$\boxed{\textbf{Exercice 1}}$ Dériver les fonctions.

a) $f(x) = (1-x)^{20}$

b) $f(x) = \left(x^2 + 1\right)^4$

c) $f(x) = \left(x^2 + 1\right)^3 \left(2 - x^3\right)^2$

d) $f(x) = \left(7x^2 - \frac{4}{x} + 6\right)^6$

e) $f(x) = \frac{1+\sqrt{x}}{1+\sqrt{2x}}$

f) $f(x) = \frac{2}{(x^2-x+1)^2}$

g) $f(x) = \frac{1}{\sqrt{a^2-x^2}}$

h) $f(x) = \left(\frac{1+x}{1-x}\right)^2$

i) $f(x) = \frac{x^2}{\sqrt{x^2+a^2}}$

$\boxed{\textbf{Corrigé 1}}$ *Correction générée par IA*

a) $f'(x) = 20(1-x)^{19} \cdot (-1) = -20(1-x)^{19}$

b) $f'(x) = 4(x^2+1)^3 \cdot 2x = 8x(x^2+1)^3$

c) $f'(x) = 3(x^2+1)^2 \cdot 2x \cdot (2-x^3)^2 + (x^2+1)^3 \cdot 2(2-x^3) \cdot (-3x^2)$
$= 6x(x^2+1)^2(2-x^3)^2 - 6x^2(x^2+1)^3(2-x^3)$

d) $f'(x) = 6\left(7x^2 - \frac{4}{x} + 6\right)^5 \cdot \left(14x + \frac{4}{x^2}\right)$

e) $f'(x) = \dfrac{\dfrac{1}{2\sqrt{x}}(1+\sqrt{2x}) - (1+\sqrt{x}) \cdot \dfrac{1}{\sqrt{2x}}}{(1+\sqrt{2x})^2}$

$= \dfrac{\dfrac{1+\sqrt{2x}}{2\sqrt{x}} - \dfrac{1+\sqrt{x}}{\sqrt{2x}}}{(1+\sqrt{2x})^2}$

f) $f'(x) = -\dfrac{2 \cdot 2(x^2-x+1) \cdot (2x-1)}{(x^2-x+1)^4} = -\dfrac{4(2x-1)}{(x^2-x+1)^3}$

g) $f'(x) = -\dfrac{1}{2}(a^2-x^2)^{-\frac{3}{2}} \cdot (-2x) = \dfrac{x}{(a^2-x^2)\sqrt{a^2-x^2}}$

h) $f'(x) = 2\left(\dfrac{1+x}{1-x}\right) \cdot \dfrac{1 \cdot (1-x) - (1+x) \cdot (-1)}{(1-x)^2}$

$= 2\left(\dfrac{1+x}{1-x}\right) \cdot \dfrac{1-x+1+x}{(1-x)^2}$

$= 2\left(\dfrac{1+x}{1-x}\right) \cdot \dfrac{2}{(1-x)^2}$

$= \dfrac{4(1+x)}{(1-x)^3}$

i) $f'(x) = \dfrac{2x\sqrt{x^2+a^2} - x^2 \cdot \dfrac{2x}{2\sqrt{x^2+a^2}}}{x^2+a^2}$

$= \dfrac{2x\sqrt{x^2+a^2} - \dfrac{x^3}{\sqrt{x^2+a^2}}}{x^2+a^2}$

$= \dfrac{2x(x^2+a^2) - x^3}{(x^2+a^2)\sqrt{x^2+a^2}}$

$= \dfrac{2x^3 + 2a^2x - x^3}{(x^2+a^2)^{\frac{3}{2}}}$

$= \dfrac{x^3 + 2a^2x}{(x^2+a^2)\sqrt{x^2+a^2}}$