



# Notebook - Maratona de Programação

Lenhadoras de Segtree

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# 1 Math

## 1.1 Ceil

```
1 long long division_ceil(long long a, long long b) {
2     return 1 + ((a - 1) / b); // if a != 0
3 }
```

## 1.2 To Decimal

```
1 const string digits { "0123456789
2     ABCDEFGHIJKLMNOPQRSTUVWXYZ" };
3
4 long long to_decimal(const string& rep, long long
5     base) {
6     long long n = 0;
7
8     for (auto c : rep) {
9         // if the number can't be represented in this
10        base
11        if (c > digits[base - 1]) return -1;
12        n *= base;
13        n += digits.find(c);
14    }
15
16    return n;
17 }
```

## 1.3 Subsets

```
1 void subsets(vector<int>& nums){
2     int n = nums.size();
3     int powSize = 1 << n;
4
5     for(int counter = 0; counter < powSize; counter++){
6         for(int j = 0; j < n; j++) {
7             if((counter & (1LL << j)) != 0) {
8                 cout << nums[j] << ' ';
9             }
10            cout << '\n';
11        }
12    }
13 }
```

## 1.4 Matrix Exponentiation

```
1 // Description:
2 // Calculate the nth term of a linear recursion
3
4 // Example Fibonacci:
5 // Given a linear recurrence, for example fibonacci
6 // F(n) = n, x <= 1
7 // F(n) = F(n - 1) + F(n - 2), x > 1
8
9 // The recurrence has two terms, so we can build a
10 // matrix 2 x 1 so that
11 // n + 1 = transition * n
12 // (2 x 1) = (2 x 2) * (2 x 1)
13 // F(n)      = a b * F(n - 1)
14 // F(n - 1)   c d   F(n - 2)
15
16 // Another Example:
17 // Given a grid 3 x n, you want to color it using 3
18 // distinct colors so that
19 // no adjacent place has the same color. In how many
20 // different ways can you do that?
21 // There are 6 ways for the first column to be
22 // colored using 3 distinct colors
23 // ans 6 ways using 2 equal colors and 1 distinct one
```

```
21
22 // Adding another column, there are:
23 // 3 ways to go from 2 equal to 2 equal
24 // 2 ways to go from 2 equal to 3 distinct
25 // 2 ways to go from 3 distinct to 2 equal
26 // 2 ways to go from 3 distinct to 3 distinct
27
28 // So we start with matrix 6 6 and multiply it by the
29 // transition 3 2 and get 18 12
30 //
31 //          2 2          6 6          12 12
32 // the we can exponentiate this matrix to find the
33 // nth column
34
35 // Problem:
36 // https://cses.fi/problemset/task/1722/
37
38 // Complexity:
39 // O(log n)
40
41 // How to use:
42 // vector<vector<ll>> v = {{1, 1}, {1, 0}};
43 // Matriz transition = Matriz(v);
44 // cout << fexp(transition, n)[0][1] << '\n';
45
46 using ll = long long;
47
48 const int MOD = 1e9+7;
49
50 struct Matriz{
51     vector<vector<ll>> mat;
52     int rows, columns;
53
54     vector<ll> operator[](int i){
55         return mat[i];
56     }
57
58     Matriz(vector<vector<ll>>& matriz){
59         mat = matriz;
60         rows = mat.size();
61         columns = mat[0].size();
62     }
63
64     Matriz(int row, int column, bool identity=false){
65         rows = row; columns = column;
66         mat.assign(rows, vector<ll>(columns, 0));
67         if(identity) {
68             for(int i = 0; i < min(rows, columns); i
69             ++){
70                 mat[i][i] = 1;
71             }
72         }
73     }
74
75     Matriz operator * (Matriz a) {
76         assert(columns == a.rows);
77         vector<vector<ll>> resp(rows, vector<ll>(a.
78         columns, 0));
79
80         for(int i = 0; i < rows; i++){
81             for(int j = 0; j < a.columns; j++){
82                 for(int k = 0; k < a.rows; k++){
83                     resp[i][j] = (resp[i][j] + (mat[i
84                     ][k] * 1LL * a[k][j]) % MOD) % MOD;
85                 }
86             }
87         }
88         return Matriz(resp);
89     }
90
91     Matriz operator + (Matriz a) {
92         assert(rows == a.rows && columns == a.columns
93         );
```

```

87     vector<vector<ll>> resp(rows, vector<ll>(
columns,0));
88     for(int i = 0; i < rows; i++){
89         for(int j = 0; j < columns; j++){
90             resp[i][j] = (resp[i][j] + mat[i][j]
+ a[i][j]) % MOD;
91         }
92     }
93     return Matriz(resp);
94 }
95 };
96
97 Matriz fexp(Matriz base, ll exponent){
98     Matriz result = Matriz(base.rows, base.rows, 1);
99     while(exponent > 0){
100         if(exponent & 1LL) result = result * base;
101         base = base * base;
102         exponent = exponent >> 1;
103     }
104     return result;
105 }

```

## 1.5 Crt

```

1 ll crt(const vector<pair<ll, ll>> &vet){
2     ll ans = 0, lcm = 1;
3     ll a, b, g, x, y;
4     for(const auto &p : vet) {
5         tie(a, b) = p;
6         tie(g, x, y) = gcd(lcm, b);
7         if((a - ans) % g != 0) return -1; // no
solution
8         ans = ans + x * ((a - ans) / g) % (b / g) *
lcm;
9         lcm = lcm * (b / g);
10        ans = (ans % lcm + lcm) % lcm;
11    }
12    return ans;
13 }

```

## 1.6 Binary To Decimal

```

1 int binary_to_decimal(long long n) {
2     int dec = 0, i = 0, rem;
3
4     while (n!=0) {
5         rem = n % 10;
6         n /= 10;
7         dec += rem * pow(2, i);
8         ++i;
9     }
10
11    return dec;
12 }
13
14 long long decimal_to_binary(int n) {
15     long long bin = 0;
16     int rem, i = 1;
17
18     while (n!=0) {
19         rem = n % 2;
20         n /= 2;
21         bin += rem * i;
22         i *= 10;
23     }
24
25    return bin;
26 }

```

## 1.7 Fast Exponentiation

```

1 ll fexp(ll b, ll e, ll mod) {
2     ll res = 1;
3     b %= mod;
4     while(e){
5         if(e & 1LL)
6             res = (res * b) % mod;
7         e = e >> 1LL;
8         b = (b * b) % mod;
9     }
10    return res;
11 }

```

## 1.8 Linear Diophantine Equation

```

1 // int a, b, c, x1, x2, y1, y2; cin >> a >> b >> c >>
x1 >> x2 >> y1 >> y2;
2 // int ans = -1;
3 // if (a == 0 && b == 0) {
4 //     if (c != 0) ans = 0;
5 //     else ans = (x2 - x1 + 1) * (y2 - y1 + 1);
6 // }
7 // else if (a == 0) {
8 //     if (c % b == 0 && y1 <= c / b && y2 >= c / b)
ans = (x2 - x1 + 1);
9 //     else ans = 0;
10 // }
11 // else if (b == 0) {
12 //     if (c % a == 0 && x1 <= c / a && x2 >= c / a)
ans = (y2 - y1 + 1);
13 //     else ans = 0;
14 // }
15
16 // Careful when a or b are negative or zero
17
18 // if (ans == -1) ans = find_all_solutions(a, b, c,
x1, x2, y1, y2);
19 // cout << ans << '\n';
20
21 // Problems:
22 // https://www.spoj.com/problems/CEQU/
23 // http://codeforces.com/problemsets/acmsguru/problem
/99999/106
24
25 // consider trivial case a or b is 0
26 int gcd(int a, int b, int& x, int& y) {
27     if (b == 0) {
28         x = 1;
29         y = 0;
30         return a;
31     }
32     int x1, y1;
33     int d = gcd(b, a % b, x1, y1);
34     x = y1;
35     y = x1 - y1 * (a / b);
36     return d;
37 }
38
39 // x and y are one solution and g is the gcd, all
passed as reference
40 // minx <= x <= maxx miny <= y <= maxy
41 bool find_any_solution(int a, int b, int c, int &x0,
int &y0, int &g) {
42     g = gcd(abs(a), abs(b), x0, y0);
43     if (c % g) {
44         return false;
45     }
46
47     x0 *= c / g;
48     y0 *= c / g;
49     if (a < 0) x0 = -x0;
50     if (b < 0) y0 = -y0;
51     return true;
52 }

```

```

53
54 void shift_solution(int & x, int & y, int a, int b,
    int cnt) {
55     x += cnt * b;
56     y -= cnt * a;
57 }
58
59 // return number of solutions in the interval
60 int find_all_solutions(int a, int b, int c, int minx,
    int maxx, int miny, int maxy) {
61     int x, y, g;
62     if (!find_any_solution(a, b, c, x, y, g))
63         return 0;
64     a /= g;
65     b /= g;
66
67     int sign_a = a > 0 ? +1 : -1;
68     int sign_b = b > 0 ? +1 : -1;
69
70     shift_solution(x, y, a, b, (minx - x) / b);
71     if (x < minx)
72         shift_solution(x, y, a, b, sign_b);
73     if (x > maxx)
74         return 0;
75     int lx1 = x;
76
77     shift_solution(x, y, a, b, (maxx - x) / b);
78     if (x > maxx)
79         shift_solution(x, y, a, b, -sign_b);
80     int rx1 = x;
81
82     shift_solution(x, y, a, b, -(miny - y) / a);
83     if (y < miny)
84         shift_solution(x, y, a, b, -sign_a);
85     if (y > maxy)
86         return 0;
87     int lx2 = x;
88
89     shift_solution(x, y, a, b, -(maxy - y) / a);
90     if (y > maxy)
91         shift_solution(x, y, a, b, sign_a);
92     int rx2 = x;
93
94     if (lx2 > rx2)
95         swap(lx2, rx2);
96     int lx = max(lx1, lx2);
97     int rx = min(rx1, rx2);
98
99     if (lx > rx)
100         return 0;
101     return (rx - lx) / abs(b) + 1;
102 }

```

## 1.9 Function Root

```

1 const ld EPS1 = 1e-9; // iteration precision error
2 const ld EPS2 = 1e-4; // output precision error
3
4 ld f(ld x) {
5     // exp(-x) == e^(-x)
6     return p * exp(-x) + q * sin(x) + r * cos(x) + s *
    tan(x) + t * x * x + u;
7 }
8
9 ld root(ld a, ld b) {
10     while (b - a >= EPS1) {
11         ld c = (a + b) / 2.0;
12         ld y = f(c);
13
14         if (y < 0) b = c;
15         else a = c;
16     }
17 }

```

```

18     return (a + b) / 2;
19 }
20
21 int main() {
22     ld ans = root(0, 1);
23     if (abs(f(ans)) <= EPS2) cout << fixed <<
        setprecision(4) << ans << '\n';
24     else cout << "No solution\n";
25
26     return 0;
27 }

```

## 1.10 Sieve Of Eratosthenes

```

1 vector<bool> is_prime(MAX, true);
2 vector<int> primes;
3
4 void sieve() {
5     is_prime[0] = is_prime[1] = false;
6     for (int i = 2; i < MAX; i++) {
7         if (is_prime[i]) {
8             primes.push_back(i);
9
10            for (int j = i + i; j < MAX; j += i)
11                is_prime[j] = false;
12        }
13    }
14 }

```

## 1.11 Horner Algorithm

```

1 // Description:
2 // Evaluates y = f(x)
3
4 // Problem:
5 // https://onlinejudge.org/index.php?option=
    com_onlinejudge&Itemid=8&page=show_problem&
    problem=439
6
7 // Complexity:
8 // O(n)
9
10 using polynomial = std::vector<int>;
11
12 polynomial p {6, -5, 2}; // p(x) = x^2 - 5x + 6;
13
14 int degree(const polynomial& p) {
15     return p.size() - 1;
16 }
17
18 int evaluate(const polynomial& p, int x) {
19     int y = 0, N = degree(p);
20
21     for (int i = N; i >= 0; --i) {
22         y *= x;
23         y += p[i];
24     }
25
26     return y;
27 }

```

## 1.12 Multiplicative Inverse

```

1 ll extend_euclid(ll a, ll b, ll &x, ll &y) {
2     if (a == 0)
3     {
4         x = 0; y = 1;
5         return b;
6     }
7     ll x1, y1;
8     ll d = extend_euclid(b%a, a, x1, y1);
9     x = y1 - (b / a) * x1;

```

```

10     y = x1;
11     return d;
12 }
13
14 // gcd(a, m) = 1 para existir solucao
15 // ax + my = 1, ou a*x = 1 (mod m)
16 ll inv_gcd(ll a, ll m) { // com gcd
17     ll x, y;
18     extend_euclid(a, m, x, y);
19     return ((x % m) + m) % m;
20 }
21
22 ll inv(ll a, ll phim) { // com phi(m), se m for primo
23     entao phi(m) = p-1
24     ll e = phim-1;
25     return fexp(a, e, MOD);
26 }

```

## 1.13 Representation Arbitrary Base

```

1 const string digits { "0123456789
2     ABCDEFGHIJKLMNOPQRSTUVWXYZ" };
3
4 string representation(int n, int b) {
5     string rep;
6
7     do {
8         rep.push_back(digits[n % b]);
9         n /= b;
10    } while (n);
11
12    reverse(rep.begin(), rep.end());
13
14    return rep;
15 }

```

## 1.14 Set Operations

```

1 // Complexity;
2 // O(n * m) being n and m the sizes of the two sets
3 // 2*(count1+count2)-1 (where countX is the distance
4 // between firstX and lastX):
5
6 vector<int> res;
7 set_union(s1.begin(), s1.end(), s2.begin(), s2.end(),
8     inserter(res, res.begin()));
9 set_intersection(s1.begin(), s1.end(), s2.begin(), s2
10     .end(), inserter(res, res.begin()));
11 // present in the first set, but not in the second
12 set_difference(s1.begin(), s1.end(), s2.begin(), s2
13     .end(), inserter(res, res.begin()));
14 // present in one of the sets, but not in the other
15 set_symmetric_difference(s1.begin(), s1.end(), s2
16     .begin(), s2.end(), inserter(res, res.begin()));

```

## 1.15 Divisors

```

1 vector<long long> all_divisors(long long n) {
2     vector<long long> ans;
3     for(long long a = 1; a*a <= n; a++){
4         if(n % a == 0) {
5             long long b = n / a;
6             ans.push_back(a);
7             if(a != b) ans.push_back(b);
8         }
9     }
10    sort(ans.begin(), ans.end());
11    return ans;
12 }

```

## 1.16 Check If Bit Is On

```

1 // msb de 0 é undefined
2 #define msb(n) (32 - __builtin_clz(n))
3 // #define msb(n) (64 - __builtin_clzll(n) )
4 // popcount
5 // turn bit off
6
7 bool bit_on(int n, int bit) {
8     if(1 & (n >> bit)) return true;
9     else return false;
10 }

```

## 1.17 Prime Factors

```

1 vector<pair<long long, int>> fatora(long long n) {
2     vector<pair<long long, int>> ans;
3     for(long long p = 2; p*p <= n; p++) {
4         if(n % p == 0) {
5             int expoente = 0;
6             while(n % p == 0) {
7                 n /= p;
8                 expoente++;
9             }
10            ans.emplace_back(p, expoente);
11        }
12    }
13    if(n > 1) ans.emplace_back(n, 1);
14    return ans;
15 }

```

## 2 DP

### 2.1 Knapsack With Index

```

1 void knapsack(int W, int wt[], int val[], int n) {
2     int i, w;
3     int K[n + 1][W + 1];
4
5     for (i = 0; i <= n; i++) {
6         for (w = 0; w <= W; w++) {
7             if (i == 0 || w == 0)
8                 K[i][w] = 0;
9             else if (wt[i - 1] <= w)
10                K[i][w] = max(val[i - 1] +
11                    K[i - 1][w - wt[i - 1]], K[i -
12                1][w]);
13            else
14                K[i][w] = K[i - 1][w];
15        }
16    }
17
18    int res = K[n][W];
19    cout << res << endl;
20
21    w = W;
22    for (i = n; i > 0 && res > 0; i--) {
23        if (res == K[i - 1][w])
24            continue;
25        else {
26            cout << " " << wt[i - 1] ;
27            res = res - val[i - 1];
28            w = w - wt[i - 1];
29        }
30    }
31
32    int main()
33    {
34        int val[] = { 60, 100, 120 };
35        int wt[] = { 10, 20, 30 };
36        int W = 50;
37        int n = sizeof(val) / sizeof(val[0]);

```

```

38     knapsack(W, wt, val, n);
39
40
41     return 0;
42 }

```

## 2.2 Substr Palindrome

```

1 // êvoc deve informar se a substring de S formada
  pelos elementos entre os índices i e j
2 // é um palindromo ou não.
3
4 char s[MAX];
5 int calculado[MAX][MAX]; // iniciado com false, ou 0
6 int tabela[MAX][MAX];
7
8 int is_palin(int i, int j){
9     if(calculado[i][j]){
10         return tabela[i][j];
11     }
12     if(i == j) return true;
13     if(i + 1 == j) return s[i] == s[j];
14
15     int ans = false;
16     if(s[i] == s[j]){
17         if(is_palin(i+1, j-1)){
18             ans = true;
19         }
20     }
21     calculado[i][j] = true;
22     tabela[i][j] = ans;
23     return ans;
24 }

```

## 2.3 Edit Distance

```

1 // Description:
2 // Minimum number of operations required to transform
  a string into another
3 // Operations allowed: add character, remove
  character, replace character
4
5 // Parameters:
6 // str1 - string to be transformed into str2
7 // str2 - string that str1 will be transformed into
8 // m - size of str1
9 // n - size of str2
10
11 // Problem:
12 // https://cses.fi/problemset/task/1639
13
14 // Complexity:
15 // O(m x n)
16
17 // How to use:
18 // memset(dp, -1, sizeof(dp));
19 // string a, b;
20 // edit_distance(a, b, (int)a.size(), (int)b.size());
21
22 // Notes:
23 // Size of dp matriz is m x n
24
25 int dp[MAX][MAX];
26
27 int edit_distance(string &str1, string &str2, int m,
  int n) {
28     if (m == 0) return n;
29     if (n == 0) return m;
30
31     if (dp[m][n] != -1) return dp[m][n];
32
33     if (str1[m - 1] == str2[n - 1]) return dp[m][n] =
  edit_distance(str1, str2, m - 1, n - 1);

```

```

34     return dp[m][n] = 1 + min({edit_distance(str1,
  str2, m, n - 1), edit_distance(str1, str2, m - 1,
  n), edit_distance(str1, str2, m - 1, n - 1)});
35 }

```

## 2.4 Knapsack

```

1 int val[MAXN], peso[MAXN], dp[MAXN][MAXS];
2
3 int knapsack(int n, int m){ // n Objetos | Peso max
4     for(int i=0; i<=n; i++){
5         for(int j=0; j<=m; j++){
6             if(i==0 || j==0)
7                 dp[i][j] = 0;
8             else if(peso[i-1]<=j)
9                 dp[i][j] = max(val[i-1]+dp[i-1][j-
  peso[i-1]], dp[i-1][j]);
10            else
11                dp[i][j] = dp[i-1][j];
12        }
13    }
14    return dp[n][m];
15 }

```

## 2.5 Digits

```

1 // achar a quantidade de numeros menores que R que
  possuem no maximo 3 digitos nao nulos
2 // a ideia eh utilizar da ordem lexicografica para
  checar isso pois se temos por exemplo
3 // o numero 8500, a gente sabe que se pegarmos o
  numero 7... qualquer digito depois do 7
4 // sera necessariamente menor q 8500
5
6 string r;
7 int tab[20][2][5];
8
9 // i - digito de R
10 // menor - ja pegou um numero menor que um digito de
  R
11 // qt - quantidade de digitos nao nulos
12 int dp(int i, bool menor, int qt){
13     if(qt > 3) return 0;
14     if(i >= r.size()) return 1;
15     if(tab[i][menor][qt] != -1) return tab[i][menor][
  qt];
16
17     int dr = r[i]-'0';
18     int res = 0;
19
20     for(int d = 0; d <= 9; d++) {
21         int dnn = qt + (d > 0);
22         if(menor == true) {
23             res += dp(i+1, true, dnn);
24         }
25         else if(d < dr) {
26             res += dp(i+1, true, dnn);
27         }
28         else if(d == dr) {
29             res += dp(i+1, false, dnn);
30         }
31     }
32
33     return tab[i][menor][qt] = res;
34 }

```

## 2.6 Coins

```

1 int tb[1005];
2 int n;
3 vector<int> moedas;
4
5 int dp(int i){

```

```

6     if(i >= n)
7         return 0;
8     if(tb[i] != -1)
9         return tb[i];
10
11     tb[i] = max(dp(i+1), dp(i+2) + moedas[i]);
12     return tb[i];
13 }
14
15 int main(){
16     memset(tb, -1, sizeof(tb));
17 }

```

## 2.7 Minimum Coin Change

```

1 int n;
2 vector<int> valores;
3
4 int tabela[1005];
5
6 int dp(int k){
7     if(k == 0){
8         return 0;
9     }
10    if(tabela[k] != -1)
11        return tabela[k];
12    int melhor = 1e9;
13    for(int i = 0; i < n; i++){
14        if(valores[i] <= k)
15            melhor = min(melhor, 1 + dp(k - valores[i]));
16    }
17    return tabela[k] = melhor;
18 }

```

## 2.8 Kadane

```

1 // achar uma subsequencia continua no array que a
2 // soma seja a maior possivel
3 // nesse caso vc precisa multiplicar exatamente 1
4 // elemento da subsequencia
5 // e achar a maior soma com isso
6
7 int n, x, arr[MAX], tab[MAX][2]; // tab[maior
8 // resposta no intervalo][foi multiplicado ou ãno]
9
10 int dp(int i, bool mult) {
11     if (i == n-1) {
12         if (!mult) return arr[n-1]*x;
13         return arr[n-1];
14     }
15     if (tab[i][mult] != -1) return tab[i][mult];
16
17     int res;
18
19     if (mult) {
20         res = max(arr[i], arr[i] + dp(i+1, 1));
21     }
22     else {
23         res = max({
24             arr[i]*x,
25             arr[i]*x + dp(i+1, 1),
26             arr[i] + dp(i+1, 0)
27         });
28     }
29
30     return tab[i][mult] = res;
31 }
32
33 int main() {
34     memset(tab, -1, sizeof(tab));
35 }

```

```

34 int ans = -oo;
35 for (int i = 0; i < n; i++) {
36     ans = max(ans, dp(i, 0));
37 }
38
39 return 0;
40 }
41
42
43
44 int ans = a[0], ans_l = 0, ans_r = 0;
45 int sum = 0, minus_pos = -1;
46
47 for (int r = 0; r < n; ++r) {
48     sum += a[r];
49     if (sum > ans) {
50         ans = sum;
51         ans_l = minus_pos + 1;
52         ans_r = r;
53     }
54     if (sum < 0) {
55         sum = 0;
56         minus_pos = r;
57     }
58 }

```

## 3 Template

### 3.1 Template

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define int long long
5 #define optimize std::ios::sync_with_stdio(false);
6 cin.tie(NULL);
7
8 #define vi vector<int>
9 #define ll long long
10 #define pb push_back
11 #define mp make_pair
12 #define ff first
13 #define ss second
14 #define pii pair<int, int>
15 #define MOD 1000000007
16 #define sqr(x) ((x) * (x))
17 #define all(x) (x).begin(), (x).end()
18 #define FOR(i, j, n) for (int i = j; i < n; i++)
19 #define qle(i, n) (i == n ? "\n" : " ")
20 #define endl "\n"
21 const int oo = 1e9;
22 const int MAX = 1e6;
23
24 int32_t main(){ optimize;
25
26     return 0;
27 }

```

### 3.2 Template Clean

```

1 // Notes:
2 // Compile and execute
3 // g++ teste.cpp -o teste -std=c++17
4 // ./teste < teste.txt
5
6 // Print with precision
7 // cout << fixed << setprecision(12) << value << endl
8 // ;
9 // File as input and output
10 // freopen("input.txt", "r", stdin);
11 // freopen("output.txt", "w", stdout);

```



```

12
13 #include <bits/stdc++.h>
14 using namespace std;
15
16 int main() {
17     ios::sync_with_stdio(false);
18     cin.tie(NULL);
19
20
21
22     return 0;
23 }

```

## 4 Strings

### 4.1 Kmp

```

1 vector<int> prefix_function(string s) {
2     int n = (int)s.length();
3     vector<int> pi(n);
4     for (int i = 1; i < n; i++) {
5         int j = pi[i-1];
6         while (j > 0 && s[i] != s[j])
7             j = pi[j-1];
8         if (s[i] == s[j])
9             j++;
10        pi[i] = j;
11    }
12    return pi;
13 }

```

### 4.2 Generate All Permutations

```

1 vector<string> generate_permutations(string s) {
2     int n = s.size();
3     vector<string> ans;
4
5     sort(s.begin(), s.end());
6
7     do {
8         ans.push_back(s);
9     } while (next_permutation(s.begin(), s.end()));
10
11    return ans;
12 }

```

### 4.3 Generate All Sequences Length K

```

1 // gera todas as possíveis sequências usando as letras
   em set (de comprimento n) e que tenham tamanho k
2 // sequence = ""
3 vector<string> generate_sequences(char set[], string
   sequence, int n, int k) {
4     if (k == 0){
5         return { sequence };
6     }
7
8     vector<string> ans;
9     for (int i = 0; i < n; i++) {
10        auto aux = generate_sequences(set, sequence +
            set[i], n, k - 1);
11        ans.insert(ans.end(), aux.begin(), aux.end())
12        ;
13        // for (auto e : aux) ans.push_back(e);
14    }
15    return ans;
16 }

```

### 4.4 Lcs

```

1 // Description:
2 // Finds the longest common subsequence between two
   string
3
4 // Problem:
5 // https://codeforces.com/gym/103134/problem/B
6
7 // Complexity:
8 // O(mn) where m and n are the length of the strings
9
10 string lcsAlgo(string s1, string s2, int m, int n) {
11     int LCS_table[m + 1][n + 1];
12
13     for (int i = 0; i <= m; i++) {
14         for (int j = 0; j <= n; j++) {
15             if (i == 0 || j == 0)
16                 LCS_table[i][j] = 0;
17             else if (s1[i - 1] == s2[j - 1])
18                 LCS_table[i][j] = LCS_table[i - 1][j - 1] +
19                 1;
20             else
21                 LCS_table[i][j] = max(LCS_table[i - 1][j],
22                     LCS_table[i][j - 1]);
23         }
24     }
25
26     int index = LCS_table[m][n];
27     char lcsAlgo[index + 1];
28     lcsAlgo[index] = '\0';
29
30     int i = m, j = n;
31     while (i > 0 && j > 0) {
32         if (s1[i - 1] == s2[j - 1]) {
33             lcsAlgo[index - 1] = s1[i - 1];
34             i--;
35             j--;
36             index--;
37         }
38         else if (LCS_table[i - 1][j] > LCS_table[i][j - 1])
39             i--;
40         else
41             j--;
42     }
43     return lcsAlgo;
44 }

```

### 4.5 Trie

```

1 const int K = 26;
2
3 struct Vertex {
4     int next[K];
5     bool output = false;
6     int p = -1;
7     char pch;
8     int link = -1;
9     int go[K];
10
11     Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
12         fill(begin(next), end(next), -1);
13         fill(begin(go), end(go), -1);
14     }
15 };
16
17 vector<Vertex> t(1);
18
19 void add_string(string const& s) {
20     int v = 0;
21     for (char ch : s) {
22         int c = ch - 'a';

```

```

23         if (t[v].next[c] == -1) {
24             t[v].next[c] = t.size();
25             t.emplace_back(v, ch);
26         }
27         v = t[v].next[c];
28     }
29     t[v].output = true;
30 }
31
32 int go(int v, char ch);
33
34 int get_link(int v) {
35     if (t[v].link == -1) {
36         if (v == 0 || t[v].p == 0)
37             t[v].link = 0;
38         else
39             t[v].link = go(get_link(t[v].p), t[v].pch
40     );
41     }
42     return t[v].link;
43 }
44
45 int go(int v, char ch) {
46     int c = ch - 'a';
47     if (t[v].go[c] == -1) {
48         if (t[v].next[c] != -1)
49             t[v].go[c] = t[v].next[c];
50         else
51             t[v].go[c] = v == 0 ? 0 : go(get_link(v),
52     ch);
53     }
54     return t[v].go[c];
55 }

```

## 4.6 Z-function

```

1 vector<int> z_function(string s) {
2     int n = (int) s.length();
3     vector<int> z(n);
4     for (int i = 1, l = 0, r = 0; i < n; ++i) {
5         if (i <= r)
6             z[i] = min(r - i + 1, z[i - l]);
7         while (i + z[i] < n && s[z[i]] == s[i + z[i]
8     ])
9             ++z[i];
10        if (i + z[i] - 1 > r)
11            l = i, r = i + z[i] - 1;
12    }
13    return z;
14 }

```

## 5 Misc

### 5.1 Split

```

1 vector<string> split(string txt, char key = ' '){
2     vector<string> ans;
3
4     string palTemp = "";
5     for(int i = 0; i < txt.size(); i++){
6
7         if(txt[i] == key){
8             if(palTemp.size() > 0){
9                 ans.push_back(palTemp);
10                palTemp = "";
11            }
12        } else{
13            palTemp += txt[i];
14        }
15    }
16 }

```

```

17         if(palTemp.size() > 0)
18             ans.push_back(palTemp);
19
20     }
21     return ans;
22 }

```

## 5.2 Int128

```

1 __int128 read() {
2     __int128 x = 0, f = 1;
3     char ch = getchar();
4     while (ch < '0' || ch > '9') {
5         if (ch == '-') f = -1;
6         ch = getchar();
7     }
8     while (ch >= '0' && ch <= '9') {
9         x = x * 10 + ch - '0';
10        ch = getchar();
11    }
12    return x * f;
13 }
14
15 void print(__int128 x) {
16     if (x < 0) {
17         putchar('-');
18         x = -x;
19     }
20     if (x > 9) print(x / 10);
21     putchar(x % 10 + '0');
22 }

```

## 6 Graphs

### 6.1 Centroid Find

```

1 // Description:
2 // Indexed at zero
3 // Find a centroid, that is a node such that when it
4 // is appointed the root of the tree,
5 // each subtree has at most floor(n/2) nodes.
6 // Problem:
7 // https://cses.fi/problemset/task/2079/
8
9 // Complexity:
10 // O(n)
11
12 // How to use:
13 // get_subtree_size(0);
14 // cout << get_centroid(0) + 1 << endl;
15
16 int n;
17 vector<int> adj[MAX];
18 int subtree_size[MAX];
19
20 int get_subtree_size(int node, int par = -1) {
21     int &res = subtree_size[node];
22     res = 1;
23     for (int i : adj[node]) {
24         if (i == par) continue;
25         res += get_subtree_size(i, node);
26     }
27     return res;
28 }
29
30 int get_centroid(int node, int par = -1) {
31     for (int i : adj[node]) {
32         if (i == par) continue;
33
34         if (subtree_size[i] * 2 > n) { return
35     get_centroid(i, node); }
36     }
37 }

```

```

35 }
36 return node;
37 }
38
39 int main() {
40     cin >> n;
41     for (int i = 0; i < n - 1; i++) {
42         int u, v; cin >> u >> v;
43         u--; v--;
44         adj[u].push_back(v);
45         adj[v].push_back(u);
46     }
47
48     get_subtree_size(0);
49     cout << get_centroid(0) + 1 << endl;
50 }

```

## 6.2 Bipartite

```

1  const int NONE = 0, BLUE = 1, RED = 2;
2  vector<vector<int>> graph(100005);
3  vector<bool> visited(100005);
4  int color[100005];
5
6  bool bfs(int s = 1){
7
8      queue<int> q;
9      q.push(s);
10     color[s] = BLUE;
11
12     while (not q.empty()){
13         auto u = q.front(); q.pop();
14
15         for (auto v : graph[u]){
16             if (color[v] == NONE){
17                 color[v] = 3 - color[u];
18                 q.push(v);
19             }
20             else if (color[v] == color[u]){
21                 return false;
22             }
23         }
24     }
25
26     return true;
27 }
28
29 bool is_bipartite(int n){
30
31     for (int i = 1; i <= n; i++)
32         if (color[i] == NONE and not bfs(i))
33             return false;
34
35     return true;
36 }

```

## 6.3 Prim

```

1  int n;
2  vector<vector<int>> adj; // adjacency matrix of graph
3  const int INF = 1000000000; // weight INF means there
   is no edge
4
5  struct Edge {
6      int w = INF, to = -1;
7  };
8
9  void prim() {
10     int total_weight = 0;
11     vector<bool> selected(n, false);
12     vector<Edge> min_e(n);
13     min_e[0].w = 0;

```

```

14
15     for (int i=0; i<n; ++i) {
16         int v = -1;
17         for (int j = 0; j < n; ++j) {
18             if (!selected[j] && (v == -1 || min_e[j].
19                 w < min_e[v].w))
20                 v = j;
21         }
22
23         if (min_e[v].w == INF) {
24             cout << "No MST!" << endl;
25             exit(0);
26         }
27
28         selected[v] = true;
29         total_weight += min_e[v].w;
30         if (min_e[v].to != -1)
31             cout << v << " " << min_e[v].to << endl;
32
33         for (int to = 0; to < n; ++to) {
34             if (adj[v][to] < min_e[to].w)
35                 min_e[to] = {adj[v][to], v};
36         }
37
38     }
39
40     cout << total_weight << endl;

```

## 6.4 Eulerian Undirected

```

1  // Description:
2  // Hierholzer's Algorithm
3  // An Eulerian path is a path that passes through
   every edge exactly once.
4  // An Eulerian circuit is an Eulerian path that
   starts and ends on the same node.
5
6  // An Eulerian path exists in an undirected graph if
   the degree of every node is even (not counting
   self-edges)
7  // except for possibly exactly two nodes that have
   and odd degree (start and end nodes).
8  // An Eulerian circuit exists in an undirected graph
   if the degree of every node is even.
9
10 // The graph has to be connected (except for isolated
   nodes which are allowed because there
11 // are no edges connected to them).
12
13 // Problem:
14 // https://cses.fi/problemset/task/1691
15
16 // Complexity:
17 //  $O(E * \log(E))$  where E is the number of edges
18
19 // How to use
20 // Check whether the path exists before trying to
   find it
21 // Find the root - any node that has at least 1
   outgoing edge
22 // (if the problem requires that you start from a
   node v, the root will be the node v)
23 // Count the degree;
24 //
25 // for (int i = 0; i < m; i++) {
26 //     int a, b; cin >> a >> b;
27 //     adj[a].pb(b); adj[b].pb(a);
28 //     root = a;
29 //     degree[a]++; degree[b]++;
30 // }
31
32 // Notes

```

```

33 // If you want to find a path start and ending nodes v and u
34 // if ((is_eulerian(n, root, start, end) != 1) || (start != v) || (end != u)) cout << "IMPOSSIBLE\n"
35
36 // It can be speed up to work on O(E) on average by using unordered_set instead of set
37
38 // It works when there are self loops, but not when there are multiple edges
39 // If the graph has multiple edges, add more notes to simulate the edges
40 // e.g
41 // 1 2
42 // 1 2
43 // 1 2
44 // becomes
45 // 3 4
46 // 4 1
47 // 1 2
48
49 vector<bool> visited;
50 vector<int> degree;
51 vector<vector<int>> adj;
52
53 void dfs(int v) {
54     visited[v] = true;
55     for (auto u : adj[v]) {
56         if (!visited[u]) dfs(u);
57     }
58 }
59
60 int is_eulerian(int n, int root, int& start, int& end) {
61     start = -1, end = -1;
62     if (n == 1) return 2; // only one node
63     visited.assign(n + 1, false);
64     dfs(root);
65
66     for (int i = 1; i <= n; i++) {
67         if (!visited[i] && degree[i] > 0) return 0;
68     }
69
70     for (int i = 1; i <= n; i++) {
71         if (start == -1 && degree[i] % 2 == 1) start = i;
72         else if (end == -1 && degree[i] % 2 == 1) end = i;
73         else if (degree[i] % 2 == 1) return 0;
74     }
75
76     if (start == -1 && end == -1) {start = root; end = root; return 2;} // has eulerian circuit and path
77     if (start != -1 && end != -1) return 1; // has eulerian path
78     return 0; // no eulerian path nor circuit
79 }
80
81 vector<int> path;
82 vector<set<int>> mark;
83
84 void dfs_path(int v) {
85     visited[v] = true;
86
87     while (degree[v] != 0) {
88         degree[v]--;
89         int u = adj[v][degree[v]];
90         if (mark[v].find(u) != mark[v].end()) continue;
91         mark[v].insert(u);
92         mark[u].insert(v);
93         int next_edge = adj[v][degree[v]];
94         dfs_path(next_edge);
95     }
96     path.pb(v);

```

```

97 }
98
99 void find_path(int n, int start) {
100     path.clear();
101     mark.resize(n + 1);
102     visited.assign(n + 1, false);
103     dfs_path(start);
104 }

```

## 6.5 Ford Fulkerson Edmonds Karp

```

1 // Description:
2 // Obtains the maximum possible flow rate given a network. A network is a graph with a single source vertex and a single sink vertex in which each edge has a capacity
3
4 // Complexity:
5 // O(V * E^2) where V is the number of vertex and E is the number of edges
6
7 int n;
8 vector<vector<int>> capacity;
9 vector<vector<int>> adj;
10
11 int bfs(int s, int t, vector<int>& parent) {
12     fill(parent.begin(), parent.end(), -1);
13     parent[s] = -2;
14     queue<pair<int, int>> q;
15     q.push({s, INF});
16
17     while (!q.empty()) {
18         int cur = q.front().first;
19         int flow = q.front().second;
20         q.pop();
21
22         for (int next : adj[cur]) {
23             if (parent[next] == -1 && capacity[cur][next]) {
24                 parent[next] = cur;
25                 int new_flow = min(flow, capacity[cur][next]);
26                 if (next == t) return new_flow;
27                 q.push({next, new_flow});
28             }
29         }
30     }
31     return 0;
32
33
34
35
36 int maxflow(int s, int t) {
37     int flow = 0;
38     vector<int> parent(n);
39     int new_flow;
40
41     while (new_flow = bfs(s, t, parent)) {
42         flow += new_flow;
43         int cur = t;
44         while (cur != s) {
45             int prev = parent[cur];
46             capacity[prev][cur] -= new_flow;
47             capacity[cur][prev] += new_flow;
48             cur = prev;
49         }
50     }
51
52     return flow;
53 }

```

## 6.6 Hld Edge

```

1 // Description:
2 // Make queries and updates between two vertexes on a
   tree
3
4 // Problem:
5 // https://www.spoj.com/problems/QTREE/
6
7 // Complexity:
8 //  $O(\log^2 n)$  for both query and update
9
10 // How to use:
11 // HLD hld = HLD(n + 1, adj)
12
13 // Notes
14 // Change the root of the tree on the constructor if
   it's different from 1
15 // Use together with Segtree
16
17 struct HLD {
18     vector<int> parent;
19     vector<int> pos;
20     vector<int> head;
21     vector<int> subtree_size;
22     vector<int> level;
23     vector<int> heavy_child;
24     vector<ftype> subtree_weight;
25     vector<ftype> path_weight;
26     vector<vector<int>>> adj;
27     vector<int> at;
28     Segtree seg = Segtree(0);
29     int cpos;
30     int n;
31     int root;
32
33     HLD() {}
34
35     HLD(int n, vector<vector<int>>>& adj, int root = 1) {
36         : adj(adj), n(n), root(root) {
37             seg = Segtree(n);
38             cpos = 0;
39             at.assign(n, 0);
40             parent.assign(n, 0);
41             pos.assign(n, 0);
42             head.assign(n, 0);
43             subtree_size.assign(n, 1);
44             level.assign(n, 0);
45             heavy_child.assign(n, -1);
46             parent[root] = -1;
47             dfs(root, -1);
48             decompose(root, -1);
49         }
50
51     void dfs(int v, int p) {
52         parent[v] = p;
53         if (p != -1) level[v] = level[p] + 1;
54         for (auto u : adj[v]) {
55             if (u != p) {
56                 dfs(u, v);
57                 subtree_size[v] += subtree_size[u];
58                 if (heavy_child[v] == -1 || subtree_size[u] >
59                     subtree_size[heavy_child[v]]) heavy_child[v] = u;
60             }
61         }
62     }
63
64     void decompose(int v, int chead) {
65         // start a new path
66         if (chead == -1) chead = v;
67
68         // consecutive ids in the hld path
69         at[cpos] = v;
70         pos[v] = cpos++;
71
72         head[v] = chead;
73
74         // if not a leaf
75         if (heavy_child[v] != -1) decompose(heavy_child[v],
76             chead);
77
78         // light child
79         for (auto u : adj[v]){
80             // start new path
81             if (u != parent[v] && u != heavy_child[v])
82                 decompose(u, -1);
83         }
84     }
85
86     ll query_path(int a, int b) {
87         if (a == b) return 0;
88         if(pos[a] < pos[b]) swap(a, b);
89
90         if(head[a] == head[b]) return seg.query(pos[b] +
91             1, pos[a]);
92         return seg.f(seg.query(pos[head[a]], pos[a]),
93             query_path(parent[head[a]], b));
94     }
95
96     ftype query_subtree(int a) {
97         if (subtree_size[a] == 1) return 0;
98         return seg.query(pos[a] + 1, pos[a] +
99             subtree_size[a] - 1);
100     }
101
102     void update_path(int a, int b, int x) {
103         if (a == b) return;
104         if(pos[a] < pos[b]) swap(a, b);
105
106         if(head[a] == head[b]) return (void)seg.update(
107             pos[b] + 1, pos[a], x);
108         seg.update(pos[head[a]], pos[a], x); update_path(
109             parent[head[a]], b, x);
110     }
111
112     void update_subtree(int a, int val) {
113         if (subtree_size[a] == 1) return;
114         seg.update(pos[a] + 1, pos[a] + subtree_size[a] -
115             1, val);
116     }
117
118     // vertex
119     void update(int a, int val) {
120         seg.update(pos[a], pos[a], val);
121     }
122
123     //edge
124     void update(int a, int b, int val) {
125         if (parent[a] == b) swap(a, b);
126         update(b, val);
127     }
128
129     int lca(int a, int b) {
130         if(pos[a] < pos[b]) swap(a, b);
131         return head[a] == head[b] ? b : lca(parent[head[a]],
132             b);
133     }
134 };

```

## 6.7 Floyd Warshall

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4 using ll = long long;
5
6 const int MAX = 507;
7 const long long INF = 0x3f3f3f3f3f3f3f3fLL;

```

```

8
9 ll dist[MAX][MAX];
10 int n;
11
12 void floyd_warshall() {
13     for (int i = 0; i < n; i++) {
14         for (int j = 0; j < n; j++) {
15             if (i == j) dist[i][j] = 0;
16             else if (!dist[i][j]) dist[i][j] = INF;
17         }
18     }
19
20     for (int k = 0; k < n; k++) {
21         for (int i = 0; i < n; i++) {
22             for (int j = 0; j < n; j++) {
23                 // trata o caso no qual o grafo tem
24                 arestas com peso negativo
25                 if (dist[i][k] < INF && dist[k][j] <
26                     INF){
27                     dist[i][j] = min(dist[i][j], dist
28                     [i][k] + dist[k][j]);
29                 }
30             }
31         }
32     }
33 }

```

## 6.8 Lca

```

1 // Description:
2 // Find the lowest common ancestor between two nodes
3 // in a tree
4
5 // Problem:
6 // https://cses.fi/problemset/task/1135
7
8 // Complexity:
9 // O(log n)
10
11 // How to use:
12 // preprocess();
13 // lca(a, b);
14
15 // Notes
16 // To calculate the distance between two nodes use
17 // the following formula
18 // level_peso[a] + level_peso[b] - 2*level_peso[lca(a
19 // , b)]
20
21 const int MAX = 2e5+10;
22 const int BITS = 30;
23
24 vector<pii> adj[MAX];
25 vector<bool> visited(MAX);
26
27 int up[MAX][BITS + 1];
28 int level[MAX];
29 int level_peso[MAX];
30
31 void find_level() {
32     queue<pii> q;
33
34     q.push(mp(1, 0));
35     visited[1] = true;
36
37     while (!q.empty()) {
38         auto [v, depth] = q.front();
39         q.pop();
40         level[v] = depth;
41
42         for (auto [u,d] : adj[v]) {
43             if (!visited[u]) {
44                 visited[u] = true;

```

```

42         up[u][0] = v;
43         q.push(mp(u, depth + 1));
44     }
45 }
46
47 void find_level_peso() {
48     queue<pii> q;
49
50     q.push(mp(1, 0));
51     visited[1] = true;
52
53     while (!q.empty()) {
54         auto [v, depth] = q.front();
55         q.pop();
56         level_peso[v] = depth;
57
58         for (auto [u,d] : adj[v]) {
59             if (!visited[u]) {
60                 visited[u] = true;
61                 up[u][0] = v;
62                 q.push(mp(u, depth + d));
63             }
64         }
65     }
66 }
67
68 int lca(int a, int b) {
69     // get the nodes to the same level
70     int mn = min(level[a], level[b]);
71
72     for (int j = 0; j <= BITS; j++) {
73         if (a != -1 && ((level[a] - mn) & (1 << j))) a
74         = up[a][j];
75         if (b != -1 && ((level[b] - mn) & (1 << j))) b
76         = up[b][j];
77     }
78
79     // special case
80     if (a == b) return a;
81
82     // binary search
83     for (int j = BITS; j >= 0; j--) {
84         if (up[a][j] != up[b][j]) {
85             a = up[a][j];
86             b = up[b][j];
87         }
88     }
89     return up[a][0];
90 }
91
92 void preprocess() {
93     visited = vector<bool>(MAX, false);
94     find_level();
95     visited = vector<bool>(MAX, false);
96     find_level_peso();
97
98     for (int j = 1; j <= BITS; j++) {
99         for (int i = 1; i <= n; i++) {
100             if (up[i][j - 1] != -1) up[i][j] = up[up[i][j -
101             1]][j - 1];
102         }
103     }

```

## 6.9 Eulerian Directed

```

1 // Description:
2 // Hierholzer's Algorithm
3 // An Eulerian path is a path that passes through
4 // every edge exactly once.

```

```

4 // An Eulerian circuit is an Eulerian path that
  starts and ends on the same node.
5
6 // An Eulerian path exists in an directed graph if
  the indegree and outdegree is equal
7 // for every node (not counting self-edges)
8 // except for possibly exactly one node that have
  outdegree - indegree = 1
9 // and one node that has indegree - outdegree = 1 (
  start and end nodes).
10 // An Eulerian circuit exists in an directed graph if
  the indegree and outdegree is equal for every
  node.
11
12 // The graph has to be connected (except for isolated
  nodes which are allowed because there
13 // are no edges connected to them).
14
15 // Problem:
16 // https://cses.fi/problemset/task/1693
17
18 // Complexity:
19 // O(E) where E is the number of edges
20
21 // How to use
22 // Check whether the path exists before trying to
  find it
23 // Find the root - any node that has at least 1
  outgoing edge
24 // (if the problem requires that you start from a
  node v, the root will be the node v)
25 // Count the degree;
26 //
27 // for (int i = 0; i < m; i++) {
28 //   int a, b; cin >> a >> b;
29 //   adj[a].pb(b);
30 //   root = a;
31 //   outdegree[a]++; indegree[b]++;
32 // }
33
34 // Notes
35 // It works when there are self loops, but not when
  there are multiple edges
36
37 vector<bool> visited;
38 vector<int> outdegree, indegree;
39 vector<vector<int>> adj, undir;
40
41 void dfs(int v) {
42     visited[v] = true;
43     for (auto u : undir[v]) {
44         if (!visited[u]) dfs(u);
45     }
46 }
47
48 int is_eulerian(int n, int root, int &start, int& end
  ) {
49     start = -1, end = -1;
50     if (n == 1) return 2; // only one node
51     visited.assign(n + 1, false);
52     dfs(root);
53
54     for (int i = 1; i <= n; i++) {
55         if (!visited[i] && (i == n || i == 1 || outdegree
          [i] + indegree[i] > 0)) return 0;
56     }
57
58     // start => node with indegree - outdegree = 1
59     // end => node with outdegree - indegree = 1
60     for (int i = 1; i <= n; i++) {
61         if (start == -1 && indegree[i] - outdegree[i] ==
          1) start = i;
62         else if (end == -1 && outdegree[i] - indegree[i]
          == 1) end = i;
63         else if (indegree[i] != outdegree[i]) return 0;
64     }
65
66     if (start == -1 && end == -1) {start = root; end =
      root; return 2;} // has eulerian circuit and path
67     if (start != -1 && end != -1) {swap(start, end);
      return 1;} // has eulerian path
68     return 0; // no eulerian path nor circuit
69 }
70
71 vector<int> path;
72
73 void dfs_path(int v) {
74     visited[v] = true;
75
76     while (outdegree[v] != 0) {
77         int u = adj[v][--outdegree[v]];
78         int next_edge = adj[v][outdegree[v]];
79         dfs_path(next_edge);
80     }
81     path.pb(v);
82 }
83
84 void find_path(int n, int start) {
85     path.clear();
86     visited.assign(n + 1, false);
87     dfs_path(start);
88     reverse(path.begin(), path.end());
89 }

```

## 6.10 Bellman Ford

```

1 // Description:
2 // Finds the shortest path from a vertex v to any
  other vertex
3
4 // Problem:
5 // https://cses.fi/problemset/task/1673
6
7 // Complexity:
8 // O(n * m)
9
10 struct Edge {
11     int a, b, cost;
12     Edge(int a, int b, int cost) : a(a), b(b), cost(
      cost) {}
13 };
14
15 int n, m;
16 vector<Edge> edges;
17 const int INF = 1e9+10;
18
19 void bellman_ford(int v, int t) {
20     vector<int> d(n + 1, INF);
21     d[v] = 0;
22     vector<int> p(n + 1, -1);
23
24     for (;;) {
25         bool any = false;
26         for (Edge e : edges) {
27             if (d[e.a] >= INF) continue;
28             if (d[e.b] > d[e.a] + e.cost) {
29                 d[e.b] = d[e.a] + e.cost;
30                 p[e.b] = e.a;
31                 any = true;
32             }
33         }
34         if (!any) break;
35     }
36
37     if (d[t] == INF)

```

```

38     cout << "No path from " << v << " to " << t << ".
39 ";
40 else {
41     vector<int> path;
42     for (int cur = t; cur != -1; cur = p[cur]) {
43         path.push_back(cur);
44     }
45     reverse(path.begin(), path.end());
46
47     cout << "Path from " << v << " to " << t << ": ";
48     for (int u : path) {
49         cout << u << ' ';
50     }
51 }

```

## 6.11 Dinic

```

1 // Description:
2 // Obtains the maximum possible flow rate given a
3 // network. A network is a graph with a single
4 // source vertex and a single sink vertex in which
5 // each edge has a capacity
6
7 // Problem:
8 // https://codeforces.com/gym/103708/problem/J
9
10 // Complexity:
11 //  $O(V^2 * E)$  where V is the number of vertex and E
12 // is the number of edges
13
14 // Unit network
15 // A unit network is a network in which for any
16 // vertex except source and sink either incoming or
17 // outgoing edge is unique and has unit capacity (
18 // matching problem).
19 // Complexity on unit networks:  $O(E * \sqrt{V})$ 
20
21 // Unity capacity networks
22 // A more generic settings when all edges have unit
23 // capacities, but the number of incoming and
24 // outgoing edges is unbounded
25 // Complexity on unity capacity networks:  $O(E * \sqrt{E})$ 
26
27 // How to use:
28 // Dinic dinic = Dinic(num_vertex, source, sink);
29 // dinic.add_edge(vertex1, vertex2, capacity);
30 // cout << dinic.max_flow() << '\n';
31
32 #include <bits/stdc++.h>
33
34 #define pb push_back
35 #define mp make_pair
36 #define pii pair<int, int>
37 #define ff first
38 #define ss second
39 #define ll long long
40
41 using namespace std;
42
43 const ll INF = 1e18+10;
44
45 struct Edge {
46     int from;
47     int to;
48     ll capacity;
49     ll flow;
50     Edge* residual;
51
52     Edge() {}

```

```

53     Edge(int from, int to, ll capacity) : from(from),
54         to(to), capacity(capacity) {
55         flow = 0;
56     }
57
58     ll get_capacity() {
59         return capacity - flow;
60     }
61
62     ll get_flow() {
63         return flow;
64     }
65
66     void augment(ll bottleneck) {
67         flow += bottleneck;
68         residual->flow -= bottleneck;
69     }
70
71     void reverse(ll bottleneck) {
72         flow -= bottleneck;
73         residual->flow += bottleneck;
74     }
75
76     bool operator<(const Edge& e) const {
77         return true;
78     }
79 };
80
81 struct Dinic {
82     int source;
83     int sink;
84     int nodes;
85     ll flow;
86     vector<vector<Edge*>> adj;
87     vector<int> level;
88     vector<int> next;
89     vector<int> reach;
90     vector<bool> visited;
91     vector<vector<int>> path;
92
93     Dinic(int source, int sink, int nodes) : source(
94         source), sink(sink), nodes(nodes) {
95         adj.resize(nodes + 1);
96     }
97
98     void add_edge(int from, int to, ll capacity) {
99         Edge* e1 = new Edge(from, to, capacity);
100         Edge* e2 = new Edge(to, from, 0);
101         // Edge* e2 = new Edge(to, from, capacity);
102         e1->residual = e2;
103         e2->residual = e1;
104         adj[from].pb(e1);
105         adj[to].pb(e2);
106     }
107
108     bool bfs() {
109         level.assign(nodes + 1, -1);
110         queue<int> q;
111         q.push(source);
112         level[source] = 0;
113
114         while (!q.empty()) {
115             int node = q.front();
116             q.pop();
117
118             for (auto e : adj[node]) {
119                 if (level[e->to] == -1 && e->
120                     get_capacity() > 0) {
121                     level[e->to] = level[e->from] +
122                         1;
123                     q.push(e->to);
124                 }
125             }
126         }

```



```

114     }
115
116     return level[sink] != -1;
117 }
118
119 ll dfs(int v, ll flow) {
120     if (v == sink)
121         return flow;
122
123     int sz = adj[v].size();
124     for (int i = next[v]; i < sz; i++) {
125         Edge* e = adj[v][i];
126         if (level[e->to] == level[e->from] + 1 && e->get_capacity() > 0) {
127             ll bottleneck = dfs(e->to, min(flow, e->get_capacity()));
128             if (bottleneck > 0) {
129                 e->augment(bottleneck);
130                 return bottleneck;
131             }
132         }
133         next[v] = i + 1;
134     }
135
136     return 0;
137 }
138
139 ll max_flow() {
140     flow = 0;
141     while (bfs()) {
142         next.assign(nodes + 1, 0);
143         ll sent = -1;
144         while (sent != 0) {
145             sent = dfs(source, INF);
146             flow += sent;
147         }
148     }
149     return flow;
150 }
151
152 void reachable(int v) {
153     visited[v] = true;
154
155     for (auto e : adj[v]) {
156         if (!visited[e->to] && e->get_capacity() > 0) {
157             reach.pb(e->to);
158             visited[e->to] = true;
159             reachable(e->to);
160         }
161     }
162 }
163
164 void print_min_cut() {
165     reach.clear();
166     visited.assign(nodes + 1, false);
167     reach.pb(source);
168     reachable(source);
169
170     for (auto v : reach) {
171         for (auto e : adj[v]) {
172             if (!visited[e->to] && e->get_capacity() == 0) {
173                 cout << e->from << ' ' << e->to << '\n';
174             }
175         }
176     }
177 }
178
179 ll build_path(int v, int id, ll flow) {
180     visited[v] = true;
181
182     if (v == sink) {
183         return flow;
184     }
185
186     for (auto e : adj[v]) {
187         if (!visited[e->to] && e->get_flow() > 0) {
188             visited[e->to] = true;
189             ll bottleneck = build_path(e->to, id, min(flow, e->get_flow()));
190             if (bottleneck > 0) {
191                 path[id].pb(e->to);
192                 e->reverse(bottleneck);
193                 return bottleneck;
194             }
195         }
196     }
197
198     return 0;
199 }
200
201 void print_flow_path() {
202     path.clear();
203     ll sent = -1;
204     int id = -1;
205     while (sent != 0) {
206         visited.assign(nodes + 1, false);
207         path.pb(vector<int>{});
208         sent = build_path(source, ++id, INF);
209         path[id].pb(source);
210     }
211     path.pop_back();
212
213     for (int i = 0; i < id; i++) {
214         cout << path[i].size() << '\n';
215         reverse(path[i].begin(), path[i].end());
216         for (auto e : path[i]) {
217             cout << e << ' ';
218         }
219         cout << '\n';
220     }
221 }
222 };
223
224 int main() {
225     ios::sync_with_stdio(false);
226     cin.tie(NULL);
227
228     int n, m; cin >> n >> m;
229
230     Dinic dinic = Dinic(1, n, n);
231
232     for (int i = 1; i <= m; i++) {
233         int v, u; cin >> v >> u;
234         dinic.add_edge(v, u, 1);
235     }
236
237     cout << dinic.max_flow() << '\n';
238     // dinic.print_min_cut();
239     // dinic.print_flow_path();
240
241     return 0;
242 }

```

## 6.12 2sat

```

1 // Description:
2 // Solves expression of the type (a v b) ^ (c v d) ^
  (e v f)
3
4 // Problem:
5 // https://cses.fi/problemset/task/1684
6

```

```

7 // Complexity:
8 //  $O(n + m)$  where  $n$  is the number of variables and  $m$ 
   is the number of clauses
9
10 #include <bits/stdc++.h>
11 #define pb push_back
12 #define mp make_pair
13 #define pii pair<int, int>
14 #define ff first
15 #define ss second
16
17 using namespace std;
18
19 struct SAT {
20     int nodes;
21     int curr = 0;
22     int component = 0;
23     vector<vector<int>> adj;
24     vector<vector<int>> rev;
25     vector<vector<int>> condensed;
26     vector<pii> departure;
27     vector<bool> visited;
28     vector<int> scc;
29     vector<int> order;
30
31     // 1 to nodes
32     // nodes + 1 to 2 * nodes
33     SAT(int nodes) : nodes(nodes) {
34         adj.resize(2 * nodes + 1);
35         rev.resize(2 * nodes + 1);
36         visited.resize(2 * nodes + 1);
37         scc.resize(2 * nodes + 1);
38     }
39
40     void add_imp(int a, int b) {
41         adj[a].pb(b);
42         rev[b].pb(a);
43     }
44
45     int get_not(int a) {
46         if (a > nodes) return a - nodes;
47         return a + nodes;
48     }
49
50     void add_or(int a, int b) {
51         add_imp(get_not(a), b);
52         add_imp(get_not(b), a);
53     }
54
55     void add_nor(int a, int b) {
56         add_or(get_not(a), get_not(b));
57     }
58
59     void add_and(int a, int b) {
60         add_or(get_not(a), b);
61         add_or(a, get_not(b));
62         add_or(a, b);
63     }
64
65     void add_nand(int a, int b) {
66         add_or(get_not(a), b);
67         add_or(a, get_not(b));
68         add_or(get_not(a), get_not(b));
69     }
70
71     void add_xor(int a, int b) {
72         add_or(a, b);
73         add_or(get_not(a), get_not(b));
74     }
75
76     void add_xnor(int a, int b) {
77         add_or(get_not(a), b);
78         add_or(a, get_not(b));
79     }
80
81     void departure_time(int v) {
82         visited[v] = true;
83
84         for (auto u : adj[v]) {
85             if (!visited[u]) departure_time(u);
86         }
87
88         departure.pb(mp(++curr, v));
89     }
90
91     void find_component(int v, int component) {
92         scc[v] = component;
93         visited[v] = true;
94
95         for (auto u : rev[v]) {
96             if (!visited[u]) find_component(u,
97 component);
98         }
99     }
100
101     void topological_order(int v) {
102         visited[v] = true;
103
104         for (auto u : condensed[v]) {
105             if (!visited[u]) topological_order(u);
106         }
107
108         order.pb(v);
109     }
110
111     bool is_possible() {
112         component = 0;
113         for (int i = 1; i <= 2 * nodes; i++) {
114             if (!visited[i]) departure_time(i);
115         }
116
117         sort(departure.begin(), departure.end(),
118 greater<pii>());
119
120         visited.assign(2 * nodes + 1, false);
121
122         for (auto [_ , node] : departure) {
123             if (!visited[node]) find_component(node,
124 ++component);
125         }
126
127         for (int i = 1; i <= nodes; i++) {
128             if (scc[i] == scc[i + nodes]) return
129 false;
130         }
131
132         return true;
133     }
134
135     int find_value(int e, vector<int> &ans) {
136         if (e > nodes && ans[e - nodes] != 2) return
137 !ans[e - nodes];
138         if (e <= nodes && ans[e + nodes] != 2) return
139 !ans[e + nodes];
140         return 0;
141     }
142
143     vector<int> find_ans() {
144         condensed.resize(component + 1);
145
146         for (int i = 1; i <= 2 * nodes; i++) {
147             for (auto u : adj[i]) {
148                 if (scc[i] != scc[u]) condensed[scc[i]
149 ]].pb(scc[u]);
150             }
151         }
152     }

```

```

145         visited.assign(component + 1, false);
146
147     for (int i = 1; i <= component; i++) {
148         if (!visited[i]) topological_order(i);
149     }
150
151     reverse(order.begin(), order.end());
152
153     // 0 - false
154     // 1 - true
155     // 2 - no value yet
156     vector<int> ans(2 * nodes + 1, 2);
157
158     vector<vector<int>> belong(component + 1);
159
160     for (int i = 1; i <= 2 * nodes; i++) {
161         belong[scc[i]].pb(i);
162     }
163
164     for (auto p : order) {
165         for (auto e : belong[p]) {
166             ans[e] = find_value(e, ans);
167         }
168     }
169
170     return ans;
171 }
172 };
173
174
175 int main() {
176     ios::sync_with_stdio(false);
177     cin.tie(NULL);
178
179     int n, m; cin >> n >> m;
180
181     SAT sat = SAT(m);
182
183     for (int i = 0; i < n; i++) {
184         char op1, op2; int a, b; cin >> op1 >> a >>
185         op2 >> b;
186         if (op1 == '+' && op2 == '+') sat.add_or(a, b);
187         if (op1 == '-' && op2 == '-') sat.add_or(sat.get_not(a), sat.get_not(b));
188         if (op1 == '+' && op2 == '-') sat.add_or(a, sat.get_not(b));
189         if (op1 == '-' && op2 == '+') sat.add_or(sat.get_not(a), b);
190     }
191
192     if (!sat.is_possible()) cout << "IMPOSSIBLE\n";
193     else {
194         vector<int> ans = sat.find_ans();
195         for (int i = 1; i <= m; i++) {
196             cout << (ans[i] == 1 ? '+' : '-') << ' ';
197         }
198         cout << '\n';
199     }
200
201     return 0;
202 }

```

## 6.13 Find Cycle

```

1  bitset<MAX> visited;
2  vector<int> path;
3  vector<int> adj[MAX];
4
5  bool dfs(int u, int p){
6
7      if (visited[u]) return false;
8

```

```

9      path.pb(u);
10     visited[u] = true;
11
12     for (auto v : adj[u]){
13         if (visited[v] and u != v and p != v){
14             path.pb(v); return true;
15         }
16         if (dfs(v, u)) return true;
17     }
18
19     path.pop_back();
20     return false;
21 }
22
23 bool has_cycle(int N){
24
25     visited.reset();
26
27     for (int u = 1; u <= N; ++u){
28         path.clear();
29         if (not visited[u] and dfs(u, -1))
30             return true;
31     }
32
33     return false;
34 }
35
36 }

```

## 6.14 Cycle Path Recovery

```

1  int n;
2  vector<vector<int>> adj;
3  vector<char> color;
4  vector<int> parent;
5  int cycle_start, cycle_end;
6
7  bool dfs(int v) {
8      color[v] = 1;
9      for (int u : adj[v]) {
10         if (color[u] == 0) {
11             parent[u] = v;
12             if (dfs(u))
13                 return true;
14         } else if (color[u] == 1) {
15             cycle_end = v;
16             cycle_start = u;
17             return true;
18         }
19     }
20     color[v] = 2;
21     return false;
22 }
23
24 void find_cycle() {
25     color.assign(n, 0);
26     parent.assign(n, -1);
27     cycle_start = -1;
28
29     for (int v = 0; v < n; v++) {
30         if (color[v] == 0 && dfs(v))
31             break;
32     }
33
34     if (cycle_start == -1) {
35         cout << "Acyclic" << endl;
36     } else {
37         vector<int> cycle;
38         cycle.push_back(cycle_start);
39         for (int v = cycle_end; v != cycle_start; v =
40             parent[v])
41             cycle.push_back(v);
42         cycle.push_back(cycle_start);

```

```

42     reverse(cycle.begin(), cycle.end());
43
44     cout << "Cycle found: ";
45     for (int v : cycle)
46         cout << v << " ";
47     cout << endl;
48 }
49 }

```

## 6.15 Centroid Decomposition

```

1  int n;
2  vector<set<int>> adj;
3  vector<char> ans;
4
5  vector<bool> removed;
6
7  vector<int> subtree_size;
8
9  int dfs(int u, int p = 0) {
10     subtree_size[u] = 1;
11
12     for(int v : adj[u]) {
13         if(v != p && !removed[v]) {
14             subtree_size[u] += dfs(v, u);
15         }
16     }
17
18     return subtree_size[u];
19 }
20
21 int get_centroid(int u, int sz, int p = 0) {
22     for(int v : adj[u]) {
23         if(v != p && !removed[v]) {
24             if(subtree_size[v]*2 > sz) {
25                 return get_centroid(v, sz, u);
26             }
27         }
28     }
29
30     return u;
31 }
32
33 char get_next(char c) {
34     if (c != 'Z') return c + 1;
35     return '$';
36 }
37
38 bool flag = true;
39
40 void solve(int node, char c) {
41     int center = get_centroid(node, dfs(node));
42     ans[center] = c;
43     removed[center] = true;
44
45     for (auto u : adj[center]) {
46         if (!removed[u]) {
47             char next = get_next(c);
48             if (next == '$') {
49                 flag = false;
50                 return;
51             }
52             solve(u, next);
53         }
54     }
55 }
56
57 int32_t main(){
58     ios::sync_with_stdio(false);
59     cin.tie(NULL);
60
61     cin >> n;
62     adj.resize(n + 1);

```

```

63     ans.resize(n + 1);
64     removed.resize(n + 1);
65     subtree_size.resize(n + 1);
66
67     for (int i = 1; i <= n - 1; i++) {
68         int u, v; cin >> u >> v;
69         adj[u].insert(v);
70         adj[v].insert(u);
71     }
72
73     solve(1, 'A');
74
75     if (!flag) cout << "Impossible!\n";
76     else {
77         for (int i = 1; i <= n; i++) {
78             cout << ans[i] << ' ';
79         }
80         cout << '\n';
81     }
82
83     return 0;
84 }

```

## 6.16 Tarjan Bridge

```

1  // Description:
2  // Find a bridge in a connected undirected graph
3  // A bridge is an edge so that if you remove that
   edge the graph is no longer connected
4
5  // Problem:
6  // https://cses.fi/problemset/task/2177/
7
8  // Complexity:
9  //  $O(V + E)$  where  $V$  is the number of vertices and  $E$ 
   is the number of edges
10
11 int n;
12 vector<vector<int>> adj;
13
14 vector<bool> visited;
15 vector<int> tin, low;
16 int timer;
17
18 void dfs(int v, int p) {
19     visited[v] = true;
20     tin[v] = low[v] = timer++;
21     for (int to : adj[v]) {
22         if (to == p) continue;
23         if (visited[to]) {
24             low[v] = min(low[v], tin[to]);
25         } else {
26             dfs(to, v);
27             low[v] = min(low[v], low[to]);
28             if (low[to] > tin[v]) {
29                 IS_BRIDGE(v, to);
30             }
31         }
32     }
33 }
34
35 void find_bridges() {
36     timer = 0;
37     visited.assign(n, false);
38     tin.assign(n, -1);
39     low.assign(n, -1);
40     for (int i = 0; i < n; ++i) {
41         if (!visited[i])
42             dfs(i, -1);
43     }
44 }

```

## 6.17 Hld Vertex

```

1 // Description:
2 // Make queries and updates between two vertexes on a
   tree
3 // Query path - query path (a, b) inclusive
4 // Update path - update path (a, b) inclusive
5 // Query subtree - query subtree of a
6 // Update subtree - update subtree of a
7 // Update - update vertex or edge
8 // Lca - get lowest common ancestor of a and b
9 // Search - perform a binary search to find the last
   node with a certain property
10 // on the path from a to the root
11
12 // Problem:
13 // https://codeforces.com/gym/101908/problem/L
14
15 // Complexity:
16 //  $O(\log^2 n)$  for both query and update
17
18 // How to use:
19 // HLD hld = HLD(n + 1, adj)
20
21 // Notes
22 // Change the root of the tree on the constructor if
   it's different from 1
23 // Use together with Segtree
24
25 typedef long long ftype;
26
27 struct HLD {
28     vector<int> parent;
29     vector<int> pos;
30     vector<int> head;
31     vector<int> subtree_size;
32     vector<int> level;
33     vector<int> heavy_child;
34     vector<ftype> subtree_weight;
35     vector<ftype> path_weight;
36     vector<vector<int>>> adj;
37     vector<int> at;
38     Segtree seg = Segtree(0);
39     int cpos;
40     int n;
41     int root;
42     vector<vector<int>>> up;
43
44     HLD() {}
45
46     HLD(int n, vector<vector<int>>>& adj, int root = 1)
       : adj(adj), n(n), root(root) {
47         seg = Segtree(n);
48         cpos = 0;
49         at.resize(n);
50         parent.resize(n);
51         pos.resize(n);
52         head.resize(n);
53         subtree_size.assign(n, 1);
54         level.assign(n, 0);
55         heavy_child.assign(n, -1);
56         parent[root] = -1;
57         dfs(root, -1);
58         decompose(root, -1);
59     }
60
61     void dfs(int v, int p) {
62         parent[v] = p;
63         if (p != -1) level[v] = level[p] + 1;
64         for (auto u : adj[v]) {
65             if (u != p) {
66                 dfs(u, v);
67                 subtree_size[v] += subtree_size[u];
68                 if (heavy_child[v] == -1 || subtree_size[u] >
                   subtree_size[heavy_child[v]]) heavy_child[v] = u;
69             }
70         }
71     }
72
73     void decompose(int v, int chead) {
74         // start a new path
75         if (chead == -1) chead = v;
76
77         // consecutive ids in the hld path
78         at[cpos] = v;
79         pos[v] = cpos++;
80         head[v] = chead;
81
82         // if not a leaf
83         if (heavy_child[v] != -1) decompose(heavy_child[v],
                                           chead);
84
85         // light child
86         for (auto u : adj[v]){
87             // start new path
88             if (u != parent[v] && u != heavy_child[v])
               decompose(u, -1);
89         }
90     }
91
92     ftype query_path(int a, int b) {
93         if(pos[a] < pos[b]) swap(a, b);
94
95         if(head[a] == head[b]) return seg.query(pos[b],
                                                  pos[a]);
96         return seg.f(seg.query(pos[head[a]], pos[a]),
                      query_path(parent[head[a]], b));
97     }
98
99     // iterative
100     /*ftype query_path(int a, int b) {
101         ftype ans = 0;
102
103         while (head[a] != head[b]) {
104             if (level[head[a]] > level[head[b]]) swap(a, b);
105             ans = seg.merge(ans, seg.query(pos[head[b]],
                                             pos[b]));
106             b = parent[head[b]];
107         }
108
109         if (level[a] > level[b]) swap(a, b);
110         ans = seg.merge(ans, seg.query(pos[a], pos[b]));
111         return ans;
112     }*/
113
114     ftype query_subtree(int a) {
115         return seg.query(pos[a], pos[a] + subtree_size[a]
                           - 1);
116     }
117
118     void update_path(int a, int b, int x) {
119         if(pos[a] < pos[b]) swap(a, b);
120
121         if(head[a] == head[b]) return (void)seg.update(
            pos[b], pos[a], x);
122         seg.update(pos[head[a]], pos[a], x); update_path(
            parent[head[a]], b, x);
123     }
124
125     void update_subtree(int a, int val) {
126         seg.update(pos[a], pos[a] + subtree_size[a] - 1,
                    val);
127     }
128

```

```

129 void update(int a, int val) {
130     seg.update(pos[a], pos[a], val);
131 }
132
133 //edge
134 void update(int a, int b, int val) {
135     if (level[a] > level[b]) swap(a, b);
136     update(b, val);
137 }
138
139 int lca(int a, int b) {
140     if(pos[a] < pos[b]) swap(a, b);
141     return head[a] == head[b] ? b : lca(parent[head[a]], b);
142 }
143
144 void search(int a) {
145     a = parent[a];
146     if (a == -1) return;
147     if (seg.query(pos[head[a]], pos[head[a]]+
148         subtree_size[head[a]]-1) + pos[a]-pos[head[a]]+1
149         == subtree_size[head[a]]) {
150         seg.update(pos[head[a]], pos[a], 1);
151         return search(parent[head[a]]);
152     }
153     int l = pos[head[a]], r = pos[a]+1;
154     while (l < r) {
155         int m = (l+r)/2;
156         if (seg.query(m, m+subtree_size[at[m]]-1) + pos
157         [a]-m+1 == subtree_size[at[m]]) {
158             r = m;
159         }
160         else l = m+1;
161     }
162     seg.update(l, pos[a], 1);
163 }
164
165 /* k-th ancestor of x
166 int x, k; cin >> x >> k;
167
168 for (int b = 0; b <= BITS; b++) {
169     if (x != -1 && (k & (1 << b))) {
170         x = up[x][b];
171     }
172 }
173
174 cout << x << '\n';
175 */
176 void preprocess() {
177     up.assign(n + 1, vector<int>(31, -1));
178
179     for (int i = 1; i < n; i++) {
180         up[i][0] = parent[i];
181     }
182
183     for (int i = 1; i < n; i++) {
184         for (int j = 1; j <= 30; j++) {
185             if (up[i][j - 1] != -1) up[i][j] = up[up[i][j
186             - 1]][j - 1];
187         }
188     }
189 }
190
191 int getKth(int p, int q, int k){
192     int a = lca(p,q), d;
193
194     if( a == p ){
195         d = level[q] - level[p] + 1;
196         swap(p,q);
197         k = d - k + 1;
198     }
199     else if( a == q );
200     else {

```

```

197         if( k > level[p] - level[a] + 1 ) {
198             d = level[p] + level[q] - 2 * level[a] +
199             1;
200             k = d - k + 1;
201             swap(p,q);
202         }
203         else ;
204     }
205     int lg ; for( lg = 1 ; (1 << lg) <= level[p] ; ++
206     lg ); lg--;
207     k--;
208     for( int i = lg ; i >= 0 ; i-- ){
209         if( (1 << i) <= k ){
210             p = up[p][i];
211             k -= ( 1 << i );
212         }
213     }
214     return p;
215 }
216 };

```

## 6.18 Small To Large

```

1 // Problem:
2 // https://codeforces.com/contest/600/problem/E
3
4 void process_colors(int curr, int parent) {
5
6     for (int n : adj[curr]) {
7         if (n != parent) {
8             process_colors(n, curr);
9
10             if (colors[curr].size() < colors[n].size
11             ()) {
12                 sum_num[curr] = sum_num[n];
13                 vmax[curr] = vmax[n];
14                 swap(colors[curr], colors[n]);
15             }
16
17             for (auto [item,vzs] : colors[n]) {
18                 if(colors[curr][item]+vzs > vmax[curr
19                 ]){
20                     vmax[curr] = colors[curr][item] +
21                     vzs;
22                     sum_num[curr] = item;
23                 }
24                 else if(colors[curr][item]+vzs ==
25                 vmax[curr]){
26                     sum_num[curr] += item;
27                 }
28                 colors[curr][item] += vzs;
29             }
30         }
31     }
32 }
33
34 int32_t main() {
35
36     int n; cin >> n;
37
38     for (int i = 1; i <= n; i++) {
39         int a; cin >> a;
40         colors[i][a] = 1;
41         vmax[i] = 1;
42         sum_num[i] = a;
43     }
44
45     for (int i = 1; i < n; i++) {
46         int a, b; cin >> a >> b;

```

```

47     adj[a].push_back(b);
48     adj[b].push_back(a);
49 }
50
51 process_colors(1, 0);
52
53 for (int i = 1; i <= n; i++) {
54     cout << sum_num[i] << (i < n ? " " : "\n");
55 }
56
57 return 0;
58 }
59 }
60

```

## 6.19 Tree Diameter

```

1 #include<bits/stdc++.h>
2
3 using namespace std;
4
5 const int MAX = 3e5+17;
6
7 vector<int> adj[MAX];
8 bool visited[MAX];
9
10 int max_depth = 0, max_node = 1;
11
12 void dfs (int v, int depth) {
13     visited[v] = true;
14
15     if (depth > max_depth) {
16         max_depth = depth;
17         max_node = v;
18     }
19
20     for (auto u : adj[v]) {
21         if (!visited[u]) dfs(u, depth + 1);
22     }
23 }
24
25 int tree_diameter() {
26     dfs(1, 0);
27     max_depth = 0;
28     for (int i = 0; i < MAX; i++) visited[i] = false;
29     dfs(max_node, 0);
30     return max_depth;
31 }

```

## 6.20 Dijkstra

```

1 const int MAX = 2e5+7;
2 const int INF = 1000000000;
3 vector<vector<pair<int, int>>> adj(MAX);
4
5 void dijkstra(int s, vector<int> & d, vector<int> & p
6 ) {
7     int n = adj.size();
8     d.assign(n, INF);
9     p.assign(n, -1);
10
11     d[s] = 0;
12     set<pair<int, int>> q;
13     q.insert({0, s});
14     while (!q.empty()) {
15         int v = q.begin()->second;
16         q.erase(q.begin());
17
18         for (auto edge : adj[v]) {
19             int to = edge.first;
20             int len = edge.second;

```

```

21         if (d[v] + len < d[to]) {
22             q.erase({d[to], to});
23             d[to] = d[v] + len;
24             p[to] = v;
25             q.insert({d[to], to});
26         }
27     }
28 }
29 }
30
31 vector<int> restore_path(int s, int t) {
32     vector<int> path;
33
34     for (int v = t; v != s; v = p[v])
35         path.push_back(v);
36     path.push_back(s);
37
38     reverse(path.begin(), path.end());
39     return path;
40 }
41
42 int adj[MAX][MAX];
43 int dist[MAX];
44 int minDistance(int dist[], bool sptSet[], int V) {
45     int min = INT_MAX, min_index;
46
47     for (int v = 0; v < V; v++)
48         if (sptSet[v] == false && dist[v] <= min)
49             min = dist[v], min_index = v;
50
51     return min_index;
52 }
53
54 void dijkstra(int src, int V) {
55     bool sptSet[V];
56     for (int i = 0; i < V; i++)
57         dist[i] = INT_MAX, sptSet[i] = false;
58
59     dist[src] = 0;
60
61     for (int count = 0; count < V - 1; count++) {
62         int u = minDistance(dist, sptSet, V);
63
64         sptSet[u] = true;
65
66         for (int v = 0; v < V; v++)
67             if (!sptSet[v] && adj[u][v]
68                 && dist[u] != INT_MAX
69                 && dist[u] + adj[u][v] < dist[v])
70                 dist[v] = dist[u] + adj[u][v];
71     }
72 }
73
74 }

```

## 6.21 Kruskall

```

1 struct DSU {
2     int n;
3     vector<int> link, sizes;
4
5     DSU(int n) {
6         this->n = n;
7         link.assign(n+1, 0);
8         sizes.assign(n+1, 1);
9
10        for (int i = 0; i <= n; i++)
11            link[i] = i;
12    }
13
14    int find(int x) {
15        while (x != link[x])
16            x = link[x];

```

```

17         return x;
18     }
19 }
20
21 bool same(int a, int b) {
22     return find(a) == find(b);
23 }
24
25 void unite(int a, int b) {
26     a = find(a);
27     b = find(b);
28
29     if (a == b) return;
30
31     if (sizes[a] < sizes[b])
32         swap(a, b);
33
34     sizes[a] += sizes[b];
35     link[b] = a;
36 }
37 };
38
39 struct Edge {
40     int u, v;
41     long long weight;
42
43     Edge() {}
44
45     Edge(int u, int v, long long weight) : u(u), v(v), weight(weight) {}
46
47     bool operator<(const Edge& other) const {
48         return weight < other.weight;
49     }
50
51     bool operator>(const Edge& other) const {
52         return weight > other.weight;
53     }
54 };
55
56 vector<Edge> kruskal(vector<Edge> edges, int n) {
57     vector<Edge> result; // arestas da MST
58     long long cost = 0;
59
60     sort(edges.begin(), edges.end());
61
62     DSU dsu(n);
63
64     for (auto e : edges) {
65         if (!dsu.same(e.u, e.v)) {
66             cost += e.weight;
67             result.push_back(e);
68             dsu.unite(e.u, e.v);
69         }
70     }
71
72     return result;
73 }

```

## 6.22 Negative Cycle

```

1 // Description
2 // Detects any cycle in which the sum of edge weights
  is negative.
3 // Alternatively, we can detect whether there is a
  negative cycle
4 // starting from a specific vertex.
5
6 // Problem:
7 // https://cses.fi/problemset/task/1197
8
9 // Complexity:
10 // O(n * m)

```

```

11
12 // Notes
13 // In order to consider only the negative cycles
  located on the path from a to b,
14 // Reverse the graph, run a dfs from node b and mark
  the visited nodes
15 // Consider only the edges that connect to visited
  nodes when running bellman-ford
16 // on the normal graph
17
18 struct Edge {
19     int a, b, cost;
20     Edge(int a, int b, int cost) : a(a), b(b), cost(
  cost) {}
21 };
22
23 int n, m;
24 vector<Edge> edges;
25 const int INF = 1e9+10;
26
27 void negative_cycle() {
28     // uncomment to find negative cycle starting from a
  vertex v
29     // vector<int> d(n + 1, INF);
30     // d[v] = 0;
31     vector<int> d(n + 1, 0);
32     vector<int> p(n + 1, -1);
33     int x;
34     // uncomment to find all negative cycles
35     // // set<int> s;
36     for (int i = 1; i <= n; ++i) {
37         x = -1;
38         for (Edge e : edges) {
39             // if (d[e.a] >= INF) continue;
40             if (d[e.b] > d[e.a] + e.cost) {
41                 // d[e.b] = max(-INF, d[e.a] + e.cost);
42                 d[e.b] = d[e.a] + e.cost;
43                 p[e.b] = e.a;
44                 x = e.b;
45                 // // s.insert(e.b);
46             }
47         }
48     }
49
50     if (x == -1)
51         cout << "NO\n";
52     else {
53         // // int y = all nodes in set s
54         int y = x;
55         for (int i = 1; i <= n; ++i) {
56             y = p[y];
57         }
58
59         vector<int> path;
60         for (int cur = y;; cur = p[cur]) {
61             path.push_back(cur);
62             if (cur == y && path.size() > 1) break;
63         }
64         reverse(path.begin(), path.end());
65
66         cout << "YES\n";
67         for (int u : path)
68             cout << u << ' ';
69         cout << '\n';
70     }
71 }

```

## 7 Geometry

### 7.1 Shoelace Boundary

```

1 // Description

```



```

2 // Shoelace formula finds the area of a polygon
3 // Boundary points return the number of integer
  points on the edges of a polygon
4 // not counting the vertexes
5
6 // Problem
7 // https://codeforces.com/gym/101873/problem/G
8
9 // Complexity
10 // O(n)
11
12 // before dividing by two
13 int shoelace(vector<point> & points) {
14     int n = points.size();
15     vector<point> v(n + 2);
16
17     for (int i = 1; i <= n; i++) {
18         v[i] = points[i - 1];
19     }
20     v[n + 1] = points[0];
21
22     int sum = 0;
23     for (int i = 1; i <= n; i++) {
24         sum += (v[i].x * v[i + 1].y - v[i + 1].x * v[
25 i].y);
26     }
27
28     sum = abs(sum);
29     return sum;
30 }
31
32 int boundary_points(vector<point> & points) {
33     int n = points.size();
34     vector<point> v(n + 2);
35
36     for (int i = 1; i <= n; i++) {
37         v[i] = points[i - 1];
38     }
39     v[n + 1] = points[0];
40
41     int ans = 0;
42     for (int i = 1; i <= n; i++) {
43         if (v[i].x == v[i + 1].x) ans += abs(v[i].y -
44 v[i + 1].y) - 1;
45         else if (v[i].y == v[i + 1].y) ans += abs(v[i
46 ].x - v[i + 1].x) - 1;
47         else ans += gcd(abs(v[i].x - v[i + 1].x), abs
48 (v[i].y - v[i + 1].y)) - 1;
49     }
50     return points.size() + ans;
51 }

```

## 7.2 Inside Polygon

```

1 // Description
2 // Checks if a given point is inside, outside or on
  the boundary of a polygon
3
4 // Problem
5 // https://cses.fi/problemset/task/2192/
6
7 // Complexity
8 // O(n)
9
10 int inside(vp &p, point pp){
11     // 1 - inside / 0 - boundary / -1 - outside
12     int n = p.size();
13     for(int i=0;i<n;i++){
14         int j = (i+1)%n;
15         if(line({p[i], p[j]}).inside_seg(pp))
16             return 0; // boundary
17     }
18     int inter = 0;

```

```

19     for(int i=0;i<n;i++){
20         int j = (i+1)%n;
21         if(p[i].x <= pp.x and pp.x < p[j].x and ccw(p
22 [i], p[j], pp)==1)
23             inter++; // up
24         else if(p[j].x <= pp.x and pp.x < p[i].x and
25 ccw(p[i], p[j], pp)==-1)
26             inter++; // down
27     }
28
29     if(inter%2==0) return -1; // outside
30     else return 1; // inside
31 }

```

## 7.3 Closest Pair Points

```

1 // Description
2 // Find the squared distance between the closest two
  points among n points
3 // Also finds which pair of points is closest (could
  be more than one)
4
5 // Problem
6 // https://cses.fi/problemset/task/2194/
7
8 // Complexity
9 // O(n log n)
10
11 ll closest_pair_points(vp &vet){
12     pair<point, point> ans;
13     int n = vet.size();
14     sort(vet.begin(), vet.end());
15     set<point> s;
16
17     ll best_dist = LLONG_MAX;
18     int j=0;
19     for(int i=0;i<n;i++){
20         ll d = ceil(sqrt(best_dist));
21         while(j<n and vet[i].x-vet[j].x >= d){
22             s.erase(point(vet[j].y, vet[j].x));
23             j++;
24         }
25
26         auto it1 = s.lower_bound({vet[i].y - d, vet[i
27 ].x});
28         auto it2 = s.upper_bound({vet[i].y + d, vet[i
29 ].x});
30
31         for(auto it=it1; it!=it2; it++){
32             ll dx = vet[i].x - it->x;
33             ll dy = vet[i].y - it->y;
34
35             if(best_dist > dx*dx + dy*dy){
36                 best_dist = dx*dx + dy*dy;
37                 // closest pair points
38                 ans = mp(vet[i], point(it->y, it->x))
39             }
40         }
41         s.insert(point(vet[i].y, vet[i].x));
42     }
43
44     // best distance squared
45     return best_dist;
46 }

```

## 7.4 2d

```

1 #define vp vector<point>
2 #define ld long double
3 const ld EPS = 1e-6;

```

```

4 const ld PI = acos(-1);
5
6 // typedef ll cod;
7 // bool eq(cod a, cod b){ return (a==b); }
8 typedef ld cod;
9 bool eq(cod a, cod b){ return abs(a - b) <= EPS; }
10
11 struct point{
12     cod x, y;
13     int id;
14     point(cod x=0, cod y=0): x(x), y(y){}
15
16     point operator+(const point &o) const{ return {x+o.x, y+o.y}; }
17     point operator-(const point &o) const{ return {x-o.x, y-o.y}; }
18     point operator*(cod t) const{ return {x*t, y*t}; }
19     point operator/(cod t) const{ return {x/t, y/t}; }
20     cod operator*(const point &o) const{ return x * o.x + y * o.y; }
21     cod operator^(const point &o) const{ return x * o.y - y * o.x; }
22     bool operator<(const point &o) const{
23         return (eq(x, o.x) ? y < o.y : x < o.x);
24     }
25     bool operator==(const point &o) const{
26         return eq(x, o.x) and eq(y, o.y);
27     }
28     friend ostream& operator<<(ostream& os, point p) {
29         return os << "(" << p.x << "," << p.y << ")"; }
30 };
31
32 int ccw(point a, point b, point e){ // -1=dir; 0=
33     collinear; 1=esq;
34     cod tmp = (b-a) ^ (e-a); // vector from a to b
35     return (tmp > EPS) - (tmp < -EPS);
36 }
37 ld norm(point a){ // Modulo
38     return sqrt(a * a);
39 }
40 cod norm2(point a){
41     return a * a;
42 }
43 bool nulo(point a){
44     return (eq(a.x, 0) and eq(a.y, 0));
45 }
46 point rotccw(point p, ld a){
47     // a = PI*a/180; // graus
48     return point((p.x*cos(a)-p.y*sin(a)), (p.y*cos(a)+p.x*sin(a)));
49 }
50 point rot90cw(point a) { return point(a.y, -a.x); }
51 point rot90ccw(point a) { return point(-a.y, a.x); }
52
53 ld proj(point a, point b){ // a sobre b
54     return a*b/norm(b);
55 }
56 ld angle(point a, point b){ // em radianos
57     ld ang = a*b / norm(a) / norm(b);
58     return acos(max(min(ang, (ld)1), (ld)-1));
59 }
60 ld angle_vec(point v){
61     // return 180/PI*atan2(v.x, v.y); // graus
62     return atan2(v.x, v.y);
63 }
64 ld order_angle(point a, point b){ // from a to b ccw
65     (a in front of b)
66     ld aux = angle(a,b)*180/PI;
67     return ((a^b)<=0 ? aux:360-aux);
68 }
69
70 bool angle_less(point a1, point b1, point a2, point
71     b2){ // ang(a1,b1) <= ang(a2,b2)
72     point p1((a1*b1), abs((a1^b1)));
73     point p2((a2*b2), abs((a2^b2)));
74     return (p1^p2) <= 0;
75 }
76
77 ld area(vp &p){ // (points sorted)
78     ld ret = 0;
79     for(int i=2;i<(int)p.size();i++)
80         ret += (p[i]-p[0])^(p[i-1]-p[0]);
81     return abs(ret/2);
82 }
83 ld areaT(point &a, point &b, point &c){
84     return abs((b-a)^(c-a))/2.0;
85 }
86
87 point center(vp &A){
88     point c = point();
89     int len = A.size();
90     for(int i=0;i<len;i++)
91         c=c+A[i];
92     return c/len;
93 }
94
95 point forca_mod(point p, ld m){
96     ld cm = norm(p);
97     if(cm<EPS) return point();
98     return point(p.x*m/cm,p.y*m/cm);
99 }
100
101 ld param(point a, point b, point v){
102     // v = t*(b-a) + a // return t;
103     // assert(line(a, b).inside_seg(v));
104     return ((v-a) * (b-a)) / ((b-a) * (b-a));
105 }
106
107 bool simetric(vp &a){ //ordered
108     int n = a.size();
109     point c = center(a);
110     if(n&1) return false;
111     for(int i=0;i<n/2;i++)
112         if(ccw(a[i], a[i+n/2], c) != 0)
113             return false;
114     return true;
115 }
116
117 point mirror(point m1, point m2, point p){
118     // mirror point p around segment m1m2
119     point seg = m2-m1;
120     ld t0 = ((p-m1)*seg) / (seg*seg);
121     point ort = m1 + seg*t0;
122     point pm = ort-(p-ort);
123     return pm;
124 }
125
126 // Line
127
128 struct line{
129     point p1, p2;
130     cod a, b, c; // ax+by+c = 0;
131     // y-y1 = ((y2-y1)/(x2-x1))(x-x1)
132     line(point p1=0, point p2=0): p1(p1), p2(p2){
133         a = p1.y - p2.y;
134         b = p2.x - p1.x;
135         c = p1 ^ p2;
136     }
137     line(cod a=0, cod b=0, cod c=0): a(a), b(b), c(c)
138     {
139         // Gera os pontos p1 p2 dados os coeficientes

```

```

139 // isso aqui eh um lixo mas quebra um galho
kkkkkk
140 if(b==0){
141     p1 = point(1, -c/a);
142     p2 = point(0, -c/a);
143 }else{
144     p1 = point(1, (-c-a*1)/b);
145     p2 = point(0, -c/b);
146 }
147 }
148
149 cod eval(point p){
150     return a*p.x+b*p.y+c;
151 }
152 bool inside(point p){
153     return eq(eval(p), 0);
154 }
155 point normal(){
156     return point(a, b);
157 }
158
159 bool inside_seg(point p){
160     return (
161         ((p1-p) ^ (p2-p)) == 0 and
162         ((p1-p) * (p2-p)) <= 0
163     );
164 }
165
166 };
167
168 // be careful with precision error
169 vp inter_line(line l1, line l2){
170     ld det = l1.a*l2.b - l1.b*l2.a;
171     if(det==0) return {};
172     ld x = (l1.b*l2.c - l1.c*l2.b)/det;
173     ld y = (l1.c*l2.a - l1.a*l2.c)/det;
174     return {point(x, y)};
175 }
176
177 // segments not collinear
178 vp inter_seg(line l1, line l2){
179     vp ans = inter_line(l1, l2);
180     if(ans.empty() or !l1.inside_seg(ans[0]) or !l2.
inside_seg(ans[0]))
181         return {};
182     return ans;
183 }
184 bool seg_has_inter(line l1, line l2){
185     // if collinear
186     if (l1.inside_seg(l2.p1) || l1.inside_seg(l2.p2)
|| l2.inside_seg(l1.p1) || l2.inside_seg(l1.p2))
187         return true;
188
189     return ccw(l1.p1, l1.p2, l2.p1) * ccw(l1.p1, l1.
p2, l2.p2) < 0 and
190         ccw(l2.p1, l2.p2, l1.p1) * ccw(l2.p1, l2.
p2, l1.p2) < 0;
191 }
192 ld dist_seg(point p, point a, point b){ // point -
seg
193     if((p-a)*(b-a) < EPS) return norm(p-a);
194     if((p-b)*(a-b) < EPS) return norm(p-b);
195     return abs((p-a)^(b-a)) / norm(b-a);
196 }
197
198 ld dist_line(point p, line l){ // point - line
199     return abs(l.eval(p))/sqrt(1.a*1.a + 1.b*1.b);
200 }
201
202 line bisector(point a, point b){
203     point d = (b-a)*2;
204     return line(d.x, d.y, a*a - b*b);
205 }
206
207 line perpendicular(line l, point p){ // passes
through p
208     return line(l.b, -l.a, -l.b*p.x + l.a*p.y);
209 }
210
211
212 // Circle //
213
214
215 struct circle{
216     point c; cod r;
217     circle() : c(0, 0), r(0){}
218     circle(const point o) : c(o), r(0){}
219     circle(const point a, const point b){
220         c = (a+b)/2;
221         r = norm(a-c);
222     }
223
224     circle(const point a, const point b, const point
cc){
225         assert(ccw(a, b, cc) != 0);
226         c = inter_line(bisector(a, b), bisector(b, cc
))[0];
227         r = norm(a-c);
228     }
229     bool inside(const point &a) const{
230         return norm(a - c) <= r + EPS;
231     }
232 };
233
234 pair<point, point> tangent_points(circle cr, point p)
{
235     ld d1 = norm(p-cr.c), theta = asin(cr.r/d1);
236     point p1 = rotccw(cr.c-p, -theta);
237     point p2 = rotccw(cr.c-p, theta);
238     assert(d1 >= cr.r);
239     p1 = p1 * (sqrt(d1*d1-cr.r*cr.r) / d1) + p;
240     p2 = p2 * (sqrt(d1*d1-cr.r*cr.r) / d1) + p;
241     return {p1, p2};
242 }
243
244 circle incircle(point p1, point p2, point p3){
245     ld m1 = norm(p2-p3);
246     ld m2 = norm(p1-p3);
247     ld m3 = norm(p1-p2);
248     point c = (p1*m1 + p2*m2 + p3*m3)*(1/(m1+m2+m3));
249     ld s = 0.5*(m1+m2+m3);
250     ld r = sqrt(s*(s-m1)*(s-m2)*(s-m3)) / s;
251     return circle(c, r);
252 }
253
254 circle circumcircle(point a, point b, point c) {
255     circle ans;
256     point u = point((b-a).y, -(b-a).x);
257     point v = point((c-a).y, -(c-a).x);
258     point n = (c-b)*0.5;
259     ld t = (u^n)/(v^u);
260     ans.c = ((a+c)*0.5) + (v*t);
261     ans.r = norm(ans.c-a);
262     return ans;
263 }
264
265 vp inter_circle_line(circle C, line L){
266     point ab = L.p2 - L.p1, p = L.p1 + ab * ((C.c-L.
p1)*(ab) / (ab*ab));
267     ld s = (L.p2-L.p1)^(C.c-L.p1), h2 = C.r*C.r - s*s
/ (ab*ab);
268     if (h2 < -EPS) return {};
269     if (eq(h2, 0)) return {p};
270     point h = (ab/norm(ab)) * sqrt(h2);
271

```

```

272     return {p - h, p + h};
273 }
274
275 vp inter_circle(circle C1, circle C2){
276     if(C1.c == C2.c) { assert(C1.r != C2.r); return
{}; }
277     point vec = C2.c - C1.c;
278     ld d2 = vec*vec, sum = C1.r+C2.r, dif = C1.r-C2.r
;
279     ld p = (d2 + C1.r*C1.r - C2.r*C2.r)/(d2*2), h2 =
C1.r*C1.r - p*p*d2;
280     if (sum*sum < d2 or dif*dif > d2) return {};
281     point mid = C1.c + vec*p, per = point(-vec.y, vec
.x) * sqrt(max((ld)0, h2) / d2);
282     if(eq(per.x, 0) and eq(per.y, 0)) return {mid};
283     return {mid + per, mid - per};
284 }
285
286 // minimum circle cover O(n) amortizado
287 circle min_circle_cover(vp v){
288     random_shuffle(v.begin(), v.end());
289     circle ans;
290     int n = v.size();
291     for(int i=0;i<n;i++){ if(!ans.inside(v[i])){
292         ans = circle(v[i]);
293         for(int j=0;j<i;j++){ if(!ans.inside(v[j])){
294             ans = circle(v[i], v[j]);
295             for(int k=0;k<j;k++){ if(!ans.inside(v[k])){
296                 ans = circle(v[i], v[j], v[k]);
297             }
298         }
299     }
300     return ans;
301 }

```

## 8 Algorithms

### 8.1 Lis

```

1 int lis(vector<int> const& a) {
2     int n = a.size();
3     vector<int> d(n, 1);
4     for (int i = 0; i < n; i++) {
5         for (int j = 0; j < i; j++) {
6             if (a[j] < a[i])
7                 d[i] = max(d[i], d[j] + 1);
8         }
9     }
10
11     int ans = d[0];
12     for (int i = 1; i < n; i++) {
13         ans = max(ans, d[i]);
14     }
15     return ans;
16 }

```

### 8.2 Delta-encoding

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 int main(){
5     int n, q;
6     cin >> n >> q;
7     int [n];
8     int delta[n+2];
9
10    while(q--){
11        int l, r, x;
12        cin >> l >> r >> x;

```

```

13        delta[l] += x;
14        delta[r+1] -= x;
15    }
16
17    int curr = 0;
18    for(int i=0; i < n; i++){
19        curr += delta[i];
20        v[i] = curr;
21    }
22
23    for(int i=0; i < n; i++){
24        cout << v[i] << ' ';
25    }
26    cout << '\n';
27
28    return 0;
29 }

```

### 8.3 Subsets

```

1 void subsets(vector<int>& nums){
2     int n = nums.size();
3     int powSize = 1 << n;
4
5     for(int counter = 0; counter < powSize; counter++){
6         for(int j = 0; j < n; j++){
7             if((counter & (1LL << j)) != 0) {
8                 cout << nums[j] << ' ';
9             }
10        }
11        cout << '\n';
12    }
13 }

```

### 8.4 Binary Search Last True

```

1 int last_true(int lo, int hi, function<bool(int)> f)
{
2     lo--;
3     while (lo < hi) {
4         int mid = lo + (hi - lo + 1) / 2;
5         if (f(mid)) {
6             lo = mid;
7         } else {
8             hi = mid - 1;
9         }
10    }
11    return lo;
12 }

```

### 8.5 Ternary Search

```

1 double ternary_search(double l, double r) {
2     double eps = 1e-9; //set the error
3     limit here
4     while (r - l > eps) {
5         double m1 = l + (r - l) / 3;
6         double m2 = r - (r - l) / 3;
7         double f1 = f(m1); //evaluates the
8         double f2 = f(m2); //evaluates the
9         function at m1
10        if (f1 < f2)
11            l = m1;
12        else
13            r = m2;
14    }
15    return f(l); //return the
16    maximum of f(x) in [l, r]

```

### 8.6 Binary Search First True

```

1 int first_true(int lo, int hi, function<bool(int)> f) 54
2 {
3     hi++;
4     while (lo < hi) {
5         int mid = lo + (hi - lo) / 2;
6         if (f(mid)) {
7             hi = mid;
8         } else {
9             lo = mid + 1;
10        }
11    }
12    return lo;
13 }

```

## 8.7 Biggest K

```

1 // Description: Gets sum of k biggest or k smallest
2 // elements in an array
3 // Problem: https://atcoder.jp/contests/abc306/tasks/
4 // abc306_e
5 // Complexity: O(log n)
6
7 struct SetSum {
8     ll s = 0;
9     multiset<ll> mt;
10    void add(ll x){
11        mt.insert(x);
12        s += x;
13    }
14    int pop(ll x){
15        auto f = mt.find(x);
16        if(f == mt.end()) return 0;
17        mt.erase(f);
18        s -= x;
19        return 1;
20    }
21 };
22
23 struct BigK {
24     int k;
25     SetSum gt, mt;
26     BigK(int _k){
27         k = _k;
28     }
29     void balancear(){
30         while((int)gt.mt.size() < k && (int)mt.mt.
31 size()){
32             auto p = (prev(mt.mt.end()));
33             gt.add(*p);
34             mt.pop(*p);
35         }
36         while((int)mt.mt.size() && (int)gt.mt.size()
37 &&
38 *(gt.mt.begin()) < *(prev(mt.mt.end())) ){
39             ll u = *(gt.mt.begin());
40             ll v = *(prev(mt.mt.end()));
41             gt.pop(u); mt.pop(v);
42             gt.add(v); mt.add(u);
43         }
44     }
45     void add(ll x){
46         mt.add(x);
47         balancear();
48     }
49     void rem(ll x){
50         //x = -x;
51         if(mt.pop(x) == 0)
52             gt.pop(x);
53         balancear();
54     }
55 };

```

```

55 int main() {
56     ios::sync_with_stdio(false);
57     cin.tie(NULL);
58
59     int n, k, q; cin >> n >> k >> q;
60
61     BigK big = BigK(k);
62
63     int arr[n] = {};
64
65     while (q--) {
66         int pos, num; cin >> pos >> num;
67         pos--;
68         big.rem(arr[pos]);
69         arr[pos] = num;
70         big.add(arr[pos]);
71
72         cout << big.gt.s << '\n';
73     }
74
75     return 0;
76 }

```

## 9 Data Structures

### 9.1 Ordered Set

```

1 // Description:
2 // insert(k) - add element k to the ordered set
3 // erase(k) - remove element k from the ordered set
4 // erase(it) - remove element it points to from the
5 // ordered set
6 // order_of_key(k) - returns number of elements
7 // strictly smaller than k
8 // find_by_order(n) - return an iterator pointing to
9 // the k-th element in the ordered set (counting
10 // from zero).
11
12 // Problem:
13 // https://cses.fi/problemset/task/2169/
14
15 // Complexity:
16 // O(log n) for all operations
17
18 // How to use:
19 // ordered_set<int> os;
20 // cout << os.order_of_key(1) << '\n';
21 // cout << os.find_by_order(1) << '\n';
22
23 // Notes
24 // The ordered set only contains different elements
25 // By using less_equal<T> instead of less<T> on using
26 // ordered_set declaration
27 // The ordered_set becomes an ordered_multiset
28 // So the set can contain elements that are equal
29
30 #include <ext/pb_ds/assoc_container.hpp>
31 #include <ext/pb_ds/tree_policy.hpp>
32
33 using namespace __gnu_pbds;
34 template <typename T>
35 using ordered_set = tree<T, null_type, less<T>,
36 rb_tree_tag, tree_order_statistics_node_update>;
37
38 void Erase(ordered_set<int>& a, int x){
39     int r = a.order_of_key(x);
40     auto it = a.find_by_order(r);
41     a.erase(it);
42 }

```

## 9.2 Priority Queue

```
1 // Description:
2 // Keeps the largest (by default) element at the top
  of the queue
3
4 // Problem:
5 // https://cses.fi/problemset/task/1164/
6
7 // Complexity:
8 // O(log n) for push and pop
9 // O(1) for looking at the element at the top
10
11 // How to use:
12 // priority_queue<int> pq;
13 // pq.push(1);
14 // pq.top();
15 // pq.pop()
16
17 // Notes
18 // To use the priority queue keeping the smallest
  element at the top
19
20 priority_queue<int, vector<int>, greater<int>> pq;
```

## 9.3 Dsu

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 const int MAX = 1e6+17;
6
7 struct DSU {
8     int n;
9     vector<int> link, sizes;
10
11     DSU(int n) {
12         this->n = n;
13         link.assign(n+1, 0);
14         sizes.assign(n+1, 1);
15
16         for (int i = 0; i <= n; i++)
17             link[i] = i;
18     }
19
20     int find(int x) {
21         while (x != link[x])
22             x = link[x];
23
24         return x;
25     }
26
27     bool same(int a, int b) {
28         return find(a) == find(b);
29     }
30
31     void unite(int a, int b) {
32         a = find(a);
33         b = find(b);
34
35         if (a == b) return;
36
37         if (sizes[a] < sizes[b])
38             swap(a, b);
39
40         sizes[a] += sizes[b];
41         link[b] = a;
42     }
43
44     int size(int x) {
45         return sizes[x];
```

```
46     }
47 };
48
49 int main() {
50     ios::sync_with_stdio(false);
51     cin.tie(NULL);
52
53     int cities, roads; cin >> cities >> roads;
54     vector<int> final_roads;
55     int ans = 0;
56     DSU dsu = DSU(cities);
57     for (int i = 0, a, b; i < roads; i++) {
58         cin >> a >> b;
59         dsu.unite(a, b);
60     }
61
62     for (int i = 2; i <= cities; i++) {
63         if (!dsu.same(1, i)) {
64             ans++;
65             final_roads.push_back(i);
66             dsu.unite(1, i);
67         }
68     }
69
70     cout << ans << '\n';
71     for (auto e : final_roads) {
72         cout << "1 " << e << '\n';
73     }
74 }
75 }
```

## 9.4 Two Sets

```
1 // Description
2 // The values are divided in two multisets so that
  one of them contain all values that are
3 // smaller than the median and the other one contains
  all values that are greater or equal to the
  median.
4
5 // Problem:
6 // https://atcoder.jp/contests/abc306/tasks/abc306_e
7 // Problem I - Maratona Feminina de 3ªProgramao da
  Unicamp 2023
8 // https://codeforces.com/group/WYIydkIPyE/contest
  /450037/attachments
9
10 // Complexity:
11 // Add and remove elements - O(log n)
12 // Return sum of biggest or smallest set or return
  the median - O(1)
13
14 using ll = long long;
15
16 struct TwoSets {
17     multiset<int> small;
18     multiset<int> big;
19     ll sums = 0;
20     ll sumb = 0;
21     int n = 0;
22
23     int size_small() {
24         return small.size();
25     }
26
27     int size_big() {
28         return big.size();
29     }
30
31     void balance() {
32         while (size_small() > n / 2) {
33             int v = *small.rbegin();
34             small.erase(prev(small.end()));
```

```

35     big.insert(v);
36     sums -= v;
37     sumb += v;
38 }
39 while (size_big() > n - n / 2) {
40     int v = *big.begin();
41     big.erase(big.begin());
42     small.insert(v);
43     sumb -= v;
44     sums += v;
45 }
46 }
47
48 void add(int x) {
49     n++;
50     small.insert(x);
51     sums += x;
52     while (!small.empty() && *small.rbegin() > *big.
begin()) {
53         int v = *small.rbegin();
54         small.erase(prev(small.end()));
55         big.insert(v);
56         sums -= v;
57         sumb += v;
58     }
59     balance();
60 }
61
62 bool rem(int x) {
63     n--;
64     auto it1 = small.find(x);
65     auto it2 = big.find(x);
66     bool flag = false;
67     if (it1 != small.end()) {
68         sums -= *it1;
69         small.erase(it1);
70         flag = true;
71     } else if (it2 != big.end()) {
72         sumb -= *it2;
73         big.erase(it2);
74         flag = true;
75     }
76     balance();
77     return flag;
78 }
79
80 ll sum_small() {
81     return sums;
82 }
83
84 ll sum_big() {
85     return sumb;
86 }
87
88 int median() {
89     return *big.begin();
90 }
91 };

```

## 9.5 Dynamic Implicit Sparse

```

1 // Description:
2 // Indexed at one
3
4 // When the indexes of the nodes are too big to be
   stored in an array
5 // and the queries need to be answered online so we
   can't sort the nodes and compress them
6 // we create nodes only when they are needed so there
   'll be (Q*log(MAX)) nodes
7 // where Q is the number of queries and MAX is the
   maximum index a node can assume
8

```

```

9 // Query - get sum of elements from range (l, r)
   inclusive
10 // Update - update element at position id to a value
   val
11
12 // Problem:
13 // https://cses.fi/problemset/task/1648
14
15 // Complexity:
16 // O(log n) for both query and update
17
18 // How to use:
19 // MAX is the maximum index a node can assume
20
21 // Segtree seg = Segtree(MAX);
22
23 typedef long long ftype;
24
25 const int MAX = 1e9+17;
26
27 struct Segtree {
28     vector<ftype> seg, d, e;
29     const ftype NEUTRAL = 0;
30     int n;
31
32     Segtree(int n) {
33         this->n = n;
34         create();
35         create();
36     }
37
38     ftype f(ftype a, ftype b) {
39         return a + b;
40     }
41
42     ftype create() {
43         seg.push_back(0);
44         e.push_back(0);
45         d.push_back(0);
46         return seg.size() - 1;
47     }
48
49     ftype query(int pos, int ini, int fim, int p, int
q) {
50         if (q < ini || p > fim) return NEUTRAL;
51         if (pos == 0) return 0;
52         if (p <= ini && fim <= q) return seg[pos];
53         int m = (ini + fim) >> 1;
54         return f(query(e[pos], ini, m, p, q), query(d
[pos], m + 1, fim, p, q));
55     }
56
57     void update(int pos, int ini, int fim, int id,
int val) {
58         if (ini > id || fim < id) {
59             return;
60         }
61
62         if (ini == fim) {
63             seg[pos] = val;
64
65             return;
66         }
67
68         int m = (ini + fim) >> 1;
69
70         if (id <= m) {
71             if (e[pos] == 0) e[pos] = create();
72             update(e[pos], ini, m, id, val);
73         } else {
74             if (d[pos] == 0) d[pos] = create();
75             update(d[pos], m + 1, fim, id, val);
76         }
77     }
78 }

```



```

77         seg[pos] = f(seg[e[pos]], seg[d[pos]]);
78     }
79 }
80
81 ftype query(int p, int q) {
82     return query(1, 1, n, p, q);
83 }
84
85 void update(int id, int val) {
86     update(1, 1, n, id, val);
87 }
88 };

```

## 9.6 Segtree2d

```

1 // Description:
2 // Indexed at zero
3 // Given a N x M grid, where i represents the row and
4 // j the column, perform the following operations
5 // update(j, i) - update the value of grid[i][j]
6 // query(j1, j2, i1, i2) - return the sum of values
7 // inside the rectangle
8 // defined by grid[i1][j1] and grid[i2][j2] inclusive
9 // Problem:
10 // https://cses.fi/problemset/task/1739/
11 // Complexity:
12 // Time complexity:
13 // O(log N * log M) for both query and update
14 // O(N * M) for build
15 // Memory complexity:
16 // 4 * M * N
17
18 // How to use:
19 // Segtree2D seg = Segtree2D(n, n);
20 // vector<vector<int>>> v(n, vector<int>(n));
21 // seg.build(v);
22
23 // Notes
24 // Indexed at zero
25
26 struct Segtree2D {
27     const int MAXN = 1025;
28     int N, M;
29
30     vector<vector<int>>> seg;
31
32     Segtree2D(int N, int M) {
33         this->N = N;
34         this->M = M;
35         seg.resize(2*MAXN, vector<int>(2*MAXN));
36     }
37
38     void buildY(int noX, int lX, int rX, int noY, int
39         lY, int rY, vector<vector<int>>> &v){
40         if(lY == rY){
41             if(lX == rX){
42                 seg[noX][noY] = v[rX][rY];
43             }else{
44                 seg[noX][noY] = seg[2*noX+1][noY] +
45                 seg[2*noX+2][noY];
46             }
47         }else{
48             int m = (lY+rY)/2;
49
50             buildY(noX, lX, rX, 2*noY+1, lY, m, v);
51             buildY(noX, lX, rX, 2*noY+2, m+1, rY, v);
52
53             seg[noX][noY] = seg[noX][2*noY+1] + seg[
54             noX][2*noY+2];
55         }
56     }
57 }

```

```

54
55 void buildX(int noX, int lX, int rX, vector<
56     vector<int>>> &v){
57     if(lX != rX){
58         int m = (lX+rX)/2;
59
60         buildX(2*noX+1, lX, m, v);
61         buildX(2*noX+2, m+1, rX, v);
62     }
63
64     buildY(noX, lX, rX, 0, 0, M - 1, v);
65 }
66
67 void updateY(int noX, int lX, int rX, int noY,
68     int lY, int rY, int y){
69     if(lY == rY){
70         if(lX == rX){
71             seg[noX][noY] = !seg[noX][noY];
72         }else{
73             seg[noX][noY] = seg[2*noX+1][noY] +
74             seg[2*noX+2][noY];
75         }
76     }else{
77         int m = (lY+rY)/2;
78
79         if(y <= m){
80             updateY(noX, lX, rX, 2*noY+1, lY, m, y
81 );
82         }else if(m < y){
83             updateY(noX, lX, rX, 2*noY+2, m+1, rY
84 , y);
85         }
86
87         seg[noX][noY] = seg[noX][2*noY+1] + seg[
88         noX][2*noY+2];
89     }
90 }
91
92 void updateX(int noX, int lX, int rX, int x, int
93     y){
94     int m = (lX+rX)/2;
95
96     if(lX != rX){
97         if(x <= m){
98             updateX(2*noX+1, lX, m, x, y);
99         }else if(m < x){
100             updateX(2*noX+2, m+1, rX, x, y);
101         }
102     }
103
104     updateY(noX, lX, rX, 0, 0, M - 1, y);
105 }
106
107 int queryY(int noX, int noY, int lY, int rY, int
108     aY, int bY){
109     if(aY <= lY && rY <= bY) return seg[noX][noY
110 ];
111
112     int m = (lY+rY)/2;
113
114     if(bY <= m) return queryY(noX, 2*noY+1, lY, m
115 , aY, bY);
116     if(m < aY) return queryY(noX, 2*noY+2, m+1,
117 rY, aY, bY);
118
119     return queryY(noX, 2*noY+1, lY, m, aY, bY) +
120     queryY(noX, 2*noY+2, m+1, rY, aY, bY);
121 }
122
123 int queryX(int noX, int lX, int rX, int aX, int
124     bX, int aY, int bY){
125     if(aX <= lX && rX <= bX) return queryY(noX,
126     0, 0, M - 1, aY, bY);
127 }

```



```

113         int m = (lX+rX)/2;
114
115         if(bX <= m) return queryX(2*noX+1, lX, m, aX,
116             bX, aY, bY);
117         if(m < aX) return queryX(2*noX+2, m+1, rX, aX
118             , bX, aY, bY);
119
120         return queryX(2*noX+1, lX, m, aX, bX, aY, bY)
121             + queryX(2*noX+2, m+1, rX, aX, bX, aY, bY);
122     }
123
124     void build(vector<vector<int>> &v) {
125         buildX(0, 0, N - 1, v);
126     }
127
128     int query(int aX, int bX, int aY, int bY) {
129         return queryX(0, 0, N - 1, aX, bX, aY, bY);
130     }
131
132     void update(int x, int y) {
133         updateX(0, 0, N - 1, x, y);
134     }
135 };

```

## 9.7 Minimum And Amount

```

1 // Description:
2 // Query - get minimum element in a range (l, r)
3 // inclusive
4 // and also the number of times it appears in that
5 // range
6 // Update - update element at position id to a value
7 // val
8
9 // Problem:
10 // https://codeforces.com/edu/course/2/lesson/4/1/
11 // practice/contest/273169/problem/C
12
13 // Complexity:
14 // O(log n) for both query and update
15
16 // How to use:
17 // Segtree seg = Segtree(n);
18 // seg.build(v);
19
20 #define pii pair<int, int>
21 #define mp make_pair
22 #define ff first
23 #define ss second
24
25 const int INF = 1e9+17;
26
27 typedef pii ftype;
28
29 struct Segtree {
30     vector<ftype> seg;
31     int n;
32     const ftype NEUTRAL = mp(INF, 0);
33
34     Segtree(int n) {
35         int sz = 1;
36         while (sz < n) sz *= 2;
37         this->n = sz;
38
39         seg.assign(2*sz, NEUTRAL);
40     }
41
42     ftype f(ftype a, ftype b) {
43         if (a.ff < b.ff) return a;
44         if (b.ff < a.ff) return b;
45
46         return mp(a.ff, a.ss + b.ss);
47     }
48 }

```

```

43     }
44
45     ftype query(int pos, int ini, int fim, int p, int
46         q) {
47         if (ini >= p && fim <= q) {
48             return seg[pos];
49         }
50
51         if (q < ini || p > fim) {
52             return NEUTRAL;
53         }
54
55         int e = 2*pos + 1;
56         int d = 2*pos + 2;
57         int m = ini + (fim - ini) / 2;
58
59         return f(query(e, ini, m, p, q), query(d, m +
60             1, fim, p, q));
61     }
62
63     void update(int pos, int ini, int fim, int id,
64         int val) {
65         if (ini > id || fim < id) {
66             return;
67         }
68
69         if (ini == id && fim == id) {
70             seg[pos] = mp(val, 1);
71         }
72
73         return;
74
75         int e = 2*pos + 1;
76         int d = 2*pos + 2;
77         int m = ini + (fim - ini) / 2;
78
79         update(e, ini, m, id, val);
80         update(d, m + 1, fim, id, val);
81
82         seg[pos] = f(seg[e], seg[d]);
83     }
84
85     void build(int pos, int ini, int fim, vector<int>
86         &v) {
87         if (ini == fim) {
88             if (ini < (int)v.size()) {
89                 seg[pos] = mp(v[ini], 1);
90             }
91             return;
92         }
93
94         int e = 2*pos + 1;
95         int d = 2*pos + 2;
96         int m = ini + (fim - ini) / 2;
97
98         build(e, ini, m, v);
99         build(d, m + 1, fim, v);
100
101         seg[pos] = f(seg[e], seg[d]);
102     }
103
104     ftype query(int p, int q) {
105         return query(0, 0, n - 1, p, q);
106     }
107
108     void update(int id, int val) {
109         update(0, 0, n - 1, id, val);
110     }
111
112     void build(vector<int> &v) {
113         build(0, 0, n - 1, v);
114     }
115 }

```

```

112 void debug() {
113     for (auto e : seg) {
114         cout << e.ff << ' ' << e.ss << '\n';
115     }
116     cout << '\n';
117 }
118 };

```

## 9.8 Lazy Addition To Segment

```

1 // Description:
2 // Query - get sum of elements from range (l, r)
   inclusive
3 // Update - add a value val to elementos from range (l, r) inclusive
4
5 // Problem:
6 // https://codeforces.com/edu/course/2/lesson/5/1/
   practice/contest/279634/problem/A
7
8 // Complexity:
9 // O(log n) for both query and update
10
11 // How to use:
12 // Segtree seg = Segtree(n);
13 // seg.build(v);
14
15 // Notes
16 // Change neutral element and f function to perform a
   different operation
17
18 const long long INF = 1e18+10;
19
20 typedef long long ftype;
21
22 struct Segtree {
23     vector<ftype> seg;
24     vector<ftype> lazy;
25     int n;
26     const ftype NEUTRAL = 0;
27     const ftype NEUTRAL_LAZY = -1; // change to -INF
   if there are negative numbers
28
29     Segtree(int n) {
30         int sz = 1;
31         while (sz < n) sz *= 2;
32         this->n = sz;
33
34         seg.assign(2*sz, NEUTRAL);
35         lazy.assign(2*sz, NEUTRAL_LAZY);
36     }
37
38     ftype apply_lazy(ftype a, ftype b, int len) {
39         if (b == NEUTRAL_LAZY) return a;
40         if (a == NEUTRAL_LAZY) return b * len;
41         else return a + b * len;
42     }
43
44     void propagate(int pos, int ini, int fim) {
45         if (ini == fim) {
46             return;
47         }
48
49         int e = 2*pos + 1;
50         int d = 2*pos + 2;
51         int m = ini + (fim - ini) / 2;
52
53         lazy[e] = apply_lazy(lazy[e], lazy[pos], 1);
54         lazy[d] = apply_lazy(lazy[d], lazy[pos], 1);
55
56         seg[e] = apply_lazy(seg[e], lazy[pos], m -
   ini + 1);

```

```

57         seg[d] = apply_lazy(seg[d], lazy[pos], fim -
   m);
58
59         lazy[pos] = NEUTRAL_LAZY;
60     }
61
62     ftype f(ftype a, ftype b) {
63         return a + b;
64     }
65
66     ftype query(int pos, int ini, int fim, int p, int
   q) {
67         propagate(pos, ini, fim);
68
69         if (ini >= p && fim <= q) {
70             return seg[pos];
71         }
72
73         if (q < ini || p > fim) {
74             return NEUTRAL;
75         }
76
77         int e = 2*pos + 1;
78         int d = 2*pos + 2;
79         int m = ini + (fim - ini) / 2;
80
81         return f(query(e, ini, m, p, q), query(d, m +
   1, fim, p, q));
82     }
83
84     void update(int pos, int ini, int fim, int p, int
   q, int val) {
85         propagate(pos, ini, fim);
86
87         if (ini > q || fim < p) {
88             return;
89         }
90
91         if (ini >= p && fim <= q) {
92             lazy[pos] = apply_lazy(lazy[pos], val, 1)
   ;
93             seg[pos] = apply_lazy(seg[pos], val, fim
   - ini + 1);
94
95             return;
96         }
97
98         int e = 2*pos + 1;
99         int d = 2*pos + 2;
100         int m = ini + (fim - ini) / 2;
101
102         update(e, ini, m, p, q, val);
103         update(d, m + 1, fim, p, q, val);
104
105         seg[pos] = f(seg[e], seg[d]);
106     }
107
108     void build(int pos, int ini, int fim, vector<int>
   &v) {
109         if (ini == fim) {
110             if (ini < (int)v.size()) {
111                 seg[pos] = v[ini];
112             }
113             return;
114         }
115
116         int e = 2*pos + 1;
117         int d = 2*pos + 2;
118         int m = ini + (fim - ini) / 2;
119
120         build(e, ini, m, v);
121         build(d, m + 1, fim, v);
122

```

```

123     seg[pos] = f(seg[e], seg[d]);
124 }
125
126 ftype query(int p, int q) {
127     return query(0, 0, n - 1, p, q);
128 }
129
130 void update(int p, int q, int val) {
131     update(0, 0, n - 1, p, q, val);
132 }
133
134 void build(vector<int> &v) {
135     build(0, 0, n - 1, v);
136 }
137
138 void debug() {
139     for (auto e : seg) {
140         cout << e << ' ';
141     }
142     cout << '\n';
143     for (auto e : lazy) {
144         cout << e << ' ';
145     }
146     cout << '\n';
147     cout << '\n';
148 }
149 };

```

## 9.9 Segment With Maximum Sum

```

1 // Description:
2 // Query - get sum of segment that is maximum among
   all segments
3 // E.g
4 // Array: 5 -4 4 3 -5
5 // Maximum segment sum: 8 because 5 + (-4) + 4 + 3 =
   8
6 // Update - update element at position id to a value
   val
7
8 // Problem:
9 // https://codeforces.com/edu/course/2/lesson/4/2/
   practice/contest/273278/problem/A
10
11 // Complexity:
12 // O(log n) for both query and update
13
14 // How to use:
15 // Segtree seg = Segtree(n);
16 // seg.build(v);
17
18 // Notes
19 // The maximum segment sum can be a negative number
20 // In that case, taking zero elements is the best
   choice
21 // So we need to take the maximum between 0 and the
   query
22 // max(0LL, seg.query(0, n).max_seg)
23
24 using ll = long long;
25
26 typedef ll ftype_node;
27
28 struct Node {
29     ftype_node max_seg;
30     ftype_node pref;
31     ftype_node suf;
32     ftype_node sum;
33
34     Node(ftype_node max_seg, ftype_node pref,
         ftype_node suf, ftype_node sum) : max_seg(max_seg),
         pref(pref), suf(suf), sum(sum) {};
35 };

```

```

36
37 typedef Node ftype;
38
39 struct Segtree {
40     vector<ftype> seg;
41     int n;
42     const ftype NEUTRAL = Node(0, 0, 0, 0);
43
44     Segtree(int n) {
45         int sz = 1;
46         // potencia de dois mais proxima
47         while (sz < n) sz *= 2;
48         this->n = sz;
49
50         // numero de nos da seg
51         seg.assign(2*sz, NEUTRAL);
52     }
53
54     ftype f(ftype a, ftype b) {
55         ftype_node max_seg = max({a.max_seg, b.
max_seg, a.suf + b.pref});
56         ftype_node pref = max(a.pref, a.sum + b.pref);
57
58         ftype_node suf = max(b.suf, b.sum + a.suf);
59         ftype_node sum = a.sum + b.sum;
60
61         return Node(max_seg, pref, suf, sum);
62     }
63
64     ftype query(int pos, int ini, int fim, int p, int
q) {
65         if (ini >= p && fim <= q) {
66             return seg[pos];
67         }
68
69         if (q < ini || p > fim) {
70             return NEUTRAL;
71         }
72
73         int e = 2*pos + 1;
74         int d = 2*pos + 2;
75         int m = ini + (fim - ini) / 2;
76
77         return f(query(e, ini, m, p, q), query(d, m +
1, fim, p, q));
78     }
79
80     void update(int pos, int ini, int fim, int id,
int val) {
81         if (ini > id || fim < id) {
82             return;
83         }
84
85         if (ini == id && fim == id) {
86             seg[pos] = Node(val, val, val, val);
87         }
88
89         return;
90     }
91
92     int e = 2*pos + 1;
93     int d = 2*pos + 2;
94     int m = ini + (fim - ini) / 2;
95
96     update(e, ini, m, id, val);
97     update(d, m + 1, fim, id, val);
98
99     seg[pos] = f(seg[e], seg[d]);
100 }
101
102 void build(int pos, int ini, int fim, vector<int>
&v) {
103     if (ini == fim) {
104         // se a posição existir no array original

```

```

103         // seg tamanho potencia de dois
104         if (ini < (int)v.size()) {
105             seg[pos] = Node(v[ini], v[ini], v[ini
106         ], v[ini]);
107         }
108         return;
109     }
110
111     int e = 2*pos + 1;
112     int d = 2*pos + 2;
113     int m = ini + (fim - ini) / 2;
114
115     build(e, ini, m, v);
116     build(d, m + 1, fim, v);
117
118     seg[pos] = f(seg[e], seg[d]);
119 }
120
121 ftype query(int p, int q) {
122     return query(0, 0, n - 1, p, q);
123 }
124
125 void update(int id, int val) {
126     update(0, 0, n - 1, id, val);
127 }
128
129 void build(vector<int> &v) {
130     build(0, 0, n - 1, v);
131 }
132
133 void debug() {
134     for (auto e : seg) {
135         cout << e.max_seg << ' ' << e.pref << ' '
136         << e.suf << ' ' << e.sum << '\n';
137     }
138     cout << '\n';
139 }
140 };

```

## 9.10 Range Query Point Update

```

1 // Description:
2 // Indexed at zero
3 // Query - get sum of elements from range (l, r)
4 // inclusive
5 // Update - update element at position id to a value
6 // val
7
8 // Problem:
9 // https://codeforces.com/edu/course/2/lesson/4/1/
10 // practice/contest/273169/problem/B
11
12 // Complexity:
13 // O(log n) for both query and update
14
15 // How to use:
16 // Segtree seg = Segtree(n);
17 // seg.build(v);
18
19 // Notes
20 // Change neutral element and f function to perform a
21 // different operation
22
23 // If you want to change the operations to point
24 // query and range update
25
26 // Use the same segtree, but perform the following
27 // operations
28
29 // Query - seg.query(0, id);
30 // Update - seg.update(l, v); seg.update(r + 1, -v);
31
32 typedef long long ftype;
33
34 struct Segtree {

```

```

27 vector<ftype> seg;
28 int n;
29 const ftype NEUTRAL = 0;
30
31 Segtree(int n) {
32     int sz = 1;
33     while (sz < n) sz *= 2;
34     this->n = sz;
35
36     seg.assign(2*sz, NEUTRAL);
37 }
38
39 ftype f(ftype a, ftype b) {
40     return a + b;
41 }
42
43 ftype query(int pos, int ini, int fim, int p, int
44 q) {
45     if (ini >= p && fim <= q) {
46         return seg[pos];
47     }
48
49     if (q < ini || p > fim) {
50         return NEUTRAL;
51     }
52
53     int e = 2*pos + 1;
54     int d = 2*pos + 2;
55     int m = ini + (fim - ini) / 2;
56
57     return f(query(e, ini, m, p, q), query(d, m +
58 1, fim, p, q));
59 }
60
61 void update(int pos, int ini, int fim, int id,
62 int val) {
63     if (ini > id || fim < id) {
64         return;
65     }
66
67     if (ini == id && fim == id) {
68         seg[pos] = val;
69     }
70
71     return;
72 }
73
74 int e = 2*pos + 1;
75 int d = 2*pos + 2;
76 int m = ini + (fim - ini) / 2;
77
78 update(e, ini, m, id, val);
79 update(d, m + 1, fim, id, val);
80
81 seg[pos] = f(seg[e], seg[d]);
82 }
83
84 void build(int pos, int ini, int fim, vector<int>
85 &v) {
86     if (ini == fim) {
87         if (ini < (int)v.size()) {
88             seg[pos] = v[ini];
89         }
90         return;
91     }
92
93     int e = 2*pos + 1;
94     int d = 2*pos + 2;
95     int m = ini + (fim - ini) / 2;
96
97     build(e, ini, m, v);
98     build(d, m + 1, fim, v);
99
100     seg[pos] = f(seg[e], seg[d]);

```

```

96     }
97
98     ftype query(int p, int q) {
99         return query(0, 0, n - 1, p, q);
100     }
101
102     void update(int id, int val) {
103         update(0, 0, n - 1, id, val);
104     }
105
106     void build(vector<int> &v) {
107         build(0, 0, n - 1, v);
108     }
109
110     void debug() {
111         for (auto e : seg) {
112             cout << e << ' ';
113         }
114         cout << '\n';
115     }
116 };

```

## 9.11 Lazy Assignment To Segment

```

1  const long long INF = 1e18+10;
2
3  typedef long long ftype;
4
5  struct Segtree {
6      vector<ftype> seg;
7      vector<ftype> lazy;
8      int n;
9      const ftype NEUTRAL = 0;
10     const ftype NEUTRAL_LAZY = -1; // Change to -INF
    if there are negative numbers
11
12     Segtree(int n) {
13         int sz = 1;
14         // potencia de dois mais proxima
15         while (sz < n) sz *= 2;
16         this->n = sz;
17
18         // numero de nos da seg
19         seg.assign(2*sz, NEUTRAL);
20         lazy.assign(2*sz, NEUTRAL_LAZY);
21     }
22
23     ftype apply_lazy(ftype a, ftype b, int len) {
24         if (b == NEUTRAL_LAZY) return a;
25         if (a == NEUTRAL_LAZY) return b * len;
26         else return b * len;
27     }
28
29     void propagate(int pos, int ini, int fim) {
30         if (ini == fim) {
31             return;
32         }
33
34         int e = 2*pos + 1;
35         int d = 2*pos + 2;
36         int m = ini + (fim - ini) / 2;
37
38         lazy[e] = apply_lazy(lazy[e], lazy[pos], 1);
39         lazy[d] = apply_lazy(lazy[d], lazy[pos], 1);
40
41         seg[e] = apply_lazy(seg[e], lazy[pos], m -
ini + 1);
42         seg[d] = apply_lazy(seg[d], lazy[pos], fim -
m);
43
44         lazy[pos] = NEUTRAL_LAZY;
45     }
46

```

```

47     ftype f(ftype a, ftype b) {
48         return a + b;
49     }
50
51     ftype query(int pos, int ini, int fim, int p, int
q) {
52         propagate(pos, ini, fim);
53
54         if (ini >= p && fim <= q) {
55             return seg[pos];
56         }
57
58         if (q < ini || p > fim) {
59             return NEUTRAL;
60         }
61
62         int e = 2*pos + 1;
63         int d = 2*pos + 2;
64         int m = ini + (fim - ini) / 2;
65
66         return f(query(e, ini, m, p, q), query(d, m +
1, fim, p, q));
67     }
68
69     void update(int pos, int ini, int fim, int p, int
q, int val) {
70         propagate(pos, ini, fim);
71
72         if (ini > q || fim < p) {
73             return;
74         }
75
76         if (ini >= p && fim <= q) {
77             lazy[pos] = apply_lazy(lazy[pos], val, 1)
;
78             seg[pos] = apply_lazy(seg[pos], val, fim
- ini + 1);
79
80             return;
81         }
82
83         int e = 2*pos + 1;
84         int d = 2*pos + 2;
85         int m = ini + (fim - ini) / 2;
86
87         update(e, ini, m, p, q, val);
88         update(d, m + 1, fim, p, q, val);
89
90         seg[pos] = f(seg[e], seg[d]);
91     }
92
93     void build(int pos, int ini, int fim, vector<int>
&v) {
94         if (ini == fim) {
95             // se a posição existir no array original
96             // seg tamanho potencia de dois
97             if (ini < (int)v.size()) {
98                 seg[pos] = v[ini];
99             }
100             return;
101         }
102
103         int e = 2*pos + 1;
104         int d = 2*pos + 2;
105         int m = ini + (fim - ini) / 2;
106
107         build(e, ini, m, v);
108         build(d, m + 1, fim, v);
109
110         seg[pos] = f(seg[e], seg[d]);
111     }
112
113     ftype query(int p, int q) {

```

```

114     return query(0, 0, n - 1, p, q);
115 }
116
117 void update(int p, int q, int val) {
118     update(0, 0, n - 1, p, q, val);
119 }
120
121 void build(vector<int> &v) {
122     build(0, 0, n - 1, v);
123 }
124
125 void debug() {
126     for (auto e : seg) {
127         cout << e << ' ';
128     }
129     cout << '\n';
130     for (auto e : lazy) {
131         cout << e << ' ';
132     }
133     cout << '\n';
134     cout << '\n';
135 }
136 };

```

## 9.12 Lazy Dynamic Implicit Sparse

```

1 // Description:
2 // Indexed at one
3
4 // When the indexes of the nodes are too big to be
5 // stored in an array
6 // and the queries need to be answered online so we
7 // can't sort the nodes and compress them
8 // we create nodes only when they are needed so there
9 // 'll be (Q*log(MAX)) nodes
10 // where Q is the number of queries and MAX is the
11 // maximum index a node can assume
12
13 // Query - get sum of elements from range (l, r)
14 // inclusive
15 // Update - update element at position id to a value
16 // val
17
18 // Problem:
19 // https://oj.uz/problem/view/IZh012_apple
20
21 // Complexity:
22 // O(log n) for both query and update
23
24 // How to use:
25 // MAX is the maximum index a node can assume
26 // Create a default null node
27 // Create a node to be the root of the segtree
28
29 // Segtree seg = Segtree(MAX);
30
31 const int MAX = 1e9+10;
32 const long long INF = 1e18+10;
33
34 typedef long long ftype;
35
36 struct Segtree {
37     vector<ftype> seg, d, e, lazy;
38     const ftype NEUTRAL = 0;
39     const ftype NEUTRAL_LAZY = -1; // change to -INF
40     // if the elements can be negative
41     int n;
42
43     Segtree(int n) {
44         this->n = n;
45         create();
46         create();
47     }
48 };

```

```

41
42 ftype apply_lazy(ftype a, ftype b, int len) {
43     if (b == NEUTRAL_LAZY) return a;
44     else return b * len; // change to a + b * len
45     // to add to an element instead of updating it
46 }
47
48 void propagate(int pos, int ini, int fim) {
49     if (seg[pos] == 0) return;
50
51     if (ini == fim) {
52         return;
53     }
54
55     int m = (ini + fim) >> 1;
56
57     if (e[pos] == 0) e[pos] = create();
58     if (d[pos] == 0) d[pos] = create();
59
60     lazy[e[pos]] = apply_lazy(lazy[e[pos]], lazy[
61     pos], 1);
62     lazy[d[pos]] = apply_lazy(lazy[d[pos]], lazy[
63     pos], 1);
64
65     seg[e[pos]] = apply_lazy(seg[e[pos]], lazy[
66     pos], m - ini + 1);
67     seg[d[pos]] = apply_lazy(seg[d[pos]], lazy[
68     pos], fim - m);
69
70     lazy[pos] = NEUTRAL_LAZY;
71 }
72
73 ftype f(ftype a, ftype b) {
74     return a + b;
75 }
76
77 ftype create() {
78     seg.push_back(0);
79     e.push_back(0);
80     d.push_back(0);
81     lazy.push_back(-1);
82     return seg.size() - 1;
83 }
84
85 ftype query(int pos, int ini, int fim, int p, int
86 q) {
87     propagate(pos, ini, fim);
88     if (q < ini || p > fim) return NEUTRAL;
89     if (pos == 0) return 0;
90     if (p <= ini && fim <= q) return seg[pos];
91     int m = (ini + fim) >> 1;
92     return f(query(e[pos], ini, m, p, q), query(d
93     [pos], m + 1, fim, p, q));
94 }
95
96 void update(int pos, int ini, int fim, int p, int
97 q, int val) {
98     propagate(pos, ini, fim);
99     if (ini > q || fim < p) {
100         return;
101     }
102
103     if (ini >= p && fim <= q) {
104         lazy[pos] = apply_lazy(lazy[pos], val, 1)
105 ;
106         seg[pos] = apply_lazy(seg[pos], val, fim
107 - ini + 1);
108
109         return;
110     }
111
112     int m = (ini + fim) >> 1;

```

```

104         if (e[pos] == 0) e[pos] = create();
105         update(e[pos], ini, m, p, q, val);
106
107         if (d[pos] == 0) d[pos] = create();
108         update(d[pos], m + 1, fim, p, q, val);
109
110         seg[pos] = f(seg[e[pos]], seg[d[pos]]);
111     }
112
113     ftype query(int p, int q) {
114         return query(1, 1, n, p, q);
115     }
116
117     void update(int p, int q, int val) {
118         update(1, 1, n, p, q, val);
119     }
120 };

```

### 9.13 Persistent

```

1 // Description:
2 // Persistent segtree allows for you to save the
3 // different versions of the segtree between each
4 // update
5 // Indexed at one
6 // Query - get sum of elements from range (l, r)
7 // inclusive
8 // Update - update element at position id to a value
9 // val
10
11 // Problem:
12 // https://cses.fi/problemset/task/1737/
13
14 // Complexity:
15 // O(log n) for both query and update
16
17 // How to use:
18 // vector<int> raiz(MAX); // vector to store the
19 // roots of each version
20 // Segtree seg = Segtree(INF);
21 // raiz[0] = seg.create(); // null node
22 // curr = 1; // keep track of the last version
23
24 // raiz[k] = seg.update(raiz[k], idx, val); //
25 // updating version k
26 // seg.query(raiz[k], l, r) // querying version k
27 // raiz[++curr] = raiz[k]; // create a new version
28 // based on version k
29
30 const int MAX = 2e5+17;
31 const int INF = 1e9+17;
32
33 typedef long long ftype;
34
35 struct Segtree {
36     vector<ftype> seg, d, e;
37     const ftype NEUTRAL = 0;
38     int n;
39 };

```

```

33 Segtree(int n) {
34     this->n = n;
35 }
36
37 ftype f(ftype a, ftype b) {
38     return a + b;
39 }
40
41 ftype create() {
42     seg.push_back(0);
43     e.push_back(0);
44     d.push_back(0);
45     return seg.size() - 1;
46 }
47
48 ftype query(int pos, int ini, int fim, int p, int
49 q) {
50     if (q < ini || p > fim) return NEUTRAL;
51     if (pos == 0) return 0;
52     if (p <= ini && fim <= q) return seg[pos];
53     int m = (ini + fim) >> 1;
54     return f(query(e[pos], ini, m, p, q), query(d
55 [pos], m + 1, fim, p, q));
56 }
57
58 int update(int pos, int ini, int fim, int id, int
59 val) {
60     int novo = create();
61
62     seg[novo] = seg[pos];
63     e[novo] = e[pos];
64     d[novo] = d[pos];
65
66     if (ini == fim) {
67         seg[novo] = val;
68         return novo;
69     }
70
71     int m = (ini + fim) >> 1;
72
73     if (id <= m) e[novo] = update(e[novo], ini, m
74 , id, val);
75     else d[novo] = update(d[novo], m + 1, fim, id
76 , val);
77
78     seg[novo] = f(seg[e[novo]], seg[d[novo]]);
79
80     return novo;
81 }
82
83 ftype query(int pos, int p, int q) {
84     return query(pos, 1, n, p, q);
85 }
86
87 int update(int pos, int id, int val) {
88     return update(pos, 1, n, id, val);
89 }
90 };

```