# Derivatives of Radicals and Rational Powers

Calculus Exercises

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## 1 Part 1: Introductory Level

Find the derivative $\frac{dy}{dx}$ for each of the following functions.

a) $y = \sqrt{7x-2}$ **Solution:**
$\frac{dy}{dx} = \frac{1}{2}(7x-2)^{-\frac{1}{2}} \cdot (7) = \frac{7}{2\sqrt{7x-2}}$

b) $y = x^2\sqrt{x}$ **Solution:**
Simplify first: $y = x^2 \cdot x^{\frac{1}{2}} = x^{\frac{5}{2}}$.
$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} = \frac{5}{2}x\sqrt{x}$

c) $y = (3x^2+1)^{\frac{1}{3}}$ **Solution:**
$\frac{dy}{dx} = \frac{1}{3}(3x^2+1)^{-\frac{2}{3}} \cdot (6x) = \frac{2x}{\sqrt[3]{(3x^2+1)^2}}$

d) $y = \frac{1}{\sqrt{x+5}}$ **Solution:**
$y = (x+5)^{-\frac{1}{2}}$.
$\frac{dy}{dx} = -\frac{1}{2}(x+5)^{-\frac{3}{2}} = -\frac{1}{2\sqrt{(x+5)^3}}$

e) $y = \sqrt{x}(2x-3)$ **Solution:**
Using Product Rule:
$\frac{dy}{dx} = \left(\frac{1}{2\sqrt{x}}\right)(2x-3) + \sqrt{x}(2) = \frac{2x-3+4x}{2\sqrt{x}} = \frac{6x-3}{2\sqrt{x}}$

## 2 Part 2: Advanced Level

a) $y = \sqrt{x^2 + \sqrt{x+1}}$ **Solution:**

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x^2+\sqrt{x+1}}} \cdot \left(2x + \frac{1}{2\sqrt{x+1}}\right)$$

b) $y = (x^2+1)^3\sqrt{3x-1}$ **Solution:**

$$\frac{dy}{dx} = 3(x^2+1)^2(2x)\sqrt{3x-1} + (x^2+1)^3\left(\frac{3}{2\sqrt{3x-1}}\right)$$

c) $y = \dfrac{\sqrt{x^2-1}}{x+2}$ **Solution:**

Using Quotient Rule:

$$\frac{dy}{dx} = \frac{\frac{x}{\sqrt{x^2-1}}(x+2) - \sqrt{x^2-1}}{(x+2)^2} = \frac{2x+1}{(x+2)^2\sqrt{x^2-1}}$$

d) $y = \left(\dfrac{x}{x^2+1}\right)^{\frac{3}{4}}$ **Solution:**

Note how LaTeX automatically aligns the large parentheses and exponents:

$$\frac{dy}{dx} = \frac{3}{4}\left(\frac{x}{x^2+1}\right)^{-\frac{1}{4}} \cdot \frac{(1)(x^2+1)-(x)(2x)}{(x^2+1)^2} = \frac{3(1-x^2)}{4(x^2+1)^{\frac{7}{4}}x^{\frac{1}{4}}}$$

e) $y = \sqrt[3]{x^2\sqrt{x^3+1}}$ **Solution:**

$y = x^{\frac{2}{3}}(x^3+1)^{\frac{1}{6}}$.

$$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}}(x^3+1)^{\frac{1}{6}} + x^{\frac{2}{3}}\left(\frac{1}{6}(x^3+1)^{-\frac{5}{6}}(3x^2)\right)$$