CS111 ASSIGNMENT 4

Problem 1: Give an asymptotic estimate, using the Θ-notation, of the number of letters printed by the algorithms given below. Give a complete justification for your answer, by providing an appropriate recurrence equation and its solution.

(a)
```
algorithm PrintAs(n)
    if n ≤ 1 then
        print("A")
    else
        for j ← 1 to n^3
            do print("A")
        for i ← 1 to 5 do
            PrintAs( ⌊n/2⌋ )
```

(b)
```
algorithm PrintBs(n)
    if n ≥ 4 then
        for j ← 1 to n^2
            do print("B")
        for i ← 1 to 6 do
            PrintBs( ⌊n/4⌋ )
        for i ← 1 to 10 do
            PrintBs( ⌈n/4⌉ )
```

(c)
```
algorithm PrintCs(n)
    if n ≤ 2 then
        print("C")
    else
        for j ← 1 to n
            do print("C")
        PrintCs( ⌊n/3⌋ )
        PrintCs( ⌊n/3⌋ )
        PrintCs( ⌊n/3⌋ )
        PrintCs( ⌊n/3⌋ )
```

(d)
```
algorithm PrintDs(n)
    if n ≥ 5 then
        print("D")
        print("D")
      if (x ≡ 0 (mod 2)) then
            PrintDs( ⌊n/5⌋ )
            PrintDs( ⌈n/5⌉ )
            x ←  x + 3
        else
            PrintDs( ⌈n/5⌉ )
            PrintDs( ⌊n/5⌋ )
            x ← 5x + 3
```

In part (d), variable x is a global variable initialized to 1.

Problem 2: We have three sets A, B, C with the following properties:

(a) $|B| = 2|A|$, $|C| = 3|A|$,

(b) $|A \cap B| = 18$, $|A \cap C| = 20$, $|B \cap C| = 24$,

(c) $|A \cap B \cap C| = 11$,

(d) $|A \cup B \cup C| = 129$.

Use the inclusion-exclusion principle to determine the number of elements in A. Show your work.

Problem 3: A company, Nice Inc., will award 45 fellowships to high-achieving UCR students from four different majors: computer science, biology, political science and history. They decided to give fellowship awards to at least 8 students majoring in computer science and at most 8 biology majors. The number of political science and history majors should be between 5 and 12 students each. How many possible lists of awardees are there? You need to give a complete derivation for the final answer, using the method developed in class. (Brute force listing of all lists will not be accepted.)

Academic integrity declaration. The homework papers must include at the end an academic integrity declaration. This should be a short paragraph where you briefly explain *in your own words* (1) whether you did the homework individually or in collaboration with a partner student (if so, provide the name), and (2) whether you used any external help or resources.

Submission. To submit the homework, you need to upload the pdf file to Gradescope. If you submit with a partner, you need to put two names on the assignment and submit it as a group assignment. Remember that only LaTeX papers are accepted.