Electrodynamics hw1

This is a homework file so it contains some of the calculations for my electrodynamics hw.
The aim for this problem is to calculate the magnetic field of a helix with radius a, parametric variable α, pitch angle β.

Problem 1e + 1f + 1g

In[∘]:=
```
(*defining the path and path integral*)
rp = {Cos[α], Sin[α], α Tan[β]};
dr = {-Sin[α], Cos[α], Tan[β]};
r = {0, 0, 0};
(*manually defining the norm as mathematica needlessly included
 abs functions which is messing with the integration step*)
norm3 = Sqrt[(r[[1]] - rp[[1]])^2 + (r[[2]] - rp[[2]])^2 + (r[[3]] - rp[[3]])^2]^3;
```

Our integral is dr' x r-r' over |r-r'|^3 so...

In[∘]:=
```
integrand = FullSimplify[Cross[dr, r - rp] / FullSimplify[norm3]]
```

Out[∘]=
$$\left\{\frac{(-\alpha\,\mathrm{Cos}[\alpha]+\mathrm{Sin}[\alpha])\,\mathrm{Tan}[\beta]}{\left(1-\alpha^2+\alpha^2\,\mathrm{Sec}[\beta]^2\right)^{3/2}},\ -\frac{(\mathrm{Cos}[\alpha]+\alpha\,\mathrm{Sin}[\alpha])\,\mathrm{Tan}[\beta]}{\left(1-\alpha^2+\alpha^2\,\mathrm{Sec}[\beta]^2\right)^{3/2}},\ \frac{1}{\left(1-\alpha^2+\alpha^2\,\mathrm{Sec}[\beta]^2\right)^{3/2}}\right\}$$

Now hopefully we can just hit this with the α integral, but we will see if that actually works..

In[∘]:=
```
res = Integrate[integrand, {α, -∞, ∞}]
```

Out[∘]=
$$\left\{0,\ \boxed{-2\,\mathrm{Cot}[\beta]\left(\mathrm{BesselK}\left[1,\frac{1}{\sqrt{\mathrm{Tan}[\beta]^2}}\right]+\frac{\mathrm{BesselK}\left[0,\frac{1}{\sqrt{\mathrm{Tan}[\beta]^2}}\right]}{\sqrt{\mathrm{Tan}[\beta]^2}}\right)\quad \text{if}\ \mathrm{Re}\left[\mathrm{Sec}[\beta]^2\right]\geq 1\ \|\ \mathrm{Sec}[\beta]^2\notin\mathbb{R}},\ \boxed{\frac{2}{\sqrt{\mathrm{Tan}[\beta]^2}}\quad \text{if}\ \mathrm{Re}\left[\mathrm{Sec}[\beta]^2\right]\geq 1\ \|\ \mathrm{Sec}[\beta]^2\notin\mathbb{R}}\right\}$$

It doesn't but apparently it doesn't need to, I just need to write the alpha integrals down for this part, not actually solve them.

The next part however does have us solve for the origin so let's see if we can actually do that (the integral above completes that that step).

Now we need to confirm our results with the numerical results given for a helix of 1000 turns and pitch angle β=0.7 radians.

The integral we just solved gives us components of the B field so now we note this is equivalent to $B_i\, 4\,\pi\, \frac{Ya}{I}$ but we can divide res by 4π to get it scaled just by constants of the system.

In[◦]:=
```
fin = res/(4 π) /. β → 0.7
```

Out[◦]=
```
{0, -0.156489, 0.188955}
```

These are exactly as anticipated.

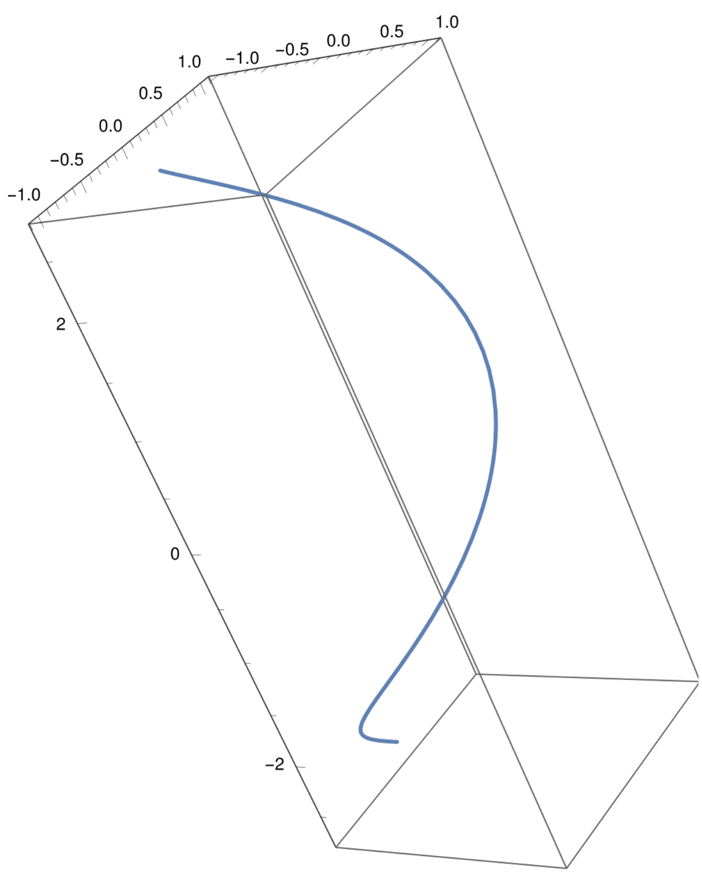
Problem 1h

In order to get the B field at any z value, we can simply consider rotations of the helix as it is infinitely long and clearly symmetric in this regard. Thus, we should be able to hit the helix with a rotation matrix in order to .

First I'm gonna plot the helix.

In[◦]:=
```
ParametricPlot3D[a {Cos[α], Sin[α], α Tan[β]} /. β → 0.7 /. a → 1, {α, -3, 3}]
```

Out[◦]=



Great, now we get to consider a rotation. We want to rotate about the Z axis here.

In[∘]:=
```
RotationMatrix[θ, {0, 0, 1}] // MatrixForm
```

Out[∘]//MatrixForm=

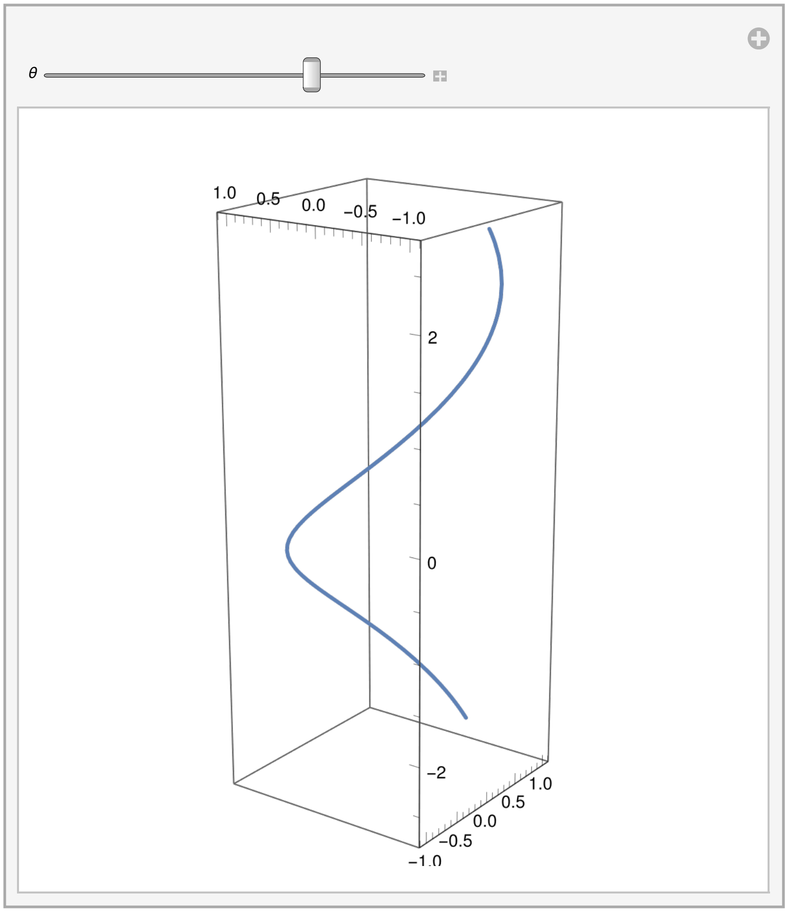
$$\begin{pmatrix} \text{Cos}[\theta] & -\text{Sin}[\theta] & 0 \\ \text{Sin}[\theta] & \text{Cos}[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[∘]:=
```
rothelix[θ_] = RotationMatrix[θ, {0, 0, 1}].rp
```

Out[∘]=
```
{Cos[α] Cos[θ] - Sin[α] Sin[θ], Cos[θ] Sin[α] + Cos[α] Sin[θ], α Tan[β]}
```

In[∘]:=
```
Manipulate[ParametricPlot3D[rothelix[θ] /. β → 0.7, {α, -3, 3}], {θ, -2 π, 2 π}]
```

Out[∘]=



In order to convert to z we consider that moving up a single turn spacing in z is equivalent to a rotation of θ=2π.

Now, in order to convert from θ to a specific z value...
We know that the turn spacing is 2πaTan(β).We want θ=0 to correlate to z=0 so consider the relationship z=θ/2πaTan(β).

At θ=0 z=0, at θ=2π we get that z=1/πaTan(β)...Does this make sense?

In[∘]:=
```
z[θ_] := θ Tan[β]
```

In[∘]:=
```
z[0] /. β → 0.7
z[π] /. β → 0.7
```

Out[∘]= 0

Out[∘]= 2.64613

Does inverting this relationship make sense?

In[∘]:=
```
θ[z_] := z/Tan[β]
θ[0]
θ[.25] /. β → 0.7
```

Out[∘]= 0

Out[∘]= 0.29681

After playing with this relationship (and the parametric plots above) I am convinced it is the right relationship, mostly by playing around with the plots. I could also try a direct z-shift on the parametric function though to compare, but this is sufficient.

Now we could write the whole integral out after substituting in for the θ function at every point in the rotation matrix.

Alternatively we could just consider a rotation of the fields as an equivalent process, so we can simply hit the B field with the rotation matrix on z. Then our new integral is simply the old integral times the rotation matrix where here the rotation angle θ must in fact be α unless we only want to consider a discrete rotation for a specific value of z. Previously we simple called the rotation angle θ for simplicity of plotting it in the parametricplot3d function as if we had tried to plot α and manipulate α it wouldn't have displayed anything.

In[∘]:=
```
newintegrand = FullSimplify[RotationMatrix[α, {0, 0, 1}].integrand /. α → θ[z]]
```

Out[∘]=

$$\left\{\frac{-z\,\mathrm{Cos}[2\,z\,\mathrm{Cot}[\beta]]+\mathrm{Sin}[2\,z\,\mathrm{Cot}[\beta]]\,\mathrm{Tan}[\beta]}{(1+z^2)^{3/2}},\ \frac{\left(-\mathrm{Cos}[z\,\mathrm{Cot}[\beta]]^2+\mathrm{Sin}[z\,\mathrm{Cot}[\beta]]^2-z\,\mathrm{Cot}[\beta]\,\mathrm{Sin}[2\,z\,\mathrm{Cot}[\beta]]\right)\mathrm{Tan}[\beta]}{(1+z^2)^{3/2}},\ \frac{1}{(1+z^2)^{3/2}}\right\}$$

One would hope that if we take z=0 the previous result of the B field at x,y,z=0 would return...

In[∘]:=
```
newres = Integrate[newintegrand, {z, -∞, ∞}]
```

Out[∘]=

$$\left\{0 \text{ if } \mathrm{Cot}[\beta]\in\mathbb{R},\ -4\,\mathrm{BesselK}[0,\,2\,\mathrm{Abs}[\mathrm{Cot}[\beta]]]\,\mathrm{Cot}[\beta]-4\,\mathrm{Abs}[\mathrm{Cot}[\beta]]\,\mathrm{BesselK}[1,\,2\,\mathrm{Abs}[\mathrm{Cot}[\beta]]]\,\mathrm{Tan}[\beta] \text{ if } \mathrm{Cot}[\beta]\in\mathbb{R},\ 2\right\}$$

In[∘]:=
```
newfin = newres /. β → 0.7
```

Out[∘]=
```
{0, -0.689584, 2}
```

So this doesn't quite work out and I haven't figured out why. If I have extra time I will come back and see if I made an error somewhere in the computation
- i did not have time :(

Problem 2a

Here we are given a problem relating to a charge moving with a constant x velocity. Part a asks for the charge density and current density which we define as follows

In[∘]:=
```
ρ = q DiracDelta[x - vt] DiracDelta[y] DiracDelta[z];
j = q v DiracDelta[xp - v tp] DiracDelta[yp] DiracDelta[zp];
distvec = Sqrt[(x - xp)^2 + (y - yp)^2 + (z - zp)^2];
```

Where here i've used xp to denote the primed coordinates. We now want to find the vector potential so we consider the integral that follows...

In[∘]:=
$$\text{vecpot} = \frac{1}{4\,\pi\,c}\,\text{Integrate}\left[j\,\frac{\text{DiracDelta}\left[tp - t + c^{-1}\,\text{distvec}\right]}{\text{distvec}},\ xp,\ yp,\ zp,\ tp\right]$$

Out[∘]=
$$\frac{\int\int\int\int \frac{q\,v\,\text{DiracDelta}\left[-tp\,v+xp\right]\,\text{DiracDelta}[yp]\,\text{DiracDelta}\left[-t+tp+\frac{\sqrt{(x-xp)^2+(y-yp)^2+(z-zp)^2}}{c}\right]\,\text{DiracDelta}[zp]}{\sqrt{(x-xp)^2+(y-yp)^2+(z-zp)^2}}\,d tp\,d zp\,d yp\,d xp}{4\,c\,\pi}$$

Seems like mathematica doesn't like doing these so much. I will do the deltas on paper.

In the process of solving, you use a delta identity (equation 9 of Schmidt's notes) which leaves you needing to solve the following.

In[∘]:=
```
Clear[tp, tpn, t]
```

```
In[•]:= (*distvecp is the distance vector after
         getting hit with the spatial dirac deltas*)
```

$$\text{distvecp} = \text{Sqrt}\left[(x - v\,tp)^2 + y^2 + z^2\right];$$

$$\text{tempsol} = \text{Solve}\left[tp - t + c^{-1}\,\text{distvecp} == 0,\ tp\right]$$

Solve: There may be values of the parameters for which some or all solutions are not valid.

$$\text{Out[•]=}\ \left\{\left\{tp \to \frac{2\,c^2\,t - 2\,v\,x - \sqrt{\left(-2\,c^2\,t + 2\,v\,x\right)^2 - 4\,(c^2 - v^2)\,\left(c^2\,t^2 - x^2 - y^2 - z^2\right)}}{2\,(c^2 - v^2)}\right\},\right.$$
$$\left.\left\{tp \to \frac{2\,c^2\,t - 2\,v\,x + \sqrt{\left(-2\,c^2\,t + 2\,v\,x\right)^2 - 4\,(c^2 - v^2)\,\left(c^2\,t^2 - x^2 - y^2 - z^2\right)}}{2\,(c^2 - v^2)}\right\}\right\}$$

We note, however, that there are two solutions to this equation! Utilization of eqn9 in Schmidt's notes requires us to sum over all the possible roots in the end so we will keep this in mind...

Take the derivative first...

(coming back from the future here, this doesn't go well, philip feel free to skip this bit, it's almost certainly wrong)

$$\text{In[•]:= help} = \text{Simplify}\left[\left(\frac{1}{\text{distvecp}}\left(D\left[tp - t + c^{-1}\,\text{distvecp},\ tp\right]\right)^{-1}\right) /.\right.$$
$$tp \to \frac{2\,c^2\,t - 2\,v\,x - \sqrt{\left(-2\,c^2\,t + 2\,v\,x\right)^2 - 4\,(c^2 - v^2)\,\left(c^2\,t^2 - x^2 - y^2 - z^2\right)}}{2\,(c^2 - v^2)} +$$
$$\left(\frac{1}{\text{distvecp}}\left(D\left[tp - t + c^{-1}\,\text{distvec},\ tp\right]\right)^{-1}\right) /.$$
$$\left. tp \to \frac{2\,c^2\,t - 2\,v\,x + \sqrt{\left(-2\,c^2\,t + 2\,v\,x\right)^2 - 4\,(c^2 - v^2)\,\left(c^2\,t^2 - x^2 - y^2 - z^2\right)}}{2\,(c^2 - v^2)}\right]$$

Out[ ]=

$$1\Bigg/\left(\sqrt{\left(y^2+z^2+\left(x+\frac{v\left(-c^2\,t+v\,x+\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)}{c^2-v^2}-\frac{v}{\sqrt{y^2+z^2+\frac{\left(c^2\left(t\,v-x\right)+v\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}}\right)^2\right)}\right.$$

$$\left(1-\left(v\left(x+\frac{v\left(-c^2\,t+v\,x+\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)}{c^2-v^2}-\frac{v}{\sqrt{y^2+z^2+\frac{\left(c^2\left(t\,v-x\right)+v\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}}\right)\right)\Bigg/\right.$$

$$\left.\left.\left(c\,\sqrt{\left(y^2+z^2+\left(x+\frac{v\left(-c^2\,t+v\,x+\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)}{c^2-v^2}-\frac{v}{\sqrt{y^2+z^2+\frac{\left(c^2\left(t\,v-x\right)+v\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}}\right)^2\right)}\right)\right)\right)$$

In[ ]:= **Simplify[D[help, x]]**

Out[ ]=

$$-\left(\left(c\left(2\,c^2\,v\left(t\,v^2-v\,x+\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)\right.\right.\right.$$

$$\left(c^2\left(t\,v-x\right)+v\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)-$$

$$2\left(c^2-v^2\right)^2\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}$$

$$\left(y^2+z^2+\frac{\left(c^2\left(t\ v-x\right)+v\ \sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}\right)^{3/2}+$$

$$v\left(c^2-v^2\right)\left(2\ c^2\left(t\ v-x\right)-2\ v\ \sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)$$

$$\left(y^2+z^2+\frac{\left(c^2\left(t\ v-x\right)+v\ \sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}\right)^{3/2}\Bigg)\Bigg(-v$$

$$\Bigg(-c^2\ v+v^3+\left(c^2-v^2\right)x$$

$$\sqrt{y^2+z^2+\frac{\left(c^2\left(t\ v-x\right)+v\ \sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}+$$

$$v\left(-c^2\ t+v\ x+\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)$$

$$\sqrt{y^2+z^2+\frac{\left(c^2\left(t\ v-x\right)+v\ \sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}\Bigg)^2+$$

$$v\left(c^2-v^2\right)^2\left(y^2+z^2+\frac{\left(c^2\left(t\ v-x\right)+v\ \sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}\right)$$

$$\Bigg(y^2+z^2+\Bigg(x+\frac{v\left(-c^2\ t+v\ x+\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)}{c^2-v^2}-$$

$$\frac{v}{\sqrt{y^2+z^2+\frac{\left(c^2\left(t\ v-x\right)+v\ \sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}}\Bigg)^2\Bigg)+\Bigg(v\left(c^2-v^2\right)-$$

$$\left(c^2-v^2\right)x\ \sqrt{y^2+z^2+\frac{\left(c^2\left(t\ v-x\right)+v\ \sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}-$$

$$v\left(-c^2\ t+v\ x+\sqrt{-v^2\ (y^2+z^2)+c^2\ (t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2)}\right)$$

$$\sqrt{y^2+z^2+\frac{\left(c^2\ (t\ v-x)+v\ \sqrt{-v^2\ (y^2+z^2)+c^2\ (t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2)}\right)^2}{(c^2-v^2)^2}}\ \Bigg)\Bigg(v$$

$$\Bigg(v\ (c^2-v^2)-(c^2-v^2)\ x$$

$$\sqrt{y^2+z^2+\frac{\left(c^2\ (t\ v-x)+v\ \sqrt{-v^2\ (y^2+z^2)+c^2\ (t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2)}\right)^2}{(c^2-v^2)^2}}\ -$$

$$v\left(-c^2\ t+v\ x+\sqrt{-v^2\ (y^2+z^2)+c^2\ (t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2)}\right)$$

$$\sqrt{y^2+z^2+\frac{\left(c^2\ (t\ v-x)+v\ \sqrt{-v^2\ (y^2+z^2)+c^2\ (t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2)}\right)^2}{(c^2-v^2)^2}}\ \Bigg)+$$

$$c\ (c^2-v^2)\ \sqrt{y^2+z^2+\frac{\left(c^2\ (t\ v-x)+v\ \sqrt{-v^2\ (y^2+z^2)+c^2\ (t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2)}\right)^2}{(c^2-v^2)^2}}$$

$$\sqrt{\Bigg(y^2+z^2+\Bigg(x+\frac{v\left(-c^2\ t+v\ x+\sqrt{-v^2\ (y^2+z^2)+c^2\ (t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2)}\right)}{c^2-v^2}\ -}$$

$$\frac{v}{\sqrt{y^2+z^2+\frac{\left(c^2\ (t\ v-x)+v\ \sqrt{-v^2\ (y^2+z^2)+c^2\ (t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2)}\right)^2}{(c^2-v^2)^2}}}\Bigg)^2\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Big/$$

$$\Bigg(2\ (c^2-v^2)^2\ \sqrt{-v^2\ (y^2+z^2)+c^2\ (t^2\ v^2-2\ t\ v\ x+x^2+y^2+z^2)}$$

$$\left(y^2+z^2+\frac{\left(c^2\left(t\,v-x\right)+v\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}\right)^{3/2}$$

$$\left(y^2+z^2+\right.$$

$$\left(x+\frac{v\left(-c^2\,t+v\,x+\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)}{c^2-v^2}-\right.$$

$$\left.\left.\left.\frac{v}{\sqrt{y^2+z^2+\frac{\left(c^2\left(t\,v-x\right)+v\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}}\right)^2\right)\right)\left(v\left(v\left(c^2-v^2\right)-\left(c^2-v^2\right)x\right.\right.$$

$$\sqrt{y^2+z^2+\frac{\left(c^2\left(t\,v-x\right)+v\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}\;-$$

$$v\left(-c^2\,t+v\,x+\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)$$

$$\left.\sqrt{y^2+z^2+\frac{\left(c^2\left(t\,v-x\right)+v\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}\right)+$$

$$c\left(c^2-v^2\right)\sqrt{y^2+z^2+\frac{\left(c^2\left(t\,v-x\right)+v\,\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)^2}{\left(c^2-v^2\right)^2}}$$

$$\sqrt{\left(y^2+z^2+\left(x+\frac{v\left(-c^2\,t+v\,x+\sqrt{-v^2\left(y^2+z^2\right)+c^2\left(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2\right)}\right)}{c^2-v^2}-\right.\right.}$$

$$\left.\left.\left.\frac{v}{\sqrt{y^2+z^2+\frac{\left(c^2\,(t\,v-x)+v\,\sqrt{-v^2\,(y^2+z^2)+c^2\,(t^2\,v^2-2\,t\,v\,x+x^2+y^2+z^2)}\right)^2}{(c^2-v^2)^2}}}\right)^2\right)^2\right)\right)$$

god help me :((((

this better simplify so help me

I went further, it doesn't, I tried doing it on a chalkboard and also couldn't get it to simplify even when substituting gamma at various points to reduce the radicals.

I will use what I have here to calculate the E and B fields, but I will be assuming everything from here on incorrect.

Scratchwork for problem 3 and 5a

In[24]:= $\texttt{intsol = Integrate}\left[\frac{a}{(a^2+(z-zp)^2)^{3/2}},\ \{zp,\ -l/2,\ l/2\}\right]$

Out[24]= $a\left(\frac{l-2\,z}{a^2\,\sqrt{4\,a^2+(l-2\,z)^2}}-\frac{-l-2\,z}{a^2\,\sqrt{4\,a^2+(l+2\,z)^2}}\right)$ if condition

In[25]:= `TrigReduce[intsol]`

Out[25]= $\left(l\,\sqrt{4\,a^2+l^2-4\,l\,z+4\,z^2}+2\,z\,\sqrt{4\,a^2+l^2-4\,l\,z+4\,z^2}+l\,\sqrt{4\,a^2+l^2+4\,l\,z+4\,z^2}-2\,z\,\sqrt{4\,a^2+l^2+4\,l\,z+4\,z^2}\right)\Big/\left(a\,\sqrt{4\,a^2+l^2-4\,l\,z+4\,z^2}\,\sqrt{4\,a^2+l^2+4\,l\,z+4\,z^2}\right)$ if condition

In[26]:= $z1 = l/2-\frac{a}{\mathrm{Tan}[\theta 2]};$

$z2 = \frac{a}{\mathrm{Tan}[\theta 1]}-l/2;$

In[28]:= $f = \frac{l-2\,z1}{a^2\,\sqrt{4\,a^2+(l-2\,z1)^2}}-\frac{-l-2\,z2}{a^2\,\sqrt{4\,a^2+(l+2\,z2)^2}}$

Out[28]= $-\frac{-l-2\left(-\frac{l}{2}+a\,\mathrm{Cot}[\theta 1]\right)}{a^2\,\sqrt{4\,a^2+\left(l+2\left(-\frac{l}{2}+a\,\mathrm{Cot}[\theta 1]\right)\right)^2}}+\frac{l-2\left(\frac{l}{2}-a\,\mathrm{Cot}[\theta 2]\right)}{a^2\,\sqrt{4\,a^2+\left(l-2\left(\frac{l}{2}-a\,\mathrm{Cot}[\theta 2]\right)\right)^2}}$

In[29]:= `FullSimplify[f]`

Out[29]=
$$\frac{\text{Cos}[\theta 1]\sqrt{a^2\,\text{Csc}[\theta 1]^2}\,\text{Sin}[\theta 1]+\text{Cos}[\theta 2]\sqrt{a^2\,\text{Csc}[\theta 2]^2}\,\text{Sin}[\theta 2]}{a^3}$$

In[•]:= `TrigReduce[%]`

Out[•]=
$$\frac{\sqrt{a^2\,\text{Csc}[\theta 1]^2}\,\text{Sin}[2\,\theta 1]+\sqrt{a^2\,\text{Csc}[\theta 2]^2}\,\text{Sin}[2\,\theta 2]}{2\,a^3}$$

In[•]:= `FullSimplify[Expand[%]]`

Out[•]=
$$\frac{l}{a\sqrt{4\,a^2+(l-2\,z)^2}}-\frac{2\,z}{a\sqrt{4\,a^2+(l-2\,z)^2}}+\frac{l}{a\sqrt{4\,a^2+(l+2\,z)^2}}+\frac{2\,z}{a\sqrt{4\,a^2+(l+2\,z)^2}}\ \text{if condition}$$

In[•]:= `FullSimplify[D[intsol, z]]`

Out[•]=
$$8\,a\left(-\frac{1}{\left(4\,a^2+(l-2\,z)^2\right)^{3/2}}+\frac{1}{\left(4\,a^2+(l+2\,z)^2\right)^{3/2}}\right)\ \text{if condition}$$

In[•]:= `FullSimplify[D[intsol, z, z]]`

Out[•]=
$$48\,a\left(-\frac{l}{\left(4\,a^2+(l-2\,z)^2\right)^{5/2}}+\frac{2\,z}{\left(4\,a^2+(l-2\,z)^2\right)^{5/2}}-\frac{l}{\left(4\,a^2+(l+2\,z)^2\right)^{5/2}}-\frac{2\,z}{\left(4\,a^2+(l+2\,z)^2\right)^{5/2}}\right)\ \text{if condition}$$

In[•]:= `FullSimplify[D[intsol, z, z, z]]`

Out[•]=
$$\frac{24}{a\left(4\,a^2+(l-2\,z)^2\right)^{3/2}}-\frac{144\,(l-2\,z)^2}{a\left(4\,a^2+(l-2\,z)^2\right)^{5/2}}+\frac{120\,(l-2\,z)^4}{a\left(4\,a^2+(l-2\,z)^2\right)^{7/2}}-\frac{120\,(l+2\,z)^4}{a\left(4\,a^2+(l+2\,z)^2\right)^{7/2}}+\frac{144\,(l+2\,z)^2}{a\left(4\,a^2+(l+2\,z)^2\right)^{5/2}}-\frac{24}{a\left(4\,a^2+(l+2\,z)^2\right)^{3/2}}\ \text{if condition}$$

In[•]:= $\texttt{intsolifn = Integrate}\left[\frac{a}{\left(a^2+(z-zp)^2\right)^{3/2}},\ \{zp,\,-\infty,\,\infty\}\right]$

Out[•]=
$$\frac{2}{a}\ \text{if condition}$$