



Notebook - Maratona de Programação

Lenhadoras de Segtree

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1 Math

1.1 Ceil

```
1 long long division_ceil(long long a, long long b) {
2     return 1 + ((a - 1) / b); // if a != 0
3 }
```

1.2 To Decimal

```
1 const string digits { "0123456789
2     ABCDEFGHIJKLMNOPQRSTUVWXYZ" };
3
4 long long to_decimal(const string& rep, long long
5     base) {
6     long long n = 0;
7
8     for (auto c : rep) {
9         // if the number can't be represented in this
10        base
11        if (c > digits[base - 1]) return -1;
12        n *= base;
13        n += digits.find(c);
14    }
15
16    return n;
17 }
```

1.3 Subsets

```
1 void subsets(vector<int>& nums){
2     int n = nums.size();
3     int powSize = 1 << n;
4
5     for(int counter = 0; counter < powSize; counter++){
6         for(int j = 0; j < n; j++) {
7             if((counter & (1LL << j)) != 0) {
8                 cout << nums[j] << ' ';
9             }
10            cout << '\n';
11        }
12    }
13 }
```

1.4 Matrix Exponentiation

```
1 // Description:
2 // Calculate the nth term of a linear recursion
3
4 // Example Fibonacci:
5 // Given a linear recurrence, for example fibonacci
6 // F(n) = n, x <= 1
7 // F(n) = F(n - 1) + F(n - 2), x > 1
8
9 // The recurrence has two terms, so we can build a
10 // matrix 2 x 1 so that
11 // n + 1 = transition * n
12 // (2 x 1) = (2 x 2) * (2 x 1)
13 // F(n) = a b * F(n - 1)
14 // F(n - 1) c d F(n - 2)
15
16 // Another Example:
17 // Given a grid 3 x n, you want to color it using 3
18 // distinct colors so that
19 // no adjacent place has the same color. In how many
20 // different ways can you do that?
21 // There are 6 ways for the first column to be
22 // colored using 3 distinct colors
23 // ans 6 ways using 2 equal colors and 1 distinct one
```

```
21
22 // Adding another column, there are:
23 // 3 ways to go from 2 equal to 2 equal
24 // 2 ways to go from 2 equal to 3 distinct
25 // 2 ways to go from 3 distinct to 2 equal
26 // 2 ways to go from 3 distinct to 3 distinct
27
28 // So we start with matrix 6 6 and multiply it by the
29 // transition 3 2 and get 18 12
30 //
31 //
32 // the we can exponentiate this matrix to find the
33 // nth column
34
35 // Problem:
36 // https://cses.fi/problemset/task/1722/
37
38 // Complexity:
39 // O(log n)
40
41 // How to use:
42 // vector<vector<ll>> v = {{1, 1}, {1, 0}};
43 // Matriz transition = Matriz(v);
44 // cout << fexp(transition, n)[0][1] << '\n';
45
46 using ll = long long;
47
48 const int MOD = 1e9+7;
49
50 struct Matriz{
51     vector<vector<ll>> mat;
52     int rows, columns;
53
54     vector<ll> operator[](int i){
55         return mat[i];
56     }
57
58     Matriz(vector<vector<ll>>& matriz){
59         mat = matriz;
60         rows = mat.size();
61         columns = mat[0].size();
62     }
63
64     Matriz(int row, int column, bool identity=false){
65         rows = row; columns = column;
66         mat.assign(rows, vector<ll>(columns, 0));
67         if(identity) {
68             for(int i = 0; i < min(rows, columns); i
69             ++){
70                 mat[i][i] = 1;
71             }
72         }
73     }
74
75     Matriz operator * (Matriz a) {
76         assert(columns == a.rows);
77         vector<vector<ll>> resp(rows, vector<ll>(a.
78         columns, 0));
79
80         for(int i = 0; i < rows; i++){
81             for(int j = 0; j < a.columns; j++){
82                 for(int k = 0; k < a.rows; k++){
83                     resp[i][j] = (resp[i][j] + (mat[i
84                     ][k] * 1LL * a[k][j]) % MOD) % MOD;
85                 }
86             }
87         }
88         return Matriz(resp);
89     }
90
91     Matriz operator + (Matriz a) {
92         assert(rows == a.rows && columns == a.columns
93         );
```

```

87     vector<vector<ll>> resp(rows, vector<ll>(
columns,0));
88     for(int i = 0; i < rows; i++){
89         for(int j = 0; j < columns; j++){
90             resp[i][j] = (resp[i][j] + mat[i][j]
+ a[i][j]) % MOD;
91         }
92     }
93     return Matriz(resp);
94 }
95 };
96
97 Matriz fexp(Matriz base, ll exponent){
98     Matriz result = Matriz(base.rows, base.rows, 1);
99     while(exponent > 0){
100         if(exponent & 1LL) result = result * base;
101         base = base * base;
102         exponent = exponent >> 1;
103     }
104     return result;
105 }

```

1.5 Crt

```

1 ll crt(const vector<pair<ll, ll>> &vet){
2     ll ans = 0, lcm = 1;
3     ll a, b, g, x, y;
4     for(const auto &p : vet) {
5         tie(a, b) = p;
6         tie(g, x, y) = gcd(lcm, b);
7         if((a - ans) % g != 0) return -1; // no
solution
8         ans = ans + x * ((a - ans) / g) % (b / g) *
lcm;
9         lcm = lcm * (b / g);
10        ans = (ans % lcm + lcm) % lcm;
11    }
12    return ans;
13 }

```

1.6 Binary To Decimal

```

1 int binary_to_decimal(long long n) {
2     int dec = 0, i = 0, rem;
3
4     while (n!=0) {
5         rem = n % 10;
6         n /= 10;
7         dec += rem * pow(2, i);
8         ++i;
9     }
10
11    return dec;
12 }
13
14 long long decimal_to_binary(int n) {
15     long long bin = 0;
16     int rem, i = 1;
17
18     while (n!=0) {
19         rem = n % 2;
20         n /= 2;
21         bin += rem * i;
22         i *= 10;
23     }
24
25    return bin;
26 }

```

1.7 Fast Exponentiation

```

1 ll fexp(ll b, ll e, ll mod) {
2     ll res = 1;
3     b %= mod;
4     while(e){
5         if(e & 1LL)
6             res = (res * b) % mod;
7         e = e >> 1LL;
8         b = (b * b) % mod;
9     }
10    return res;
11 }

```

1.8 Linear Diophantine Equation

```

1 // int a, b, c, x1, x2, y1, y2; cin >> a >> b >> c >>
x1 >> x2 >> y1 >> y2;
2 // int ans = -1;
3 // if (a == 0 && b == 0) {
4 //     if (c != 0) ans = 0;
5 //     else ans = (x2 - x1 + 1) * (y2 - y1 + 1);
6 // }
7 // else if (a == 0) {
8 //     if (c % b == 0 && y1 <= c / b && y2 >= c / b)
ans = (x2 - x1 + 1);
9 //     else ans = 0;
10 // }
11 // else if (b == 0) {
12 //     if (c % a == 0 && x1 <= c / a && x2 >= c / a)
ans = (y2 - y1 + 1);
13 //     else ans = 0;
14 // }
15
16 // Careful when a or b are negative or zero
17
18 // if (ans == -1) ans = find_all_solutions(a, b, c,
x1, x2, y1, y2);
19 // cout << ans << '\n';
20
21 // Problems:
22 // https://www.spoj.com/problems/CEQU/
23 // http://codeforces.com/problemsets/acmsguru/problem
/99999/106
24
25 // consider trivial case a or b is 0
26 int gcd(int a, int b, int& x, int& y) {
27     if (b == 0) {
28         x = 1;
29         y = 0;
30         return a;
31     }
32     int x1, y1;
33     int d = gcd(b, a % b, x1, y1);
34     x = y1;
35     y = x1 - y1 * (a / b);
36     return d;
37 }
38
39 // x and y are one solution and g is the gcd, all
passed as reference
40 // minx <= x <= maxx miny <= y <= maxy
41 bool find_any_solution(int a, int b, int c, int &x0,
int &y0, int &g) {
42     g = gcd(abs(a), abs(b), x0, y0);
43     if (c % g) {
44         return false;
45     }
46
47     x0 *= c / g;
48     y0 *= c / g;
49     if (a < 0) x0 = -x0;
50     if (b < 0) y0 = -y0;
51     return true;
52 }

```

```

53
54 void shift_solution(int & x, int & y, int a, int b,
    int cnt) {
55     x += cnt * b;
56     y -= cnt * a;
57 }
58
59 // return number of solutions in the interval
60 int find_all_solutions(int a, int b, int c, int minx,
    int maxx, int miny, int maxy) {
61     int x, y, g;
62     if (!find_any_solution(a, b, c, x, y, g))
63         return 0;
64     a /= g;
65     b /= g;
66
67     int sign_a = a > 0 ? +1 : -1;
68     int sign_b = b > 0 ? +1 : -1;
69
70     shift_solution(x, y, a, b, (minx - x) / b);
71     if (x < minx)
72         shift_solution(x, y, a, b, sign_b);
73     if (x > maxx)
74         return 0;
75     int lx1 = x;
76
77     shift_solution(x, y, a, b, (maxx - x) / b);
78     if (x > maxx)
79         shift_solution(x, y, a, b, -sign_b);
80     int rx1 = x;
81
82     shift_solution(x, y, a, b, -(miny - y) / a);
83     if (y < miny)
84         shift_solution(x, y, a, b, -sign_a);
85     if (y > maxy)
86         return 0;
87     int lx2 = x;
88
89     shift_solution(x, y, a, b, -(maxy - y) / a);
90     if (y > maxy)
91         shift_solution(x, y, a, b, sign_a);
92     int rx2 = x;
93
94     if (lx2 > rx2)
95         swap(lx2, rx2);
96     int lx = max(lx1, lx2);
97     int rx = min(rx1, rx2);
98
99     if (lx > rx)
100         return 0;
101     return (rx - lx) / abs(b) + 1;
102 }

```

1.9 Function Root

```

1 const ld EPS1 = 1e-9; // iteration precision error
2 const ld EPS2 = 1e-4; // output precision error
3
4 ld f(ld x) {
5     // exp(-x) == e^(-x)
6     return p * exp(-x) + q * sin(x) + r * cos(x) + s *
    tan(x) + t * x * x + u;
7 }
8
9 ld root(ld a, ld b) {
10     while (b - a >= EPS1) {
11         ld c = (a + b) / 2.0;
12         ld y = f(c);
13
14         if (y < 0) b = c;
15         else a = c;
16     }
17 }

```

```

18     return (a + b) / 2;
19 }
20
21 int main() {
22     ld ans = root(0, 1);
23     if (abs(f(ans)) <= EPS2) cout << fixed <<
        setprecision(4) << ans << '\n';
24     else cout << "No solution\n";
25
26     return 0;
27 }

```

1.10 Sieve Of Eratosthenes

```

1 vector<bool> is_prime(MAX, true);
2 vector<int> primes;
3
4 void sieve() {
5     is_prime[0] = is_prime[1] = false;
6     for (int i = 2; i < MAX; i++) {
7         if (is_prime[i]) {
8             primes.push_back(i);
9
10            for (int j = i + i; j < MAX; j += i)
11                is_prime[j] = false;
12        }
13    }
14 }

```

1.11 Horner Algorithm

```

1 // Description:
2 // Evaluates y = f(x)
3
4 // Problem:
5 // https://onlinejudge.org/index.php?option=
    com_onlinejudge&Itemid=8&page=show_problem&
    problem=439
6
7 // Complexity:
8 // O(n)
9
10 using polynomial = std::vector<int>;
11
12 polynomial p {6, -5, 2}; // p(x) = x^2 - 5x + 6;
13
14 int degree(const polynomial& p) {
15     return p.size() - 1;
16 }
17
18 int evaluate(const polynomial& p, int x) {
19     int y = 0, N = degree(p);
20
21     for (int i = N; i >= 0; --i) {
22         y *= x;
23         y += p[i];
24     }
25
26     return y;
27 }

```

1.12 Multiplicative Inverse

```

1 ll extend_euclid(ll a, ll b, ll &x, ll &y) {
2     if (a == 0)
3     {
4         x = 0; y = 1;
5         return b;
6     }
7     ll x1, y1;
8     ll d = extend_euclid(b%a, a, x1, y1);
9     x = y1 - (b / a) * x1;

```

```

10     y = x1;
11     return d;
12 }
13
14 // gcd(a, m) = 1 para existir solucao
15 // ax + my = 1, ou a*x = 1 (mod m)
16 ll inv_gcd(ll a, ll m) { // com gcd
17     ll x, y;
18     extend_euclid(a, m, x, y);
19     return ((x % m) + m) % m;
20 }
21
22 ll inv(ll a, ll phim) { // com phi(m), se m for primo
23     entao phi(m) = p-1
24     ll e = phim-1;
25     return fexp(a, e, MOD);
26 }

```

1.13 Representation Arbitrary Base

```

1 const string digits { "0123456789
2     ABCDEFGHIJKLMNOPQRSTUVWXYZ" };
3
4 string representation(int n, int b) {
5     string rep;
6
7     do {
8         rep.push_back(digits[n % b]);
9         n /= b;
10    } while (n);
11
12    reverse(rep.begin(), rep.end());
13
14    return rep;
15 }

```

1.14 Set Operations

```

1 // Complexity;
2 // O(n * m) being n and m the sizes of the two sets
3 // 2*(count1+count2)-1 (where countX is the distance
4 // between firstX and lastX):
5
6 vector<int> res;
7 set_union(s1.begin(), s1.end(), s2.begin(), s2.end(),
8     inserter(res, res.begin()));
9 set_intersection(s1.begin(), s1.end(), s2.begin(), s2
10     .end(), inserter(res, res.begin()));
11 // present in the first set, but not in the second
12 set_difference(s1.begin(), s1.end(), s2.begin(), s2
13     .end(), inserter(res, res.begin()));
14 // present in one of the sets, but not in the other
15 set_symmetric_difference(s1.begin(), s1.end(), s2
16     .begin(), s2.end(), inserter(res, res.begin()));

```

1.15 Divisors

```

1 vector<long long> all_divisors(long long n) {
2     vector<long long> ans;
3     for(long long a = 1; a*a <= n; a++){
4         if(n % a == 0) {
5             long long b = n / a;
6             ans.push_back(a);
7             if(a != b) ans.push_back(b);
8         }
9     }
10    sort(ans.begin(), ans.end());
11    return ans;
12 }

```

1.16 Check If Bit Is On

```

1 // msb de 0 é undefined
2 #define msb(n) (32 - __builtin_clz(n))
3 // #define msb(n) (64 - __builtin_clzll(n) )
4 // popcount
5 // turn bit off
6
7 bool bit_on(int n, int bit) {
8     if(1 & (n >> bit)) return true;
9     else return false;
10 }

```

1.17 Prime Factors

```

1 vector<pair<long long, int>> fatora(long long n) {
2     vector<pair<long long, int>> ans;
3     for(long long p = 2; p*p <= n; p++) {
4         if(n % p == 0) {
5             int expoente = 0;
6             while(n % p == 0) {
7                 n /= p;
8                 expoente++;
9             }
10            ans.emplace_back(p, expoente);
11        }
12    }
13    if(n > 1) ans.emplace_back(n, 1);
14    return ans;
15 }

```

2 DP

2.1 Knapsack With Index

```

1 void knapsack(int W, int wt[], int val[], int n) {
2     int i, w;
3     int K[n + 1][W + 1];
4
5     for (i = 0; i <= n; i++) {
6         for (w = 0; w <= W; w++) {
7             if (i == 0 || w == 0)
8                 K[i][w] = 0;
9             else if (wt[i - 1] <= w)
10                K[i][w] = max(val[i - 1] +
11                    K[i - 1][w - wt[i - 1]], K[i -
12                1][w]);
13            else
14                K[i][w] = K[i - 1][w];
15        }
16    }
17
18    int res = K[n][W];
19    cout << res << endl;
20
21    w = W;
22    for (i = n; i > 0 && res > 0; i--) {
23        if (res == K[i - 1][w])
24            continue;
25        else {
26            cout << " " << wt[i - 1] ;
27            res = res - val[i - 1];
28            w = w - wt[i - 1];
29        }
30    }
31
32    int main()
33    {
34        int val[] = { 60, 100, 120 };
35        int wt[] = { 10, 20, 30 };
36        int W = 50;
37        int n = sizeof(val) / sizeof(val[0]);

```

```

38     knapsack(W, wt, val, n);
39
40     return 0;
41 }
42 }

```

2.2 Substr Palindrome

```

1 // êvoc deve informar se a substring de S formada
  pelos elementos entre os índices i e j
2 // é um palindromo ou não.
3
4 char s[MAX];
5 int calculado[MAX][MAX]; // iniciado com false, ou 0
6 int tabela[MAX][MAX];
7
8 int is_palin(int i, int j){
9     if(calculado[i][j]){
10         return tabela[i][j];
11     }
12     if(i == j) return true;
13     if(i + 1 == j) return s[i] == s[j];
14
15     int ans = false;
16     if(s[i] == s[j]){
17         if(is_palin(i+1, j-1)){
18             ans = true;
19         }
20     }
21     calculado[i][j] = true;
22     tabela[i][j] = ans;
23     return ans;
24 }

```

2.3 Edit Distance

```

1 // Description:
2 // Minimum number of operations required to transform
  a string into another
3 // Operations allowed: add character, remove
  character, replace character
4
5 // Parameters:
6 // str1 - string to be transformed into str2
7 // str2 - string that str1 will be transformed into
8 // m - size of str1
9 // n - size of str2
10
11 // Problem:
12 // https://cses.fi/problemset/task/1639
13
14 // Complexity:
15 // O(m x n)
16
17 // How to use:
18 // memset(dp, -1, sizeof(dp));
19 // string a, b;
20 // edit_distance(a, b, (int)a.size(), (int)b.size());
21
22 // Notes:
23 // Size of dp matriz is m x n
24
25 int dp[MAX][MAX];
26
27 int edit_distance(string &str1, string &str2, int m,
  int n) {
28     if (m == 0) return n;
29     if (n == 0) return m;
30
31     if (dp[m][n] != -1) return dp[m][n];
32
33     if (str1[m - 1] == str2[n - 1]) return dp[m][n] =
  edit_distance(str1, str2, m - 1, n - 1);

```

```

34     return dp[m][n] = 1 + min({edit_distance(str1,
  str2, m, n - 1), edit_distance(str1, str2, m - 1,
  n), edit_distance(str1, str2, m - 1, n - 1)});
35 }

```

2.4 Knapsack

```

1 int val[MAXN], peso[MAXN], dp[MAXN][MAXS];
2
3 int knapsack(int n, int m){ // n Objetos | Peso max
4     for(int i=0; i<=n; i++){
5         for(int j=0; j<=m; j++){
6             if(i==0 || j==0)
7                 dp[i][j] = 0;
8             else if(peso[i-1]<=j)
9                 dp[i][j] = max(val[i-1]+dp[i-1][j-
  peso[i-1]], dp[i-1][j]);
10            else
11                dp[i][j] = dp[i-1][j];
12        }
13    }
14    return dp[n][m];
15 }

```

2.5 Digits

```

1 // achar a quantidade de numeros menores que R que
  possuem no maximo 3 digitos nao nulos
2 // a ideia eh utilizar da ordem lexicografica para
  checar isso pois se temos por exemplo
3 // o numero 8500, a gente sabe que se pegarmos o
  numero 7... qualquer digito depois do 7
4 // sera necessariamente menor q 8500
5
6 string r;
7 int tab[20][2][5];
8
9 // i - digito de R
10 // menor - ja pegou um numero menor que um digito de
  R
11 // qt - quantidade de digitos nao nulos
12 int dp(int i, bool menor, int qt){
13     if(qt > 3) return 0;
14     if(i >= r.size()) return 1;
15     if(tab[i][menor][qt] != -1) return tab[i][menor][
  qt];
16
17     int dr = r[i]-'0';
18     int res = 0;
19
20     for(int d = 0; d <= 9; d++) {
21         int dnn = qt + (d > 0);
22         if(menor == true) {
23             res += dp(i+1, true, dnn);
24         }
25         else if(d < dr) {
26             res += dp(i+1, true, dnn);
27         }
28         else if(d == dr) {
29             res += dp(i+1, false, dnn);
30         }
31     }
32
33     return tab[i][menor][qt] = res;
34 }

```

2.6 Coins

```

1 int tb[1005];
2 int n;
3 vector<int> moedas;
4
5 int dp(int i){

```

```

6     if(i >= n)
7         return 0;
8     if(tb[i] != -1)
9         return tb[i];
10
11     tb[i] = max(dp(i+1), dp(i+2) + moedas[i]);
12     return tb[i];
13 }
14
15 int main(){
16     memset(tb, -1, sizeof(tb));
17 }

```

2.7 Minimum Coin Change

```

1 int n;
2 vector<int> valores;
3
4 int tabela[1005];
5
6 int dp(int k){
7     if(k == 0){
8         return 0;
9     }
10    if(tabela[k] != -1)
11        return tabela[k];
12    int melhor = 1e9;
13    for(int i = 0; i < n; i++){
14        if(valores[i] <= k)
15            melhor = min(melhor, 1 + dp(k - valores[i]));
16    }
17    return tabela[k] = melhor;
18 }

```

2.8 Kadane

```

1 // achar uma subsequencia continua no array que a
2 // soma seja a maior possivel
3 // nesse caso vc precisa multiplicar exatamente 1
4 // elemento da subsequencia
5 // e achar a maior soma com isso
6
7 int n, x, arr[MAX], tab[MAX][2]; // tab[maior
8 // resposta no intervalo][foi multiplicado ou ão]
9
10 int dp(int i, bool mult) {
11     if (i == n-1) {
12         if (!mult) return arr[n-1]*x;
13         return arr[n-1];
14     }
15     if (tab[i][mult] != -1) return tab[i][mult];
16
17     int res;
18
19     if (mult) {
20         res = max(arr[i], arr[i] + dp(i+1, 1));
21     }
22     else {
23         res = max({
24             arr[i]*x,
25             arr[i]*x + dp(i+1, 1),
26             arr[i] + dp(i+1, 0)
27         });
28     }
29
30     return tab[i][mult] = res;
31 }
32
33 int main() {
34     memset(tab, -1, sizeof(tab));
35 }

```

```

34 int ans = -oo;
35 for (int i = 0; i < n; i++) {
36     ans = max(ans, dp(i, 0));
37 }
38
39 return 0;
40 }
41
42
43
44 int ans = a[0], ans_l = 0, ans_r = 0;
45 int sum = 0, minus_pos = -1;
46
47 for (int r = 0; r < n; ++r) {
48     sum += a[r];
49     if (sum > ans) {
50         ans = sum;
51         ans_l = minus_pos + 1;
52         ans_r = r;
53     }
54     if (sum < 0) {
55         sum = 0;
56         minus_pos = r;
57     }
58 }

```

3 Template

3.1 Template

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define int long long
5 #define optimize std::ios::sync_with_stdio(false);
6 cin.tie(NULL);
7
8 #define vi vector<int>
9 #define ll long long
10 #define pb push_back
11 #define mp make_pair
12 #define ff first
13 #define ss second
14 #define pii pair<int, int>
15 #define MOD 1000000007
16 #define sqr(x) ((x) * (x))
17 #define all(x) (x).begin(), (x).end()
18 #define FOR(i, j, n) for (int i = j; i < n; i++)
19 #define qle(i, n) (i == n ? "\n" : " ")
20 #define endl "\n"
21 const int oo = 1e9;
22 const int MAX = 1e6;
23
24 int32_t main(){ optimize;
25
26     return 0;
27 }

```

3.2 Template Clean

```

1 // Notes:
2 // Compile and execute
3 // g++ teste.cpp -o teste -std=c++17
4 // ./teste < teste.txt
5
6 // Print with precision
7 // cout << fixed << setprecision(12) << value << endl
8 // ;
9 // File as input and output
10 // freopen("input.txt", "r", stdin);
11 // freopen("output.txt", "w", stdout);

```



```

12
13 #include <bits/stdc++.h>
14 using namespace std;
15
16 #define pb push_back
17 #define mp make_pair
18 #define mt make_tuple
19 #define ff first
20 #define ss second
21 #define ld long double
22 #define ll long long
23 #define int long long
24 #define pii pair<int, int>
25 #define tii tuple<int, int, int>
26
27 int main() {
28     ios::sync_with_stdio(false);
29     cin.tie(NULL);
30
31
32
33     return 0;
34 }

```

4 Strings

4.1 Hash

```

1 // Description:
2 // Turns a string into a integer.
3 // If the hash is different then the strings are
4 // different.
5 // If the hash is the same the strings may be
6 // different.
7
8 // Problem:
9 // https://codeforces.com/gym/104518/problem/I
10
11 // Complexity:
12 // O(n) to calculate the hash
13 // O(1) to query
14
15 // Notes:
16 // Primes 1000000007, 1000041323, 100663319,
17 // 201326611, 1000015553, 1000028537
18
19 struct Hash {
20     const ll P = 31;
21     int n; string s;
22     vector<ll> h, hi, p;
23     Hash() {}
24     Hash(string s): s(s), n(s.size()), h(n), hi(n), p
25     (n) {
26         for (int i=0;i<n;i++) p[i] = (i ? P*p[i-1]:1)
27         % MOD;
28         for (int i=0;i<n;i++)
29             h[i] = (s[i] + (i ? h[i-1]:0) * P) % MOD;
30         for (int i=n-1;i>=0;i--)
31             hi[i] = (s[i] + (i+1<n ? hi[i+1]:0) * P)
32             % MOD;
33     }
34     int query(int l, int r) {
35         ll hash = (h[r] - (l ? h[l-1]*p[r-l+1]:MOD :
36         0));
37         return hash < 0 ? hash + MOD : hash;
38     }
39     int query_inv(int l, int r) {
40         ll hash = (hi[l] - (r+1 < n ? hi[r+1]*p[r-l
41         +1] % MOD : 0));
42         return hash < 0 ? hash + MOD : hash;
43     }
44 };

```

4.2 Kmp

```

1 vector<int> prefix_function(string s) {
2     int n = (int)s.length();
3     vector<int> pi(n);
4     for (int i = 1; i < n; i++) {
5         int j = pi[i-1];
6         while (j > 0 && s[i] != s[j])
7             j = pi[j-1];
8         if (s[i] == s[j])
9             j++;
10        pi[i] = j;
11    }
12    return pi;
13 }

```

4.3 Generate All Permutations

```

1 vector<string> generate_permutations(string s) {
2     int n = s.size();
3     vector<string> ans;
4
5     sort(s.begin(), s.end());
6
7     do {
8         ans.push_back(s);
9     } while (next_permutation(s.begin(), s.end()));
10
11     return ans;
12 }

```

4.4 Generate All Sequences Length K

```

1 // gera todas as ípossveis êsequencias usando as letras
2 // em set (de comprimento n) e que tenham tamanho k
3 // sequence = ""
4 vector<string> generate_sequences(char set[], string
5 sequence, int n, int k) {
6     if (k == 0){
7         return { sequence };
8     }
9
10    vector<string> ans;
11    for (int i = 0; i < n; i++) {
12        auto aux = generate_sequences(set, sequence +
13        set[i], n, k - 1);
14        ans.insert(ans.end(), aux.begin(), aux.end())
15        ;
16        // for (auto e : aux) ans.push_back(e);
17    }
18
19    return ans;
20 }

```

4.5 Suffix Array

```

1 // Description:
2 // Suffix array is an array with the indexes of the
3 // starting letter of every
4 // suffix in an array sorted in lexicographical order
5
6 // Problem:
7 // https://codeforces.com/edu/course/2/lesson/2/1/
8 // practice/contest/269100/problem/A
9
10 // Complexity:
11 // O(n log n) with radix sort
12 // O(n log ^ 2 n) with regular sort
13
14 // Notes:
15 // Relevant Problems

```

```

14 // Substring search: Queries to know whether a given
    substring is present in a string
15 // Binary search for the first suffix that is greater
    or equal
16 //  $O(\log n |p|)$  where  $|p|$  is the total size of the
    substrings queried
17 //
18 // Substring size: Queries to know how many times a
    given substring appears in a string
19 // Binary search both for first and last that is
    greater or equal
20 //
21 // Number of different substrings:
22 // A given suffix gives sz new substrings being sz
    the size of the suffix
23 // We can subtract the lcp (longest common prefix) to
    remove substrings
24 // that were already counted.
25 //
26 // Longest common substring between two strings:
27 // We can calculate the suffix array and lcp array of
    the two strings
28 // concatenated with a character greater than $ and
    smaller than A (like '&')
29 // The answer will be the lcp between two consecutive
    suffixes that belong to different strings
30 // (index at suffix array <= size of the first array)
31
32 void radix_sort(vector<pair<pair<int, int>, int>>& a)
    {
33     int n = a.size();
34     vector<pair<pair<int, int>, int>> ans(n);
35
36     vector<int> count(n);
37
38     for (int i = 0; i < n; i++) {
39         count[a[i].first.second]++;
40     }
41
42     vector<int> p(n);
43
44     p[0] = 0;
45     for (int i = 1; i < n; i++) {
46         p[i] = p[i - 1] + count[i - 1];
47     }
48
49     for (int i = 0; i < n; i++) {
50         ans[p[a[i].first.second]++] = a[i];
51     }
52
53     a = ans;
54
55     count.assign(n, 0);
56
57     for (int i = 0; i < n; i++) {
58         count[a[i].first.first]++;
59     }
60
61     p.assign(n, 0);
62
63     p[0] = 0;
64     for (int i = 1; i < n; i++) {
65         p[i] = p[i - 1] + count[i - 1];
66     }
67
68     for (int i = 0; i < n; i++) {
69         ans[p[a[i].first.first]++] = a[i];
70     }
71
72     a = ans;
73 }
74
75 vector<int> p, c;
76
77 vector<int> suffix_array(string s) {
78     int n = s.size();
79     vector<pair<char, int>> a(n);
80     p.assign(n, 0);
81     c.assign(n, 0);
82
83     for (int i = 0; i < n; i++) {
84         a[i] = mp(s[i], i);
85     }
86
87     sort(a.begin(), a.end());
88
89     for (int i = 0; i < n; i++) {
90         p[i] = a[i].second;
91     }
92
93     c[p[0]] = 0;
94     for (int i = 1; i < n; i++) {
95         if (a[i].first == a[i - 1].first) c[p[i]] = c[p[i
            - 1]];
96         else c[p[i]] = c[p[i - 1]] + 1;
97     }
98
99     int k = 0;
100     while ((1 << k) < n) {
101         vector<pair<pair<int, int>, int>> a(n);
102         for (int i = 0; i < n; i++) {
103             a[i] = mp(mp(c[i], c[(i + (1 << k)) % n]), i);
104         }
105
106         radix_sort(a);
107
108         for (int i = 0; i < n; i++) {
109             p[i] = a[i].second;
110         }
111
112         c[p[0]] = 0;
113         for (int i = 1; i < n; i++) {
114             if (a[i].first == a[i - 1].first) c[p[i]] = c[p
                [i - 1]];
115             else c[p[i]] = c[p[i - 1]] + 1;
116         }
117
118         k++;
119     }
120
121     /* for (int i = 0; i < n; i++) {
122         for (int j = p[i]; j < n; j++) {
123             cout << s[j];
124         }
125         cout << '\n';
126     } */
127
128     return p;
129 }
130
131 // the first suffix will always be $ the (n - 1)th
    character in the string
132 vector<int> lcp_array(string s) {
133     int n = s.size();
134     vector<int> ans(n);
135     // minimum lcp
136     int k = 0;
137     for (int i = 0; i < n - 1; i++) {
138         // indice in the suffix array p of suffix
            starting in i
139         int pi = c[i];
140         // start index of the previous suffix in suffix
            array
141         int j = p[pi - 1];
142         while (s[i + k] == s[j + k]) k++;
143         ans[pi] = k;

```

```

144     k = max(k - 1, 0);
145 }
146
147 return ans;
148 }

```

4.6 Lcs

```

1 // Description:
2 // Finds the longest common subsequence between two
  string
3
4 // Problem:
5 // https://codeforces.com/gym/103134/problem/B
6
7 // Complexity:
8 // O(mn) where m and n are the length of the strings
9
10 string lcsAlgo(string s1, string s2, int m, int n) {
11     int LCS_table[m + 1][n + 1];
12
13     for (int i = 0; i <= m; i++) {
14         for (int j = 0; j <= n; j++) {
15             if (i == 0 || j == 0)
16                 LCS_table[i][j] = 0;
17             else if (s1[i - 1] == s2[j - 1])
18                 LCS_table[i][j] = LCS_table[i - 1][j - 1] +
19 1;
20             else
21                 LCS_table[i][j] = max(LCS_table[i - 1][j],
22 LCS_table[i][j - 1]);
23         }
24     }
25
26     int index = LCS_table[m][n];
27     char lcsAlgo[index + 1];
28     lcsAlgo[index] = '\0';
29
30     int i = m, j = n;
31     while (i > 0 && j > 0) {
32         if (s1[i - 1] == s2[j - 1]) {
33             lcsAlgo[index - 1] = s1[i - 1];
34             i--;
35             j--;
36             index--;
37         }
38         else if (LCS_table[i - 1][j] > LCS_table[i][j - 1])
39             i--;
40         else
41             j--;
42     }
43
44     return lcsAlgo;
45 }

```

4.7 Trie

```

1 const int K = 26;
2
3 struct Vertex {
4     int next[K];
5     bool output = false;
6     int p = -1;
7     char pch;
8     int link = -1;
9     int go[K];
10
11     Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
12         fill(begin(next), end(next), -1);
13         fill(begin(go), end(go), -1);

```

```

14     }
15 };
16
17 vector<Vertex> t(1);
18
19 void add_string(string const& s) {
20     int v = 0;
21     for (char ch : s) {
22         int c = ch - 'a';
23         if (t[v].next[c] == -1) {
24             t[v].next[c] = t.size();
25             t.emplace_back(v, ch);
26         }
27         v = t[v].next[c];
28     }
29     t[v].output = true;
30 }
31
32 int go(int v, char ch);
33
34 int get_link(int v) {
35     if (t[v].link == -1) {
36         if (v == 0 || t[v].p == 0)
37             t[v].link = 0;
38         else
39             t[v].link = go(get_link(t[v].p), t[v].pch
40 );
41     }
42     return t[v].link;
43 }
44
45 int go(int v, char ch) {
46     int c = ch - 'a';
47     if (t[v].go[c] == -1) {
48         if (t[v].next[c] != -1)
49             t[v].go[c] = t[v].next[c];
50         else
51             t[v].go[c] = v == 0 ? 0 : go(get_link(v),
52 ch);
53     }
54     return t[v].go[c];
55 }

```

4.8 Z-function

```

1 vector<int> z_function(string s) {
2     int n = (int) s.length();
3     vector<int> z(n);
4     for (int i = 1, l = 0, r = 0; i < n; ++i) {
5         if (i <= r)
6             z[i] = min(r - i + 1, z[i - l]);
7         while (i + z[i] < n && s[z[i]] == s[i + z[i]
8 ]])
9             ++z[i];
10         if (i + z[i] - 1 > r)
11             l = i, r = i + z[i] - 1;
12     }
13     return z;
14 }

```

5 Misc

5.1 Split

```

1 vector<string> split(string txt, char key = ' '){
2     vector<string> ans;
3
4     string palTemp = "";
5     for(int i = 0; i < txt.size(); i++){
6
7         if(txt[i] == key){

```

```

8         if(palTemp.size() > 0){
9             ans.push_back(palTemp);
10            palTemp = "";
11        }
12    } else{
13        palTemp += txt[i];
14    }
15 }
16
17 if(palTemp.size() > 0)
18     ans.push_back(palTemp);
19
20 return ans;
21 }
22 }

```

5.2 Int128

```

1  __int128 read() {
2      __int128 x = 0, f = 1;
3      char ch = getchar();
4      while (ch < '0' || ch > '9') {
5          if (ch == '-') f = -1;
6          ch = getchar();
7      }
8      while (ch >= '0' && ch <= '9') {
9          x = x * 10 + ch - '0';
10         ch = getchar();
11     }
12     return x * f;
13 }
14 void print(__int128 x) {
15     if (x < 0) {
16         putchar('-');
17         x = -x;
18     }
19     if (x > 9) print(x / 10);
20     putchar(x % 10 + '0');
21 }

```

6 Graphs

6.1 Centroid Find

```

1  // Description:
2  // Indexed at zero
3  // Find a centroid, that is a node such that when it
4  // is appointed the root of the tree,
5  // each subtree has at most floor(n/2) nodes.
6
7  // Problem:
8  // https://cses.fi/problemset/task/2079/
9
10 // Complexity:
11 // O(n)
12
13 // How to use:
14 // get_subtree_size(0);
15 // cout << get_centroid(0) + 1 << endl;
16
17 int n;
18 vector<int> adj[MAX];
19 int subtree_size[MAX];
20
21 int get_subtree_size(int node, int par = -1) {
22     int &res = subtree_size[node];
23     res = 1;
24     for (int i : adj[node]) {
25         if (i == par) continue;
26         res += get_subtree_size(i, node);
27     }
28 }

```

```

27     return res;
28 }
29
30 int get_centroid(int node, int par = -1) {
31     for (int i : adj[node]) {
32         if (i == par) continue;
33
34         if (subtree_size[i] * 2 > n) { return
35             get_centroid(i, node); }
36     }
37     return node;
38 }
39
40 int main() {
41     cin >> n;
42     for (int i = 0; i < n - 1; i++) {
43         int u, v; cin >> u >> v;
44         u--; v--;
45         adj[u].push_back(v);
46         adj[v].push_back(u);
47     }
48     get_subtree_size(0);
49     cout << get_centroid(0) + 1 << endl;
50 }

```

6.2 Bipartite

```

1  const int NONE = 0, BLUE = 1, RED = 2;
2  vector<vector<int>> graph(100005);
3  vector<bool> visited(100005);
4  int color[100005];
5
6  bool bfs(int s = 1){
7
8      queue<int> q;
9      q.push(s);
10     color[s] = BLUE;
11
12     while (not q.empty()){
13         auto u = q.front(); q.pop();
14
15         for (auto v : graph[u]){
16             if (color[v] == NONE){
17                 color[v] = 3 - color[u];
18                 q.push(v);
19             }
20             else if (color[v] == color[u]){
21                 return false;
22             }
23         }
24     }
25     return true;
26 }
27
28 bool is_bipartite(int n){
29     for (int i = 1; i <= n; i++)
30         if (color[i] == NONE and not bfs(i))
31             return false;
32     return true;
33 }
34
35 }

```

6.3 Prim

```

1  int n;
2  vector<vector<int>> adj; // adjacency matrix of graph
3  const int INF = 1000000000; // weight INF means there
4  // is no edge

```

```

5 struct Edge {
6     int w = INF, to = -1;
7 };
8
9 void prim() {
10     int total_weight = 0;
11     vector<bool> selected(n, false);
12     vector<Edge> min_e(n);
13     min_e[0].w = 0;
14
15     for (int i=0; i<n; ++i) {
16         int v = -1;
17         for (int j = 0; j < n; ++j) {
18             if (!selected[j] && (v == -1 || min_e[j].
19 w < min_e[v].w))
20                 v = j;
21
22             if (min_e[v].w == INF) {
23                 cout << "No MST!" << endl;
24                 exit(0);
25             }
26
27             selected[v] = true;
28             total_weight += min_e[v].w;
29             if (min_e[v].to != -1)
30                 cout << v << " " << min_e[v].to << endl;
31
32             for (int to = 0; to < n; ++to) {
33                 if (adj[v][to] < min_e[to].w)
34                     min_e[to] = {adj[v][to], v};
35             }
36         }
37
38         cout << total_weight << endl;
39 }

```

6.4 Eulerian Undirected

```

1 // Description:
2 // Hierholzer's Algorithm
3 // An Eulerian path is a path that passes through
4 // every edge exactly once.
5 // An Eulerian circuit is an Eulerian path that
6 // starts and ends on the same node.
7
8 // An Eulerian path exists in an undirected graph if
9 // the degree of every node is even (not counting
10 // self-edges)
11 // except for possibly exactly two nodes that have
12 // and odd degree (start and end nodes).
13 // An Eulerian circuit exists in an undirected graph
14 // if the degree of every node is even.
15
16 // The graph has to be connected (except for isolated
17 // nodes which are allowed because there
18 // are no edges connected to them).
19
20 // Problem:
21 // https://cses.fi/problemset/task/1691
22
23 // Complexity:
24 //  $O(E \cdot \log(E))$  where E is the number of edges
25
26 // How to use
27 // Check whether the path exists before trying to
28 // find it
29 // Find the root - any node that has at least 1
30 // outgoing edge
31 // (if the problem requires that you start from a
32 // node v, the root will be the node v)
33 // Count the degree;
34 //

```

```

25 // for (int i = 0; i < m; i++) {
26 //     int a, b; cin >> a >> b;
27 //     adj[a].pb(b); adj[b].pb(a);
28 //     root = a;
29 //     degree[a]++; degree[b]++;
30 // }
31
32 // Notes
33 // If you want to find a path start and ending nodes
34 // v and u
35 // if ((is_eulerian(n, root, start, end) != 1) || (
36 //     start != v) || (end != u)) cout << "IMPOSSIBLE\n"
37
38 // It can be speed up to work on  $O(E)$  on average by
39 // using unordered_set instead of set
40
41 // It works when there are self loops, but not when
42 // there are multiple edges
43 // If the graph has multiple edges, add more notes to
44 // simulate the edges
45 // e.g
46 // 1 2
47 // 1 2
48 // 1 2
49 // becomes
50 // 3 4
51 // 4 1
52 // 1 2
53
54 vector<bool> visited;
55 vector<int> degree;
56 vector<vector<int>> adj;
57
58 void dfs(int v) {
59     visited[v] = true;
60     for (auto u : adj[v]) {
61         if (!visited[u]) dfs(u);
62     }
63 }
64
65 int is_eulerian(int n, int root, int& start, int& end)
66 {
67     start = -1, end = -1;
68     if (n == 1) return 2; // only one node
69     visited.assign(n + 1, false);
70     dfs(root);
71
72     for (int i = 1; i <= n; i++) {
73         if (!visited[i] && degree[i] > 0) return 0;
74     }
75
76     for (int i = 1; i <= n; i++) {
77         if (start == -1 && degree[i] % 2 == 1) start = i;
78         else if (end == -1 && degree[i] % 2 == 1) end = i;
79         else if (degree[i] % 2 == 1) return 0;
80     }
81
82     if (start == -1 && end == -1) {start = root; end =
83         root; return 2;} // has eulerian circuit and path
84     if (start != -1 && end != -1) return 1; // has
85         eulerian path
86     return 0; // no eulerian path nor circuit
87 }
88
89 vector<int> path;
90 vector<set<int>> mark;
91
92 void dfs_path(int v) {
93     visited[v] = true;
94
95     while (degree[v] != 0) {
96         degree[v]--;
97     }
98 }

```

```

89     int u = adj[v][degree[v]];
90     if (mark[v].find(u) != mark[v].end()) continue;
91     mark[v].insert(u);
92     mark[u].insert(v);
93     int next_edge = adj[v][degree[v]];
94     dfs_path(next_edge);
95 }
96 path.pb(v);
97 }
98
99 void find_path(int n, int start) {
100     path.clear();
101     mark.resize(n + 1);
102     visited.assign(n + 1, false);
103     dfs_path(start);
104 }

```

6.5 Ford Fulkerson Edmonds Karp

```

1 // Description:
2 // Obtains the maximum possible flow rate given a
  network. A network is a graph with a single
  source vertex and a single sink vertex in which
  each edge has a capacity
3
4 // Complexity:
5 //  $O(V * E^2)$  where V is the number of vertex and E
  is the number of edges
6
7 int n;
8 vector<vector<int>> capacity;
9 vector<vector<int>> adj;
10
11 int bfs(int s, int t, vector<int>& parent) {
12     fill(parent.begin(), parent.end(), -1);
13     parent[s] = -2;
14     queue<pair<int, int>> q;
15     q.push({s, INF});
16
17     while (!q.empty()) {
18         int cur = q.front().first;
19         int flow = q.front().second;
20         q.pop();
21
22         for (int next : adj[cur]) {
23             if (parent[next] == -1 && capacity[cur][
next]) {
24                 parent[next] = cur;
25                 int new_flow = min(flow, capacity[cur
][next]);
26                 if (next == t)
27                     return new_flow;
28                 q.push({next, new_flow});
29             }
30         }
31     }
32
33     return 0;
34 }
35
36 int maxflow(int s, int t) {
37     int flow = 0;
38     vector<int> parent(n);
39     int new_flow;
40
41     while (new_flow = bfs(s, t, parent)) {
42         flow += new_flow;
43         int cur = t;
44         while (cur != s) {
45             int prev = parent[cur];
46             capacity[prev][cur] -= new_flow;
47             capacity[cur][prev] += new_flow;
48             cur = prev;

```

```

49         }
50     }
51
52     return flow;
53 }

```

6.6 Hld Edge

```

1 // Description:
2 // Make queries and updates between two vertexes on a
  tree
3
4 // Problem:
5 // https://www.spoj.com/problems/QTREE/
6
7 // Complexity:
8 //  $O(\log^2 n)$  for both query and update
9
10 // How to use:
11 // HLD hld = HLD(n + 1, adj)
12
13 // Notes
14 // Change the root of the tree on the constructor if
  it's different from 1
15 // Use together with Segtree
16
17 struct HLD {
18     vector<int> parent;
19     vector<int> pos;
20     vector<int> head;
21     vector<int> subtree_size;
22     vector<int> level;
23     vector<int> heavy_child;
24     vector<ftype> subtree_weight;
25     vector<ftype> path_weight;
26     vector<vector<int>> adj;
27     vector<int> at;
28     Segtree seg = Segtree(0);
29     int cpos;
30     int n;
31     int root;
32
33     HLD() {}
34
35     HLD(int n, vector<vector<int>>& adj, int root = 1)
      : adj(adj), n(n), root(root) {
36         seg = Segtree(n);
37         cpos = 0;
38         at.assign(n, 0);
39         parent.assign(n, 0);
40         pos.assign(n, 0);
41         head.assign(n, 0);
42         subtree_size.assign(n, 1);
43         level.assign(n, 0);
44         heavy_child.assign(n, -1);
45         parent[root] = -1;
46         dfs(root, -1);
47         decompose(root, -1);
48     }
49
50     void dfs(int v, int p) {
51         parent[v] = p;
52         if (p != -1) level[v] = level[p] + 1;
53         for (auto u : adj[v]) {
54             if (u != p) {
55                 dfs(u, v);
56                 subtree_size[v] += subtree_size[u];
57                 if (heavy_child[v] == -1 || subtree_size[u] >
subtree_size[heavy_child[v]]) heavy_child[v] = u
58             }
59         }
60     }

```

```

61 void decompose(int v, int chead) {
62     // start a new path
63     if (chead == -1) chead = v;
64
65     // consecutive ids in the hld path
66     at[cpos] = v;
67     pos[v] = cpos++;
68     head[v] = chead;
69
70     // if not a leaf
71     if (heavy_child[v] != -1) decompose(heavy_child[v], chead);
72
73     // light child
74     for (auto u : adj[v]){
75         // start new path
76         if (u != parent[v] && u != heavy_child[v])
77             decompose(u, -1);
78     }
79 }
80
81 ll query_path(int a, int b) {
82     if (a == b) return 0;
83     if (pos[a] < pos[b]) swap(a, b);
84
85     if (head[a] == head[b]) return seg.query(pos[b] + 1, pos[a]);
86     return seg.f(seg.query(pos[head[a]], pos[a]),
87         query_path(parent[head[a]], b));
88 }
89
90 ftype query_subtree(int a) {
91     if (subtree_size[a] == 1) return 0;
92     return seg.query(pos[a] + 1, pos[a] + subtree_size[a] - 1);
93 }
94
95 void update_path(int a, int b, int x) {
96     if (a == b) return;
97     if (pos[a] < pos[b]) swap(a, b);
98
99     if (head[a] == head[b]) return (void)seg.update(
100         pos[b] + 1, pos[a], x);
101     seg.update(pos[head[a]], pos[a], x); update_path(
102         parent[head[a]], b, x);
103 }
104
105 void update_subtree(int a, int val) {
106     if (subtree_size[a] == 1) return;
107     seg.update(pos[a] + 1, pos[a] + subtree_size[a] - 1, val);
108 }
109
110 // vertex
111 void update(int a, int val) {
112     seg.update(pos[a], pos[a], val);
113 }
114
115 // edge
116 void update(int a, int b, int val) {
117     if (parent[a] == b) swap(a, b);
118     update(b, val);
119 }
120
121 int lca(int a, int b) {
122     if (pos[a] < pos[b]) swap(a, b);
123     return head[a] == head[b] ? b : lca(parent[head[a]], b);
124 }
125 }

```

6.7 Floyd Warshall

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4 using ll = long long;
5
6 const int MAX = 507;
7 const long long INF = 0x3f3f3f3f3f3f3fLL;
8
9 ll dist[MAX][MAX];
10 int n;
11
12 void floyd_warshall() {
13     for (int i = 0; i < n; i++) {
14         for (int j = 0; j < n; j++) {
15             if (i == j) dist[i][j] = 0;
16             else if (!dist[i][j]) dist[i][j] = INF;
17         }
18     }
19
20     for (int k = 0; k < n; k++) {
21         for (int i = 0; i < n; i++) {
22             for (int j = 0; j < n; j++) {
23                 // trata o caso no qual o grafo tem
24                 // arestas com peso negativo
25                 if (dist[i][k] < INF && dist[k][j] <
26                     INF){
27                     dist[i][j] = min(dist[i][j], dist
28                         [i][k] + dist[k][j]);
29                 }
30             }
31         }
32     }
33 }

```

6.8 Lca

```

1 // Description:
2 // Find the lowest common ancestor between two nodes
3 // in a tree
4
5 // Problem:
6 // https://cses.fi/problemset/task/1135
7
8 // Complexity:
9 // O(log n)
10
11 // How to use:
12 // preprocess();
13 // lca(a, b);
14
15 // Notes
16 // To calculate the distance between two nodes use
17 // the following formula
18 // level_peso[a] + level_peso[b] - 2*level_peso[lca(a, b)]
19
20 const int MAX = 2e5+10;
21 const int BITS = 30;
22
23 vector<pii> adj[MAX];
24 vector<bool> visited(MAX);
25
26 int up[MAX][BITS + 1];
27 int level[MAX];
28 int level_peso[MAX];
29
30 void find_level() {
31     queue<pii> q;
32     q.push(mp(1, 0));
33 }

```

```

32     visited[1] = true;
33
34     while (!q.empty()) {
35         auto [v, depth] = q.front();
36         q.pop();
37         level[v] = depth;
38
39         for (auto [u,d] : adj[v]) {
40             if (!visited[u]) {
41                 visited[u] = true;
42                 up[u][0] = v;
43                 q.push(mp(u, depth + 1));
44             }
45         }
46     }
47 }
48
49 void find_level_peso() {
50     queue<pii> q;
51
52     q.push(mp(1, 0));
53     visited[1] = true;
54
55     while (!q.empty()) {
56         auto [v, depth] = q.front();
57         q.pop();
58         level_peso[v] = depth;
59
60         for (auto [u,d] : adj[v]) {
61             if (!visited[u]) {
62                 visited[u] = true;
63                 up[u][0] = v;
64                 q.push(mp(u, depth + d));
65             }
66         }
67     }
68 }
69
70 int lca(int a, int b) {
71     // get the nodes to the same level
72     int mn = min(level[a], level[b]);
73
74     for (int j = 0; j <= BITS; j++) {
75         if (a != -1 && ((level[a] - mn) & (1 << j))) a
76         = up[a][j];
77         if (b != -1 && ((level[b] - mn) & (1 << j))) b
78         = up[b][j];
79     }
80
81     // special case
82     if (a == b) return a;
83
84     // binary search
85     for (int j = BITS; j >= 0; j--) {
86         if (up[a][j] != up[b][j]) {
87             a = up[a][j];
88             b = up[b][j];
89         }
90     }
91     return up[a][0];
92 }
93
94 void preprocess() {
95     visited = vector<bool>(MAX, false);
96     find_level();
97     visited = vector<bool>(MAX, false);
98     find_level_peso();
99
100     for (int j = 1; j <= BITS; j++) {
101         for (int i = 1; i <= n; i++) {
102             if (up[i][j - 1] != -1) up[i][j] = up[up[i][j - 1]][j - 1];
103         }
104     }

```

```

102     }
103 }

```

6.9 Kuhn

```

1 // Description
2 // Matching algorithm for unweighted bipartite graph
3 //
4 // Problem:
5 // https://codeforces.com/gym/104252/problem/H
6 //
7 // Complexity:
8 // O(V * E) in which V is the number of vertexes and
9 // E is the number of edges
10
11 // Notes:
12 // Indexed at zero
13
14 int n, k;
15 // adjacency list
16 vector<vector<int>> g;
17 vector<int> mt;
18 vector<bool> used;
19
20 bool try_kuhn(int v) {
21     if (used[v])
22         return false;
23     used[v] = true;
24     for (int to : g[v]) {
25         if (mt[to] == -1 || try_kuhn(mt[to])) {
26             mt[to] = v;
27             return true;
28         }
29     }
30     return false;
31 }
32
33 int main() {
34     // ... reading the graph g ...
35
36     mt.assign(k, -1);
37     vector<bool> used1(n, false);
38     for (int v = 0; v < n; ++v) {
39         for (int to : g[v]) {
40             if (mt[to] == -1) {
41                 mt[to] = v;
42                 used1[v] = true;
43                 break;
44             }
45         }
46     }
47     for (int v = 0; v < n; ++v) {
48         if (used1[v])
49             continue;
50         used.assign(n, false);
51         try_kuhn(v);
52     }
53     for (int i = 0; i < k; ++i)
54         if (mt[i] != -1)
55             printf("%d %d\n", mt[i] + 1, i + 1);
56 }

```

6.10 Eulerian Directed

```

1 // Description:
2 // Hierholzer's Algorithm
3 // An Eulerian path is a path that passes through
4 // every edge exactly once.
5 // An Eulerian circuit is an Eulerian path that
6 // starts and ends on the same node.

```



```

5
6 // An Eulerian path exists in an directed graph if
7 // the indegree and outdegree is equal
8 // for every node (not counting self-edges)
9 // except for possibly exactly one node that have
10 // outdegree - indegree = 1
11 // and one node that has indegree - outdegree = 1 (
12 // start and end nodes).
13 // An Eulerian circuit exists in an directed graph if
14 // the indegree and outdegree is equal for every
15 // node.
16
17 // The graph has to be conected (except for isolated
18 // nodes which are allowed because there
19 // are no edges connected to them).
20
21 // Problem:
22 // https://cses.fi/problemset/task/1693
23
24 // Complexity:
25 // O(E) where E is the number of edges
26
27 // How to use
28 // Check whether the path exists before trying to
29 // find it
30 // Find the root - any node that has at least 1
31 // outgoing edge
32 // (if the problem requires that you start from a
33 // node v, the root will be the node v)
34 // Count the degree;
35 //
36 // for (int i = 0; i < m; i++) {
37 //   int a, b; cin >> a >> b;
38 //   adj[a].pb(b);
39 //   root = a;
40 //   outdegree[a]++; indegree[b]++;
41 // }
42
43 // Notes
44 // It works when there are self loops, but not when
45 // there are multiple edges
46
47 vector<bool> visited;
48 vector<int> outdegree, indegree;
49 vector<vector<int>> adj, undir;
50
51 void dfs(int v) {
52     visited[v] = true;
53     for (auto u : undir[v]) {
54         if (!visited[u]) dfs(u);
55     }
56 }
57
58 int is_eulerian(int n, int root, int &start, int& end
59 ) {
60     start = -1, end = -1;
61     if (n == 1) return 2; // only one node
62     visited.assign(n + 1, false);
63     dfs(root);
64
65     for (int i = 1; i <= n; i++) {
66         if (!visited[i] && (i == n || i == 1 || outdegree
67         [i] + indegree[i] > 0)) return 0;
68     }
69
70     // start => node with indegree - outdegree = 1
71     // end => node with outdegree - indegree = 1
72     for (int i = 1; i <= n; i++) {
73         if (start == -1 && indegree[i] - outdegree[i] ==
74         1) start = i;
75         else if (end == -1 && outdegree[i] - indegree[i]
76         == 1) end = i;
77         else if (indegree[i] != outdegree[i]) return 0;
78     }
79
80     if (start == -1 && end == -1) {start = root; end =
81     root; return 2;} // has eulerian circuit and path
82     if (start != -1 && end != -1) {swap(start, end);
83     return 1;} // has eulerian path
84     return 0; // no eulerian path nor circuit
85 }
86
87 vector<int> path;
88
89 void dfs_path(int v) {
90     visited[v] = true;
91
92     while (outdegree[v] != 0) {
93         int u = adj[v][--outdegree[v]];
94         int next_edge = adj[v][outdegree[v]];
95         dfs_path(next_edge);
96     }
97     path.pb(v);
98 }
99
100 void find_path(int n, int start) {
101     path.clear();
102     visited.assign(n + 1, false);
103     dfs_path(start);
104     reverse(path.begin(), path.end());
105 }

```

6.11 Bellman Ford

```

1 // Description:
2 // Finds the shortest path from a vertex v to any
3 // other vertex
4
5 // Problem:
6 // https://cses.fi/problemset/task/1673
7
8 // Complexity:
9 // O(n * m)
10
11 struct Edge {
12     int a, b, cost;
13     Edge(int a, int b, int cost) : a(a), b(b), cost(
14     cost) {}
15 };
16
17 int n, m;
18 vector<Edge> edges;
19 const int INF = 1e9+10;
20
21 void bellman_ford(int v, int t) {
22     vector<int> d(n + 1, INF);
23     d[v] = 0;
24     vector<int> p(n + 1, -1);
25
26     for (;;) {
27         bool any = false;
28         for (Edge e : edges) {
29             if (d[e.a] >= INF) continue;
30             if (d[e.b] > d[e.a] + e.cost) {
31                 d[e.b] = d[e.a] + e.cost;
32                 p[e.b] = e.a;
33                 any = true;
34             }
35         }
36         if (!any) break;
37     }
38
39     if (d[t] == INF)
40         cout << "No path from " << v << " to " << t << ".
41         ";
42     else {

```

```

40     vector<int> path;
41     for (int cur = t; cur != -1; cur = p[cur]) {
42         path.push_back(cur);
43     }
44     reverse(path.begin(), path.end());
45
46     cout << "Path from " << v << " to " << t << ": ";
47     for (int u : path) {
48         cout << u << ' ';
49     }
50 }
51 }

```

6.12 Dinic

```

1 // Description:
2 // Obtains the maximum possible flow rate given a
   network. A network is a graph with a single
   source vertex and a single sink vertex in which
   each edge has a capacity
3
4 // Problem:
5 // https://codeforces.com/gym/103708/problem/J
6
7 // Complexity:
8 //  $O(V^2 * E)$  where  $V$  is the number of vertex and  $E$ 
   is the number of edges
9
10 // Unit network
11 // A unit network is a network in which for any
   vertex except source and sink either incoming or
   outgoing edge is unique and has unit capacity (
   matching problem).
12 // Complexity on unit networks:  $O(E * \sqrt{V})$ 
13
14 // Unity capacity networks
15 // A more generic settings when all edges have unit
   capacities, but the number of incoming and
   outgoing edges is unbounded
16 // Complexity on unity capacity networks:  $O(E * \sqrt{E})$ 
17
18 // How to use:
19 // Dinic dinic = Dinic(num_vertex, source, sink);
20 // dinic.add_edge(vertex1, vertex2, capacity);
21 // cout << dinic.max_flow() << '\n';
22
23 #include <bits/stdc++.h>
24
25 #define pb push_back
26 #define mp make_pair
27 #define pii pair<int, int>
28 #define ff first
29 #define ss second
30 #define ll long long
31
32 using namespace std;
33
34 const ll INF = 1e18+10;
35
36 struct Edge {
37     int from;
38     int to;
39     ll capacity;
40     ll flow;
41     Edge* residual;
42
43     Edge() {}
44
45     Edge(int from, int to, ll capacity) : from(from),
   to(to), capacity(capacity) {
46         flow = 0;
47     }

```

```

48
49     ll get_capacity() {
50         return capacity - flow;
51     }
52
53     ll get_flow() {
54         return flow;
55     }
56
57     void augment(ll bottleneck) {
58         flow += bottleneck;
59         residual->flow -= bottleneck;
60     }
61
62     void reverse(ll bottleneck) {
63         flow -= bottleneck;
64         residual->flow += bottleneck;
65     }
66
67     bool operator<(const Edge& e) const {
68         return true;
69     }
70 };
71
72 struct Dinic {
73     int source;
74     int sink;
75     int nodes;
76     ll flow;
77     vector<vector<Edge*>> adj;
78     vector<int> level;
79     vector<int> next;
80     vector<int> reach;
81     vector<bool> visited;
82     vector<vector<int>> path;
83
84     Dinic(int source, int sink, int nodes) : source(
   source), sink(sink), nodes(nodes) {
85         adj.resize(nodes + 1);
86     }
87
88     void add_edge(int from, int to, ll capacity) {
89         Edge* e1 = new Edge(from, to, capacity);
90         Edge* e2 = new Edge(to, from, 0);
91         // Edge* e2 = new Edge(to, from, capacity);
92         e1->residual = e2;
93         e2->residual = e1;
94         adj[from].pb(e1);
95         adj[to].pb(e2);
96     }
97
98     bool bfs() {
99         level.assign(nodes + 1, -1);
100         queue<int> q;
101         q.push(source);
102         level[source] = 0;
103
104         while (!q.empty()) {
105             int node = q.front();
106             q.pop();
107
108             for (auto e : adj[node]) {
109                 if (level[e->to] == -1 && e->
   get_capacity() > 0) {
110                     level[e->to] = level[e->from] +
111                     1;
112                     q.push(e->to);
113                 }
114             }
115         }
116         return level[sink] != -1;
117     }

```

```

118 ll dfs(int v, ll flow) {
119     if (v == sink)
120         return flow;
121
122     int sz = adj[v].size();
123     for (int i = next[v]; i < sz; i++) {
124         Edge* e = adj[v][i];
125         if (level[e->to] == level[e->from] + 1 &&
126             e->get_capacity() > 0) {
127             ll bottleneck = dfs(e->to, min(flow,
128             e->get_capacity()));
129             if (bottleneck > 0) {
130                 e->augment(bottleneck);
131                 return bottleneck;
132             }
133             next[v] = i + 1;
134         }
135     }
136     return 0;
137 }
138
139 ll max_flow() {
140     flow = 0;
141     while(bfs()) {
142         next.assign(nodes + 1, 0);
143         ll sent = -1;
144         while (sent != 0) {
145             sent = dfs(source, INF);
146             flow += sent;
147         }
148     }
149     return flow;
150 }
151
152 void reachable(int v) {
153     visited[v] = true;
154
155     for (auto e : adj[v]) {
156         if (!visited[e->to] && e->get_capacity()
157 > 0) {
158             reach.pb(e->to);
159             visited[e->to] = true;
160             reachable(e->to);
161         }
162     }
163 }
164
165 void print_min_cut() {
166     reach.clear();
167     visited.assign(nodes + 1, false);
168     reach.pb(source);
169     reachable(source);
170
171     for (auto v : reach) {
172         for (auto e : adj[v]) {
173             if (!visited[e->to] && e->
174 get_capacity() == 0) {
175                 cout << e->from << ' ' << e->to
176 << '\n';
177             }
178         }
179     }
180
181 ll build_path(int v, int id, ll flow) {
182     visited[v] = true;
183     if (v == sink) {
184         return flow;
185     }
186     for (auto e : adj[v]) {
187         if (!visited[e->to] && e->get_flow() > 0)
188         {
189             visited[e->to] = true;
190             ll bottleneck = build_path(e->to, id,
191 min(flow, e->get_flow()));
192             if (bottleneck > 0) {
193                 path[id].pb(e->to);
194                 e->reverse(bottleneck);
195                 return bottleneck;
196             }
197         }
198     }
199     return 0;
200 }
201
202 void print_flow_path() {
203     path.clear();
204     ll sent = -1;
205     int id = -1;
206     while (sent != 0) {
207         visited.assign(nodes + 1, false);
208         path.pb(vector<int>{});
209         sent = build_path(source, ++id, INF);
210         path[id].pb(source);
211     }
212     path.pop_back();
213
214     for (int i = 0; i < id; i++) {
215         cout << path[i].size() << '\n';
216         reverse(path[i].begin(), path[i].end());
217         for (auto e : path[i]) {
218             cout << e << ' ';
219         }
220         cout << '\n';
221     }
222 }
223
224 int main() {
225     ios::sync_with_stdio(false);
226     cin.tie(NULL);
227
228     int n, m; cin >> n >> m;
229
230     Dinic dinic = Dinic(1, n, n);
231
232     for (int i = 1; i <= m; i++) {
233         int v, u; cin >> v >> u;
234         dinic.add_edge(v, u, 1);
235     }
236
237     cout << dinic.max_flow() << '\n';
238     // dinic.print_min_cut();
239     // dinic.print_flow_path();
240
241     return 0;
242 }

```

6.13 2sat

```

1 // Description:
2 // Solves expression of the type (a v b) ^ (c v d) ^
   (e v f)
3
4 // Problem:
5 // https://cses.fi/problemset/task/1684
6
7 // Complexity:
8 // O(n + m) where n is the number of variables and m
   is the number of clauses
9

```

```

10 #include <bits/stdc++.h>
11 #define pb push_back
12 #define mp make_pair
13 #define pii pair<int, int>
14 #define ff first
15 #define ss second
16
17 using namespace std;
18
19 struct SAT {
20     int nodes;
21     int curr = 0;
22     int component = 0;
23     vector<vector<int>> adj;
24     vector<vector<int>> rev;
25     vector<vector<int>> condensed;
26     vector<pii> departure;
27     vector<bool> visited;
28     vector<int> scc;
29     vector<int> order;
30
31     // 1 to nodes
32     // nodes + 1 to 2 * nodes
33     SAT(int nodes) : nodes(nodes) {
34         adj.resize(2 * nodes + 1);
35         rev.resize(2 * nodes + 1);
36         visited.resize(2 * nodes + 1);
37         scc.resize(2 * nodes + 1);
38     }
39
40     void add_imp(int a, int b) {
41         adj[a].pb(b);
42         rev[b].pb(a);
43     }
44
45     int get_not(int a) {
46         if (a > nodes) return a - nodes;
47         return a + nodes;
48     }
49
50     void add_or(int a, int b) {
51         add_imp(get_not(a), b);
52         add_imp(get_not(b), a);
53     }
54
55     void add_nor(int a, int b) {
56         add_or(get_not(a), get_not(b));
57     }
58
59     void add_and(int a, int b) {
60         add_or(get_not(a), b);
61         add_or(a, get_not(b));
62         add_or(a, b);
63     }
64
65     void add_nand(int a, int b) {
66         add_or(get_not(a), b);
67         add_or(a, get_not(b));
68         add_or(get_not(a), get_not(b));
69     }
70
71     void add_xor(int a, int b) {
72         add_or(a, b);
73         add_or(get_not(a), get_not(b));
74     }
75
76     void add_xnor(int a, int b) {
77         add_or(get_not(a), b);
78         add_or(a, get_not(b));
79     }
80
81     void departure_time(int v) {
82         visited[v] = true;

```

```

83
84         for (auto u : adj[v]) {
85             if (!visited[u]) departure_time(u);
86         }
87
88         departure.pb(mp(++curr, v));
89     }
90
91     void find_component(int v, int component) {
92         scc[v] = component;
93         visited[v] = true;
94
95         for (auto u : rev[v]) {
96             if (!visited[u]) find_component(u,
97                 component);
98         }
99     }
100
101     void topological_order(int v) {
102         visited[v] = true;
103
104         for (auto u : condensed[v]) {
105             if (!visited[u]) topological_order(u);
106         }
107
108         order.pb(v);
109     }
110
111     bool is_possible() {
112         component = 0;
113         for (int i = 1; i <= 2 * nodes; i++) {
114             if (!visited[i]) departure_time(i);
115         }
116
117         sort(departure.begin(), departure.end(),
118             greater<pii>());
119
120         visited.assign(2 * nodes + 1, false);
121
122         for (auto [_ , node] : departure) {
123             if (!visited[node]) find_component(node,
124                 ++component);
125         }
126
127         for (int i = 1; i <= nodes; i++) {
128             if (scc[i] == scc[i + nodes]) return
129                 false;
130         }
131
132         return true;
133     }
134
135     int find_value(int e, vector<int> &ans) {
136         if (e > nodes && ans[e - nodes] != 2) return
137             !ans[e - nodes];
138         if (e <= nodes && ans[e + nodes] != 2) return
139             !ans[e + nodes];
140         return 0;
141     }
142
143     vector<int> find_ans() {
144         condensed.resize(component + 1);
145
146         for (int i = 1; i <= 2 * nodes; i++) {
147             for (auto u : adj[i]) {
148                 if (scc[i] != scc[u]) condensed[scc[i]

```

```

149         if (!visited[i]) topological_order(i);
150     }
151
152     reverse(order.begin(), order.end());
153
154     // 0 - false
155     // 1 - true
156     // 2 - no value yet
157     vector<int> ans(2 * nodes + 1, 2);
158
159     vector<vector<int>> belong(component + 1);
160
161     for (int i = 1; i <= 2 * nodes; i++) {
162         belong[scc[i]].pb(i);
163     }
164
165     for (auto p : order) {
166         for (auto e : belong[p]) {
167             ans[e] = find_value(e, ans);
168         }
169     }
170
171     return ans;
172 }
173 };
174
175 int main() {
176     ios::sync_with_stdio(false);
177     cin.tie(NULL);
178
179     int n, m; cin >> n >> m;
180
181     SAT sat = SAT(m);
182
183     for (int i = 0; i < n; i++) {
184         char op1, op2; int a, b; cin >> op1 >> a >>
185         op2 >> b;
186         if (op1 == '+' && op2 == '+') sat.add_or(a, b);
187         if (op1 == '-' && op2 == '-') sat.add_or(sat.get_not(a), sat.get_not(b));
188         if (op1 == '+' && op2 == '-') sat.add_or(a, sat.get_not(b));
189         if (op1 == '-' && op2 == '+') sat.add_or(sat.get_not(a), b);
190     }
191
192     if (!sat.is_possible()) cout << "IMPOSSIBLE\n";
193     else {
194         vector<int> ans = sat.find_ans();
195         for (int i = 1; i <= m; i++) {
196             cout << (ans[i] == 1 ? '+' : '-') << ' ';
197         }
198         cout << '\n';
199     }
200
201     return 0;
202 }

```

6.14 Find Cycle

```

1  bitset<MAX> visited;
2  vector<int> path;
3  vector<int> adj[MAX];
4
5  bool dfs(int u, int p){
6
7      if (visited[u]) return false;
8
9      path.pb(u);
10     visited[u] = true;
11
12     for (auto v : adj[u]){

```

```

13         if (visited[v] and u != v and p != v){
14             path.pb(v); return true;
15         }
16
17         if (dfs(v, u)) return true;
18     }
19
20     path.pop_back();
21     return false;
22 }
23
24 bool has_cycle(int N){
25
26     visited.reset();
27
28     for (int u = 1; u <= N; ++u){
29         path.clear();
30         if (not visited[u] and dfs(u,-1))
31             return true;
32     }
33
34     return false;
35 }
36 }

```

6.15 Cycle Path Recovery

```

1  int n;
2  vector<vector<int>> adj;
3  vector<char> color;
4  vector<int> parent;
5  int cycle_start, cycle_end;
6
7  bool dfs(int v) {
8      color[v] = 1;
9      for (int u : adj[v]) {
10         if (color[u] == 0) {
11             parent[u] = v;
12             if (dfs(u))
13                 return true;
14         } else if (color[u] == 1) {
15             cycle_end = v;
16             cycle_start = u;
17             return true;
18         }
19     }
20     color[v] = 2;
21     return false;
22 }
23
24 void find_cycle() {
25     color.assign(n, 0);
26     parent.assign(n, -1);
27     cycle_start = -1;
28
29     for (int v = 0; v < n; v++) {
30         if (color[v] == 0 && dfs(v))
31             break;
32     }
33
34     if (cycle_start == -1) {
35         cout << "Acyclic" << endl;
36     } else {
37         vector<int> cycle;
38         cycle.push_back(cycle_start);
39         for (int v = cycle_end; v != cycle_start; v =
40             parent[v])
41             cycle.push_back(v);
42         cycle.push_back(cycle_start);
43         reverse(cycle.begin(), cycle.end());
44
45         cout << "Cycle found: ";
46         for (int v : cycle)

```

```

46         cout << v << " ";
47     cout << endl;
48 }
49 }

```

6.16 Blossom

```

1 // Description:
2 // Matching algorithm for general graphs (non-
   bipartite)
3
4 // Problem:
5 // https://acm.timus.ru/problem.aspx?space=1&num=1099
6
7 // Complexity:
8 // O(n^3)
9
10 // vector<pii> Blossom(vector<vector<int>>& graph) {
11 vector<int> Blossom(vector<vector<int>>& graph) {
12     int n = graph.size(), timer = -1;
13     vector<int> mate(n, -1), label(n), parent(n),
14         orig(n), aux(n, -1), q;
15     auto lca = [&](int x, int y) {
16         for (timer++; ; swap(x, y)) {
17             if (x == -1) continue;
18             if (aux[x] == timer) return x;
19             aux[x] = timer;
20             x = (mate[x] == -1 ? -1 : orig[parent[mate[x]
21 ]]);
22         }
23     };
24     auto blossom = [&](int v, int w, int a) {
25         while (orig[v] != a) {
26             parent[v] = w; w = mate[v];
27             if (label[w] == 1) label[w] = 0, q.push_back(w)
28         };
29         orig[v] = orig[w] = a; v = parent[w];
30     };
31     auto augment = [&](int v) {
32         while (v != -1) {
33             int pv = parent[v], nv = mate[pv];
34             mate[v] = pv; mate[pv] = v; v = nv;
35         }
36     };
37     auto bfs = [&](int root) {
38         fill(label.begin(), label.end(), -1);
39         iota(orig.begin(), orig.end(), 0);
40         q.clear();
41         label[root] = 0; q.push_back(root);
42         for (int i = 0; i < (int)q.size(); ++i) {
43             int v = q[i];
44             for (auto x : graph[v]) {
45                 if (label[x] == -1) {
46                     label[x] = 1; parent[x] = v;
47                     if (mate[x] == -1)
48                         return augment(x), 1;
49                     label[mate[x]] = 0; q.push_back(mate[x]);
50                 } else if (label[x] == 0 && orig[v] != orig[x]
51 ) {
52                     int a = lca(orig[v], orig[x]);
53                     blossom(x, v, a); blossom(v, x, a);
54                 }
55             }
56         }
57         return 0;
58     };
59     // Time halves if you start with (any) maximal
60     matching.
61     for (int i = 0; i < n; i++)
62         if (mate[i] == -1)
63             bfs(i);
64     return mate;

```

```

62
63 /*
64 vector<bool> used(n, false);
65 vector<pii> ans;
66 for (int i = 0; i < n; i++) {
67     if (matching[i] == -1 || used[i]) continue;
68     used[i] = true;
69     used[matching[i]] = true;
70     ans.emplace_back(i, matching[i]);
71 }
72 return ans;
73 */
74 }

```

6.17 Centroid Decomposition

```

1 int n;
2 vector<set<int>> adj;
3 vector<char> ans;
4
5 vector<bool> removed;
6
7 vector<int> subtree_size;
8
9 int dfs(int u, int p = 0) {
10     subtree_size[u] = 1;
11
12     for(int v : adj[u]) {
13         if(v != p && !removed[v]) {
14             subtree_size[u] += dfs(v, u);
15         }
16     }
17
18     return subtree_size[u];
19 }
20
21 int get_centroid(int u, int sz, int p = 0) {
22     for(int v : adj[u]) {
23         if(v != p && !removed[v]) {
24             if(subtree_size[v]*2 > sz) {
25                 return get_centroid(v, sz, u);
26             }
27         }
28     }
29
30     return u;
31 }
32
33 char get_next(char c) {
34     if (c != 'Z') return c + 1;
35     return '$';
36 }
37
38 bool flag = true;
39
40 void solve(int node, char c) {
41     int center = get_centroid(node, dfs(node));
42     ans[center] = c;
43     removed[center] = true;
44
45     for (auto u : adj[center]) {
46         if (!removed[u]) {
47             char next = get_next(c);
48             if (next == '$') {
49                 flag = false;
50                 return;
51             }
52             solve(u, next);
53         }
54     }
55 }
56
57 int32_t main(){

```

```

58 ios::sync_with_stdio(false);
59 cin.tie(NULL);
60
61 cin >> n;
62 adj.resize(n + 1);
63 ans.resize(n + 1);
64 removed.resize(n + 1);
65 subtree_size.resize(n + 1);
66
67 for (int i = 1; i <= n - 1; i++) {
68     int u, v; cin >> u >> v;
69     adj[u].insert(v);
70     adj[v].insert(u);
71 }
72
73 solve(1, 'A');
74
75 if (!flag) cout << "Impossible!\n";
76 else {
77     for (int i = 1; i <= n; i++) {
78         cout << ans[i] << ' ';
79     }
80     cout << '\n';
81 }
82
83 return 0;
84 }

```

6.18 Tarjan Bridge

```

1 // Description:
2 // Find a bridge in a connected undirected graph
3 // A bridge is an edge so that if you remove that
4 // edge the graph is no longer connected
5
6 // Problem:
7 // https://cses.fi/problemset/task/2177/
8
9 // Complexity:
10 // O(V + E) where V is the number of vertices and E
11 // is the number of edges
12
13 int n;
14 vector<vector<int>>> adj;
15
16 vector<bool> visited;
17 vector<int> tin, low;
18 int timer;
19
20 void dfs(int v, int p) {
21     visited[v] = true;
22     tin[v] = low[v] = timer++;
23     for (int to : adj[v]) {
24         if (to == p) continue;
25         if (visited[to]) {
26             low[v] = min(low[v], tin[to]);
27         } else {
28             dfs(to, v);
29             low[v] = min(low[v], low[to]);
30             if (low[to] > tin[v]) {
31                 IS_BRIDGE(v, to);
32             }
33         }
34     }
35 }
36
37 void find_bridges() {
38     timer = 0;
39     visited.assign(n, false);
40     tin.assign(n, -1);
41     low.assign(n, -1);
42     for (int i = 0; i < n; ++i) {
43         if (!visited[i])

```

```

42         dfs(i, -1);
43     }
44 }

```

6.19 Hld Vertex

```

1 // Description:
2 // Make queries and updates between two vertexes on a
3 // tree
4 // Query path - query path (a, b) inclusive
5 // Update path - update path (a, b) inclusive
6 // Query subtree - query subtree of a
7 // Update subtree - update subtree of a
8 // Update - update vertex or edge
9 // Lca - get lowest common ancestor of a and b
10 // Search - perform a binary search to find the last
11 // node with a certain property
12 // on the path from a to the root
13
14 // Problem:
15 // https://codeforces.com/gym/101908/problem/L
16
17 // Complexity:
18 // O(log2 n) for both query and update
19
20 // How to use:
21 // HLD hld = HLD(n + 1, adj)
22
23 // Notes
24 // Change the root of the tree on the constructor if
25 // it's different from 1
26 // Use together with Segtree
27
28 typedef long long ftype;
29
30 struct HLD {
31     vector<int> parent;
32     vector<int> pos;
33     vector<int> head;
34     vector<int> subtree_size;
35     vector<int> level;
36     vector<int> heavy_child;
37     vector<ftype> subtree_weight;
38     vector<ftype> path_weight;
39     vector<vector<int>>> adj;
40     vector<int> at;
41     Segtree seg = Segtree(0);
42     int cpos;
43     int n;
44     int root;
45     vector<vector<int>>> up;
46
47     HLD() {}
48
49     HLD(int n, vector<vector<int>>>& adj, int root = 1)
50         : adj(adj), n(n), root(root) {
51         seg = Segtree(n);
52         cpos = 0;
53         at.resize(n);
54         parent.resize(n);
55         pos.resize(n);
56         head.resize(n);
57         subtree_size.assign(n, 1);
58         level.assign(n, 0);
59         heavy_child.assign(n, -1);
60         parent[root] = -1;
61         dfs(root, -1);
62         decompose(root, -1);
63     }
64
65     void dfs(int v, int p) {
66         parent[v] = p;
67         if (p != -1) level[v] = level[p] + 1;

```

```

64     for (auto u : adj[v]) {
65         if (u != p) {
66             dfs(u, v);
67             subtree_size[v] += subtree_size[u];
68             if (heavy_child[v] == -1 || subtree_size[u] > subtree_size[heavy_child[v]]) heavy_child[v] = u;
69         }
70     }
71 }
72
73 void decompose(int v, int head) {
74     // start a new path
75     if (head == -1) head = v;
76
77     // consecutive ids in the hld path
78     at[cpos] = v;
79     pos[v] = cpos++;
80     head[v] = head;
81
82     // if not a leaf
83     if (heavy_child[v] != -1) decompose(heavy_child[v], head);
84
85     // light child
86     for (auto u : adj[v]){
87         // start new path
88         if (u != parent[v] && u != heavy_child[v])
89             decompose(u, -1);
90     }
91
92     ftype query_path(int a, int b) {
93         if(pos[a] < pos[b]) swap(a, b);
94
95         if(head[a] == head[b]) return seg.query(pos[b], pos[a]);
96         return seg.f(seg.query(pos[head[a]], pos[a]), query_path(parent[head[a]], b));
97     }
98
99     // iterative
100     /*ftype query_path(int a, int b) {
101         ftype ans = 0;
102
103         while (head[a] != head[b]) {
104             if (level[head[a]] > level[head[b]]) swap(a, b);
105             ans = seg.merge(ans, seg.query(pos[head[b]], pos[b]));
106             b = parent[head[b]];
107         }
108
109         if (level[a] > level[b]) swap(a, b);
110         ans = seg.merge(ans, seg.query(pos[a], pos[b]));
111         return ans;
112     }*/
113
114     ftype query_subtree(int a) {
115         return seg.query(pos[a], pos[a] + subtree_size[a] - 1);
116     }
117
118     void update_path(int a, int b, int x) {
119         if(pos[a] < pos[b]) swap(a, b);
120
121         if(head[a] == head[b]) return (void)seg.update(pos[b], pos[a], x);
122         seg.update(pos[head[a]], pos[a], x); update_path(parent[head[a]], b, x);
123     }
124
125     void update_subtree(int a, int val) {
126         seg.update(pos[a], pos[a] + subtree_size[a] - 1, val);
127     }
128
129     void update(int a, int val) {
130         seg.update(pos[a], pos[a], val);
131     }
132
133     //edge
134     void update(int a, int b, int val) {
135         if (level[a] > level[b]) swap(a, b);
136         update(b, val);
137     }
138
139     int lca(int a, int b) {
140         if(pos[a] < pos[b]) swap(a, b);
141         return head[a] == head[b] ? b : lca(parent[head[a]], b);
142     }
143
144     void search(int a) {
145         a = parent[a];
146         if (a == -1) return;
147         if (seg.query(pos[head[a]], pos[head[a]] + subtree_size[head[a]] - 1) + pos[a] - pos[head[a]] + 1 == subtree_size[head[a]]) {
148             seg.update(pos[head[a]], pos[a], 1);
149             return search(parent[head[a]]);
150         }
151         int l = pos[head[a]], r = pos[a] + 1;
152         while (l < r) {
153             int m = (l + r) / 2;
154             if (seg.query(m, m + subtree_size[at[m]] - 1) + pos[a] - m + 1 == subtree_size[at[m]]) {
155                 r = m;
156             }
157             else l = m + 1;
158         }
159         seg.update(l, pos[a], 1);
160     }
161
162     /* k-th ancestor of x
163     int x, k; cin >> x >> k;
164
165     for (int b = 0; b <= BITS; b++) {
166         if (x != -1 && (k & (1 << b))) {
167             x = up[x][b];
168         }
169     }
170
171     cout << x << '\n';
172     */
173     void preprocess() {
174         up.assign(n + 1, vector<int>(31, -1));
175
176         for (int i = 1; i < n; i++) {
177             up[i][0] = parent[i];
178         }
179
180         for (int i = 1; i < n; i++) {
181             for (int j = 1; j <= 30; j++) {
182                 if (up[i][j - 1] != -1) up[i][j] = up[up[i][j - 1]][j - 1];
183             }
184         }
185     }
186
187     int getKth(int p, int q, int k){
188         int a = lca(p, q), d;
189
190         if( a == p ){
191             d = level[q] - level[p] + 1;
192             swap(p, q);

```



```

193     k = d - k + 1 ;
194 }
195 else if( a == q ) ;
196 else {
197     if( k > level[p] - level[a] + 1 ) {
198         d = level[p] + level[q] - 2 * level[a] +
199         1 ;
200         k = d - k + 1 ;
201         swap(p,q);
202     }
203     else ;
204 }
205 int lg ; for( lg = 1 ; (1 << lg) <= level[p] ; ++
206 lg ); lg--;
207 k--;
208 for( int i = lg ; i >= 0 ; i-- ){
209     if( (1 << i) <= k ){
210         p = up[p][i];
211         k -= ( 1 << i );
212     }
213 }
214 return p;
215 }
216 };

```

6.20 Small To Large

```

1 // Problem:
2 // https://codeforces.com/contest/600/problem/E
3
4 void process_colors(int curr, int parent) {
5
6     for (int n : adj[curr]) {
7         if (n != parent) {
8             process_colors(n, curr);
9
10            if (colors[curr].size() < colors[n].size
11            ()) {
12                sum_num[curr] = sum_num[n];
13                vmax[curr] = vmax[n];
14                swap(colors[curr], colors[n]);
15            }
16
17            for (auto [item,vzs] : colors[n]) {
18                if(colors[curr][item]+vzs > vmax[curr
19                ]){
20                    vmax[curr] = colors[curr][item] +
21                    vzs;
22                    sum_num[curr] = item;
23                }
24                else if(colors[curr][item]+vzs ==
25                vmax[curr]){
26                    sum_num[curr] += item;
27                }
28                colors[curr][item] += vzs;
29            }
30        }
31    }
32
33    int32_t main() {
34
35        int n; cin >> n;
36
37        for (int i = 1; i <= n; i++) {
38            int a; cin >> a;
39            colors[i][a] = 1;
40            vmax[i] = 1;
41            sum_num[i] = a;
42        }

```

```

43
44     for (int i = 1; i < n; i++) {
45         int a, b; cin >> a >> b;
46
47         adj[a].push_back(b);
48         adj[b].push_back(a);
49     }
50
51     process_colors(1, 0);
52
53     for (int i = 1; i <= n; i++) {
54         cout << sum_num[i] << (i < n ? " " : "\n");
55     }
56
57     return 0;
58 }
59
60

```

6.21 Tree Diameter

```

1 #include<bits/stdc++.h>
2
3 using namespace std;
4
5 const int MAX = 3e5+17;
6
7 vector<int> adj[MAX];
8 bool visited[MAX];
9
10 int max_depth = 0, max_node = 1;
11
12 void dfs (int v, int depth) {
13     visited[v] = true;
14
15     if (depth > max_depth) {
16         max_depth = depth;
17         max_node = v;
18     }
19
20     for (auto u : adj[v]) {
21         if (!visited[u]) dfs(u, depth + 1);
22     }
23 }
24
25 int tree_diameter() {
26     dfs(1, 0);
27     max_depth = 0;
28     for (int i = 0; i < MAX; i++) visited[i] = false;
29     dfs(max_node, 0);
30     return max_depth;
31 }

```

6.22 Dijkstra

```

1 const int MAX = 2e5+7;
2 const int INF = 1000000000;
3 vector<vector<pair<int, int>>> adj(MAX);
4
5 void dijkstra(int s, vector<int> & d, vector<int> & p
6 ) {
7     int n = adj.size();
8     d.assign(n, INF);
9     p.assign(n, -1);
10
11     d[s] = 0;
12     set<pair<int, int>> q;
13     q.insert({0, s});
14     while (!q.empty()) {
15         int v = q.begin()->second;
16         q.erase(q.begin());

```

```

17     for (auto edge : adj[v]) {
18         int to = edge.first;
19         int len = edge.second;
20
21         if (d[v] + len < d[to]) {
22             q.erase({d[to], to});
23             d[to] = d[v] + len;
24             p[to] = v;
25             q.insert({d[to], to});
26         }
27     }
28 }
29
30 vector<int> restore_path(int s, int t) {
31     vector<int> path;
32
33     for (int v = t; v != s; v = p[v])
34         path.push_back(v);
35     path.push_back(s);
36
37     reverse(path.begin(), path.end());
38     return path;
39 }
40
41 int adj[MAX][MAX];
42 int dist[MAX];
43 int minDistance(int dist[], bool sptSet[], int V) {
44     int min = INT_MAX, min_index;
45
46     for (int v = 0; v < V; v++)
47         if (sptSet[v] == false && dist[v] <= min)
48             min = dist[v], min_index = v;
49
50     return min_index;
51 }
52
53 void dijkstra(int src, int V) {
54     bool sptSet[V];
55     for (int i = 0; i < V; i++)
56         dist[i] = INT_MAX, sptSet[i] = false;
57
58     dist[src] = 0;
59
60     for (int count = 0; count < V - 1; count++) {
61         int u = minDistance(dist, sptSet, V);
62
63         sptSet[u] = true;
64
65         for (int v = 0; v < V; v++)
66             if (!sptSet[v] && adj[u][v]
67                 && dist[u] != INT_MAX
68                 && dist[u] + adj[u][v] < dist[v])
69                 dist[v] = dist[u] + adj[u][v];
70     }
71 }
72
73 }
74

```

6.23 Kruskal

```

1 struct DSU {
2     int n;
3     vector<int> link, sizes;
4
5     DSU(int n) {
6         this->n = n;
7         link.assign(n+1, 0);
8         sizes.assign(n+1, 1);
9
10        for (int i = 0; i <= n; i++)
11            link[i] = i;
12    }

```

```

13
14 int find(int x) {
15     while (x != link[x])
16         x = link[x];
17
18     return x;
19 }
20
21 bool same(int a, int b) {
22     return find(a) == find(b);
23 }
24
25 void unite(int a, int b) {
26     a = find(a);
27     b = find(b);
28
29     if (a == b) return;
30
31     if (sizes[a] < sizes[b])
32         swap(a, b);
33
34     sizes[a] += sizes[b];
35     link[b] = a;
36 }
37 };
38
39 struct Edge {
40     int u, v;
41     long long weight;
42
43     Edge() {}
44
45     Edge(int u, int v, long long weight) : u(u), v(v),
46         weight(weight) {}
47
48     bool operator<(const Edge& other) const {
49         return weight < other.weight;
50     }
51
52     bool operator>(const Edge& other) const {
53         return weight > other.weight;
54     }
55 };
56
57 vector<Edge> kruskal(vector<Edge> edges, int n) {
58     vector<Edge> result; // arestas da MST
59     long long cost = 0;
60
61     sort(edges.begin(), edges.end());
62
63     DSU dsu(n);
64
65     for (auto e : edges) {
66         if (!dsu.same(e.u, e.v)) {
67             cost += e.weight;
68             result.push_back(e);
69             dsu.unite(e.u, e.v);
70         }
71     }
72
73     return result;
74 }

```

6.24 Hungarian

```

1 // Description:
2 // A matching algorithm for weighted bipartite graphs
3 // that returns
4 // a perfect match
5
6 // Problem:
7 // https://codeforces.com/gym/103640/problem/H

```

```

8 // Complexity:
9 //  $O(V^3)$  in which  $V$  is the number of vertices
10
11 // Notes:
12 // Indexed at 1
13
14 //  $n$  is the number of items on the right side and  $m$ 
15 // the number of items
16 // on the left side of the graph
17 // Returns minimum assignment cost and which items
18 // were matched
19
20 pair<int, vector<pii>> hungarian(int n, int m, vector
21 <vector<int>> A) {
22     vector<int> u (n+1), v (m+1), p (m+1), way (m+1);
23     for (int i=1; i<=n; ++i) {
24         p[0] = i;
25         int j0 = 0;
26         vector<int> minv (m+1, INF);
27         vector<char> used (m+1, false);
28         do {
29             used[j0] = true;
30             int i0 = p[j0], delta = INF, j1;
31             for (int j=1; j<=m; ++j)
32                 if (!used[j]) {
33                     int cur = A[i0][j]-u[i0]-v[j];
34                     if (cur < minv[j])
35                         minv[j] = cur, way[j] = j0;
36                     if (minv[j] < delta)
37                         delta = minv[j], j1 = j;
38                 }
39             for (int j=0; j<=m; ++j)
40                 if (used[j])
41                     u[p[j]] += delta, v[j] -= delta;
42                 else
43                     minv[j] -= delta;
44             j0 = j1;
45         } while (p[j0] != 0);
46         do {
47             int j1 = way[j0];
48             p[j0] = p[j1];
49             j0 = j1;
50         } while (j0);
51     }
52     vector<pair<int, int>> result;
53     for (int i = 1; i <= m; ++i){
54         result.push_back(make_pair(p[i], i));
55     }
56     int C = -v[0];
57     return mp(C, result);
58 }

```

6.25 Negative Cycle

```

1 // Description
2 // Detects any cycle in which the sum of edge weights
3 // is negative.
4 // Alternatively, we can detect whether there is a
5 // negative cycle
6 // starting from a specific vertex.
7
8 // Problem:
9 // https://cses.fi/problemset/task/1197
10
11 // Complexity:
12 //  $O(n * m)$ 
13
14 // Notes

```

```

13 // In order to consider only the negative cycles
14 // located on the path from a to b,
15 // Reverse the graph, run a dfs from node b and mark
16 // the visited nodes
17 // Consider only the edges that connect to visited
18 // nodes when running bellman-ford
19 // on the normal graph
20
21 struct Edge {
22     int a, b, cost;
23     Edge(int a, int b, int cost) : a(a), b(b), cost(
24         cost) {}
25 };
26
27 int n, m;
28 vector<Edge> edges;
29 const int INF = 1e9+10;
30
31 void negative_cycle() {
32     // uncomment to find negative cycle starting from a
33     // vertex v
34     // vector<int> d(n + 1, INF);
35     // d[v] = 0;
36     vector<int> d(n + 1, 0);
37     vector<int> p(n + 1, -1);
38     int x;
39     // uncomment to find all negative cycles
40     // // set<int> s;
41     for (int i = 1; i <= n; ++i) {
42         x = -1;
43         for (Edge e : edges) {
44             // if (d[e.a] >= INF) continue;
45             if (d[e.b] > d[e.a] + e.cost) {
46                 // d[e.b] = max(-INF, d[e.a] + e.cost);
47                 d[e.b] = d[e.a] + e.cost;
48                 p[e.b] = e.a;
49                 x = e.b;
50                 // // s.insert(e.b);
51             }
52         }
53     }
54     if (x == -1)
55         cout << "NO\n";
56     else {
57         // // int y = all nodes in set s
58         int y = x;
59         for (int i = 1; i <= n; ++i) {
60             y = p[y];
61         }
62         vector<int> path;
63         for (int cur = y; cur = p[cur]) {
64             path.push_back(cur);
65             if (cur == y && path.size() > 1) break;
66         }
67         reverse(path.begin(), path.end());
68
69         cout << "YES\n";
70         for (int u : path)
71             cout << u << ' ';
72         cout << '\n';
73     }
74 }

```

7 Geometry

7.1 Shoelace Boundary

```

1 // Description
2 // Shoelace formula finds the area of a polygon

```

```

3 // Boundary points return the number of integer
  points on the edges of a polygon
4 // not counting the vertexes
5
6 // Problem
7 // https://codeforces.com/gym/101873/problem/G
8
9 // Complexity
10 // O(n)
11
12 // before dividing by two
13 int shoelace(vector<point> & points) {
14     int n = points.size();
15     vector<point> v(n + 2);
16
17     for (int i = 1; i <= n; i++) {
18         v[i] = points[i - 1];
19     }
20     v[n + 1] = points[0];
21
22     int sum = 0;
23     for (int i = 1; i <= n; i++) {
24         sum += (v[i].x * v[i + 1].y - v[i + 1].x * v[i].y);
25     }
26
27     sum = abs(sum);
28     return sum;
29 }
30
31 int boundary_points(vector<point> & points) {
32     int n = points.size();
33     vector<point> v(n + 2);
34
35     for (int i = 1; i <= n; i++) {
36         v[i] = points[i - 1];
37     }
38     v[n + 1] = points[0];
39
40     int ans = 0;
41     for (int i = 1; i <= n; i++) {
42         if (v[i].x == v[i + 1].x) ans += abs(v[i].y - v[i + 1].y) - 1;
43         else if (v[i].y == v[i + 1].y) ans += abs(v[i].x - v[i + 1].x) - 1;
44         else ans += gcd(abs(v[i].x - v[i + 1].x), abs(v[i].y - v[i + 1].y)) - 1;
45     }
46     return points.size() + ans;
47 }

```

7.2 Inside Polygon

```

1 // Description
2 // Checks if a given point is inside, outside or on
  the boundary of a polygon
3
4 // Problem
5 // https://cses.fi/problemset/task/2192/
6
7 // Complexity
8 // O(n)
9
10 int inside(vp &p, point pp){
11     // 1 - inside / 0 - boundary / -1 - outside
12     int n = p.size();
13     for(int i=0;i<n;i++){
14         int j = (i+1)%n;
15         if(line({p[i], p[j]}).inside_seg(pp))
16             return 0; // boundary
17     }
18     int inter = 0;
19     for(int i=0;i<n;i++){

```

```

20         int j = (i+1)%n;
21         if(p[i].x <= pp.x and pp.x < p[j].x and ccw(p
[i], p[j], pp)==1)
22             inter++; // up
23         else if(p[j].x <= pp.x and pp.x < p[i].x and
ccw(p[i], p[j], pp)==-1)
24             inter++; // down
25     }
26
27     if(inter%2==0) return -1; // outside
28     else return 1; // inside
29 }

```

7.3 Closest Pair Points

```

1 // Description
2 // Find the squared distance between the closest two
  points among n points
3 // Also finds which pair of points is closest (could
  be more than one)
4
5 // Problem
6 // https://cses.fi/problemset/task/2194/
7
8 // Complexity
9 // O(n log n)
10
11 ll closest_pair_points(vp &vet){
12     pair<point, point> ans;
13     int n = vet.size();
14     sort(vet.begin(), vet.end());
15     set<point> s;
16
17     ll best_dist = LLONG_MAX;
18     int j=0;
19     for(int i=0;i<n;i++){
20         ll d = ceil(sqrt(best_dist));
21         while(j<n and vet[i].x-vet[j].x >= d){
22             s.erase(point(vet[j].y, vet[j].x));
23             j++;
24         }
25
26         auto it1 = s.lower_bound({vet[i].y - d, vet[i].x});
27         auto it2 = s.upper_bound({vet[i].y + d, vet[i].x});
28
29         for(auto it=it1; it!=it2; it++){
30             ll dx = vet[i].x - it->y;
31             ll dy = vet[i].y - it->x;
32
33             if(best_dist > dx*dx + dy*dy){
34                 best_dist = dx*dx + dy*dy;
35                 // closest pair points
36                 ans = mp(vet[i], point(it->y, it->x));
37             }
38         }
39
40         s.insert(point(vet[i].y, vet[i].x));
41     }
42
43     // best distance squared
44     return best_dist;
45 }

```

7.4 2d

```

1 #define vp vector<point>
2 #define ld long double
3 const ld EPS = 1e-6;
4 const ld PI = acos(-1);

```

```

5
6 // typedef ll cod;
7 // bool eq(cod a, cod b){ return (a==b); }
8 typedef ld cod;
9 bool eq(cod a, cod b){ return abs(a - b) <= EPS; }
10
11 struct point{
12     cod x, y;
13     int id;
14     point(cod x=0, cod y=0): x(x), y(y){}
15
16     point operator+(const point &o) const{ return {x+o.x, y+o.y}; }
17     point operator-(const point &o) const{ return {x-o.x, y-o.y}; }
18     point operator*(cod t) const{ return {x*t, y*t}; }
19     point operator/(cod t) const{ return {x/t, y/t}; }
20     cod operator*(const point &o) const{ return x * o.x + y * o.y; }
21     cod operator^(const point &o) const{ return x * o.y - y * o.x; }
22     bool operator<(const point &o) const{
23         return (eq(x, o.x) ? y < o.y : x < o.x);
24     }
25     bool operator==(const point &o) const{
26         return eq(x, o.x) and eq(y, o.y);
27     }
28     friend ostream& operator<<(ostream& os, point p) {
29         return os << "(" << p.x << "," << p.y << ")"; }
30 };
31
32 int ccw(point a, point b, point e){ // -1=dir; 0=
33     collinear; 1=esq;
34     cod tmp = (b-a) ^ (e-a); // vector from a to b
35     return (tmp > EPS) - (tmp < -EPS);
36 }
37 ld norm(point a){ // Modulo
38     return sqrt(a * a);
39 }
40 cod norm2(point a){
41     return a * a;
42 }
43 bool nulo(point a){
44     return (eq(a.x, 0) and eq(a.y, 0));
45 }
46 point rotccw(point p, ld a){
47     // a = PI*a/180; // graus
48     return point((p.x*cos(a)-p.y*sin(a)), (p.y*cos(a)+p.x*sin(a)));
49 }
50 point rot90cw(point a) { return point(a.y, -a.x); }
51 point rot90ccw(point a) { return point(-a.y, a.x); }
52
53 ld proj(point a, point b){ // a sobre b
54     return a*b/norm(b);
55 }
56 ld angle(point a, point b){ // em radianos
57     ld ang = a*b / norm(a) / norm(b);
58     return acos(max(min(ang, (ld)1), (ld)-1));
59 }
60 ld angle_vec(point v){
61     // return 180/PI*atan2(v.x, v.y); // graus
62     return atan2(v.x, v.y);
63 }
64 ld order_angle(point a, point b){ // from a to b ccw
65     (a in front of b)
66     ld aux = angle(a,b)*180/PI;
67     return ((a^b)<=0 ? aux:360-aux);
68 }
69 bool angle_less(point a1, point b1, point a2, point
b2){ // ang(a1,b1) <= ang(a2,b2)
70     point p1((a1*b1), abs((a1^b1)));
71     point p2((a2*b2), abs((a2^b2)));
72     return (p1^p2) <= 0;
73 }
74 ld area(vp &p){ // (points sorted)
75     ld ret = 0;
76     for(int i=2;i<(int)p.size();i++)
77         ret += (p[i]-p[0])^(p[i-1]-p[0]);
78     return abs(ret/2);
79 }
80 ld areaT(point &a, point &b, point &c){
81     return abs((b-a)^(c-a))/2.0;
82 }
83
84 point center(vp &A){
85     point c = point();
86     int len = A.size();
87     for(int i=0;i<len;i++)
88         c=c+A[i];
89     return c/len;
90 }
91
92 point forca_mod(point p, ld m){
93     ld cm = norm(p);
94     if(cm<EPS) return point();
95     return point(p.x*m/cm,p.y*m/cm);
96 }
97
98 ld param(point a, point b, point v){
99     // v = t*(b-a) + a // return t;
100     // assert(line(a, b).inside_seg(v));
101     return ((v-a) * (b-a)) / ((b-a) * (b-a));
102 }
103
104 bool simetric(vp &a){ //ordered
105     int n = a.size();
106     point c = center(a);
107     if(n&1) return false;
108     for(int i=0;i<n/2;i++)
109         if(ccw(a[i], a[i+n/2], c) != 0)
110             return false;
111     return true;
112 }
113
114 point mirror(point m1, point m2, point p){
115     // mirror point p around segment m1m2
116     point seg = m2-m1;
117     ld t0 = ((p-m1)*seg) / (seg*seg);
118     point ort = m1 + seg*t0;
119     point pm = ort-(p-ort);
120     return pm;
121 }
122
123
124 ///////////////
125 // Line //
126 ///////////////
127
128 struct line{
129     point p1, p2;
130     cod a, b, c; // ax+by+c = 0;
131     // y-y1 = ((y2-y1)/(x2-x1))(x-x1)
132     line(point p1=0, point p2=0): p1(p1), p2(p2){
133         a = p1.y - p2.y;
134         b = p2.x - p1.x;
135         c = p1 ^ p2;
136     }
137     line(cod a=0, cod b=0, cod c=0): a(a), b(b), c(c)
138     {
139         // Gera os pontos p1 p2 dados os coeficientes
140         // isso aqui eh um lixo mas quebra um galho

```

```

140 kkkkkk
141     if(b==0){
142         p1 = point(1, -c/a);
143         p2 = point(0, -c/a);
144     }else{
145         p1 = point(1, (-c-a*1)/b);
146         p2 = point(0, -c/b);
147     }
148
149     cod eval(point p){
150         return a*p.x+b*p.y+c;
151     }
152     bool inside(point p){
153         return eq(eval(p), 0);
154     }
155     point normal(){
156         return point(a, b);
157     }
158
159     bool inside_seg(point p){
160         return (
161             ((p1-p) ^ (p2-p)) == 0 and
162             ((p1-p) * (p2-p)) <= 0
163         );
164     }
165
166 };
167
168 // be careful with precision error
169 vp inter_line(line l1, line l2){
170     ld det = l1.a*l2.b - l1.b*l2.a;
171     if(det==0) return {};
172     ld x = (l1.b*l2.c - l1.c*l2.b)/det;
173     ld y = (l1.c*l2.a - l1.a*l2.c)/det;
174     return {point(x, y)};
175 }
176
177 // segments not collinear
178 vp inter_seg(line l1, line l2){
179     vp ans = inter_line(l1, l2);
180     if(ans.empty() or !l1.inside_seg(ans[0]) or !l2.
181         inside_seg(ans[0]))
182         return {};
183     return ans;
184 }
185
186 bool seg_has_inter(line l1, line l2){
187     // if collinear
188     if (l1.inside_seg(l2.p1) || l1.inside_seg(l2.p2)
189         || l2.inside_seg(l1.p1) || l2.inside_seg(l1.p2))
190         return true;
191
192     return ccw(l1.p1, l1.p2, l2.p1) * ccw(l1.p1, l1.
193         p2, l2.p2) < 0 and
194         ccw(l2.p1, l2.p2, l1.p1) * ccw(l2.p1, l2.
195         p2, l1.p2) < 0;
196 }
197
198 ld dist_seg(point p, point a, point b){ // point -
199     seg
200     if((p-a)*(b-a) < EPS) return norm(p-a);
201     if((p-b)*(a-b) < EPS) return norm(p-b);
202     return abs((p-a)^(b-a)) / norm(b-a);
203 }
204
205 ld dist_line(point p, line l){ // point - line
206     return abs(l.eval(p))/sqrt(1.a*1.a + 1.b*1.b);
207 }
208
209 line bisector(point a, point b){
210     point d = (b-a)*2;
211     return line(d.x, d.y, a*a - b*b);
212 }
213
214 line perpendicular(line l, point p){ // passes
215     through p
216     return line(1.b, -1.a, -1.b*p.x + 1.a*p.y);
217 }
218
219 // Circle
220
221 struct circle{
222     point c; cod r;
223     circle() : c(0, 0), r(0){}
224     circle(const point o) : c(o), r(0){}
225     circle(const point a, const point b){
226         c = (a+b)/2;
227         r = norm(a-c);
228     }
229     circle(const point a, const point b, const point
230         cc){
231         assert(ccw(a, b, cc) != 0);
232         c = inter_line(bisector(a, b), bisector(b, cc
233         ))[0];
234         r = norm(a-c);
235     }
236     bool inside(const point &a) const{
237         return norm(a - c) <= r + EPS;
238     }
239 }
240
241 pair<point, point> tangent_points(circle cr, point p)
242 {
243     ld d1 = norm(p-cr.c), theta = asin(cr.r/d1);
244     point p1 = rotccw(cr.c-p, -theta);
245     point p2 = rotccw(cr.c-p, theta);
246     assert(d1 >= cr.r);
247     p1 = p1 * (sqrt(d1*d1-cr.r*cr.r) / d1) + p;
248     p2 = p2 * (sqrt(d1*d1-cr.r*cr.r) / d1) + p;
249     return {p1, p2};
250 }
251
252 circle incircle(point p1, point p2, point p3){
253     ld m1 = norm(p2-p3);
254     ld m2 = norm(p1-p3);
255     ld m3 = norm(p1-p2);
256     point c = (p1*m1 + p2*m2 + p3*m3)*(1/(m1+m2+m3));
257     ld s = 0.5*(m1+m2+m3);
258     ld r = sqrt(s*(s-m1)*(s-m2)*(s-m3)) / s;
259     return circle(c, r);
260 }
261
262 circle circumcircle(point a, point b, point c) {
263     circle ans;
264     point u = point((b-a).y, -(b-a).x);
265     point v = point((c-a).y, -(c-a).x);
266     point n = (c-b)*0.5;
267     ld t = (u^n)/(v^u);
268     ans.c = ((a+c)*0.5) + (v*t);
269     ans.r = norm(ans.c-a);
270     return ans;
271 }
272
273 vp inter_circle_line(circle C, line L){
274     point ab = L.p2 - L.p1, p = L.p1 + ab * ((C.c-L.
275         p1)*(ab) / (ab*ab));
276     ld s = (L.p2-L.p1)^(C.c-L.p1), h2 = C.r*C.r - s*s
277         / (ab*ab);
278     if (h2 < -EPS) return {};
279     if (eq(h2, 0)) return {p};
280     point h = (ab/norm(ab)) * sqrt(h2);
281     return {p - h, p + h};
282 }

```

```

273 }
274
275 vp inter_circle(circle C1, circle C2){
276     if(C1.c == C2.c) { assert(C1.r != C2.r); return
        {};}
277     point vec = C2.c - C1.c;
278     ld d2 = vec*vec, sum = C1.r+C2.r, dif = C1.r-C2.r
        ;
279     ld p = (d2 + C1.r*C1.r - C2.r*C2.r)/(d2*2), h2 =
        C1.r*C1.r - p*p*d2;
280     if (sum*sum < d2 or dif*dif > d2) return {};
281     point mid = C1.c + vec*p, per = point(-vec.y, vec
        .x) * sqrt(max((ld)0, h2) / d2);
282     if(eq(per.x, 0) and eq(per.y, 0)) return {mid};
283     return {mid + per, mid - per};
284 }
285
286 // minimum circle cover O(n) amortizado
287 circle min_circle_cover(vp v){
288     random_shuffle(v.begin(), v.end());
289     circle ans;
290     int n = v.size();
291     for(int i=0; i<n; i++){ if(!ans.inside(v[i])){
292         ans = circle(v[i]);
293         for(int j=0; j<i; j++){ if(!ans.inside(v[j])){
294             ans = circle(v[i], v[j]);
295             for(int k=0; k<j; k++){ if(!ans.inside(v[k])){
296                 ans = circle(v[i], v[j], v[k]);
297             }
298         }
299     }
300     return ans;
301 }

```

8 Algorithms

8.1 Lis

```

1 int lis(vector<int> const& a) {
2     int n = a.size();
3     vector<int> d(n, 1);
4     for (int i = 0; i < n; i++) {
5         for (int j = 0; j < i; j++) {
6             if (a[j] < a[i])
7                 d[i] = max(d[i], d[j] + 1);
8         }
9     }
10
11     int ans = d[0];
12     for (int i = 1; i < n; i++) {
13         ans = max(ans, d[i]);
14     }
15     return ans;
16 }

```

8.2 Delta-encoding

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 int main(){
5     int n, q;
6     cin >> n >> q;
7     int [n];
8     int delta[n+2];
9
10    while(q--){
11        int l, r, x;
12        cin >> l >> r >> x;
13        delta[l] += x;

```

```

14        delta[r+1] -= x;
15    }
16
17    int curr = 0;
18    for(int i=0; i < n; i++){
19        curr += delta[i];
20        v[i] = curr;
21    }
22
23    for(int i=0; i < n; i++){
24        cout << v[i] << ' ';
25    }
26    cout << '\n';
27
28    return 0;
29 }

```

8.3 Subsets

```

1 void subsets(vector<int>& nums){
2     int n = nums.size();
3     int powSize = 1 << n;
4
5     for(int counter = 0; counter < powSize; counter++){
6         for(int j = 0; j < n; j++){
7             if((counter & (1LL << j)) != 0) {
8                 cout << nums[j] << ' ';
9             }
10        }
11        cout << '\n';
12    }
13 }

```

8.4 Binary Search Last True

```

1 int last_true(int lo, int hi, function<bool(int)> f)
2 {
3     lo--;
4     while (lo < hi) {
5         int mid = lo + (hi - lo + 1) / 2;
6         if (f(mid)) {
7             lo = mid;
8         } else {
9             hi = mid - 1;
10        }
11    }
12    return lo;

```

8.5 Ternary Search

```

1 double ternary_search(double l, double r) {
2     double eps = 1e-9; //set the error
3     limit here
4     while (r - l > eps) {
5         double m1 = l + (r - l) / 3;
6         double m2 = r - (r - l) / 3;
7         double f1 = f(m1); //evaluates the
8         function at m1
9         double f2 = f(m2); //evaluates the
10        function at m2
11        if (f1 < f2)
12            l = m1;
13        else
14            r = m2;
15    }
16    return f(l); //return the
17    maximum of f(x) in [l, r]
18 }

```

8.6 Binary Search First True

```

1 int first_true(int lo, int hi, function<bool(int)> f) 54
2 {
3     hi++;
4     while (lo < hi) {
5         int mid = lo + (hi - lo) / 2;
6         if (f(mid)) {
7             hi = mid;
8         } else {
9             lo = mid + 1;
10        }
11    }
12    return lo;
13 }

```

8.7 Biggest K

```

1 // Description: Gets sum of k biggest or k smallest
  elements in an array
2
3 // Problem: https://atcoder.jp/contests/abc306/tasks/
  abc306_e
4
5 // Complexity: O(log n)
6
7 struct SetSum {
8     ll s = 0;
9     multiset<ll> mt;
10    void add(ll x){
11        mt.insert(x);
12        s += x;
13    }
14    int pop(ll x){
15        auto f = mt.find(x);
16        if(f == mt.end()) return 0;
17        mt.erase(f);
18        s -= x;
19        return 1;
20    }
21 };
22
23 struct BigK {
24     int k;
25     SetSum gt, mt;
26     BigK(int _k){
27         k = _k;
28     }
29     void balancear(){
30         while((int)gt.mt.size() < k && (int)mt.mt.
size()){
31             auto p = (prev(mt.mt.end()));
32             gt.add(*p);
33             mt.pop(*p);
34         }
35         while((int)mt.mt.size() && (int)gt.mt.size()
&&
36             *(gt.mt.begin()) < *(prev(mt.mt.end())) ){
37             ll u = *(gt.mt.begin());
38             ll v = *(prev(mt.mt.end()));
39             gt.pop(u); mt.pop(v);
40             gt.add(v); mt.add(u);
41         }
42     }
43     void add(ll x){
44         mt.add(x);
45         balancear();
46     }
47     void rem(ll x){
48         //x = -x;
49         if(mt.pop(x) == 0)
50             gt.pop(x);
51         balancear();
52     }
53 };

```

```

55 int main() {
56     ios::sync_with_stdio(false);
57     cin.tie(NULL);
58
59     int n, k, q; cin >> n >> k >> q;
60
61     BigK big = BigK(k);
62
63     int arr[n] = {};
64
65     while (q--) {
66         int pos, num; cin >> pos >> num;
67         pos--;
68         big.rem(arr[pos]);
69         arr[pos] = num;
70         big.add(arr[pos]);
71
72         cout << big.gt.s << '\n';
73     }
74
75     return 0;
76 }

```

9 Data Structures

9.1 Sparse Table

```

1 // Description:
2 // Data structure to query for minimum and maximum
3
4 // Problem:
5 // https://cses.fi/problemset/task/1647/
6
7 // Complexity:
8 // Build O(n log n)
9 // Query O(1)
10
11 #include <bits/stdc++.h>
12
13 using namespace std;
14
15 const int MAX = 2e5+17;
16 const int INF = 1e9+17;
17
18 struct SparseTable {
19     int n;
20     vector<int> arr;
21     vector<vector<int>> st;
22     vector<int> log_2;
23
24     SparseTable(vector<int>& arr, int& n) : arr(arr), n
(n) {
25         build();
26     }
27
28     void build() {
29         log_2.resize(MAX + 1);
30
31         log_2[1] = 0;
32         for (int i = 2; i <= MAX; i++) {
33             log_2[i] = log_2[i/2] + 1;
34         }
35
36         int K = log_2[n + 1];
37
38         st.resize(MAX, vector<int>(K + 1));
39
40         for (int i = 0; i < MAX; i++) {
41             for (int j = 0; j < K + 1; j++) {
42                 st[i][j] = INF;
43             }
44         }
45     }
46 }

```



```

44     }
45
46     for (int i = 0; i < n; i++) {
47         st[i][0] = arr[i];
48     }
49
50     for (int j = 1; j <= K; j++) {
51         for (int i = 0; i + (1 << j) < MAX; i++) {
52             st[i][j] = min(st[i][j-1], st[i + (1 << (j -
53             1))][j - 1]);
54         }
55     }
56
57     int query(int l, int r) {
58         int j = log_2[r - l + 1];
59         return min(st[l][j], st[r - (1 << j) + 1][j]);
60     }
61 };

```

9.2 Ordered Set

```

1 // Description:
2 // insert(k) - add element k to the ordered set
3 // erase(k) - remove element k from the ordered set
4 // erase(it) - remove element it points to from the
   ordered set
5 // order_of_key(k) - returns number of elements
   strictly smaller than k
6 // find_by_order(n) - return an iterator pointing to
   the k-th element in the ordered set (counting
   from zero).
7
8 // Problem:
9 // https://cses.fi/problemset/task/2169/
10
11 // Complexity:
12 // O(log n) for all operations
13
14 // How to use:
15 // ordered_set<int> os;
16 // cout << os.order_of_key(1) << '\n';
17 // cout << os.find_by_order(1) << '\n';
18
19 // Notes
20 // The ordered set only contains different elements
21 // By using less_equal<T> instead of less<T> on using
   ordered_set declaration
22 // The ordered_set becomes an ordered_multiset
23 // So the set can contain elements that are equal
24
25 #include <ext/pb_ds/assoc_container.hpp>
26 #include <ext/pb_ds/tree_policy.hpp>
27
28 using namespace __gnu_pbds;
29 template <typename T>
30 using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
31
32 void Erase(ordered_set<int>& a, int x){
33     int r = a.order_of_key(x);
34     auto it = a.find_by_order(r);
35     a.erase(it);
36 }

```

9.3 Priority Queue

```

1 // Description:
2 // Keeps the largest (by default) element at the top
   of the queue
3
4 // Problem:

```

```

5 // https://cses.fi/problemset/task/1164/
6
7 // Complexity:
8 // O(log n) for push and pop
9 // O(1) for looking at the element at the top
10
11 // How to use:
12 // priority_queue<int> pq;
13 // pq.push(1);
14 // pq.top();
15 // pq.pop()
16
17 // Notes
18 // To use the priority queue keeping the smallest
   element at the top
19
20 priority_queue<int, vector<int>, greater<int>> pq;

```

9.4 Dsu

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 const int MAX = 1e6+17;
6
7 struct DSU {
8     int n;
9     vector<int> link, sizes;
10
11     DSU(int n) {
12         this->n = n;
13         link.assign(n+1, 0);
14         sizes.assign(n+1, 1);
15
16         for (int i = 0; i <= n; i++)
17             link[i] = i;
18     }
19
20     int find(int x) {
21         while (x != link[x])
22             x = link[x];
23
24         return x;
25     }
26
27     bool same(int a, int b) {
28         return find(a) == find(b);
29     }
30
31     void unite(int a, int b) {
32         a = find(a);
33         b = find(b);
34
35         if (a == b) return;
36
37         if (sizes[a] < sizes[b])
38             swap(a, b);
39
40         sizes[a] += sizes[b];
41         link[b] = a;
42     }
43
44     int size(int x) {
45         return sizes[x];
46     }
47 };
48
49 int main() {
50     ios::sync_with_stdio(false);
51     cin.tie(NULL);
52
53     int cities, roads; cin >> cities >> roads;

```

```

54     vector<int> final_roads;
55     int ans = 0;
56     DSU dsu = DSU(cities);
57     for (int i = 0, a, b; i < roads; i++) {
58         cin >> a >> b;
59         dsu.unite(a, b);
60     }
61
62     for (int i = 2; i <= cities; i++) {
63         if (!dsu.same(1, i)) {
64             ans++;
65             final_roads.push_back(i);
66             dsu.unite(1, i);
67         }
68     }
69
70     cout << ans << '\n';
71     for (auto e : final_roads) {
72         cout << "1 " << e << '\n';
73     }
74 }
75 }

```

9.5 Two Sets

```

1 // Description
2 // The values are divided in two multisets so that
3 // one of them contain all values that are
4 // smaller than the median and the other one contains
5 // all values that are greater or equal to the
6 // median.
7
8 // Problem:
9 // https://atcoder.jp/contests/abc306/tasks/abc306_e
10 // Problem I - Maratona Feminina de çãProgramao da
11 // Unicamp 2023
12 // https://codeforces.com/group/WYIydkIPyE/contest
13 // /450037/attachments
14
15 // Complexity:
16 // Add and remove elements - O(log n)
17 // Return sum of biggest or smallest set or return
18 // the median - O(1)
19
20 using ll = long long;
21
22 struct TwoSets {
23     multiset<int> small;
24     multiset<int> big;
25     ll sums = 0;
26     ll sumb = 0;
27     int n = 0;
28
29     int size_small() {
30         return small.size();
31     }
32
33     int size_big() {
34         return big.size();
35     }
36
37     void balance() {
38         while (size_small() > n / 2) {
39             int v = *small.rbegin();
40             small.erase(prev(small.end()));
41             big.insert(v);
42             sums -= v;
43             sumb += v;
44         }
45         while (size_big() > n - n / 2) {
46             int v = *big.begin();
47             big.erase(big.begin());
48             small.insert(v);
49             sumb -= v;
50             sums += v;
51         }
52     }
53
54     bool rem(int x) {
55         n--;
56         auto it1 = small.find(x);
57         auto it2 = big.find(x);
58         bool flag = false;
59         if (it1 != small.end()) {
60             sums -= *it1;
61             small.erase(it1);
62             flag = true;
63         } else if (it2 != big.end()) {
64             sumb -= *it2;
65             big.erase(it2);
66             flag = true;
67         }
68         balance();
69         return flag;
70     }
71
72     ll sum_small() {
73         return sums;
74     }
75
76     ll sum_big() {
77         return sumb;
78     }
79
80     int median() {
81         return *big.begin();
82     }
83 };

```

```

43     sumb -= v;
44     sums += v;
45 }
46
47 void add(int x) {
48     n++;
49     small.insert(x);
50     sums += x;
51     while (!small.empty() && *small.rbegin() > *big.
52         begin()) {
53         int v = *small.rbegin();
54         small.erase(prev(small.end()));
55         big.insert(v);
56         sums -= v;
57         sumb += v;
58     }
59     balance();
60 }
61
62 bool rem(int x) {
63     n--;
64     auto it1 = small.find(x);
65     auto it2 = big.find(x);
66     bool flag = false;
67     if (it1 != small.end()) {
68         sums -= *it1;
69         small.erase(it1);
70         flag = true;
71     } else if (it2 != big.end()) {
72         sumb -= *it2;
73         big.erase(it2);
74         flag = true;
75     }
76     balance();
77     return flag;
78 }
79
80 ll sum_small() {
81     return sums;
82 }
83
84 ll sum_big() {
85     return sumb;
86 }
87
88 int median() {
89     return *big.begin();
90 }
91 };

```

9.6 Psum2d

```

1 // Description:
2 // Queries the sum of a rectangle that goes from grid
3 // [from_row][from_col] to grid[to_row][to_col]
4
5 // Problem:
6 // https://cses.fi/problemset/task/1652/
7
8 // Complexity:
9 // O(n) build
10 // O(1) query
11
12 for (int i = 1; i <= n; i++) {
13     for (int j = 1; j <= n; j++) {
14         psum[i][j] = grid[i][j] + psum[i - 1][j] + psum[i
15             ][j - 1] - psum[i - 1][j - 1];
16     }
17 }
18
19 while (q--) {
20     int from_row, to_row, from_col, to_col;

```

```

19  cin >> from_row >> from_col >> to_row >> to_col;
20  cout << psum[to_row][to_col] - psum[from_row - 1][
    to_col] -
21  psum[to_row][from_col - 1] + psum[from_row - 1][
    from_col - 1] << '\n';
22 }

```

9.7 Dynamic Implicit Sparse

```

1  // Description:
2  // Indexed at one
3
4  // When the indexes of the nodes are too big to be
    stored in an array
5  // and the queries need to be answered online so we
    can't sort the nodes and compress them
6  // we create nodes only when they are needed so there
    'll be (Q*log(MAX)) nodes
7  // where Q is the number of queries and MAX is the
    maximum index a node can assume
8
9  // Query - get sum of elements from range (l, r)
    inclusive
10 // Update - update element at position id to a value
    val
11
12 // Problem:
13 // https://cses.fi/problemset/task/1648
14
15 // Complexity:
16 // O(log n) for both query and update
17
18 // How to use:
19 // MAX is the maximum index a node can assume
20
21 // Segtree seg = Segtree(MAX);
22
23 typedef long long ftype;
24
25 const int MAX = 1e9+17;
26
27 struct Segtree {
28     vector<ftype> seg, d, e;
29     const ftype NEUTRAL = 0;
30     int n;
31
32     Segtree(int n) {
33         this->n = n;
34         create();
35         create();
36     }
37
38     ftype f(ftype a, ftype b) {
39         return a + b;
40     }
41
42     ftype create() {
43         seg.push_back(0);
44         e.push_back(0);
45         d.push_back(0);
46         return seg.size() - 1;
47     }
48
49     ftype query(int pos, int ini, int fim, int p, int
    q) {
50         if (q < ini || p > fim) return NEUTRAL;
51         if (pos == 0) return 0;
52         if (p <= ini && fim <= q) return seg[pos];
53         int m = (ini + fim) >> 1;
54         return f(query(e[pos], ini, m, p, q), query(d
    [pos], m + 1, fim, p, q));
55     }
56

```

```

57 void update(int pos, int ini, int fim, int id,
    int val) {
58     if (ini > id || fim < id) {
59         return;
60     }
61
62     if (ini == fim) {
63         seg[pos] = val;
64
65         return;
66     }
67
68     int m = (ini + fim) >> 1;
69
70     if (id <= m) {
71         if (e[pos] == 0) e[pos] = create();
72         update(e[pos], ini, m, id, val);
73     } else {
74         if (d[pos] == 0) d[pos] = create();
75         update(d[pos], m + 1, fim, id, val);
76     }
77
78     seg[pos] = f(seg[e[pos]], seg[d[pos]]);
79 }
80
81 ftype query(int p, int q) {
82     return query(1, 1, n, p, q);
83 }
84
85 void update(int id, int val) {
86     update(1, 1, n, id, val);
87 }
88 };

```

9.8 Segtree2d

```

1  // Description:
2  // Indexed at zero
3  // Given a N x M grid, where i represents the row and
    j the column, perform the following operations
4  // update(i, j) - update the value of grid[i][j]
5  // query(i1, j1, i2, j2) - return the sum of values
    inside the rectangle
6  // defined by grid[i1][j1] and grid[i2][j2] inclusive
7
8  // Problem:
9  // https://cses.fi/problemset/task/1739/
10
11 // Complexity:
12 // Time complexity:
13 // O(log N * log M) for both query and update
14 // O(N * M) for build
15 // Memory complexity:
16 // 4 * M * N
17
18 // How to use:
19 // Segtree2D seg = Segtree2D(n, m);
20 // vector<vector<int>> v(n, vector<int>(m));
21 // seg.build(v);
22
23 struct Segtree2D {
24     const int MAXN = 1025;
25     const int NEUTRAL = 0;
26     int N, M;
27
28     vector<vector<int>> seg;
29
30     Segtree2D(int N, int M) {
31         this->N = N;
32         this->M = M;
33         seg.assign(4*MAXN, vector<int>(4*MAXN,
    NEUTRAL));
34     }

```

```

35 int f(int a, int b) {
36     return max(a, b);
37 }
38
39 void buildY(int noX, int lX, int rX, int noY, int
40     lY, int rY, vector<vector<int>>> &v){
41     if(lY == rY){
42         if(lX == rX){
43             seg[noX][noY] = v[rX][rY];
44         }else{
45             seg[noX][noY] = f(seg[2*noX+1][noY],
46 seg[2*noX+2][noY]);
47         }
48     }else{
49         int m = (lY+rY)/2;
50
51         buildY(noX, lX, rX, 2*noY+1, lY, m, v);
52         buildY(noX, lX, rX, 2*noY+2, m+1, rY, v);
53
54         seg[noX][noY] = f(seg[noX][2*noY+1], seg[noX][2*noY+2]);
55     }
56 }
57
58 void buildX(int noX, int lX, int rX, vector<
59 vector<int>>> &v){
60     if(lX != rX){
61         int m = (lX+rX)/2;
62
63         buildX(2*noX+1, lX, m, v);
64         buildX(2*noX+2, m+1, rX, v);
65     }
66
67     buildY(noX, lX, rX, 0, 0, M - 1, v);
68 }
69
70 void updateY(int noX, int lX, int rX, int noY,
71 int lY, int rY, int y){
72     if(lY == rY){
73         if(lX == rX){
74             seg[noX][noY] = !seg[noX][noY];
75         }else{
76             seg[noX][noY] = seg[2*noX+1][noY] +
77 seg[2*noX+2][noY];
78         }
79     }else{
80         int m = (lY+rY)/2;
81
82         if(y <= m){
83             updateY(noX, lX, rX, 2*noY+1, lY, m, y);
84         }else if(m < y){
85             updateY(noX, lX, rX, 2*noY+2, m+1, rY,
86 y);
87         }
88
89         seg[noX][noY] = seg[noX][2*noY+1] + seg[noX][2*noY+2];
90     }
91 }
92
93 void updateX(int noX, int lX, int rX, int x, int
94 y){
95     int m = (lX+rX)/2;
96
97     if(lX != rX){
98         if(x <= m){
99             updateX(2*noX+1, lX, m, x, y);
100         }else if(m < x){
101             updateX(2*noX+2, m+1, rX, x, y);
102         }
103     }
104 }
105
106 int queryY(int noX, int noY, int lY, int rY, int
107 aY, int bY){
108     if(aY <= lY && rY <= bY) return seg[noX][noY];
109
110     int m = (lY+rY)/2;
111
112     if(bY <= m) return queryY(noX, 2*noY+1, lY, m,
113 aY, bY);
114     if(m < aY) return queryY(noX, 2*noY+2, m+1, rY,
115 aY, bY);
116
117     return f(queryY(noX, 2*noY+1, lY, m, aY, bY),
118 queryY(noX, 2*noY+2, m+1, rY, aY, bY));
119 }
120
121 int queryX(int noX, int lX, int rX, int aX, int
122 bX, int aY, int bY){
123     if(aX <= lX && rX <= bX) return queryY(noX,
124 0, 0, M - 1, aY, bY);
125
126     int m = (lX+rX)/2;
127
128     if(bX <= m) return queryX(2*noX+1, lX, m, aX,
129 bX, aY, bY);
130     if(m < aX) return queryX(2*noX+2, m+1, rX, aX,
131 bX, aY, bY);
132
133     return f(queryX(2*noX+1, lX, m, aX, bX, aY,
134 bY), queryX(2*noX+2, m+1, rX, aX, bX, aY, bY));
135 }
136
137 void build(vector<vector<int>>> &v) {
138     buildX(0, 0, N - 1, v);
139 }
140
141 int query(int aX, int aY, int bX, int bY) {
142     return queryX(0, 0, N - 1, aX, bX, aY, bY);
143 }
144
145 void update(int x, int y) {
146     updateX(0, 0, N - 1, x, y);
147 }
148
149 };
150
151 // Description:
152 // Query - get minimum element in a range (l, r)
153 // inclusive
154 // and also the number of times it appears in that
155 // range
156 // Update - update element at position id to a value
157 // val
158
159 // Problem:
160 // https://codeforces.com/edu/course/2/lesson/4/1/
161 // practice/contest/273169/problem/C
162
163 // Complexity:
164 // O(log n) for both query and update
165
166 // How to use:
167 // Segtree seg = Segtree(n);
168 // seg.build(v);
169
170 #define pii pair<int, int>
171 #define mp make_pair
172 #define ff first

```

```

19 #define ss second
20
21 const int INF = 1e9+17;
22
23 typedef pii ftype;
24
25 struct Segtree {
26     vector<ftype> seg;
27     int n;
28     const ftype NEUTRAL = mp(INF, 0);
29
30     Segtree(int n) {
31         int sz = 1;
32         while (sz < n) sz *= 2;
33         this->n = sz;
34
35         seg.assign(2*sz, NEUTRAL);
36     }
37
38     ftype f(ftype a, ftype b) {
39         if (a.ff < b.ff) return a;
40         if (b.ff < a.ff) return b;
41
42         return mp(a.ff, a.ss + b.ss);
43     }
44
45     ftype query(int pos, int ini, int fim, int p, int
46     q) {
47         if (ini >= p && fim <= q) {
48             return seg[pos];
49         }
50
51         if (q < ini || p > fim) {
52             return NEUTRAL;
53         }
54
55         int e = 2*pos + 1;
56         int d = 2*pos + 2;
57         int m = ini + (fim - ini) / 2;
58
59         return f(query(e, ini, m, p, q), query(d, m +
60         1, fim, p, q));
61     }
62
63     void update(int pos, int ini, int fim, int id,
64     int val) {
65         if (ini > id || fim < id) {
66             return;
67         }
68
69         if (ini == id && fim == id) {
70             seg[pos] = mp(val, 1);
71             return;
72         }
73
74         int e = 2*pos + 1;
75         int d = 2*pos + 2;
76         int m = ini + (fim - ini) / 2;
77
78         update(e, ini, m, id, val);
79         update(d, m + 1, fim, id, val);
80
81         seg[pos] = f(seg[e], seg[d]);
82     }
83
84     void build(int pos, int ini, int fim, vector<int>
85     &v) {
86         if (ini == fim) {
87             if (ini < (int)v.size()) {
88                 seg[pos] = mp(v[ini], 1);
89             }
90             return;
91         }

```

```

88     }
89
90     int e = 2*pos + 1;
91     int d = 2*pos + 2;
92     int m = ini + (fim - ini) / 2;
93
94     build(e, ini, m, v);
95     build(d, m + 1, fim, v);
96
97     seg[pos] = f(seg[e], seg[d]);
98 }
99
100 ftype query(int p, int q) {
101     return query(0, 0, n - 1, p, q);
102 }
103
104 void update(int id, int val) {
105     update(0, 0, n - 1, id, val);
106 }
107
108 void build(vector<int> &v) {
109     build(0, 0, n - 1, v);
110 }
111
112 void debug() {
113     for (auto e : seg) {
114         cout << e.ff << ' ' << e.ss << '\n';
115     }
116     cout << '\n';
117 }
118 };

```

9.10 Lazy Addition To Segment

```

1 // Description:
2 // Query - get sum of elements from range (l, r)
3 // inclusive
4 // Update - add a value val to elements from range (
5 // l, r) inclusive
6
7 // Problem:
8 // https://codeforces.com/edu/course/2/lesson/5/1/
9 // practice/contest/279634/problem/A
10
11 // Complexity:
12 // O(log n) for both query and update
13
14 // How to use:
15 // Segtree seg = Segtree(n);
16 // seg.build(v);
17
18 // Notes
19 // Change neutral element and f function to perform a
20 // different operation
21
22 const long long INF = 1e18+10;
23
24 typedef long long ftype;
25
26 struct Segtree {
27     vector<ftype> seg;
28     vector<ftype> lazy;
29     int n;
30     const ftype NEUTRAL = 0;
31     const ftype NEUTRAL_LAZY = -1; // change to -INF
32     if there are negative numbers
33
34     Segtree(int n) {
35         int sz = 1;
36         while (sz < n) sz *= 2;
37         this->n = sz;
38
39         seg.assign(2*sz, NEUTRAL);

```

```

35     lazy.assign(2*sz, NEUTRAL_LAZY);
36 }
37
38 ftype apply_lazy(ftype a, ftype b, int len) {
39     if (b == NEUTRAL_LAZY) return a;
40     if (a == NEUTRAL_LAZY) return b * len;
41     else return a + b * len;
42 }
43
44 void propagate(int pos, int ini, int fim) {
45     if (ini == fim) {
46         return;
47     }
48
49     int e = 2*pos + 1;
50     int d = 2*pos + 2;
51     int m = ini + (fim - ini) / 2;
52
53     lazy[e] = apply_lazy(lazy[e], lazy[pos], 1);
54     lazy[d] = apply_lazy(lazy[d], lazy[pos], 1);
55
56     seg[e] = apply_lazy(seg[e], lazy[pos], m -
57 ini + 1);
58     seg[d] = apply_lazy(seg[d], lazy[pos], fim -
59 m);
60
61     lazy[pos] = NEUTRAL_LAZY;
62 }
63
64 ftype f(ftype a, ftype b) {
65     return a + b;
66 }
67
68 ftype query(int pos, int ini, int fim, int p, int
69 q) {
70     propagate(pos, ini, fim);
71
72     if (ini >= p && fim <= q) {
73         return seg[pos];
74     }
75
76     if (q < ini || p > fim) {
77         return NEUTRAL;
78     }
79
80     int e = 2*pos + 1;
81     int d = 2*pos + 2;
82     int m = ini + (fim - ini) / 2;
83
84     return f(query(e, ini, m, p, q), query(d, m
85 + 1, fim, p, q));
86 }
87
88 void update(int pos, int ini, int fim, int p, int
89 q, int val) {
90     propagate(pos, ini, fim);
91
92     if (ini > q || fim < p) {
93         return;
94     }
95
96     if (ini >= p && fim <= q) {
97         lazy[pos] = apply_lazy(lazy[pos], val, 1)
98 ;
99         seg[pos] = apply_lazy(seg[pos], val, fim
100 - ini + 1);
101
102         return;
103     }
104
105     int e = 2*pos + 1;
106     int d = 2*pos + 2;
107     int m = ini + (fim - ini) / 2;
108
109     update(e, ini, m, p, q, val);
110     update(d, m + 1, fim, p, q, val);
111
112     seg[pos] = f(seg[e], seg[d]);
113 }
114
115 void build(int pos, int ini, int fim, vector<int>
116 &v) {
117     if (ini == fim) {
118         if (ini < (int)v.size()) {
119             seg[pos] = v[ini];
120         }
121         return;
122     }
123
124     int e = 2*pos + 1;
125     int d = 2*pos + 2;
126     int m = ini + (fim - ini) / 2;
127
128     build(e, ini, m, v);
129     build(d, m + 1, fim, v);
130
131     seg[pos] = f(seg[e], seg[d]);
132 }
133
134 ftype query(int p, int q) {
135     return query(0, 0, n - 1, p, q);
136 }
137
138 void update(int p, int q, int val) {
139     update(0, 0, n - 1, p, q, val);
140 }
141
142 void build(vector<int> &v) {
143     build(0, 0, n - 1, v);
144 }
145
146 void debug() {
147     for (auto e : seg) {
148         cout << e << ' ';
149     }
150     cout << '\n';
151     for (auto e : lazy) {
152         cout << e << ' ';
153     }
154     cout << '\n';
155     cout << '\n';
156 }

```

9.11 Segment With Maximum Sum

```

1 // Description:
2 // Query - get sum of segment that is maximum among
3 // all segments
4 // E.g
5 // Array: 5 -4 4 3 -5
6 // Maximum segment sum: 8 because 5 + (-4) + 4 + 3 =
7 // 8
8 // Update - update element at position id to a value
9 // val
10
11 // Problem:
12 // https://codeforces.com/edu/course/2/lesson/4/2/
13 // practice/contest/273278/problem/A
14
15 // Complexity:
16 // O(log n) for both query and update
17
18 // How to use:
19 // Segtree seg = Segtree(n);
20 // seg.build(v);

```

```

17
18 // Notes
19 // The maximum segment sum can be a negative number
20 // In that case, taking zero elements is the best
   choice
21 // So we need to take the maximum between 0 and the
   query
22 // max(OLL, seg.query(0, n).max_seg)
23
24 using ll = long long;
25
26 typedef ll ftype_node;
27
28 struct Node {
29     ftype_node max_seg;
30     ftype_node pref;
31     ftype_node suf;
32     ftype_node sum;
33
34     Node(ftype_node max_seg, ftype_node pref,
35         ftype_node suf, ftype_node sum) : max_seg(max_seg
36         ), pref(pref), suf(suf), sum(sum) {};
37 };
38
39 typedef Node ftype;
40
41 struct Segtree {
42     vector<ftype> seg;
43     int n;
44     const ftype NEUTRAL = Node(0, 0, 0, 0);
45
46     Segtree(int n) {
47         int sz = 1;
48         // potencia de dois mais proxima
49         while (sz < n) sz *= 2;
50         this->n = sz;
51
52         // numero de nos da seg
53         seg.assign(2*sz, NEUTRAL);
54     }
55
56     ftype f(ftype a, ftype b) {
57         ftype_node max_seg = max({a.max_seg, b.
58         max_seg, a.suf + b.pref});
59         ftype_node pref = max(a.pref, a.sum + b.pref);
60
61         ftype_node suf = max(b.suf, b.sum + a.suf);
62         ftype_node sum = a.sum + b.sum;
63
64         return Node(max_seg, pref, suf, sum);
65     }
66
67     ftype query(int pos, int ini, int fim, int p, int
68     q) {
69         if (ini >= p && fim <= q) {
70             return seg[pos];
71         }
72
73         if (q < ini || p > fim) {
74             return NEUTRAL;
75         }
76
77         int e = 2*pos + 1;
78         int d = 2*pos + 2;
79         int m = ini + (fim - ini) / 2;
80
81         return f(query(e, ini, m, p, q), query(d, m +
82         1, fim, p, q));
83     }
84
85     void update(int pos, int ini, int fim, int id,
86     int val) {
87         if (ini > id || fim < id) {
88             return;
89         }
90
91         if (ini == id && fim == id) {
92             seg[pos] = Node(val, val, val, val);
93
94             return;
95         }
96
97         int e = 2*pos + 1;
98         int d = 2*pos + 2;
99         int m = ini + (fim - ini) / 2;
100
101         update(e, ini, m, id, val);
102         update(d, m + 1, fim, id, val);
103
104         seg[pos] = f(seg[e], seg[d]);
105     }
106
107     void build(int pos, int ini, int fim, vector<int>
108     &v) {
109         if (ini == fim) {
110             // se a posição existir no array original
111             // seg tamanho potencia de dois
112             if (ini < (int)v.size()) {
113                 seg[pos] = Node(v[ini], v[ini], v[ini
114                 ], v[ini]);
115             }
116             return;
117         }
118
119         int e = 2*pos + 1;
120         int d = 2*pos + 2;
121         int m = ini + (fim - ini) / 2;
122
123         build(e, ini, m, v);
124         build(d, m + 1, fim, v);
125
126         seg[pos] = f(seg[e], seg[d]);
127     }
128
129     ftype query(int p, int q) {
130         return query(0, 0, n - 1, p, q);
131     }
132
133     void update(int id, int val) {
134         update(0, 0, n - 1, id, val);
135     }
136
137     void build(vector<int> &v) {
138         build(0, 0, n - 1, v);
139     }
140
141     void debug() {
142         for (auto e : seg) {
143             cout << e.max_seg << ' ' << e.pref << ' '
144             << e.suf << ' ' << e.sum << '\n';
145         }
146         cout << '\n';
147     }
148 };

```

9.12 Range Query Point Update

```

1 // Description:
2 // Indexed at zero
3 // Query - get sum of elements from range (l, r)
   inclusive
4 // Update - update element at position id to a value
   val
5
6 // Problem:

```

```

7 // https://codeforces.com/edu/course/2/lesson/4/1/
  practice/contest/273169/problem/B
8
9 // Complexity:
10 // O(log n) for both query and update
11
12 // How to use:
13 // Segtree seg = Segtree(n);
14 // seg.build(v);
15
16 // Notes
17 // Change neutral element and f function to perform a
  different operation
18
19 // If you want to change the operations to point
  query and range update
20 // Use the same segtree, but perform the following
  operations
21 // Query - seg.query(0, id);
22 // Update - seg.update(1, v); seg.update(r + 1, -v);
23
24 typedef long long ftype;
25
26 struct Segtree {
27     vector<ftype> seg;
28     int n;
29     const ftype NEUTRAL = 0;
30
31     Segtree(int n) {
32         int sz = 1;
33         while (sz < n) sz *= 2;
34         this->n = sz;
35
36         seg.assign(2*sz, NEUTRAL);
37     }
38
39     ftype f(ftype a, ftype b) {
40         return a + b;
41     }
42
43     ftype query(int pos, int ini, int fim, int p, int
  q) {
44         if (ini >= p && fim <= q) {
45             return seg[pos];
46         }
47
48         if (q < ini || p > fim) {
49             return NEUTRAL;
50         }
51
52         int e = 2*pos + 1;
53         int d = 2*pos + 2;
54         int m = ini + (fim - ini) / 2;
55
56         return f(query(e, ini, m, p, q), query(d, m +
  1, fim, p, q));
57     }
58
59     void update(int pos, int ini, int fim, int id,
  int val) {
60         if (ini > id || fim < id) {
61             return;
62         }
63
64         if (ini == id && fim == id) {
65             seg[pos] = val;
66
67             return;
68         }
69
70         int e = 2*pos + 1;
71         int d = 2*pos + 2;
72         int m = ini + (fim - ini) / 2;
73
74         update(e, ini, m, id, val);
75         update(d, m + 1, fim, id, val);
76
77         seg[pos] = f(seg[e], seg[d]);
78     }
79
80     void build(int pos, int ini, int fim, vector<int>
  &v) {
81         if (ini == fim) {
82             if (ini < (int)v.size()) {
83                 seg[pos] = v[ini];
84             }
85             return;
86         }
87
88         int e = 2*pos + 1;
89         int d = 2*pos + 2;
90         int m = ini + (fim - ini) / 2;
91
92         build(e, ini, m, v);
93         build(d, m + 1, fim, v);
94
95         seg[pos] = f(seg[e], seg[d]);
96     }
97
98     ftype query(int p, int q) {
99         return query(0, 0, n - 1, p, q);
100     }
101
102     void update(int id, int val) {
103         update(0, 0, n - 1, id, val);
104     }
105
106     void build(vector<int> &v) {
107         build(0, 0, n - 1, v);
108     }
109
110     void debug() {
111         for (auto e : seg) {
112             cout << e << ' ';
113         }
114         cout << '\n';
115     }
116 };

```

9.13 Lazy Assignment To Segment

```

1 const long long INF = 1e18+10;
2
3 typedef long long ftype;
4
5 struct Segtree {
6     vector<ftype> seg;
7     vector<ftype> lazy;
8     int n;
9     const ftype NEUTRAL = 0;
10    const ftype NEUTRAL_LAZY = -1; // Change to -INF
  if there are negative numbers
11
12    Segtree(int n) {
13        int sz = 1;
14        // potencia de dois mais proxima
15        while (sz < n) sz *= 2;
16        this->n = sz;
17
18        // numero de nos da seg
19        seg.assign(2*sz, NEUTRAL);
20        lazy.assign(2*sz, NEUTRAL_LAZY);
21    }
22
23    ftype apply_lazy(ftype a, ftype b, int len) {
24        if (b == NEUTRAL_LAZY) return a;

```



```

25         if (a == NEUTRAL_LAZY) return b * len;
26         else return b * len;
27     }
28
29     void propagate(int pos, int ini, int fim) {
30         if (ini == fim) {
31             return;
32         }
33
34         int e = 2*pos + 1;
35         int d = 2*pos + 2;
36         int m = ini + (fim - ini) / 2;
37
38         lazy[e] = apply_lazy(lazy[e], lazy[pos], 1);
39         lazy[d] = apply_lazy(lazy[d], lazy[pos], 1);
40
41         seg[e] = apply_lazy(seg[e], lazy[pos], m -
42         ini + 1);
43         seg[d] = apply_lazy(seg[d], lazy[pos], fim -
44         m);
45
46         lazy[pos] = NEUTRAL_LAZY;
47     }
48
49     ftype f(ftype a, ftype b) {
50         return a + b;
51     }
52
53     ftype query(int pos, int ini, int fim, int p, int
54     q) {
55         propagate(pos, ini, fim);
56
57         if (ini >= p && fim <= q) {
58             return seg[pos];
59         }
60
61         if (q < ini || p > fim) {
62             return NEUTRAL;
63         }
64
65         int e = 2*pos + 1;
66         int d = 2*pos + 2;
67         int m = ini + (fim - ini) / 2;
68
69         return f(query(e, ini, m, p, q), query(d, m
70         + 1, fim, p, q));
71     }
72
73     void update(int pos, int ini, int fim, int p, int
74     q, int val) {
75         propagate(pos, ini, fim);
76
77         if (ini > q || fim < p) {
78             return;
79         }
80
81         if (ini >= p && fim <= q) {
82             lazy[pos] = apply_lazy(lazy[pos], val, 1)
83             ;
84             seg[pos] = apply_lazy(seg[pos], val, fim
85             - ini + 1);
86
87             return;
88         }
89
90         int e = 2*pos + 1;
91         int d = 2*pos + 2;
92         int m = ini + (fim - ini) / 2;
93
94         update(e, ini, m, p, q, val);
95         update(d, m + 1, fim, p, q, val);
96
97         seg[pos] = f(seg[e], seg[d]);
98     }
99
100     void build(int pos, int ini, int fim, vector<int>
101     &v) {
102         if (ini == fim) {
103             // se a posição existir no array original
104             // seg tamanho potencia de dois
105             if (ini < (int)v.size()) {
106                 seg[pos] = v[ini];
107             }
108             return;
109         }
110
111         int e = 2*pos + 1;
112         int d = 2*pos + 2;
113         int m = ini + (fim - ini) / 2;
114
115         build(e, ini, m, v);
116         build(d, m + 1, fim, v);
117
118         seg[pos] = f(seg[e], seg[d]);
119     }
120
121     ftype query(int p, int q) {
122         return query(0, 0, n - 1, p, q);
123     }
124
125     void update(int p, int q, int val) {
126         update(0, 0, n - 1, p, q, val);
127     }
128
129     void build(vector<int> &v) {
130         build(0, 0, n - 1, v);
131     }
132
133     void debug() {
134         for (auto e : seg) {
135             cout << e << ' ';
136         }
137         cout << '\n';
138         for (auto e : lazy) {
139             cout << e << ' ';
140         }
141         cout << '\n';
142         cout << '\n';
143     }
144 }
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```

9.14 Lazy Dynamic Implicit Sparse

```

1 // Description:
2 // Indexed at one
3
4 // When the indexes of the nodes are too big to be
5 // stored in an array
6 // and the queries need to be answered online so we
7 // can't sort the nodes and compress them
8 // we create nodes only when they are needed so there
9 // 'll be (Q*log(MAX)) nodes
10 // where Q is the number of queries and MAX is the
11 // maximum index a node can assume
12
13 // Query - get sum of elements from range (l, r)
14 // inclusive
15 // Update - update element at position id to a value
16 // val
17
18 // Problem:
19 // https://oj.uz/problem/view/IZh012_apple
20
21 // Complexity:
22 // O(log n) for both query and update
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```

```

18 // How to use:
19 // MAX is the maximum index a node can assume
20 // Create a default null node
21 // Create a node to be the root of the segtree
22
23 // Segtree seg = Segtree(MAX);
24
25 const int MAX = 1e9+10;
26 const long long INF = 1e18+10;
27
28 typedef long long ftype;
29
30 struct Segtree {
31     vector<ftype> seg, d, e, lazy;
32     const ftype NEUTRAL = 0;
33     const ftype NEUTRAL_LAZY = -1; // change to -INF
34     if the elements can be negative
35     int n;
36
37     Segtree(int n) {
38         this->n = n;
39         create();
40         create();
41     }
42
43     ftype apply_lazy(ftype a, ftype b, int len) {
44         if (b == NEUTRAL_LAZY) return a;
45         else return b * len; // change to a + b * len
46         to add to an element instead of updating it
47     }
48
49     void propagate(int pos, int ini, int fim) {
50         if (seg[pos] == 0) return;
51
52         if (ini == fim) {
53             return;
54         }
55
56         int m = (ini + fim) >> 1;
57
58         if (e[pos] == 0) e[pos] = create();
59         if (d[pos] == 0) d[pos] = create();
60
61         lazy[e[pos]] = apply_lazy(lazy[e[pos]], lazy[
62 pos], 1);
63         lazy[d[pos]] = apply_lazy(lazy[d[pos]], lazy[
64 pos], 1);
65
66         seg[e[pos]] = apply_lazy(seg[e[pos]], lazy[
67 pos], m - ini + 1);
68         seg[d[pos]] = apply_lazy(seg[d[pos]], lazy[
69 pos], fim - m);
70
71         lazy[pos] = NEUTRAL_LAZY;
72     }
73
74     ftype f(ftype a, ftype b) {
75         return a + b;
76     }
77
78     ftype create() {
79         seg.push_back(0);
80         e.push_back(0);
81         d.push_back(0);
82         lazy.push_back(-1);
83         return seg.size() - 1;
84     }
85
86     ftype query(int pos, int ini, int fim, int p, int
87 q) {
88         propagate(pos, ini, fim);
89         if (q < ini || p > fim) return NEUTRAL;
90         if (pos == 0) return 0;

```

```

84         if (p <= ini && fim <= q) return seg[pos];
85         int m = (ini + fim) >> 1;
86         return f(query(e[pos], ini, m, p, q), query(d
87 [pos], m + 1, fim, p, q));
88     }
89
90     void update(int pos, int ini, int fim, int p, int
91 q, int val) {
92         propagate(pos, ini, fim);
93         if (ini > q || fim < p) {
94             return;
95         }
96
97         if (ini >= p && fim <= q) {
98             lazy[pos] = apply_lazy(lazy[pos], val, 1)
99 ;
100             seg[pos] = apply_lazy(seg[pos], val, fim
101 - ini + 1);
102             return;
103         }
104
105         int m = (ini + fim) >> 1;
106
107         if (e[pos] == 0) e[pos] = create();
108         update(e[pos], ini, m, p, q, val);
109
110         if (d[pos] == 0) d[pos] = create();
111         update(d[pos], m + 1, fim, p, q, val);
112
113         seg[pos] = f(seg[e[pos]], seg[d[pos]]);
114     }
115
116     ftype query(int p, int q) {
117         return query(1, 1, n, p, q);
118     }
119
120     void update(int p, int q, int val) {
121         update(1, 1, n, p, q, val);
122     }
123 };

```

9.15 Persistent

```

1 // Description:
2 // Persistent segtree allows for you to save the
3 // different versions of the segtree between each
4 // update
5 // Indexed at one
6 // Query - get sum of elements from range (l, r)
7 // inclusive
8 // Update - update element at position id to a value
9 // val
10
11 // Problem:
12 // https://cses.fi/problemset/task/1737/
13
14 // Complexity:
15 // O(log n) for both query and update
16
17 // How to use:
18 // vector<int> raiz(MAX); // vector to store the
19 // roots of each version
20 // Segtree seg = Segtree(INF);
21 // raiz[0] = seg.create(); // null node
22 // curr = 1; // keep track of the last version
23
24 // raiz[k] = seg.update(raiz[k], idx, val); //
25 // updating version k
26 // seg.query(raiz[k], l, r) // querying version k
27 // raiz[++curr] = raiz[k]; // create a new version
28 // based on version k

```

```

23 const int MAX = 2e5+17;
24 const int INF = 1e9+17;
25
26 typedef long long ftype;
27
28 struct Segtree {
29     vector<ftype> seg, d, e;
30     const ftype NEUTRAL = 0;
31     int n;
32
33     Segtree(int n) {
34         this->n = n;
35     }
36
37     ftype f(ftype a, ftype b) {
38         return a + b;
39     }
40
41     ftype create() {
42         seg.push_back(0);
43         e.push_back(0);
44         d.push_back(0);
45         return seg.size() - 1;
46     }
47
48     ftype query(int pos, int ini, int fim, int p, int
49 q) {
50     if (q < ini || p > fim) return NEUTRAL;
51     if (pos == 0) return 0;
52     if (p <= ini && fim <= q) return seg[pos];
53     int m = (ini + fim) >> 1;
54     return f(query(e[pos], ini, m, p, q), query(d
55 [pos], m + 1, fim, p, q));
56 }
57
58 int update(int pos, int ini, int fim, int id, int
59 val) {
60     int novo = create();
61
62     seg[novo] = seg[pos];
63     e[novo] = e[pos];
64     d[novo] = d[pos];
65
66     if (ini == fim) {
67         seg[novo] = val;
68         return novo;
69     }
70
71     int m = (ini + fim) >> 1;
72
73     if (id <= m) e[novo] = update(e[novo], ini, m
74 , id, val);
75     else d[novo] = update(d[novo], m + 1, fim, id
76 , val);
77
78     seg[novo] = f(seg[e[novo]], seg[d[novo]]);
79
80     return novo;
81 }
82
83 ftype query(int pos, int p, int q) {
84     return query(pos, 1, n, p, q);
85 }
86
87 int update(int pos, int id, int val) {
88     return update(pos, 1, n, id, val);
89 }
90 };
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9.16 Sparse Table2d

```

1 // Description
2 // Minimum queries in a 2D grid

```

```

3
4 // Problem:
5 // https://codeforces.com/group/YgJnumGtHD/contest
6 // 103794/problem/D
7
8 // Complexity:
9 // Build O(N * M * log(N) * log(M))
10 // Query O(1)
11 // Memory CComplexity: O(N * M * log(N) * log(M))
12
13 const int MAX = 410;
14
15 struct SparseTable2D {
16     vector<vector<int>>> matrix;
17     vector<vector<vector<vector<int>>>> table;
18     int n, m;
19
20     SparseTable2D(vector<vector<int>>>& matrix, int n,
21 int m) : matrix(matrix), n(n), m(m) {
22     table.resize(MAX, vector<vector<vector<int>>>(MAX
23 , vector<vector<int>>>(log2(MAX) + 1, vector<int>(
24 log2(MAX) + 1))));
25     build();
26 }
27
28 int f(int a, int b) {
29     return max(a, b);
30 }
31
32 void build() {
33     for (int i = 0; i < n; i++) {
34         for (int j = 0; j < m; j++) {
35             table[i][j][0][0] = matrix[i][j];
36         }
37     }
38
39     for (int k = 1; k <= (int)(log2(n)); k++) {
40         for (int i = 0; i + (1 << k) - 1 < n; i++) {
41             for (int j = 0; j + (1 << k) - 1 < m; j++) {
42                 table[i][j][k][0] = f(
43                     table[i][j][k - 1][0],
44                     table[i + (1 << (k - 1))][j][k - 1][0]);
45             }
46         }
47     }
48
49     for (int k = 1; k <= (int)(log2(m)); k++) {
50         for (int i = 0; i < n; i++) {
51             for (int j = 0; j + (1 << k) - 1 < m; j++) {
52                 table[i][j][0][k] = f(
53                     table[i][j][0][k - 1],
54                     table[i][j + (1 << (k - 1))][0][k - 1]);
55             }
56         }
57     }
58
59     for (int k = 1; k <= (int)(log2(n)); k++) {
60         for (int l = 1; l <= (int)(log2(m)); l++) {
61             for (int i = 0; i + (1 << k) - 1 < n; i++) {
62                 for (int j = 0; j + (1 << l) - 1 < m; j++) {
63                     table[i][j][k][l] = f(
64                         f(
65                             table[i][j][k - 1][l - 1],
66                             table[i + (1 << (k - 1))][j][k - 1][l
67 - 1]
68                         ),
69                         f(
70                             table[i][j + (1 << (l - 1))][k - 1][l
71 - 1],
72                             table[i + (1 << (k - 1))][j + (1 << (
73 l - 1))][k - 1][l - 1])
74                         );
75                 }
76             }
77         }
78     }
79 }
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68     }
69   }
70 }
71 }
72 }
73
74 int query(int x1, int y1, int x2, int y2) {
75     int k = log2(x2 - x1 + 1);
76     int l = log2(y2 - y1 + 1);
77
78     return f(
79         f(

```

```

80         table[x1][y1][k][l],
81         table[x2 - (1 << k) + 1][y1][k][l]
82     ),
83     f(
84         table[x1][y2 - (1 << l) + 1][k][l],
85         table[x2 - (1 << k) + 1][y2 - (1 << l) + 1][k
86         ][l]
87     )
88 );
89 };

```