CEGE0023: Offshore and Coastal Engineering
Laboratory Report - Wave-flume experiment: wave generation and wave-data analysis

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Risk Assessment

Department: Civil, Environmental & Geomatic Engineering **Risk Assessment Form**

WORK/PROJECT TITLE: CEGE0023 Lab Visit.

LOCATION(S): Fluids Lab (B09)

DESCRIPTION OF WORK:

Observation of wave generation and measurement in a wave flume

PERSONS INVOLVED: CEGE0023 Students, PGTA (Konstantinos Chasapis)

HAZARD IDENTIFICATION (*state the hazards involved in the work*)

Moving parts of the wave generator, electrical equipment and cables, open water in flumes, laser devices, general fluids laboratory hazards

RISK ASSESSMENT (*make an assessment of the risks involved in the work and where possible state high, medium or low risk*)

A low risk of physical injury by moving parts, a low risk of falling, a low risk of electrical shock.

CONTROL MEASURES (*state the control measures that are in place for the protection of staff*)

Follow instructions of lab technicians and a PGTA demonstrating the experiment.
Do not open the electrical cupboards or any other electrical equipment.
Stay away from moving parts of the wave generators.
In the case of the fire alarm leave the lab and the building following designated fire escape routes.
Take care when walking around the lab, step over cables and other obstacles.
Don't approach areas marked as dangerous.
Do not touch or move any objects without permission of a lab technician or a PGTA

SUPERVISOR DECLARATION
I, the undersigned, have assessed the work, titled above, and declare that ~~there is no significant risk~~/the risks will be controlled by the methods stated on this form (*delete as applicable*) and that the work will be carried out in accordance with Departmental codes of practice.

Name: Eugeny Buldakov

Signed: Date: 28 Oct 2019

STUDENT DECLARATION
I, the undersigned, have assessed the work, titled above, and agree to work carefully and thoughtfully, to take the measures designed to protect my safety and that of others as described on this form, and to advise my supervisor of any additional risks or safety concerns that come to my attention.

Name: Alex (Nathalie Alexandra) Tcherdakoff

Signed: Date: 18 December 2020

1 Abstract

This report contains an analysis of the wave-data from recorded waves generated in a laboratory wave-flume experiment. The analysis covers both regular waves (with frequencies $f = 0.5$Hz, 1Hz, and 1.5Hz and respective nominal amplitudes $A = 4$cm, 3cm, and 2cm), as well as random waves (specifically, a 5 minute long sequence simulating a random sea state).

2 Experiment Description

This experiment is performed in a wave-flume, of which the dimensions are:

- 13m length (working section)
- 45cm width
- 40cm depth (water height at rest)

At either end of the wave-flume are piston-type wavemakers: the right-side wavemaker generates the waves, while the left-side wavemaker is to absorb the waves in order to eliminate reflection. However, the wavemaker is more efficient at absorbing long waves; in order to reduce overall reflections and absorb short waves, a horizontal flat metal perforated plate is placed just below water surface, in front of the wave absorber piston. While it is impossible to eliminate all reflections, the aim is to limit them as much as possible to limit experimental error. The wavermakers are connected to a control system, the input to which is the amplitude and frequency of generated regular waves, multiple of which can be superposed in order to create random waves or wave groupings. Wave generation is done using 2 pieces of software: the wave synthesizer designs specific experiments, which can be fed as input to the wave making program, which directly controls the wavemaker movement.
There are 5 wave probes placed throughout the flume in order to measure wave data: these must be calibrated in order to provide a 1 Volt/cm water level change signal to their connected wave monitor. If they are not calibrated properly, this can also lead to experimental error. The data logger then translates the analogue signal from the wave monitor to digital signal, and tranfers the data to the data acquisition software which can both save and inspect the data.

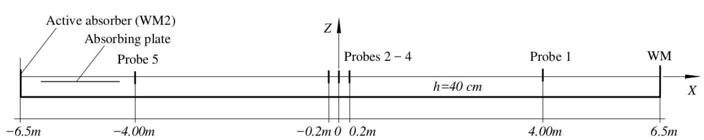

Figure 1: Wave flume layout and positions of wave probes [1]

3 Results

3.1 Regular wave record analysis

The measured wave data for probes 2, 3, and 4 can be represented using Python [2] (applies to all the graphs in this paper):

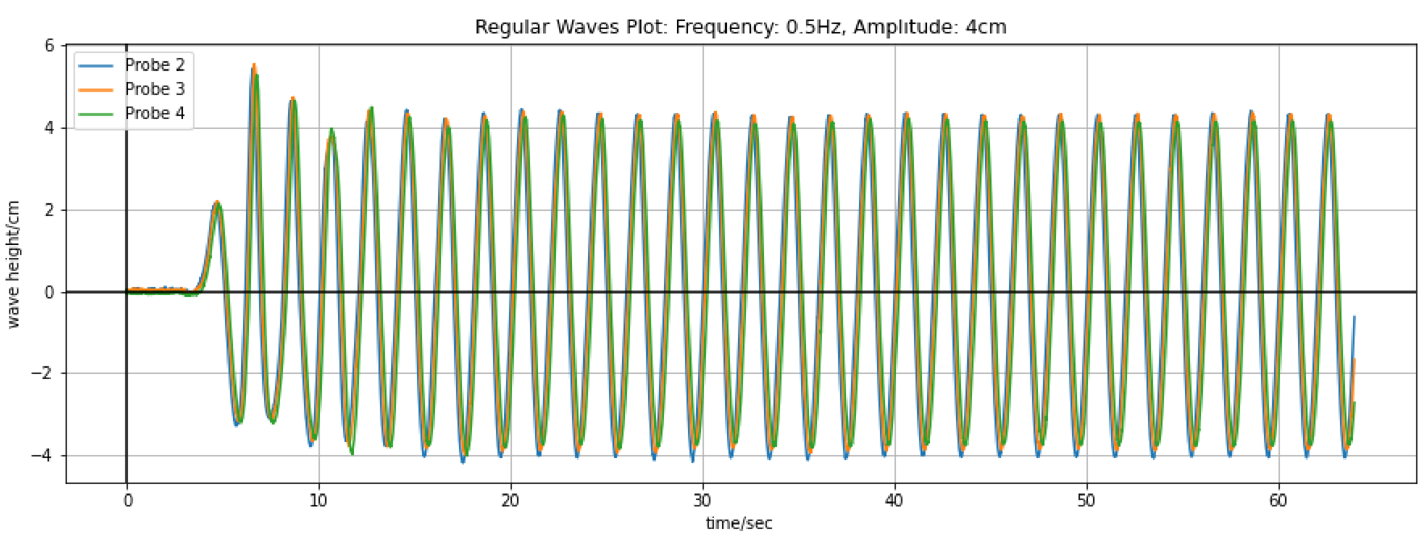

Figure 2: Regular Waves (Frequency = 0.5Hz, Amplitude = 4cm) Profile Representation measured from probes 2 through 4

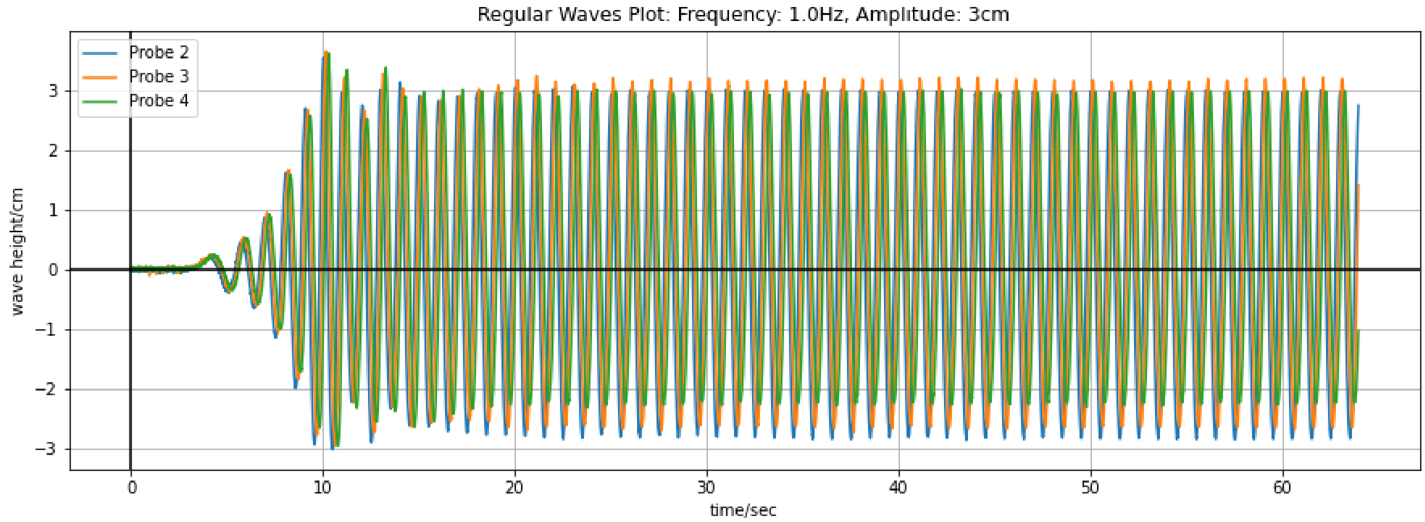

Figure 3: Regular Waves (Frequency = 1.0Hz, Amplitude = 3cm) Profile Representation measured from probes 2 through 4

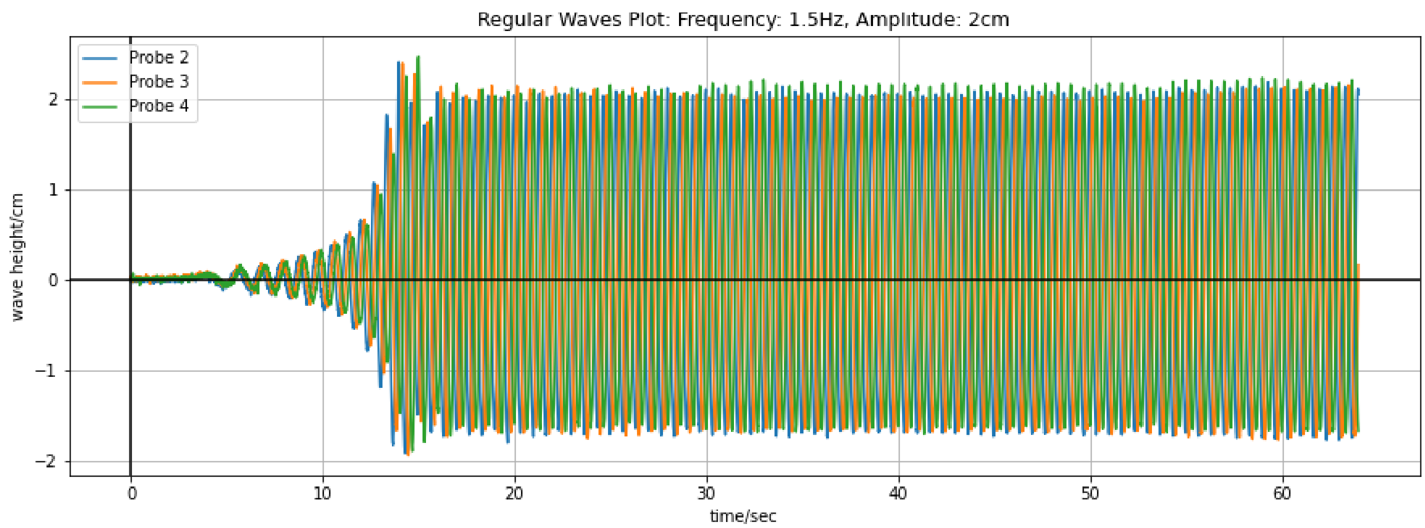

Figure 4: Regular Waves (Frequency = 1.5Hz, Amplitude = 2cm) Profile Representation measured from probes 2 through 4

The previous plots show an overview of the data, including the transitional period at the beginning. In order to analyse the wave record and compare the data, it is more appropriate to graph 1 period (not from the beginning) per record. The following periods were taken from approximately the middle of the records:

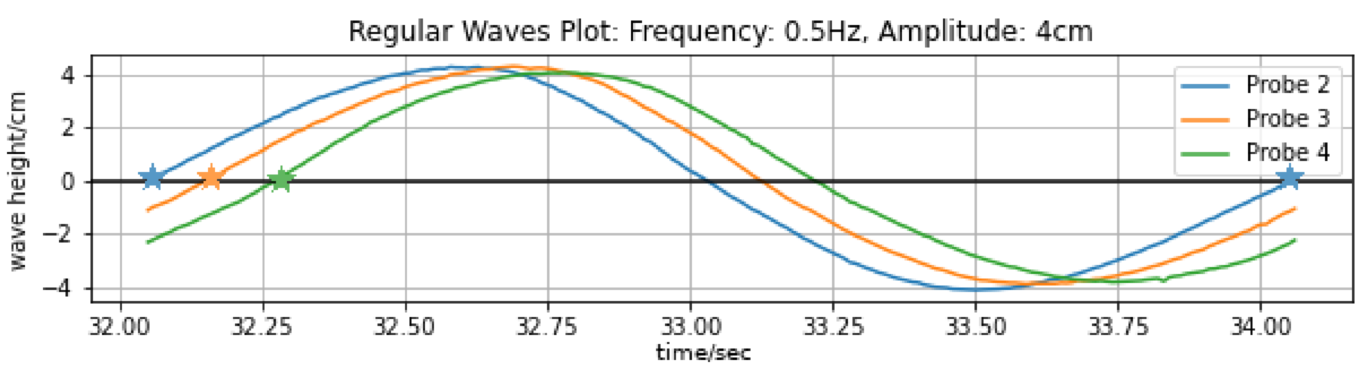

Figure 5: Regular Waves (Frequency = 0.5Hz, Amplitude = 4cm) Period Representation measured from probes 2 through 4

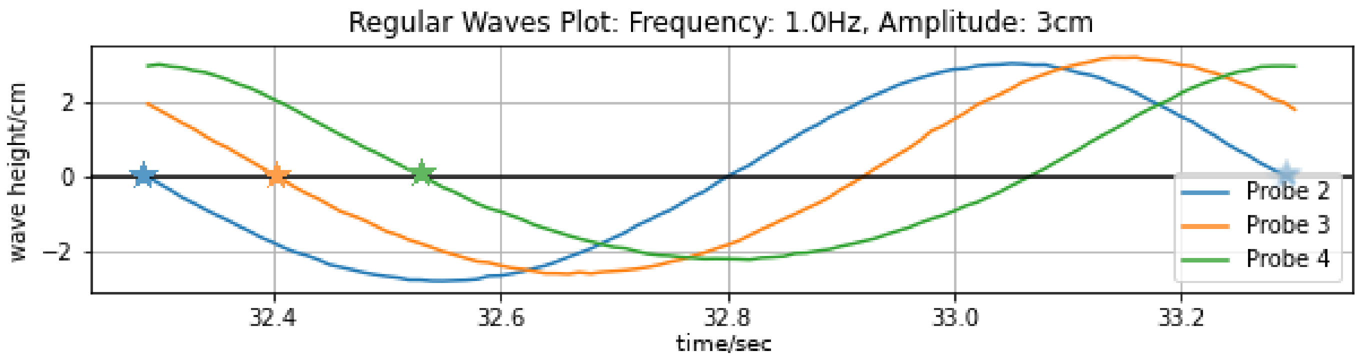

Figure 6: Regular Waves (Frequency = 1.0Hz, Amplitude = 3cm) Period Representation measured from probes 2 through 4

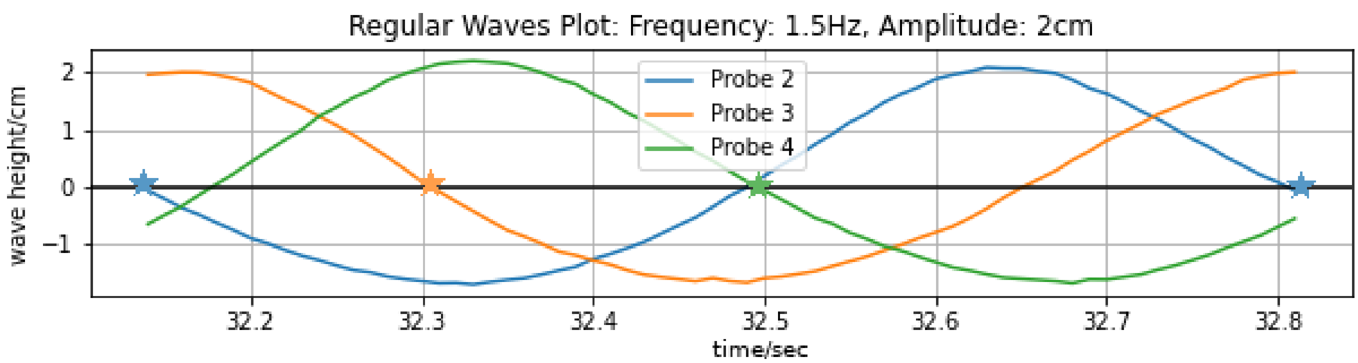

Figure 7: Regular Waves (Frequency = 1.5Hz, Amplitude = 2cm) Period Representation measured from probes 2 through 4

The following can be calcuated from the data, using the same Python program to measure times between different phases for different probes as was used to generate the previous 3 plots: (theoretical period : $T = 1/f$)

Given Data			**Measured Period (determined programmatically)**		
Wave Frequency	Amplitude	Theoretical Period	Probe 2	Probe 3	Probe 4
$f = 0.5Hz$	$A = 4cm$	$T = 1/0.5 = 2.00s$	$34.06 - 32.05 = 2.01s$	$34.16 - 32.16 = 2.00s$	$34.28 - 32.28 = 2.00s$
$f = 1.0Hz$	$A = 3cm$	$T = 1/1.0 = 1.00s$	$33.30 - 32.29 = 1.01s$	$33.41 - 32.41 = 1.00s$	$33.57 - 32.57 = 1.00s$
$f = 1.5Hz$	$A = 2cm$	$T = 1/1.5 = 0.67s$	$32.81 - 32.14 = 0.67$	$32.98 - 32.31 = 0.67s$	$33.15 - 32.48 = 0.67$

Table 1: Regular Wave Record Analysis: table of results

The difference between the theoretical and the measured periods is negligible: they are nearly identical. However, now the comparison can be made between the measured celerities: the theoretical celerity can be determined, as in theory the regular waves can be represented as sine waves where the surface elevation $\eta(x,t) = 2\pi A * sin(t/T - x/L)$ and the theoretical celerity $C = L/T$ (the theoretical celerity in Table 2 was determined using the provided calculator). The distance between probes 2 and 4 is 0.4m, this as well as the delay between can be used to calculate the measured celerity: $C_M = 0.4/(P_4 - P_2)$ m/s where P_2 and P_4 are the starting times of the measured periods from Table 1.

Wave Frequency	Amplitude	Theoretical Period	**Theoretical Celerity**	**Measured Celerity**
$f = 0.5Hz$	$A = 4cm$	$T = 2.00s$	$C = 1.84747746$ m/s	$C_M = 0.4/(34.28 - 34.06) = 1.81818$ m/s
$f = 1.0Hz$	$A = 3cm$	$T = 1.00s$	$C = 1.46373472$ m/s	$C_M = 0.4/(33.57 - 33.30) = 1.481481$ m/s
$f = 1.5Hz$	$A = 2cm$	$T = 0.67s$	$C = 1.04448977$ m/s	$C_M = 0.4/(33.15 - 32.81) = 1.176471$ m/s

Table 2: Regular Wave Record Analysis: table of celerities

The measured celerities of the first 2 waves are identical to the theoretical celerity values to 10^{-1} degrees of precision. These differences can be attributed to the inevitable experimental errors (coming from refections and probe calibration for example). The 3rd wave's measured and theoretical celerities are more different, only identical to 1 degree of precision.

3.2 Wave length calculation

Using the measured and theoretical periods and celerities, the measured and theoretical lengths of each wave can be calculated as $L = C * T$ m; these measured and theoretical values can then be compared:

Wave Frequency	Amplitude	**Theoretical Wave Length** $L = C * T$	**Measured Wave Length**
$f = 0.5Hz$	$A = 4cm$	$L = 1.84747746 * 2.00 = 3.69495492$ m	$L_M = 1.81818 * 2.00 = 3.63636$ m
$f = 1.0Hz$	$A = 3cm$	$L = 1.46373472 * 1.00 = 1.46373472$ m	$L_M = 1.481481 * 1.00 = 1.481481$ m
$f = 1.5Hz$	$A = 2cm$	$L = 1.04448977 * 0.67 = 0.699808146$ m	$L_M = 1.176471 * 0.67 = 0.7882356$ m

Table 3: Regular Wave Record Analysis: table of lengths

As expected (given the results for the measured celerity), the theoretical and measured values of the first 2 waves are identical to 10^{-2} degrees of precision, and the 3rd to 1 degree. From these results, it can be determined for each wave if it corresponds to shallow-water, intermediate-water or deep-water regime: given the water depth (1) $d = 40$cm or $d = 0.40$ m, and that:

- shallow water: $d/L < 1/25$
- transitional water: $1/25 < d/L < 1/2$
- deep water: $d/L > 1/2$

Wave Frequency	Amplitude	d/L	**Depth**
$f = 0.5Hz$	$A = 4cm$	$0.40/3.69495492 = 0.1082557$	$0.04 < 0.1082557 < 0.5$: intermediate water
$f = 1.0Hz$	$A = 3cm$	$0.40/1.46373472 = 0.27327356$	$0.04 < 0.27327356 < 0.5$: intermediate water
$f = 1.5Hz$	$A = 2cm$	$0.40/0.699808146 = 0.57158522$	$0.57158522 > 0.5$: deep water

Table 4: Regular Wave Record Analysis: table of depths

The 3rd wave being in the deep-water regime is possibly the reason the measured and theoretical values of wave-length and celerity are more different than for the other 2 waves.

3.3 Celerity and wave length: theoretical relationship

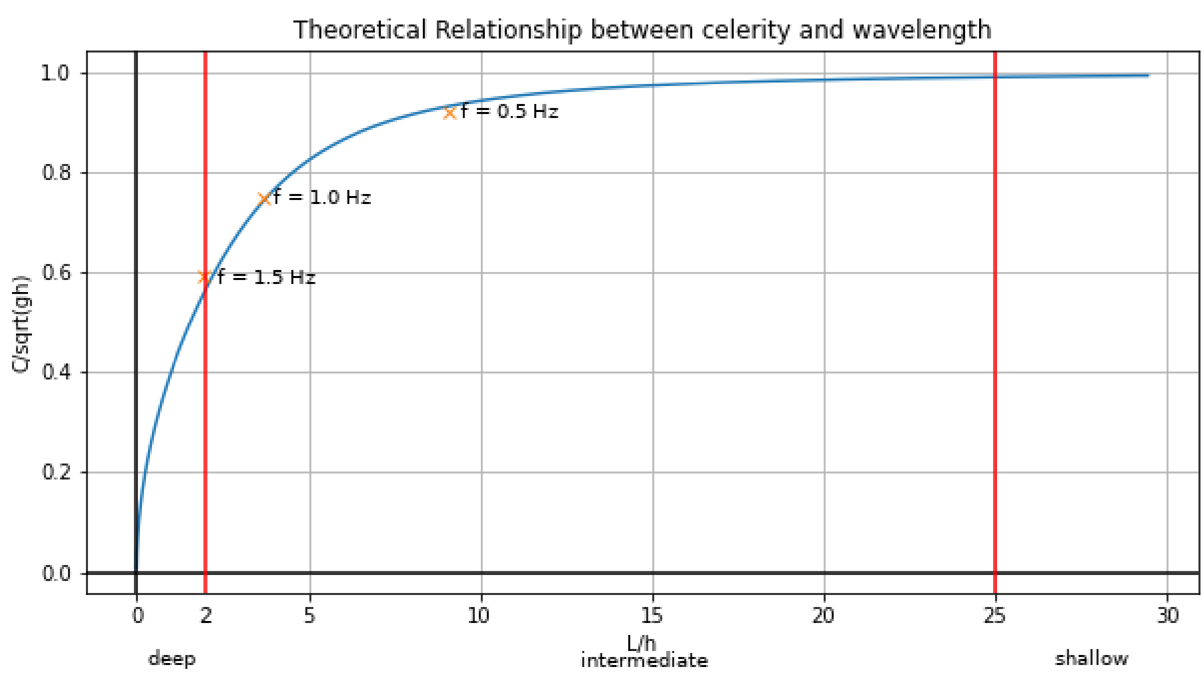

Figure 8: Theoretical relationship between celerity and wave length

3.4 Measured vs. theoretical waves

Using 1 wave period measured by probe 3 for each wave, the wave height can be calculated and compared to the nominal wave height:

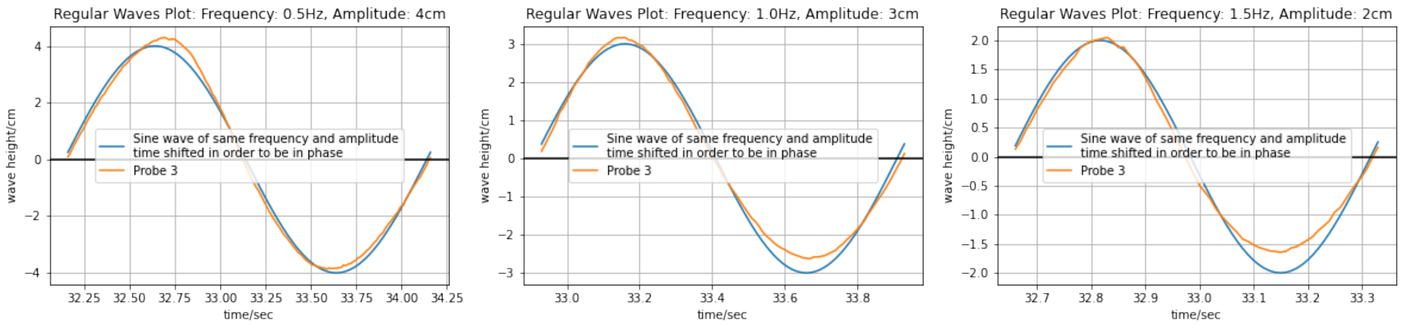

Figure 9: Regular Wave Periods measured by Probe 3 compared to sine wave (frequencies f = 0.5 Hz, 1.0 Hz, and 1.5 Hz respectively, from left to right)

For all of the experimental waves, probe 3 measures a higher peak and trough than the expected sinusoidal wave; this can be observed in the following plot with scaled values, and the wave periods overlaid:

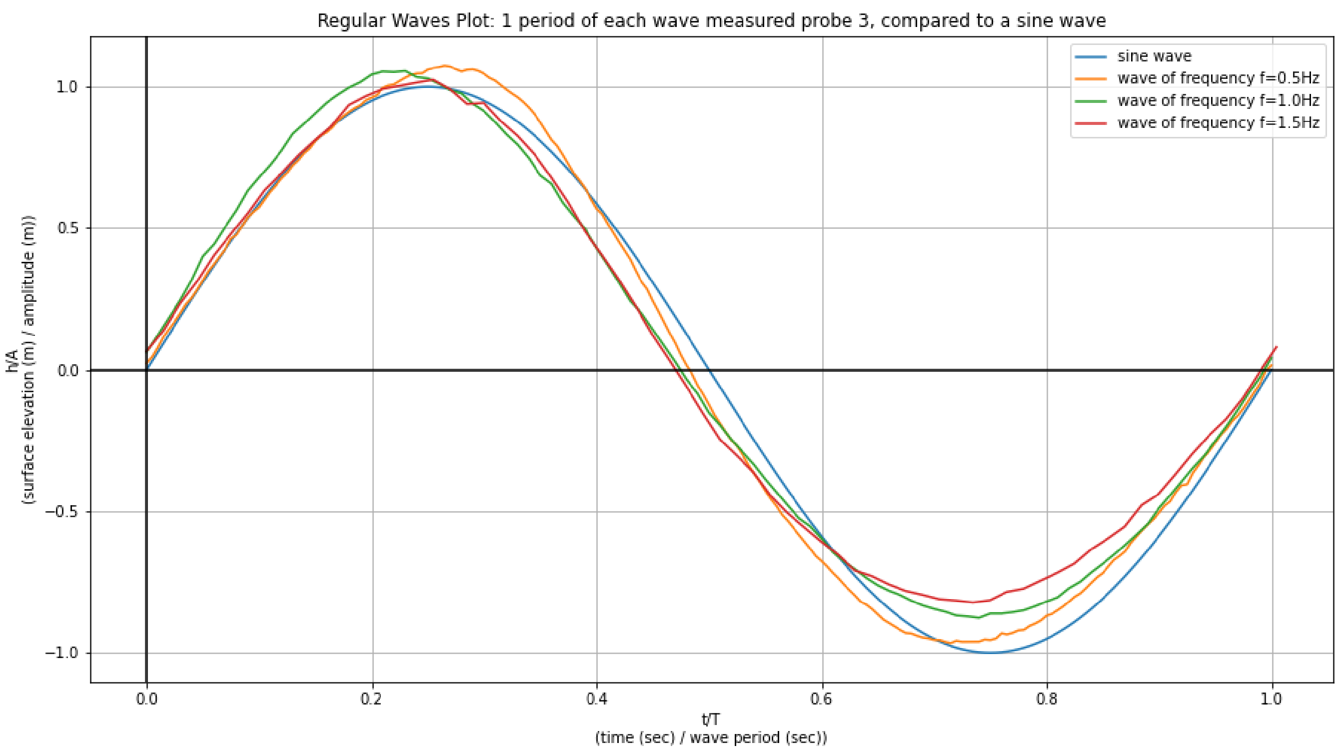

Figure 10: Single wave period measured by probe 3 for each experimental wave, scaled and compared to a sine wave

The higher peaks and shallower troughs as well as assymetry of the wave shapes in the experimental wave data measured by probe 3 can be explained by the unavoidable experimental errors (such as probe calibration errors, reflections due to wave absorption being imperfect), and a mismatch between the linear wave generation and the nonlinear nature of the experimental components.

3.5 Wave amplitude variations

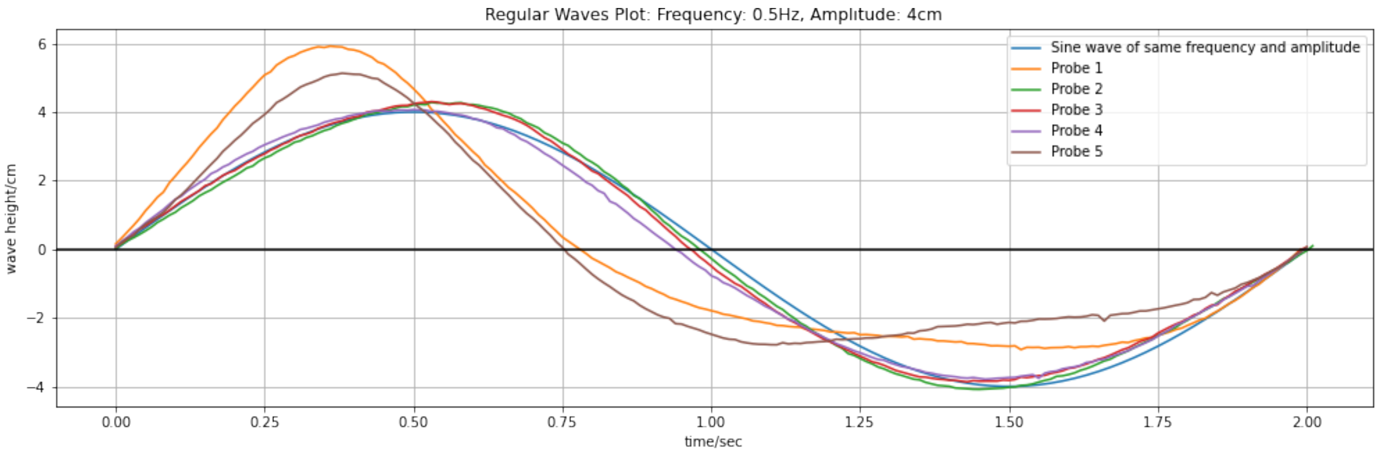

Figure 11: Regular Waves (Frequency = 0.5Hz, Amplitude = 4cm): time-shifted period-comparison of all probes and a sine wave

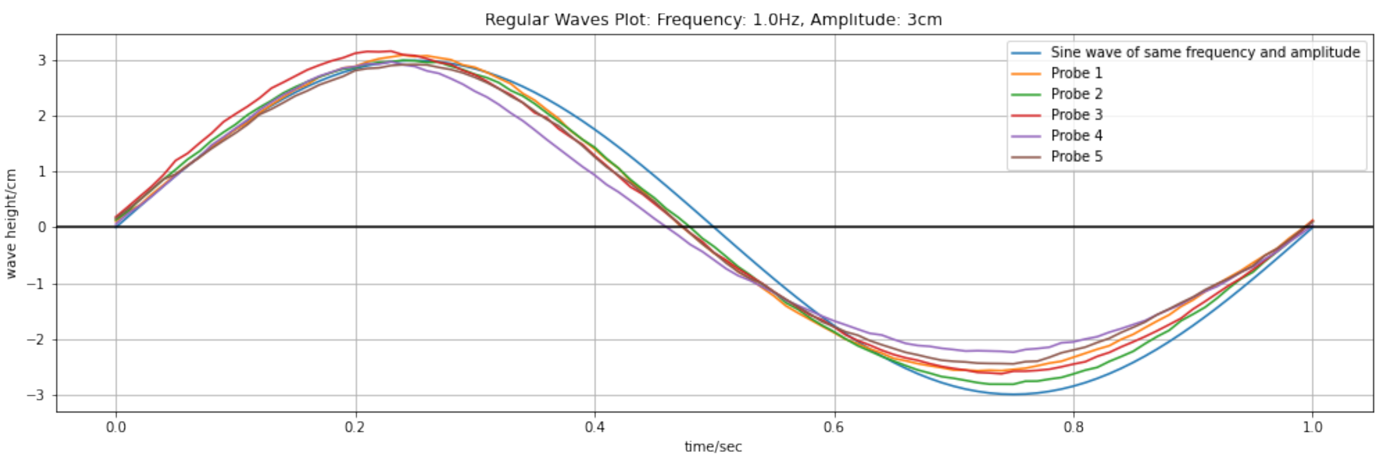

Figure 12: Regular Waves (Frequency = 1.0Hz, Amplitude = 3cm): time-shifted period-comparison of all probes and a sine wave

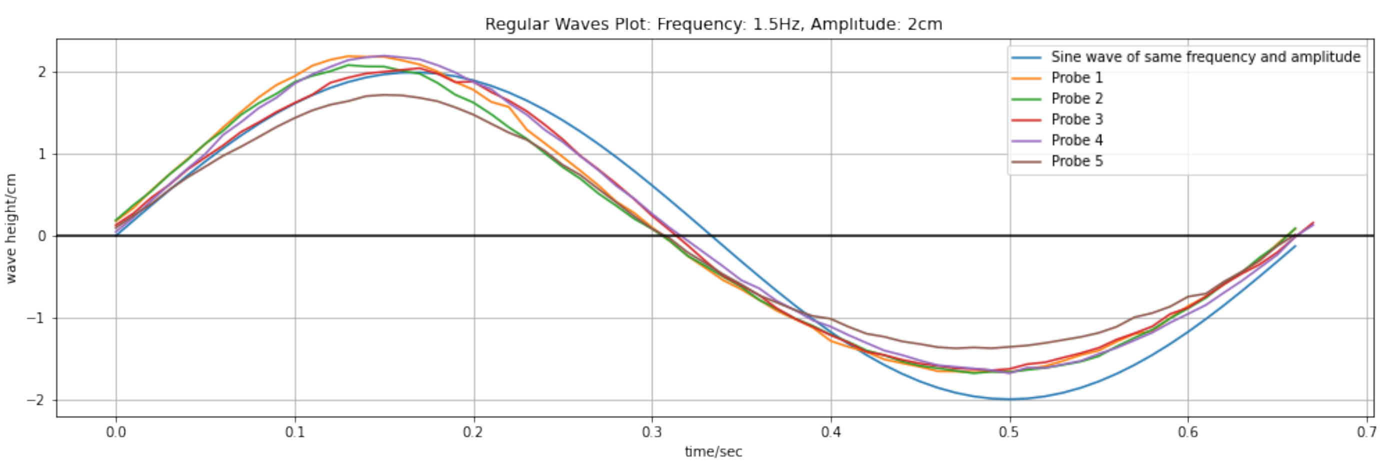

Figure 13: Regular Waves (Frequency = 1.5Hz, Amplitude = 2cm): time-shifted period-comparison of all probes and a sine wave

The previous Figures 11, 12, and 13 show analyses of developed periodic wave records for all probes compared to each other and to a regular sine wave of the same freqency and amplitude. These graphs show that the wave amplitudes are indeed **not** constant in space or time, as probes 1 and 5 for the 1st wave show higher peaks and shallower troughs than the rest of the probes as well as the sine wave, for example, and probe 5 for the 3rd waves shows overall lower peaks as well as shallower troughs. The inconsistencies with the first and last probes can be associated with the probes' proximity to the 2 wavemakers, and overall amplitude inconsistencies in the experiment can also be associated with experimental error (such as wave probe calibration imperfection, residual wave reflections, etc..) as well as, again, the mismatch between the linear wave generation, and the non-linear nature of the waves (assymetry in wave shape also leads to different wave amplitudes at different locations).

3.6 Wave front evolution

The wave front evolution can be analysed using probes 1 and 5's recordings for all the waves, specifically observing the beginning of the waves. The peaks and troughs of the values are encompassed by envelopes:

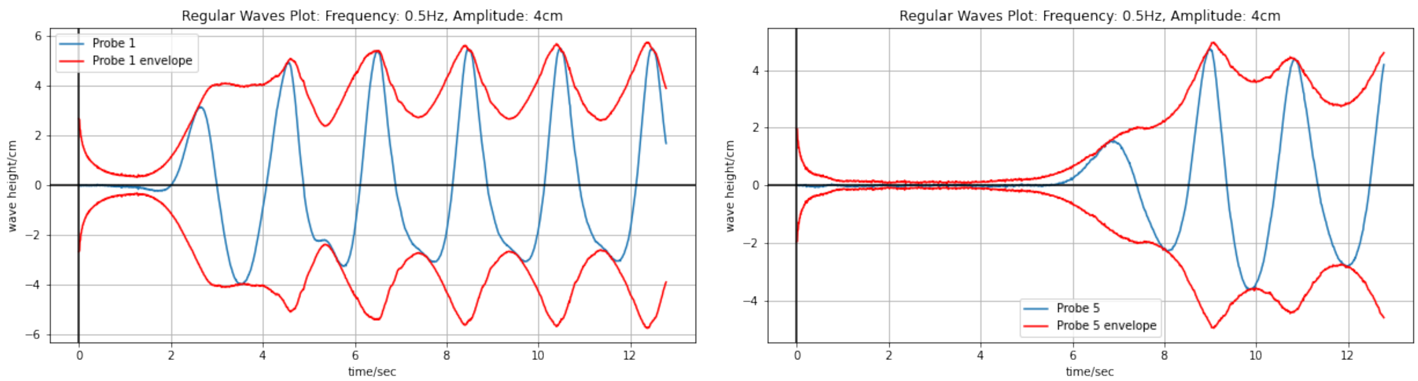

Figure 14: Wave Front Evolution for wave of frequency f = 0.5 Hz as measured by Probe 1 and 5 with wave envelopes

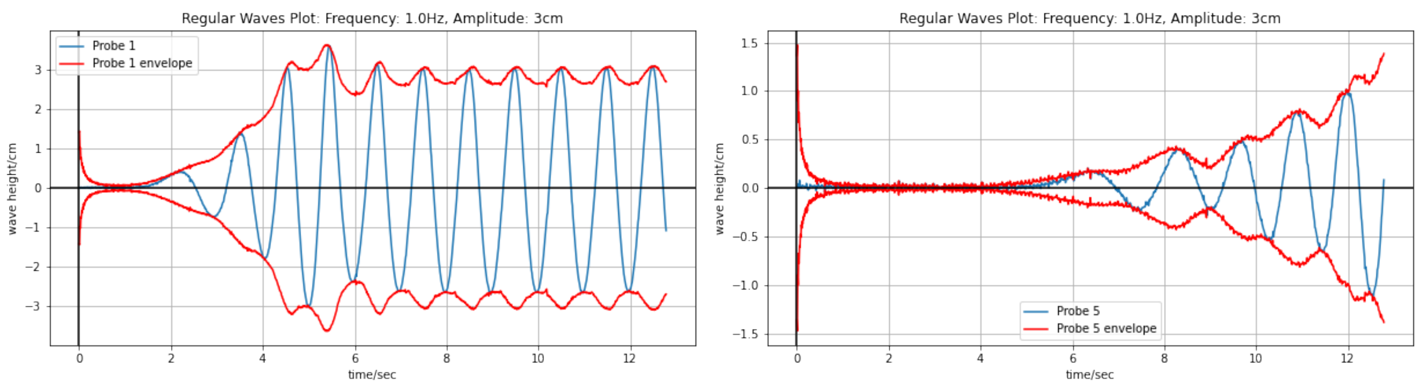

Figure 15: Wave Front Evolution for wave of frequency f = 1.0 Hz as measured by Probe 1 and 5 with wave envelopes

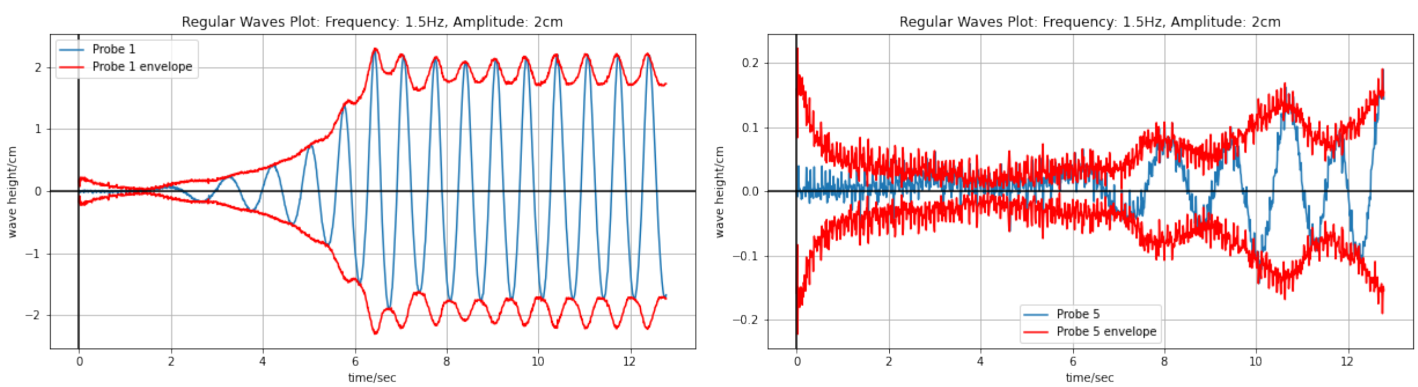

Figure 16: Wave Front Evolution for wave of frequency f = 1.5 Hz as measured by Probe 1 and 5 with wave envelopes

These envelopes can then be directly compared to each other (as seen in Figures 17, 18, and 19):

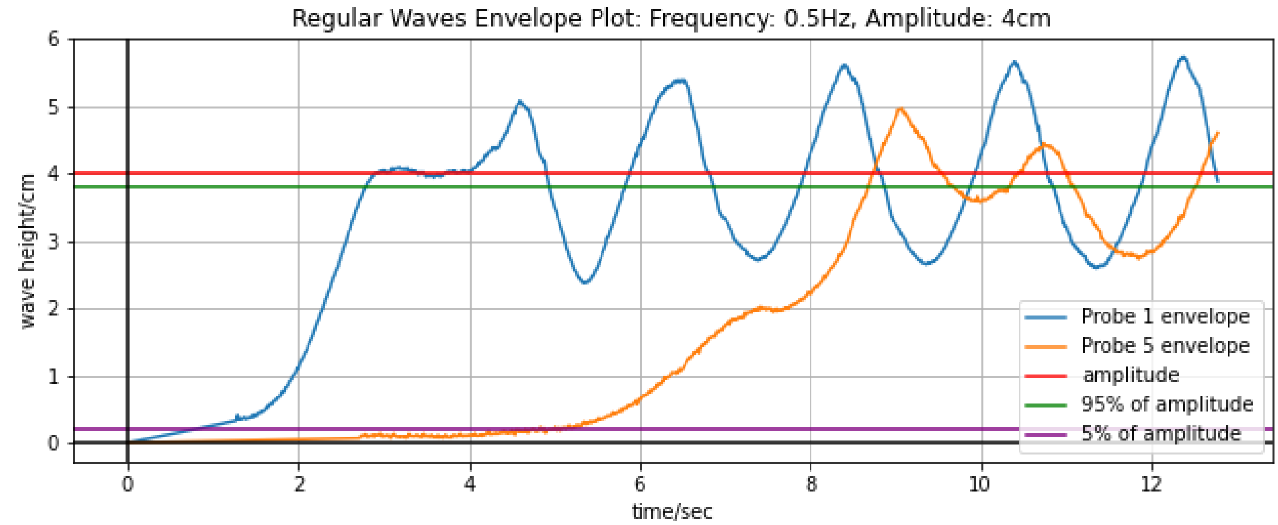

Figure 17: Regular Waves (Frequency = 0.5Hz, Amplitude = 4cm): Probes 1 and 5 envelopes comparison

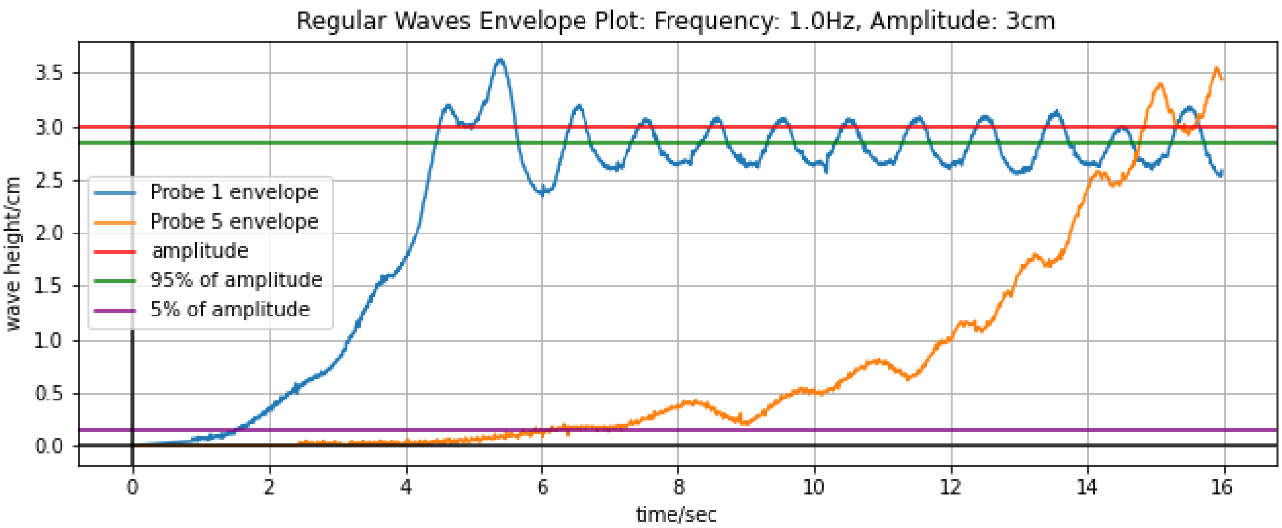

Figure 18: Regular Waves (Frequency = 1.0Hz, Amplitude = 3cm): Probes 1 and 5 envelopes comparison

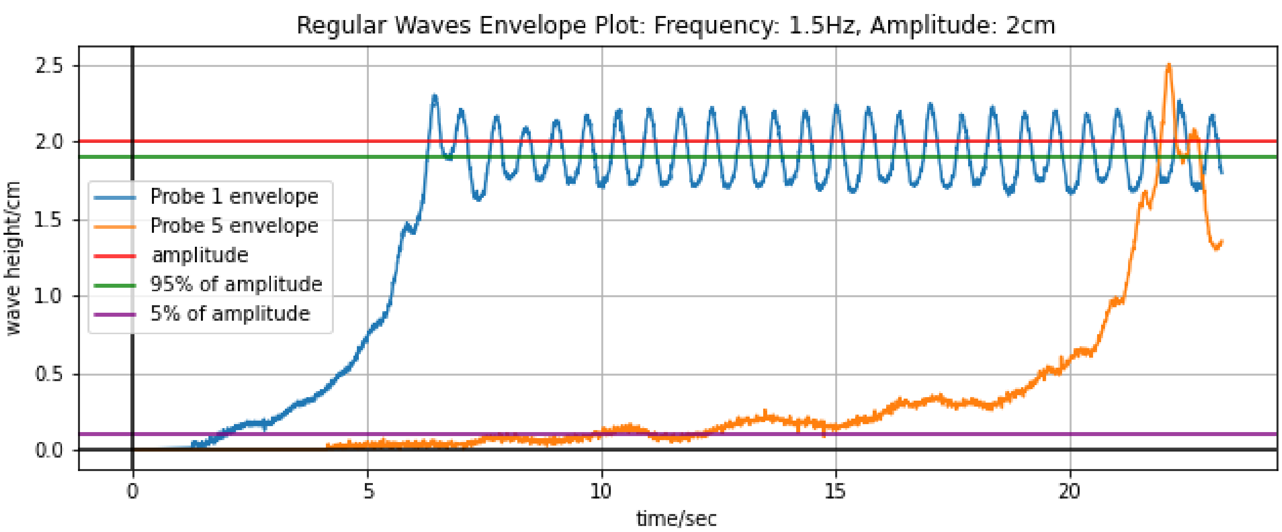

Figure 19: Regular Waves (Frequency = 1.5Hz, Amplitude = 2cm): Probes 1 and 5 envelopes comparison

For Figures 17, 18, and 19, the wave height at the location of each probe grew to a small value (at 5% of the height of a developed regular wave) is identified by the blue and orange curves crossing the purple line (the exact values determined

programatically [2]). There are 8m between probes and 5 1, so $C_M = 8/(P_5 - P_1)$ where P_5 and P_1 are the probe 5 and 1 crossings, respectively. The values calculated for theoretical celerity C in previous sections still hold. 5% of each of these waves' amplitudes qualifies as shallow water, therefore the group velocity C_g for all these waves is the same as shallow water celerity $C_s = \sqrt{gh} = \sqrt{9.80665 * 0.40} = 1.98057$ m/s for all waves.
The following values in Table 5 can therefore be derived in order to compare them:

Amplitude	P_1	P_5	**Measured Celerity**	Theoretical Celerity	Group Velocity = Shallow Water Celerity
$A = 4$ cm	0.78s	4.67s	$C_M = 2.05655527$ m/s	$C = 1.84747746$ m/s	
$A = 3$ cm	1.51s	5.91s	$C_M = 1.81818182$ m/s	$C = 1.4637472$ m/s	$C_g = C_s = 1.98057$ m/s
$A = 2$ cm	1.85s	7.83s	$C_M = 1.33779264$ m/s	$C = 1.04448977$ m/s	

Table 5: Table of celerity calculations: probes 1 and 5 first recording of 5% of the wave amplitude

For the first 2 waves, the measured celerities are closest to the group velocity (or the shallow-water celerity), whereas for the third wave, the measured celerity is nearest to the theoretical celerity (this could be due to experimental error, or partially to the fact that out of the 3 sets of waves, the 3rd one corresponds to a deeper-water regime).

3.7 Different speeds of energy propagation in the wave front

For Figures 17, 18, and 19, the wave height at the location of each probe grew to a large value (at 95% of the height of a developed regular wave) is identified by the blue and orange curves crossing the green line (the exact values determined programatically [2]). The values for measured celerity, theoretical celerity, and shallow water celerity from the previous section still hold. However, the waves here are not shallow, they are of the depth-regime previously specified: the first 2 waves qualified previously as transitional water and the 3rd wave as deep water. Therefore, the group velocity C_g for the first 2 waves is :

- $C_g = \dfrac{1 + \frac{4\pi h/L}{sinh(4\pi h/L)}}{2} * C$ for the first 2 waves (using the previously calculated theoretical wavelengths)
  - $C_g = \dfrac{1 + \frac{4\pi 0.40/3.69495492}{sinh(4\pi 0.40/3.69495492)}}{2} * C = 0.873617 * C = 1.61398772$ m/s for wave 1
  - $C_g = \dfrac{1 + \frac{4\pi 0.40/}{sinh(4\pi 0.40/1.46373472)}}{2} * C = 0.610884 * C = 0.894179745$ m/s for wave 2
- $C_g = \frac{C}{2} = 0.522244885$ m/s for the 3rd wave

The following values in Table 6 can therefore be derived:

Amplitude	P_1	P_5	**Measured Celerity**	Theoretical Celerity	Group Velocity	Shallow Water Celerity
$A = 4$ cm	2.8s	8.68s	$C_M = 1.36054422$ m/s	$C = 1.84747746$ m/s	$C_g = 1.61398772$ m/s	
$A = 3$ cm	4.46s	14.78s	$C_M = 0.775193799$ m/s	$C = 1.4637472$ m/s	$C_g = 0.894179745$ m/s	$C_s = 1.98057$ m/s
$A = 2$ cm	6.3s	21.92s	$C_M = 0.512163893$ m/s	$C = 1.04448977$ m/s	$C_g = 0.522244885$ m/s	

Table 6: Table of celerity calculations: probes 1 and 5 first recording of 95% of the wave amplitude

Here, all the measured celerities are nearest in value to the group velocity.
Different speeds of energy propagation in the wave front are observed as the more reliable of all the theoretical celerities was shown here to be the one that fluctuates depending on the depth regime of the water observed: the group velocity. Reasons for this include probes 1 and 5's proximities to the wavemakers performing opposing actions (and the absorbing plate), as well as the mismatch between the nonlinear nature of the waves and the supposedly linear wave generation, and finally, experimental errors. Certain fluctuations are also more likely to have inaccuracies, as the absorbing mechanism is more effective at reducing the longer waves: shorter waves are therefore more likely to be counteracted by non-absorbed reflections, possibly explaining the 3rd wave's measured celerity in the previous section.

3.8 Random wave record analysis

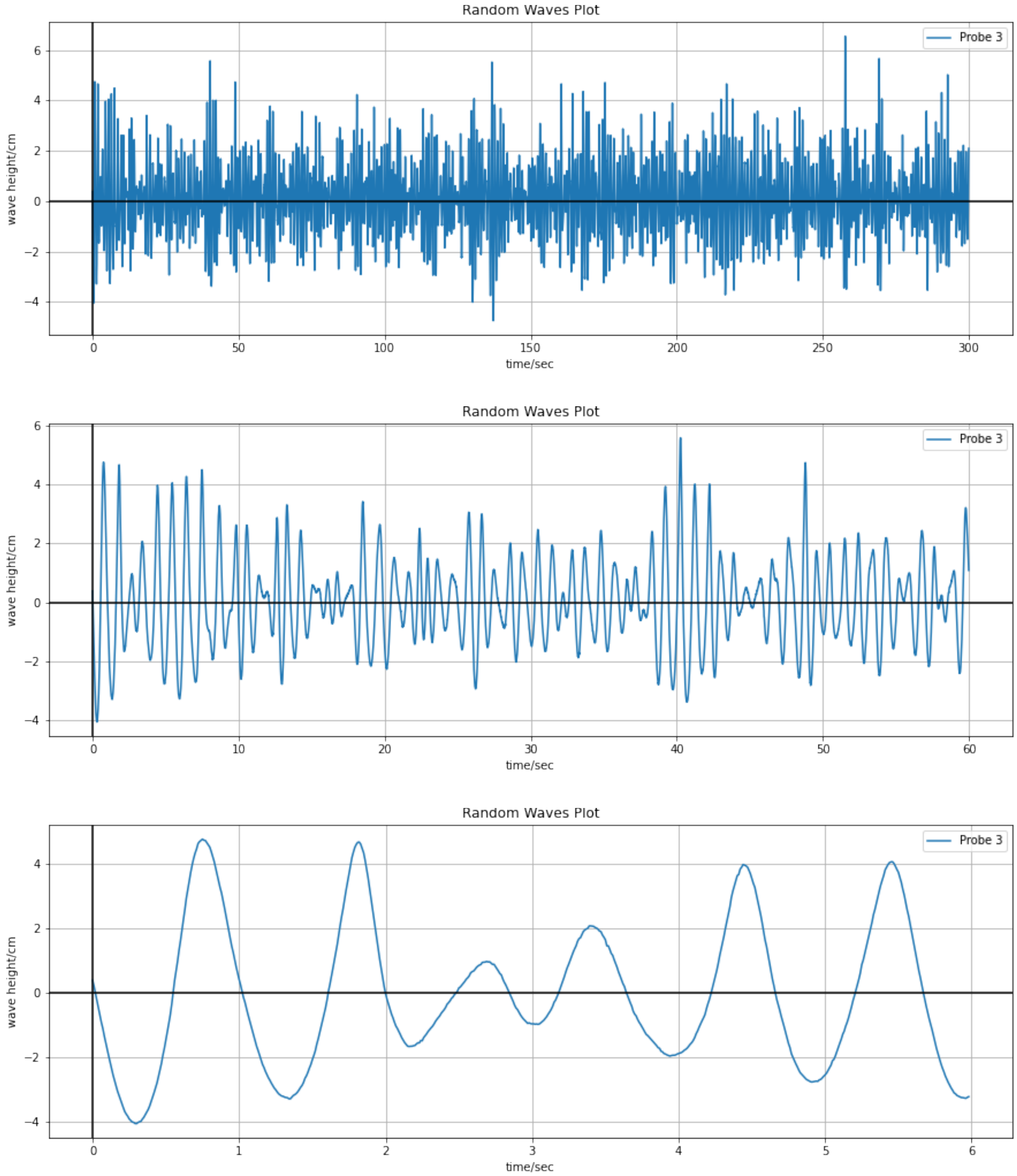

Figure 20: Random Sea State: randomly generated waves - analysis at different scales

Each period and height in the random wave sequence represented in Figure 20 was collected programatically, from which the root mean square wave height H_{RMS}, the significant wave height $H_s = H_{1/3}$ and the mean zero-crossing period $T_z = T_0$ have been computed using the following formulae where N is the number of periods in the record, total:

- $H_{RMS} = (\frac{1}{2}\sum_{i=1}^{N} H_i^2)^{1/2}$ = **4.199038328054618 cm**

- $H_{1/3} = \frac{1}{N/3}\sum_{j=1} N/3 H_j = 3.735569331325301$

  or if Rayleigh distribution is assumed: $H_s = \sqrt{2} * H_{RMS}$ =**5.938336952459287 cm**

- $T_0 = \frac{1}{N}\sum_{i=1}^{N} T_{0,i}$ = **0.8984638554216868 s**

3.9 Maximum wave height for 3 hour storm at project site

Using the Froude similarity law with the following formulae, the random wave experimental data can be scaled to full-scale conditions on a real-world site:

- Froude Number: $Fr = \frac{C}{\sqrt{gh}}$
- $L = C * T$
- $Fr_1 = Fr_2 \rightarrow \frac{C_1}{C_2} = \sqrt{\frac{h_1}{h_2}} \rightarrow \frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}$
- $C_2 = \frac{C_1}{\sqrt{\frac{h_1}{h_2}}}$
- $\frac{h_1}{L_1} = \frac{h_2}{L_2} \rightarrow L_2 = \frac{L_1 h_2}{h_1} \rightarrow T_2 = \frac{L_2}{C_2}$
- $\frac{A_1}{L_1} = \frac{A_2}{L_2} \rightarrow A_2 = \frac{L_2 A_1}{L_1}$

The wave flume has water depth of $h_1 = 0.40$ m. Using a real-world site at coordinates (50.50998, -0.58361) (or Eastings: 671327 Northings: 5598124) with a water depth of $h = 63m$ [3]:

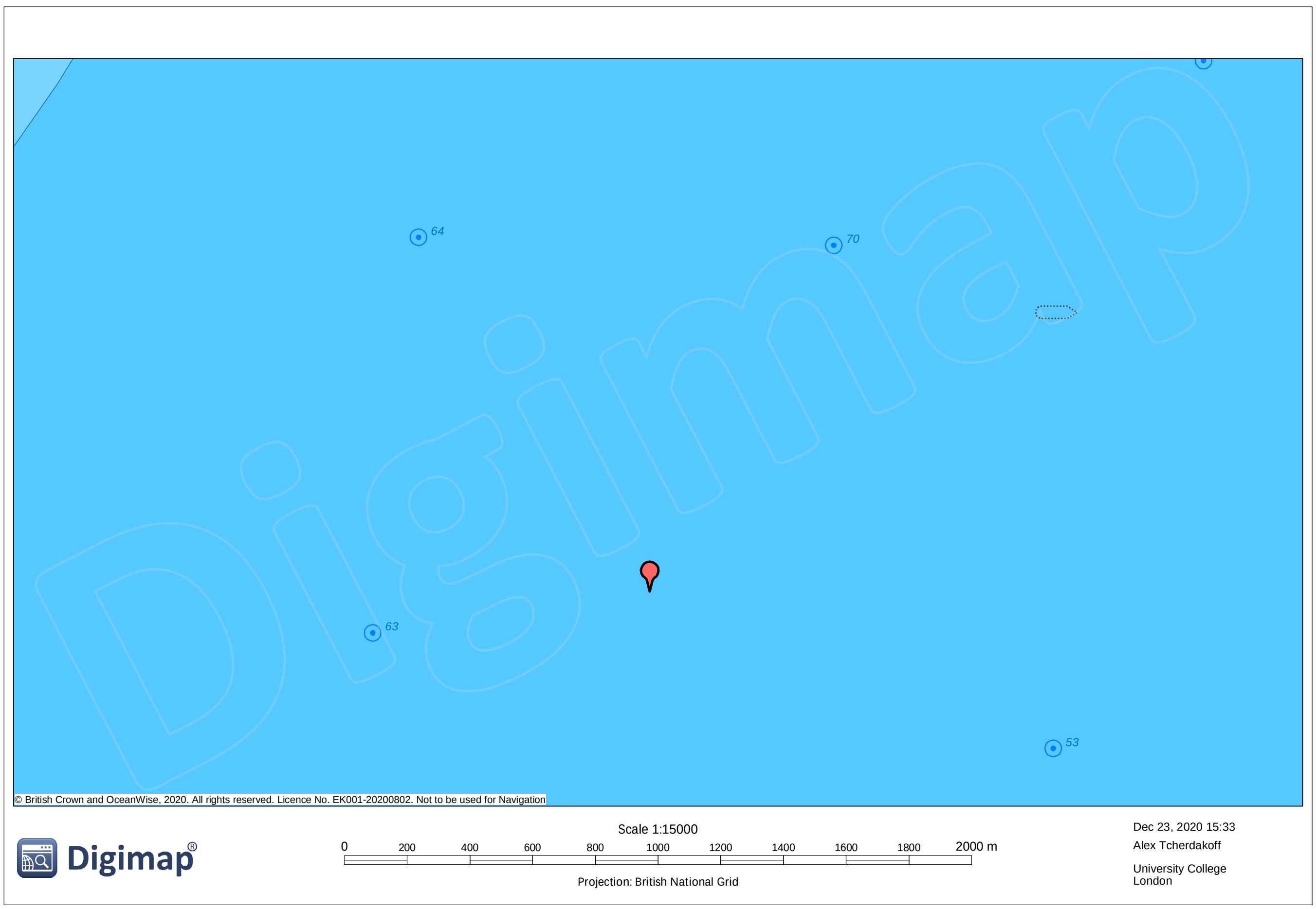

Figure 21: Bathymetrics of the selected real-world location, from Digimaps [3]

The measured celerity is needed, so it was calculated per period, as well as the Froude number. These were thereafter used in order to calculate the scaled celerities, wavelengths, periods, and amplitudes per experimental period for the real-world site selected, and establish a scaled timeline (see Figures 22 and 23):

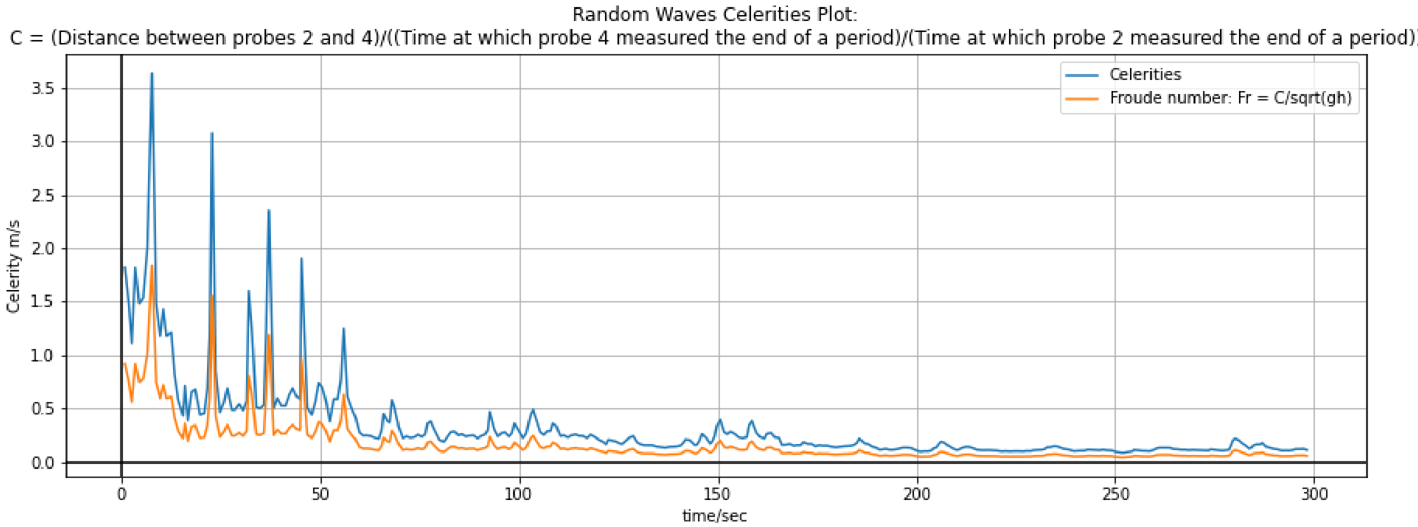

Figure 22: Randomly Generated Waves Plot: Celerity and Froude number per period over time

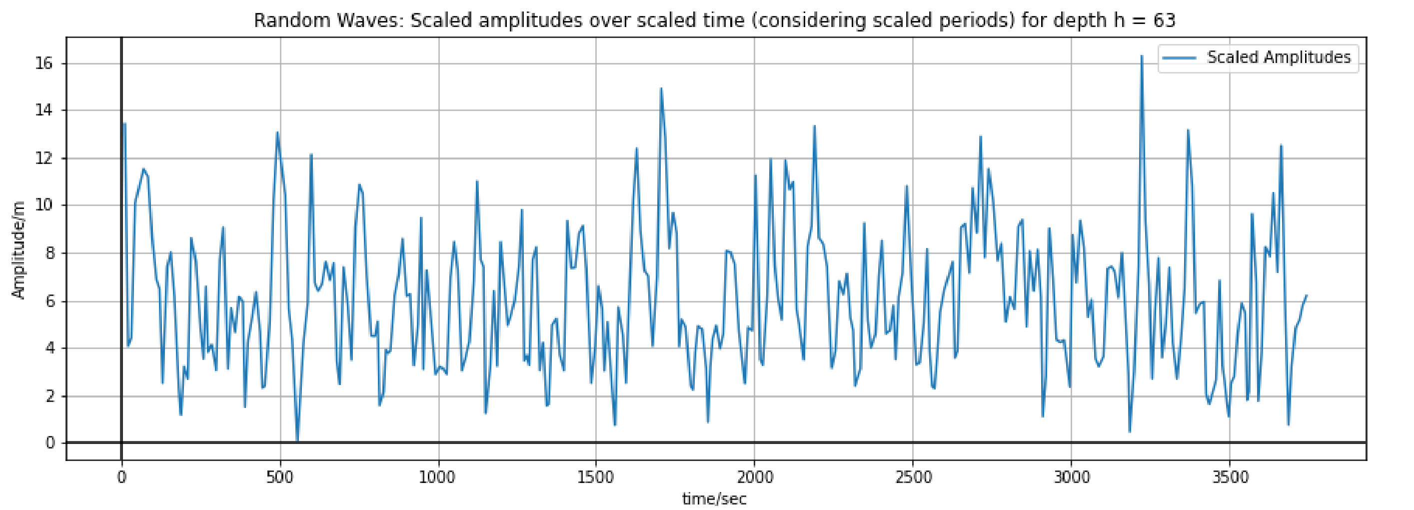

Figure 23: Randomly Generated Waves Plot: Scaled amplitudes over a scaled timeline per experimental period over time for real-world location with depth h = 63m

There are approximately 62.4 minutes of data for the real world site with depth $h = 63$m. To simulate a 3-hour storm at the real-world site, the length of the experiment would have to be tripled. Therefore, Rayleigh distribution shall be applied to find expected maximum wave height for a 3-hour storm using the following formulae with $N = x*n$ where n is the number of peaks and x is approximately 3 (assuming that repeating the experiment 3 times would yield approximately 3 times as many peaks):

- $H_s = \sqrt{2} * H_{RMS} = 9.32845193095314$ m (the formula for H_{RMS} being the same as previously mentioned)
- $H_{max} = \sqrt{ln(N)} * H_{RMS} = \mathbf{17.278685294962006}$ m

Given that the highest amplitude in the scaled data is 16.261418249999995m: the value found by the Rayleigh distribution here was not very far from it. The rule of thumb for Rayleigh distributions is that the maximum wave height is generally twice the significant wave height (which would be 18.6569038619): the value found here is in between the actual maximum wave height in the data, and twice the significant wave height. The Rayleigh distribution is very much applicable for random wave sequence analyses, and is therefore applicable here.

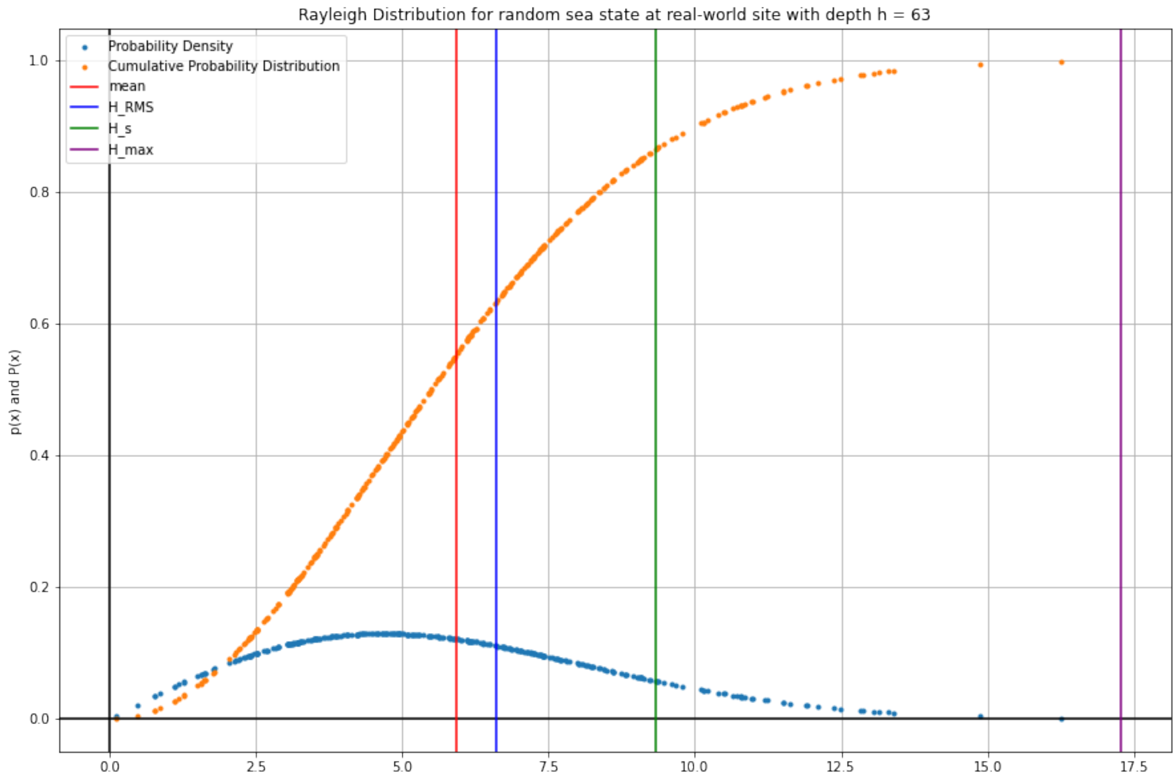

Figure 24: Rayleigh Distribution for Random Sea State at location with depth h = 63

4 Conclusions

In summary, these experiments showed that the generation of regular waves does not produce perfectly sinusoidal waves, that the speeds of energy propagation at the wave front are different based on where in the wave flume they are measured as well as the development (percentage of the significant height) of the wave measured, and that experimental errors are inevitable as the laboratory setup is imperfect (reflections, probe calibration...)
The random wave generation also allowed for the modelling of a random sea state that could be scaled to a real-world site, as well as probabilistic analyses of the scaled data in order to predict storm conditions.

References

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https://github.com/nathalex/Wave-Flume-Lab-Report

[3] Digimap Marine Roam
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