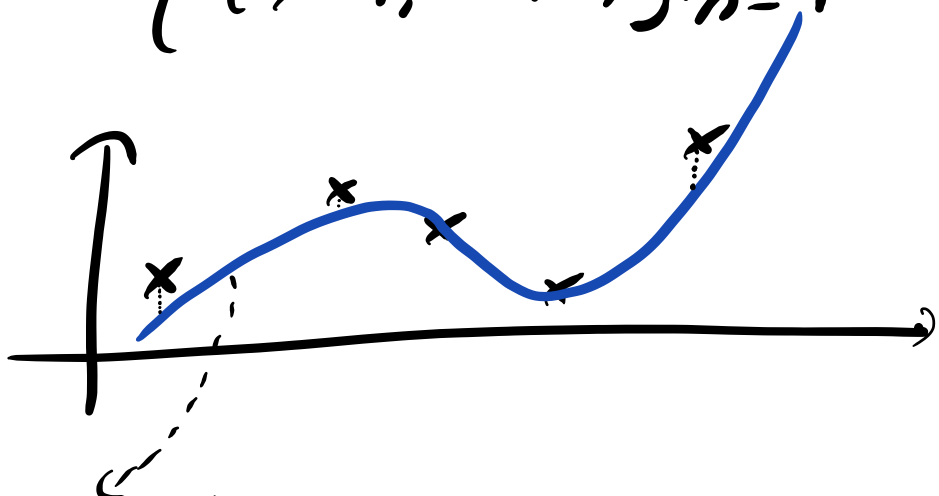
Repaso.

- Ajuste polinomial

$$D = \{(x_n, t_n)\}_{n=1}^{N}$$



(1) $y(x; \vec{w}) = w_0 + w_1 x + \cdots + w_T x^T$

(2) Minimizar el error usando $\nabla_w E$

Ejemplo: $T = 2$

$$y(x; w_0, w_1, w_2) = w_0 + w_1 x + w_2 x^2$$

Definimos un error

$$\mathcal{E}(\vec{w}) = \frac{1}{2} \underbrace{\sum_n (y_n - t_n)^2}_{\substack{\text{"mínimos cuadrados"} \\ \text{"sum-of-squares"}}}$$

$$\mathcal{E}(w_0, w_1, w_2)$$

$$= \frac{1}{2} \sum_n (w_0 + w_1 x_n + w_2 x_n^2 - t_n)^2$$

Queremos $\nabla_{\vec{w}} \mathcal{E} = 0$

$$\nabla_{\vec{w}} \mathcal{E} = \left(\frac{\partial \mathcal{E}}{\partial w_0}, \frac{\partial \mathcal{E}}{\partial w_1}, \frac{\partial \mathcal{E}}{\partial w_2} \right)$$

$$\frac{\partial \mathcal{E}}{\partial w_1} = \frac{1}{2} \sum_n 2(y_n - t_n)(x_n)$$

$$= \sum_n (y_n - t_n) x_n$$

$$= \sum_n \left(w_0 + w_1 x_n + w_2 x_n^2 - t_n\right) x_n$$

$$= \sum_n \left(w_0 x_n + w_1 x_n^2 + w_2 x_n^3 - t_n x_n\right)$$

$$\frac{\partial \varepsilon}{\partial w_1} = w_0 \left(\sum_n x_n^1\right) + w_1 \left(\sum_n x_n^2\right) + w_2 \left(\sum_n x_n^3\right) - \sum_n t_n x_n^1$$

$$\frac{\partial \varepsilon}{\partial w_0} = w_0 \overbrace{\left(\sum_n \overset{x_n^0}{1}\right)}^{N} + w_1 \left(\sum_n x_n\right) + w_2 \left(\sum_n x_n^2\right) - \sum_n t_n$$

$$\frac{\partial \varepsilon}{\partial w_2} = w_0 \left(\sum_n x_n^2\right) + w_1 \left(\sum_n x_n^3\right) + w_2 \left(\sum_n x_n^4\right) - \sum_n t_n x_n^2$$

$$
\begin{bmatrix} N & \sum x_n & \sum x_n^2 \\ \sum x_n & \sum x_n^2 & \sum x_n^3 \\ \sum x_n^2 & \sum x_n^3 & \sum x_n^4 \end{bmatrix}
\begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix}
=
\begin{bmatrix} \sum t_n \\ \sum t_n x_n \\ \sum t_n x_n^2 \end{bmatrix}
$$

$$
X = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}, \quad t = \begin{bmatrix} t_1 \\ \vdots \\ t_n \end{bmatrix}
$$

Tarea:

1. Polinomios de grado arbitrario.
2. Revisar alguna base de datos.