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CS 111 ASSIGNMENT 2

due February 5

Problem 1: Prove the following statement:

If $p > 5$ and $gcd(p, 20) = 1$, then $(p^2 - 21)(p^2 + 16) \equiv 0 \pmod{20}$.

Hint: The product of any k consecutive integers is divisible by k.

Solution 1: SOLUTION 1 GOES HERE

Problem 2: PROBLEM 2 GOES HERE

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Problem 3:

(a) Compute $5^{1627} \pmod{12}$. Show your work.

(b) Compute $8^{-1} \pmod{17}$ by listing the multiples. Show your work.

(c) Compute $8^{-1} \pmod{17}$ using Fermat's Little Theorem. Show your work.

(d) Compute $8^{-11} \pmod{17}$ using Fermat's Little Theorem. Show your work.

(e) Find an integer x, $0 \leq x \leq 40$, that satisfies the following congruence: $31x + 54 \equiv 16 \pmod{41}$. Show your work. You should not use brute force approach.

Solution 3:

(a) $5^{1627} \pmod{12} \equiv 5 \cdot (5^2)^{813} \pmod{12} \equiv 5 \cdot (25)^{813} \pmod{12} \equiv 5 \cdot (1)^{813} \pmod{12} \equiv 5 \pmod{12}$.

(b) $8^{-1} \pmod{17} \implies 8a \equiv 1 \pmod{17} \implies 8a = 17b + 1$. We need to find an a to make this equation true.
Multiples of 8: $8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120$
Multiples of 17 (and then +1): $18, 35, 52, 69, 86, 103, 120$
We can see that the equation is true when $a = 15$ and $b = 7$. This means that $8^{-1} \pmod{17} \equiv 15$.

(c) Accodring to Fermat's Little Theorem, $a^{p-1} \equiv 1 \pmod{p}$, where p is prime. We can rewrite $8^{-1} \pmod{17}$ as $8a \equiv 1 \pmod{17}$
