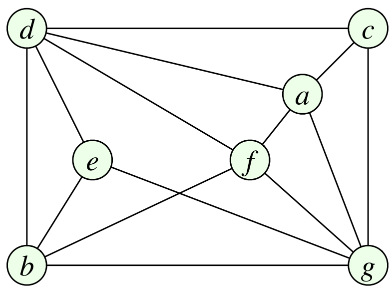
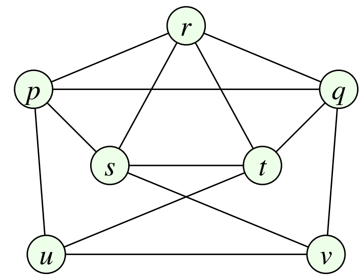
CS111 ASSIGNMENT 5

Problem 1. Determine whether the two graphs below are planar or not. To show planarity, give a planar embedding. To show that a graph is not planar, use Kuratowski's theorem.

Graph H:

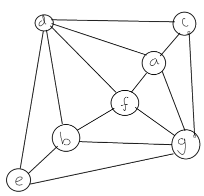


Graph G:

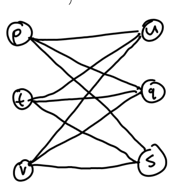


Solution 1:

For graph H, you can make a planar embedding by moving vertex e, therefore, it is planar.



For graph G, we can find a subgraph that is homeomorphic to $k_{3,3}$, therefore, according to Kuratowski's theorem, it is not planar.



Problem 2. (a) For each degree sequence below, determine whether there is a graph with 6 vertices where vertices have these degrees. If a graph exists, (i) draw it, (ii) find the chromatic number and justify. If it doesn't, justify that it doesn't exist.

Note. To give a justification for the chromatic number, you need to give a coloring and explain why it's not possible to use fewer colors.

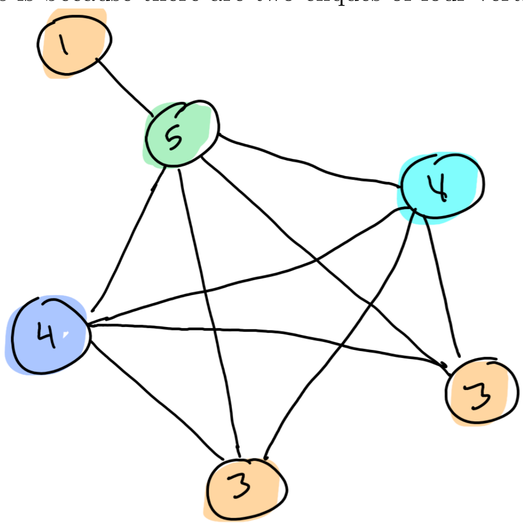
(a1) $5, 4, 4, 3, 3, 1$.
(a2) $5, 4, 3, 2, 2, 1$.
(a3) $4, 4, 4, 3, 3, 2$.

(b) For each degree sequence below, determine whether there is a planar graph with 6 vertices where vertices have these degrees. If a graph exists, (i) draw it, (ii) find the chromatic number and justify. If it doesn't, justify that it doesn't exist.

(b1) $5, 5, 4, 4, 4, 2$.
(b2) $3, 3, 3, 3, 3, 3$.

Solution 2:

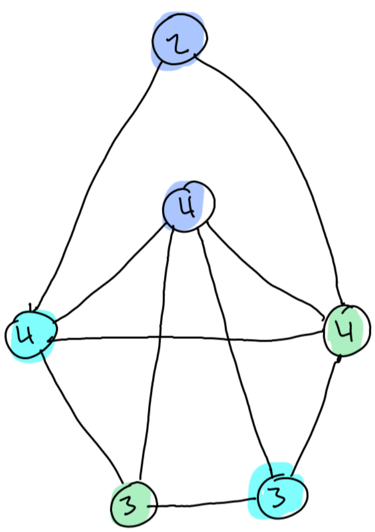
a1) This graph does exist. Its chromatic number is 4. You can color it with 4 colors, but cannot color it with 3. This is because there are two cliques of four vertices: on the picture, it is the vertices labeled 3, 4, 5.



a_1

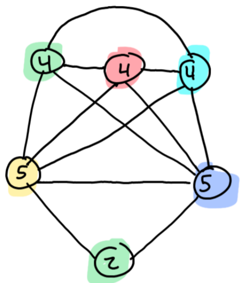
a2) This graph doesn't exist because the sum of degrees is not even. This violates the handshake lemma.

a3) This graph does exist. Its chromatic number is 3. It cannot be colored with 2 colors because there is a clique of 3: the three vertices labeled 4.



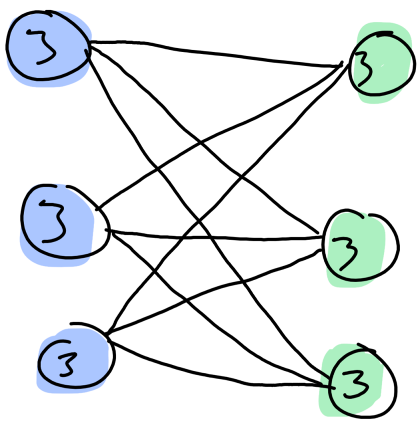
a_3

b1) This graph exists. The chromatic number is 5. As shown below, you can draw it with 5 colors, but it cannot be drawn with 4 because there is a clique of 5: the vertices labeled 4 and 5.



b_1

b2) This graph exists. Its chromatic number is two because it is a bipartite graph. This is because that you can draw one side with one color, and the other side with another color.

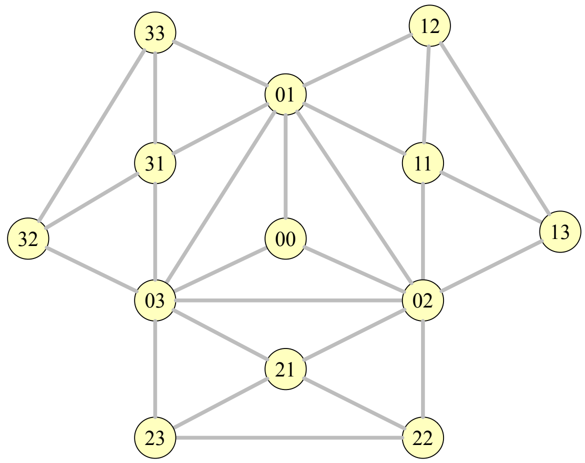


b_2

Problem 3.

a) Does the graph shown below have an Euler tour? Give a complete justification for your answer.

b) Does the graph shown below have a Hamiltonian cycle? Give a complete justification for your answer.



Solution 3:
a) This graph does not have an Euler tour because there are vertices with odd degrees. For example, 33 is degree 3.
b) This graph does not contain a Hamiltonian cycle. Imagine the graph as three rectangles surrounding a triangle. The vertices 01, 02, and 03 act like a funnel because you will have to go through them in order to move from one shape to the other. Since there are four shapes (three rectangles, one triangle), you will have to go through one of the three "funnel" vertices twice in order to get back to where you started. Therefore, this graph does not contain a Hamiltonian cycle.

Academic integrity declaration.