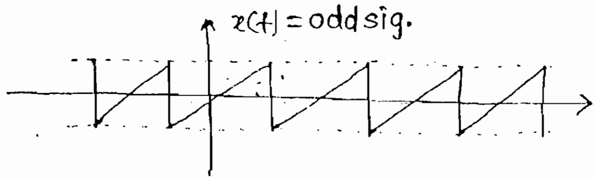
$$= dc + \text{odd harmonics of cos}$$

Que.→



Sol^n→

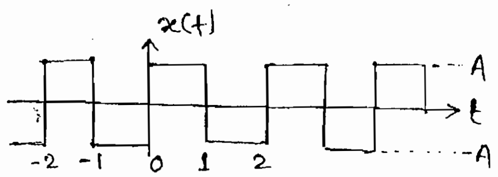
$x(t) = \text{odd sig.}$



HWS ≠ sawtooth

$$y(t) = -10 + x(t)$$

Que.→



(a.) $\frac{A}{n\pi}[1-(-1)^n]$ (c.) $\frac{A}{jn\pi}[1-(-1)^n]$

(b.) $\frac{A}{n\pi}[1+(-1)^n]$ (d.) $\frac{A}{jn\pi}[1+(-1)^n]$

$C_n = ?$

Soln→

$x(t) = R + O$

So; $c_n = I + O$

$x(t) = HWS$

$\downarrow$

Only odd harmonics are present.

$$c_n = \begin{cases} \neq 0, & n = \text{odd} \\ = 0, & n = \text{even} \end{cases}$$

from option (c.) $\frac{A}{jn\pi}[1-(-1)^n] \longrightarrow 0, n = \text{even}$
$\neq 0, n = \text{odd}$

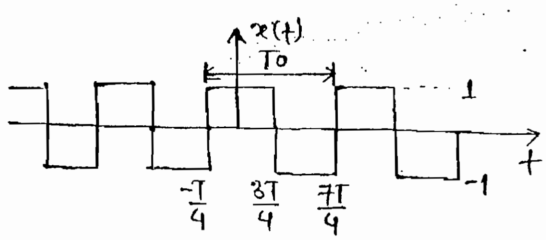
Que→ A sig. $x(t)$ is given by

$$x(t) = \begin{cases} 1, & -T/4 < t < 3T/4 \\ -1 & 3T/4 < t \le 7T/4 \\ {}^{*}-x(t+T) \end{cases}$$

which of the following gives the fundamental Fourier term of $x(t)$?

(a.) $\frac{\pi}{4}\cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$ (b.) $\frac{\pi}{4}\cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$ (c.) $\frac{4}{\pi}\sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$ (d.) $\frac{\pi}{4}\sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

Soln→



$T_0 = \frac{T}{4} + \frac{7T}{4} = 2T$

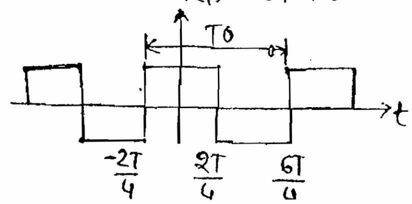
$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T}$

for HWS→

$$x(t) = -x\left(t + \frac{T_0}{2}\right) = {}^{*}-x(t+T)$$

* The above signal is NENO (Neither even nor odd)

$y(t) = E + HWS.$



$$y(t) = a_1 \cos\omega_0 t + a_3 \cos 3\omega_0 t + \cdots\cdots$$

$$x(t) = y\left(t - \frac{T}{4}\right)$$

$$= a_1 \cos\omega_0\left(t - \frac{T}{4}\right) + a_3 \cos 3\omega_0\left(t - \frac{T}{4}\right) + \cdots\cdots$$

$$= a_1 \cos\left[\frac{\pi}{T}\left(t - \frac{T}{4}\right)\right]$$

$$\boxed{x(t) = a_1 \cos\left[\frac{\pi t}{T} - \frac{\pi}{4}\right]}$$ Ans.

Fundamental Fourier term

$$= a_1 \cos\omega_0 t + b_1 \sin\omega_0 t$$

$$\& = c_1 e^{j\omega_0 t} + c_{-1} e^{-j\omega_0 t}$$

Note → Polarity of periodic signal at any time instant is decided by polarity of its fundamental Fourier term ~~which~~ because this term is dominant as compare to all other terms in the expansion of periodic signal (This rule is applicable for those periodic signals in which ∞ no. of harmonics are present).

from the above options :-

(a.) $\frac{\pi}{4} \cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right) \xrightarrow{t=0}$ +ve

~~(b.)~~ $\frac{4}{\pi} \sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right) \xrightarrow{t=0}$ −ve

* Properties of FS →

(1.) Linearity →

$$a_1 x_1(t) + a_2 x_2(t) \rightleftharpoons a_1 c_{1n} + a_2 c_{2n}$$

where; $x_1(t) = c_{1n}$

$x_2(t) = c_{2n}$

(2.) Time-reverse →

$$x(-t) \rightleftharpoons c_{(-n)}$$

(3.) Congugation →

$$x^*(t) \rightleftharpoons c^*_{-n}$$

(4.) Time-shifting →

$$x(t-T_0) \rightleftharpoons C_n e^{-jn\omega_0 t_0}$$

(5.) Freq. shifting →

$$e^{+jm\omega_0 t} x(t) \rightleftharpoons C_{n-m}$$

(6.) Convolution in time →

$$x_1(t) * x_2(t) \rightleftharpoons T_0[C_{1n} * C_{2n}]$$

(7.) multiplication in time →

$$x_1(t) \cdot x_2(t) \rightleftharpoons [C_{1n} * C_{2n}]$$

(8.) Diffenetiation →

$$\frac{d^m x(t)}{dt^m} \rightleftharpoons (jn\omega_0)^m C_n$$

(9.) Integration in time →

$$\int_{-\infty}^{t} x(t)dt \rightleftharpoons \frac{C_n}{jn\omega_0}$$

(10.) Passeval's power theorem →

$$P = \sum_{n=-\infty}^{\infty} |C_n|^2$$

Que → find c_n' in term of c_n

where; $y(t) \rightleftharpoons c_n'$

$x(t) \rightleftharpoons c_n$

(i) $y(t) = e^{-j2\omega_0 t} \cdot x(t)$ (ii) $y(t) = x(t-t_0) + x(t+t_0)$ (iii) $y(t) = \frac{d^2x(t)}{dt^2}$

(iv) $y(t) = \text{even}[x(t)]$

(v) $y(t) = \text{Real}[x(t)]$

Sol^n → (i) $c_n' = c_{n+2}$

(ii) $c_n' = c_n e^{-jn\omega_0 t_0} + c_n e^{jn\omega_0 t_0}$

$= c_n[e^{jn\omega_0 t_0} + e^{-jn\omega_0 t_0}]$

$c_n' = 2c_n \cos(n\omega_0 t_0)$

(iii) $c_n' = (jn\omega_0)^2 c_n = -n^2\omega_0^2 c_n$

(iv) $y(t) = \dfrac{x(t) + x(-t)}{2}$ (time reversal)

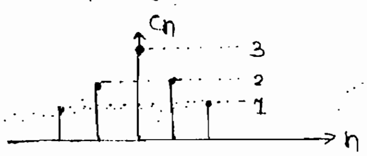
$c_n' = \dfrac{c_n + c_{-n}}{2}$

(v) $y(t) = \text{Real}[x(t)]$

$= \dfrac{x(t) + x^*(t)}{2}$ (conjugation)

$c_n' = \dfrac{c_n + c_{-n}^*}{2}$

Que.→ Calculate power of signal $x(t)$



Soln→ $p = \sum_{n=-\infty}^{\infty} |c_n|^2$

$= 1^2 + 2^2 + 3^2 + 2^2 + 1^2$

$= 19$

Que.→ Let $x(t)$ be the periodic signal with 'T_0' &

$y(t) = x(t-t_0) + x(t+t_0)$

$\left.\begin{array}{l} y(t) = b_k \\ x(t) = c_k \end{array}\right\}$ exp. FS coefficients.

If $b_k = 0$ for odd integer 'k' then 't_0' can be equal to

(a.) $\frac{T}{8}$ (b) $\frac{T}{4}$ (c) $\frac{T}{2}$ (d) $2T$.

Soln→

$y(t) = x(t-t_0) + x(t+t_0)$

$b_k = 2c_k \cos k\omega_0 t_0$ -------(i)

Given that;

$b_k = 0$; k = an odd-integer

$= 2m+1$

$m = \text{int}$

$b_{2m+1} = 0$

$$\Rightarrow 2C_{2m+1} \cdot \cos\left[(2m+1)\omega_0 t_0\right] = 0 \text{ ------ from (i)}$$

$$\cos\left[(2m+1)\omega_0 t_0\right] = 0$$

$$\cancel{\cos}(2m+1)\omega_0 t_0 = \cancel{\cos}(2m+1)\frac{\pi}{2}$$

$$\frac{2\pi}{\cancel{T}} t_0 = \frac{\pi}{2}$$

$$\boxed{t_0 = \frac{\pi}{4}}$$

*** FS for LTI system →**

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{-jn\omega_0 t} \longrightarrow \boxed{\begin{array}{c} \text{LTI} \\ \text{system} \end{array}}_{H(\omega)} \longrightarrow y(t) = \sum c_n' e^{jn\omega_0 t}$$

$$\boxed{c_n' = c_n H(n\omega_0)}$$

Que.→ Consider a contineous time LTI sys. whose i/p $x(t)$ & o/p $y(t)$ are related by the following DE.

$$\frac{dy(t)}{dt} + 4y(t) = x(t)$$

find c_n' for o/p $y(t)$ if i/p $x(t) = \cos\omega_0 t$, $\omega_0 = 2\pi$

Soln→

$$sY(s) + 4Y(s) = X(s)$$

$$Y(s)[s+4] = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{(s+4)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+4}$$

$$\& \ H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{j\omega+4}$$

$$\because x(t) = \cos\omega_0 t$$

$$= \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$$

$C_1 = 1/2$ & $C_{-1} = 1/2$

$C_n' = H(n\omega_0) C_n$

$n = 1,\ C_1' = H(\omega_0) C_1$

$= \frac{1}{4+j\omega_0} \times \left(\frac{1}{2}\right)$

$= \frac{1}{4+j\times 2\pi} \times \left(\frac{1}{2}\right)$

$n = -1,\ C_{-1}' = H(-\omega_0) C_{-1}$

$= \frac{1}{4-j2\pi} \times \left(\frac{1}{2}\right)$

$C_1' = \frac{1}{4+j2\pi}\left(\frac{1}{2}\right) \quad C_{-1}' = \frac{1}{4-j2\pi}\left(\frac{1}{2}\right)$

Que.→ Suppose we have given following information about a sig. $x(t)$

(1.) $x(t)$ is real & odd.

(2.) $x(t)$ is periodic with $T_0 = 2$

(3.) fourier coefficients

$C_n = 0, \ |n| > 1$

(4.) $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1$

The sig. that satiesfy this condn is.

(a.) $\sqrt{2}\sin\pi t$ & unique

(b.) $\sqrt{2}\sin\pi t$ & but not unique

(c.) $2\sin\pi t$ & unique

(d.) $2\sin\pi t$ & but not unique.

Soln→

$T_0 = 2, \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi$

– The avg. of odd signal is 0.

$C_n = 0,\ |n| > 1$

C_1 & $C_{-1} \neq 0$

$C_0 = 0 \ \leftarrow x(t)$ is odd.

$x(t) = A_0 \sin\omega_0 t = A_0 \sin\pi t$

$\frac{A_0^2}{2} = 1, \quad A_0 = \pm\sqrt{2}$ (Because of power signal)

Ans. (B).

Chapter-05
Fourier Transform

* FT is a mathematical tool for freq. analysis of sig. where as LT is a convenient mathematical tool for ckt analysis.
* FT exists for energy & power signals where as LT also exists for NENP signals. (upto certain extent only)
* In the category of NENP signal unit impulse is the only f^n for which FT also exists.

$$u(t) \xrightarrow{LT} \frac{1}{s} \xrightarrow{s=j\omega\,(FT)} \frac{1}{j\omega} + \pi\delta(\omega)$$

$$e^{2t}u(t) \xrightarrow{LT} \frac{1}{s-2} \quad \text{(FT does not exists)}$$

* The replacement $(s=j\omega)$ is used for Laplace to fourier conversion only for absolutely integrable signal.
* Impulse f^n & energy signals are absolutely integrable signals.

<u>fourier Xform →</u>

$$x(t) \rightleftharpoons X(\omega) \quad (\omega \rightarrow \text{rad/sec})$$

$$\boxed{X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt}$$

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,e^{j\omega t}\,d\omega}$$

<u>Conditions for existance of FT:-</u> (Dirichet's cond^n)

(1.) sig. should have finite no. of maxima & minima <u>over finite</u> interval.

(2) sig. should have finite no. of discontinuties over finite interval.

(3.) sig. should be absolutely integrable

i.e. $\boxed{\int_{-\infty}^{\infty} |x(t)|\,dt < \infty}$ → Impulse sig. → Energy sig.

* Dirichelt's condⁿ are sufficient but not necessary.

Que.→ Cal. FT for sig. $x(t) = e^{-at}u(t), a>0$

Solⁿ→

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty}$$

$$* = \frac{e^{-(a+j\omega)\infty} - e^{0}}{-(a+j\omega)}$$

$$e^{-(a+j\omega)\infty} = e^{-a\infty} \cdot e^{-j\omega\infty}$$

$$= \cancel{e^{-a\infty}}^{\,0} \cdot e^{-j\infty} \text{ (Undefined)}$$

$$e^{-a\infty} = 0, \; a>0$$

$e^{-j\infty} = \cos\infty - j\sin\infty$
These cos & sin are not defined in the given Range.

$$* = \frac{0-1}{-(a+j\omega)}$$

$$\boxed{X(\omega) = \frac{1}{a+j\omega}}$$

* At $t = \pm\infty$, complex exponantials & sinusoidal f^n are undefined.