

Exercice 1

Calculer les limites suivantes :

a) $\lim_{x \rightarrow 0} \frac{\sin(ax)}{x}$

b) $\lim_{x \rightarrow 0} \frac{\tan(ax)}{\sin(bx)}, a, b \in \mathbb{R}, b \neq 0$

Corrigé 1a) Si $a = 0$, alors la limite vaut 0. Autrement :

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin(ax)}{x} \\ &= \lim_{x \rightarrow 0} \frac{\sin(ax)}{\frac{ax}{a}} \\ &= \lim_{ax \rightarrow 0} \frac{\sin(ax)}{ax} \cdot a \\ &= a \end{aligned}$$

b) On a $\tan(x) = \frac{\sin(x)}{\cos(x)}$ et $\lim_{x \rightarrow 0} \frac{\sin(ax)}{x} = a$.

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\tan(ax)}{\sin(bx)} \\ &= \lim_{x \rightarrow 0} \frac{\sin(ax)}{\cos(ax) \sin(bx)} \\ &= \lim_{x \rightarrow 0} \frac{\sin(ax)}{x} \frac{x}{\sin(bx)} \frac{1}{\cos(ax)} \\ &= \lim_{x \rightarrow 0} \frac{\sin(ax)}{x} \lim_{x \rightarrow 0} \frac{x}{\sin(bx)} \lim_{x \rightarrow 0} \frac{1}{\cos(ax)} \\ &= a \cdot \frac{1}{b} \cdot \frac{1}{1} \\ &= \frac{a}{b} \end{aligned}$$