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CS 111 ASSIGNMENT 4

due February 19, 2024

Problem 1: Give an asymptotic estimate, using the Θ-notation, of the number of letters printed by the algorithms given below. Give a complete justification for your answer, by providing an appropriate recurrence equation and its solution.

(a)
```
algorithm PrintAs(n)
    if n ≤ 1 then
        print("A")
    else
        for j ← 1 to n^3
            do print("A")
        for i ← 1 to 5 do
            PrintAs(⌊n/2⌋)
```

(b)
```
algorithm PrintBs(n)
    if n ≥ 4 then
        for j ← 1 to n^2
            do print("B")
        for i ← 1 to 6 do
            PrintBs(⌊n/4⌋)
        for i ← 1 to 10 do
            PrintBs(⌈n/4⌉)
```

(c)
```
algorithm PrintCs(n)
    if n ≤ 2 then
        print("C")
    else
        for j ← 1 to n
            do print("C")
        PrintCs(⌊n/3⌋)
        PrintCs(⌊n/3⌋)
        PrintCs(⌊n/3⌋)
        PrintCs(⌊n/3⌋)
```

(d)
```
algorithm PrintDs(n)
    if n ≥ 5 then
        print("D")
        print("D")
        if (x ≡ 0 (mod 2)) then
            PrintDs(⌊n/5⌋)
            PrintDs(⌈n/5⌉)
            x ← x + 3
        else
            PrintDs(⌈n/5⌉)
            PrintDs(⌊n/5⌋)
            x ← 5x + 3
```

In part (d), variable x is a global variable initialized to 1.

The solutions for all the problems will use the master theorem, so I will just state it here. Let $a \geq 1, b > 1, c > 0$ and $d \geq 0$. If $T(n)$ satisfies the recurrence $T(n) = aT(n/b) + cn^d$.

$$T(x) = \begin{cases} \Theta(n^{log_b(a)}) & a > b^d \\ \Theta(n^d log(a)) & a = b^d \\ \Theta(n^d) & a < b^d \end{cases}$$

Solution 1:
(a) This algorithm can be rewritten as the recurrence: $A(n) = n^3 + 5A(\lfloor n/2 \rfloor)$. Applying the master theorem, with $a = 5, b = 2, c = 1, d = 3$. Since $5 < 2^3 = 8$, $T(n) = \Theta(n^3)$.

(b) This algorithm can be rewritten as the recurrence: $B(n) = n^2 + 6B(\lfloor n/4 \rfloor) + 10B(\lceil n/4 \rceil)$. Since I can't figure out how to justify adding up $6B(\lfloor n/4 \rfloor) + 10B(\lceil n/4 \rceil)$, I will consider each one separately, and add up the Big-Θ of each. Let's say $B'(n)$. TODO

(c) This algorithm can be rewritten as the recurrence: $C(n) = 4C(\lfloor n/3 \rfloor) + n$. Applying the master theorem, with $a = 4, b = 3, c = 1, d = 1 \implies a > b^1 \implies C(n) = \Theta(n)$.

(d)

Problem 2: We have three sets A, B, C with the following properties:

(a) $|B| = 2|A|$, $|C| = 3|A|$,

(b) $|A \cap B| = 18$, $|A \cap C| = 20$, $|B \cap C| = 24$,

(c) $|A \cap B \cap C| = 11$,

(d) $|A \cup B \cup C| = 129$.

Use the inclusion-exclusion principle to determine the number of elements in A. Show your work.

Solution 2: According to the inclusion exclusion principle, we have:
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.
We can substitute the values from (b), (c), (d) into the principle:
$129 = |A| + |B| + |C| - 18 - 20 - 24 + 11 \implies |A| + |B| + |C| = 180$. To solve this, we want to put everything in terms of a single variable. From (a), we know the relationships between the three variables. I will choose to express everything in terms of $|A|$. Plugging this in, we get
$|A| + 2|A| + 3|A| = 180 \implies |A| = 30$.
From (a), we can find the rest of the values: $|B| = 60, |C| = 90$.

Problem 3: A company, Nice Inc., will award 45 fellowships to high-achieving UCR students from four different majors: computer science, biology, political science and history. They decided to give fellowship awards to at least 8 students majoring in computer science and at most 8 biology majors. The number of political science and history majors should be between 5 and 12 students each. How many possible lists of awardees are there? You need to give a complete derivation for the final answer, using the method developed in class. (Brute force listing of all lists will not be accepted.)

Solution 3:
Let's say that the number of Computer Science awards is C, biology is B, and political science and history is P. We know that there are a total of 45 students, so $C + B + P = 45$. There are at least 8 CS awards, so $C \geq 8$, and there will be at most 8 biology majors, so $B \leq 8$. The number of political science students will be between 5 and 12, so $5 \leq P \leq 12 \implies P' \leq 7$. We can condense these conditions into $C \geq 8 \wedge B \leq 8 \wedge P' \leq 7$. Let the number of possible number of lists of awardees be S($C \geq 8 \wedge B \leq 8 \wedge P' \leq 7$), and S() be the number of lists with no restrictions. To find the number of valid lists, we can take the number of lists with no restrictions and subtract the number of invalid lists: $S() - S(C \leq 7 \vee B \geq 9 \vee P' \geq 8)$. According to the theorem regarding integer partitions, we know that $S() = \binom{45+3-1}{3-1} = 1081$. To break up the second term, we have to use the inclusion exclusion principle:
$S(C \leq 7 \vee B \geq 9 \vee P' \geq 8) =$
$S(C \leq 7) + S(B \geq 9) + S(P' \geq 8) - S(C \leq 7 \wedge B \geq 9) - S(C \leq 7 \wedge P' \geq 8)$
$- S(P' \geq 8 \wedge B \geq 9) + S(C \leq 7 \wedge B \geq 9 \wedge P' \geq 8)$

Academic integrity declaration. The homework papers must include at the end an academic integrity declaration. This should be a short paragraph where you briefly explain *in your own words* (1) whether you did the homework individually or in collaboration with a partner student (if so, provide the name), and (2) whether you used any external help or resources.