Theoretical Guide
Lenhadoras de Segtree

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Contents

1 Misc

1.1 Input by file

```
freopen("input.txt","r",stdin);
freopen("output.txt","w",stdout);
```

2 Geometry

2.1 Distances

2.1.1 Euclidean

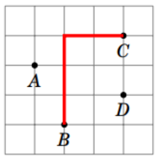
$$d(p,q) = \sqrt{(q.x - p.x)^2 + (q.y - p.y)^2}$$

2.1.2 Manhattan

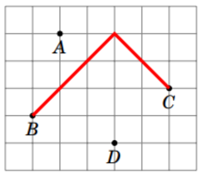
$$|p.x - q.x| + |p.y - q.y|$$

2.2 Maximum possible manhattan distance between two points given n points

Given n points, for instance:



Rotate all coordinates 45^o do that (x, y) becomes $(x+y, y-x)$, so, p becomes p' and q becomes q'.



The maximum manhattan distance is obtaining by choosing the two points that maxime:

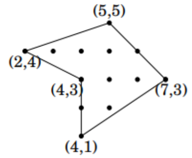
$$max(|p'.x - q'.x|, |p'.y - q'.y|)$$

2.3 Boundary points

The number of integer points in the boundary of a polygon is:

$$B = v + b$$

where v is the number of vertices (integer points as well) and b is the number of integer points situated between two vertices, like in the following figure:



b can be calculated for every line connecting two points (including the line between the last and the first point) as follows:

$$boundary\_points(p,q) = \begin{cases} |p.y - q.y| - 1 & \text{p.x = q.x} \\ |p.x - q.x| - 1 & \text{p.y = q.y} \\ gcd(|p.x - q.x|, |p.y - q.y|) - 1 & \end{cases}$$

2.4 Pick's Theorem

$$A = a + \frac{b}{2} + 1$$

where A is the area of the polygon, a is the number of integer points inside the polygon and b is the number of integer points in the boundary of the polygon

2.5 Triangle Existence Condition

$$a + b \geq c$$

$$a + c \geq b$$

$$b + c \geq a$$

2.6 Perimeter

2.6.1 Circle

$$2\pi r$$

2.7 Areas

2.7.1 Circle

$$\pi r^2$$

2.7.2 Triangle

$$\frac{b*h}{2}$$

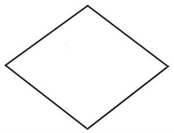
2.7.3 Square

$$l^2$$

2.7.4 Rectangle

$$hr$$

2.7.5 Rhombus



D is the biggest diagonal and d is the smallest diagonal

$$A = \frac{1}{2} * D * d$$

2.8 Volumes

2.8.1 Sphere

$$\frac{4}{3}\pi r^3$$

2.8.2 Prism

$$V = bh$$

2.8.3 Pyramid

$$\frac{bh}{3}$$

2.8.4 Cone

$$\frac{\pi r^2 h}{3}$$

2.9 Sines Rule

$$\frac{a}{sin(\alpha)} = \frac{b}{sin(\beta)} = \frac{c}{sin(\gamma)}$$

2.10 Cossines Rule

$$a^2 = b^2 + c^2 - 2bccos(\alpha)$$

2.11 Shoelace Formula

$$A = \frac{1}{2}|\sum_{i=1}^{n-1}(p_i \times p_{i+1}) = \frac{1}{2}|\sum_{i=1}^{n-1}(x_i y_{i+1} - x_{i+1} y_i)|$$

Where the points $p_1, pn, \ldots$ are in adjecent order and the first and last vertex is the same, that is, $p_1 = pn$

3 Bitwise

3.1 Turn bit on and off

Turn on bit i x & (1 << i)
Turn off bit i x & (~(1 << i))

3.2 XOR from 1 to n

$$f(n) = \begin{cases} n & n \equiv 0 \pmod 4 \\ 1 & n \equiv 1 \pmod 4 \\ n+1 & n \equiv 2 \pmod 4 \\ 0 & n \equiv 3 \pmod 4 \end{cases}$$

4 Basic Math

4.1 Logarithm

$$\log_b mn = \log_b m + \log_b n \qquad log_b \frac{m}{n} = \log_b m - \log_n n \qquad \log_b n^p = p \log_b n$$

$$\log_b \sqrt[q]{n} = \frac{1}{q} \log_b n \qquad \log_b n = \log_a n \log_b a \qquad b^{\log_b k} = k$$

$$\log_b a = \frac{\log_c a}{\log_c b} \qquad \log_b a = \frac{1}{\log_a b} \qquad \log_b a \ \log_a c = \log_b c$$

$$\log_b 1 = 0 \qquad \log_b b = 1$$

5 C++

5.1 Ordered set and multiset

```
typedef tree<pair<ll, ll>, null_type, less<pair<ll, ll>>,
rb_tree_tag, tree_order_statistics_node_update> ordered_set;
```

To change to multiset switch equal to less_equal.

5.2 Priority Queue

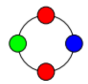
```
template<class T> using min_priority_queue =
priority_queue<T, vector<T>, greater<T>>;
```

6 Combinatorics

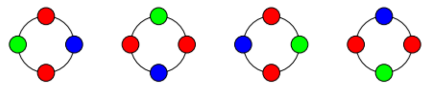
6.1 Burnside's Lemma

Burnside's lemma can be used to count the number of combinations so that only one representative is counted for each group of symmetric combinations. For example, if we have a necklace with different colored pearls and we want to know how many different combinations we can make.

For example, if we have a necklace colored like this:



This variations are the same if we consider that we can rotate the necklace:



When the number of steps is k, the number of necklaces that remain the same are:

$$m^{gcd(k,n)}$$

The number of diferent combinations for m colors and a necklace of size n is

$$\sum_{i=0}^{n-1} \frac{m^{gcd(i,n)}}{n}$$

So a necklace of length 4 with 3 colors has

$$\frac{3^4 + 3 + 3^2 + 3}{4} = 24$$

7 Constants

```
LLINF = 0x3f3f3f3f3f3f3f3fLL
```

```
PI = acos(-1)
```

8 Number Theory

$$(a+b) \bmod m = (a \bmod m + b \bmod m) \bmod m$$

$$(a-b) \bmod m = (a \bmod m - b \bmod m) \bmod m$$

$$(a \times b) \bmod m = ((a \bmod m) \times (b \bmod m)) \bmod m$$

$$a^b \bmod m = (a \bmod m)^b \bmod m$$

$$a \equiv b \pmod m \iff (b-a)|m$$

$$\gcd(a_1, a_2, a_3, a_4) = \gcd(a_1, \gcd(a_2, \gcd(a_3, a_4)))$$

$$\operatorname{lcm}(a,b) \times \gcd(a,b) = a \times b$$

$$\operatorname{lcm}(a,b) = \frac{a \times b}{\gcd(a,b)} = \frac{a}{\gcd(a,b)} \times b$$

$$gcd(a,b) = b?gcd(b, a\%b) : a$$

8.1 Number of Divisors

The number of divisors of n is about $\sqrt[3]{n}$.

n	6	60	360	5040	55440	720720	4324320	21621600
$d(n)$	4	12	24	60	120	240	384	576

8.2 Number of digits of n! in base b

$$\log_b n! = \log_b(1 \times 2 \times 3 \times ... \times n) = \log_b 1 + \log_b 2 + \log_b 3 + ... + \log_b n$$

8.3 Prime counting function - $\pi(x)$

Expected to have $\frac{x}{\log x}$ primes within $[1, x]$. The prime counting function is asymptotic to $\frac{x}{\log x}$, by the prime number theorem.

x	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
$\pi(x)$	4	25	168	1 229	9 592	78 498	664 579	5 761 455

8.4 Sum of digits of n in base b

$$f(n,b) = \begin{cases} n & n < b \\ f\left(n, \left\lfloor \frac{n}{b} \right\rfloor + (n \mod b)\right) & n \geq b \end{cases}$$

8.5 Fermat's Theorems

Let P be a prime number and a an integer, then:

$$a^p \equiv a \pmod p$$

$$a^{p-1} \equiv 1 \pmod p$$

Lemma: Let p be a prime number and a and b integers, then:

$$(a+b)^p \equiv a^p + b^p \pmod p$$

Lemma: Let p be a prime number and a an integer. The inverse of a modulo p is a^{p-2}:

$$a^{-1} \equiv a^{p-2} \pmod p$$

9 Identities

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=1}^{n} \frac{1}{i} \approx \log n \qquad \sum_{i=0}^{\infty} \frac{1}{2^i} = 2$$

10 Progressions

10.1 Arithmetic Progression

10.1.1 General Term

$$a_1 + (n-1)r$$

10.1.2 Sum

$$\frac{(a_1 + a_n)n}{2}$$

10.1.3 Sum of Second Order Arithmetic Progression

a_1 is the first element of the original progression, b_1 is the first element of the derived progression, n is the number of elements of the original progression and r is the ratio of the derived progression

$$a_1 n + \frac{(b_1 n(n-1)}{2} + \frac{rn(n-1)(n-2)}{6}$$

10.2 Geometric Progression

10.2.1 General Term

$$a_1 q^{n-1}$$

10.2.2 Sum

$$\frac{a_1(q^n - 1)}{q - 1}$$

10.2.3 Infinite Sum

$$-1 < q < 1$$
$$\frac{a_1}{1 - q}$$