CS111, Winter 24
ASSIGNMENT 1

Problem 1: Give an asymptotic estimate for the number $h(n)$ of "Hello"s printed by Algorithm PRINT-HELLOS below. Your solution *must* consist of the following steps:

(a) First express $h(n)$ using the summation notation $\sum$.

(b) Next, give a closed-form expression[1] for $h(n)$.

(c) Finally, give the asymptotic value of $h(n)$ using the Θ-notation.

Show your work and include justification for each step.

Algorithm PRINTHELLOS $(n : \textbf{integer})$
    for $i \leftarrow 1$ **to** n **do**
        for $j \leftarrow 1$ **to** $3i^2 + i$ **do** print("Hello")
    for $i \leftarrow 1$ **to** $2n^2$ **do**
        for $j \leftarrow 1$ **to** i **do** print("Hello")

Note: If you need any summation formulas for this problem, you are allowed to look them up. You do not need to prove them, you can just state in the assignment when you use them.

Problem 2:
(a) Use properties of quadratic functions to prove that $3x^2 \geq (x+1)^2$ for all real $x \geq 4$.
(b) Use mathematical induction and the inequality from part (a) to prove that $3^n \geq 2^n + 3n^2$ for all integers $n \geq 4$.
(c) Let $g(n) = 2^n + 3n^2$ and $h(n) = 3^n$. Using the inequality from part (b), prove that $g(n) = O(h(n))$. You need to give a rigorous proof derived directly from the definition of the O-notation, without using any theorems from class. (First, give a complete statement of the definition. Next, show how $g(n) = O(h(n))$ follows from this definition.)

Problem 3: Give asymptotic estimates, using the Θ-notation, for the following functions:

(a) $3n^3 - 15n^2 + 2n + 4$
(b) $3n^2 \log n + 2n^2 \sqrt{n} + n^2$
(c) $n \log^3 n - 5n + \dfrac{n^2}{\log n}$
(d) $7 \cdot n^5 + n^3 \log^2 n + 2^n$
(e) $\log^9 n + n^3 4^n + n 5^n$

Academic integrity declaration. The homework papers must include at the end an academic integrity declaration. This should be a brief paragraph where you state *in your own words* (1) whether you did the homework individually or in collaboration with a partner student (if so, provide the name), and (2) whether you used any external help or resources.

Submission. To submit the homework, you need to upload the pdf file to Gradescope and the cpp file on canvas (in Assignments). Before uploading your code to canvas, test it through Codeforces. The score that you received on Codeforces will be your score for the RSA code. If you submit with a partner, you need to put two names on the assignment and submit it as a group assignment.

Reminders. Remember that only LATEX papers are accepted.

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[1] A closed-form expression is a formula that can be evaluated in some fixed number of arithmetic operations, independent of n. For example, $3n^5 + n - 1$ and $n2^n + 5n^2$ are closed-form expressions, but $\sum_{i=1}^{n} i^2$ is not, as it involves $n - 1$ additions.