This document is for hw1 problem 5

The problem is asking us to consider the B field expansion around a point near the axis of a solenoid. We consider the result from problems 3 and 4 for this question.

The first part of the problem simply asks for the B field near the center of a long solenoid, which we can give by using the power series derived in question 4 which goes as follows.

Part a

In[1]:=

$$\text{bz[bz0\_]} := \text{bz0} - \frac{\rho^2}{4}\,\text{D[bz0, z, z]};$$

$$\text{b}\rho\text{[bz0\_]} := \frac{-\rho}{2}\,\text{D[bz0, z]} + \frac{\rho^3}{16}\,\text{D[bz0, z, z, z]};$$

Which are the leading terms in the series expansion. We can then identify bz0 as the field derived in part 3 and define a new function for it in terms of z (as ρ here is assumed zero).

In[3]:=

$$\text{bz0} = a\,n\,i\left(\frac{l-2z}{a^2\sqrt{4a^2+(l-2z)^2}} - \frac{-l-2z}{a^2\sqrt{4a^2+(l+2z)^2}}\right);$$

Where here it is assumed that z << l/2, the length scale of the solenoid...

Now we should be able to substitute in the appropriate bz0.

In[10]:=

```
bzres = FullSimplify[bz[bz0]]
```

Out[10]=

$$a\,i\,n\left(\frac{l-2z}{a^2\sqrt{4a^2+(l-2z)^2}} + \frac{l+2z}{a^2\sqrt{4a^2+(l+2z)^2}}\right) + 12\,a\,i\,n\left(2z\left(-\frac{1}{\left(4a^2+(l-2z)^2\right)^{5/2}} + \frac{1}{\left(4a^2+(l+2z)^2\right)^{5/2}}\right) + l\left(\frac{1}{\left(4a^2+(l-2z)^2\right)^{5/2}} + \frac{1}{\left(4a^2+(l+2z)^2\right)^{5/2}}\right)\right)\rho^2$$

In[9]:= `bρres = FullSimplify[bρ[bz0]]`

Out[9]=
$$4\,a\,i\,n\left(\frac{1}{\left(4\,a^2+(l-2\,z)^2\right)^{3/2}}-\frac{1}{\left(4\,a^2+(l+2\,z)^2\right)^{3/2}}\right)\rho+$$
$$24\,a\,i\,n\left(-\frac{1}{\left(4\,a^2+(l-2\,z)^2\right)^{5/2}}+\frac{1}{\left(4\,a^2+(l+2\,z)^2\right)^{5/2}}+5\,a^2\left(\frac{1}{\left(4\,a^2+(l-2\,z)^2\right)^{7/2}}-\frac{1}{\left(4\,a^2+(l+2\,z)^2\right)^{7/2}}\right)\right)\rho^3$$

We could also take this in the limit that we are at the center or z=0, or more interestingly as ρ approaches a...

In[12]:= `FullSimplify[bρres /. ρ → a]`

Out[12]=
$$4\,a^2\,i\,n\left(\frac{1}{\left(4\,a^2+(l-2\,z)^2\right)^{3/2}}-\frac{1}{\left(4\,a^2+(l+2\,z)^2\right)^{3/2}}\right)+$$
$$24\,a^4\,i\,n\left(-\frac{1}{\left(4\,a^2+(l-2\,z)^2\right)^{5/2}}+\frac{1}{\left(4\,a^2+(l+2\,z)^2\right)^{5/2}}+5\,a^2\left(\frac{1}{\left(4\,a^2+(l-2\,z)^2\right)^{7/2}}-\frac{1}{\left(4\,a^2+(l+2\,z)^2\right)^{7/2}}\right)\right)$$

In[13]:= `FullSimplify[bzres /. ρ → a]`

Out[13]=
$$a\,i\,n\left(\frac{l-2\,z}{a^2\sqrt{4\,a^2+(l-2\,z)^2}}+\frac{l+2\,z}{a^2\sqrt{4\,a^2+(l+2\,z)^2}}\right)+12\,a^3\,i\,n$$
$$\left(2\,z\left(-\frac{1}{\left(4\,a^2+(l-2\,z)^2\right)^{5/2}}+\frac{1}{\left(4\,a^2+(l+2\,z)^2\right)^{5/2}}\right)+l\left(\frac{1}{\left(4\,a^2+(l-2\,z)^2\right)^{5/2}}+\frac{1}{\left(4\,a^2+(l+2\,z)^2\right)^{5/2}}\right)\right)$$

Part b

Part c

We want to show that the ends of the solenoids we get specific B fields so let’s just calculate them using the original angles given in 5.3...
We know at one end we will find that one of the theta angles vanishes since it will be 90 degrees

In[18]:= `Clear[bz, bz0, bρ]`

$$\mathbf{bz} = \mu_0\,\mathbf{n}\,\frac{\mathbf{i}}{\mathbf{2}}\,\mathbf{Cos[\theta 1]}$$

Out[19]=
$$\frac{1}{2}\,i\,n\,\mathrm{Cos}[\theta 1]\,\mu_0$$

However, since we consider the solenoid to be very long, then the angle is small and thus can be written as it's taylor expansion...

In[21]:=
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Series[Cos[θ1], {θ1, 0, 1}]
```

Out[21]= $1 + O[\theta 1]^2$

So clearly in one case we get bz = $\mu_0\, n\, \frac{i}{2}$

Getting the bρ result is generally more complicated though, we know the divergence of B is zero but also that the divergence of B is given be

$$\text{div[B]} = \frac{1}{\rho}\,\frac{d}{\text{d}\rho}\,(\text{b}\rho * \rho) + \frac{d}{\text{dz}}\,\text{bz} = 0$$

Thus we find that bρ = $+/-\ \mu_0\, n\, \frac{i}{4}\, \frac{\rho}{a}$ which we get just by solving the above.