Homework 2 Problem 3

In this problem we focus on trying to find the self capacitance of a prolate and oblate spheroid. Phillip I will not lie to you, I do not know how to do this and I kinda want to focus on other things in my life so what I did was I found a compendium of self capacitances in the "Formulas for Computing Capacitance and Inductance" by the United States Department of Commerce (https://nvlpubs.nist.gov/nistpubs/Legacy/circ/nbscircular544.pdf). I used the equations here (1.0-1.20) to make the plots you'll see here.

Part a

We start with the capacitance of an oblate spheroid. Equation 1.17 in the cited compendium yields

$$\frac{\sqrt{a^2 - b^2}}{\sin^{-1}\left(\frac{\sqrt{a^2-b^2}}{a}\right)}$$

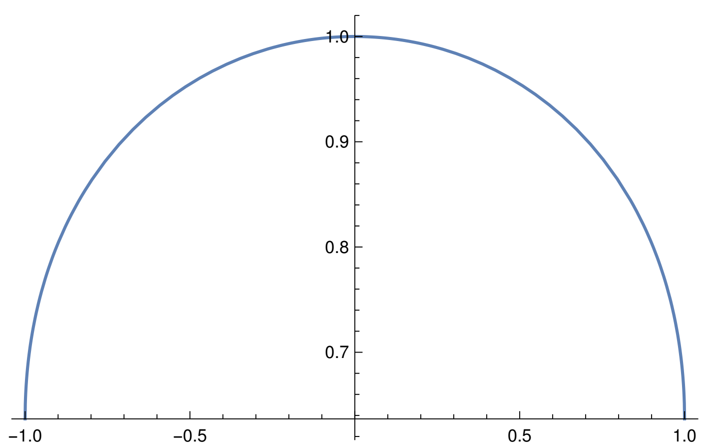
We know that we want to plot as a function of eccentricity which is $e = \sqrt{1 - \frac{b^2}{a^2}}$ so we can factor that out of the above expression and that will give us...

$$\frac{e}{\sin^{-1}(e)}$$

So we can plot this pretty quickly just from -1 to 1 because eccentricity shouldn't be larger.

In[24]:=
```
Plot[e/ArcSin[e], {e, -1, 1}]
```

Out[24]=



We can do the same thing using equation 1.18 for the prolate case...

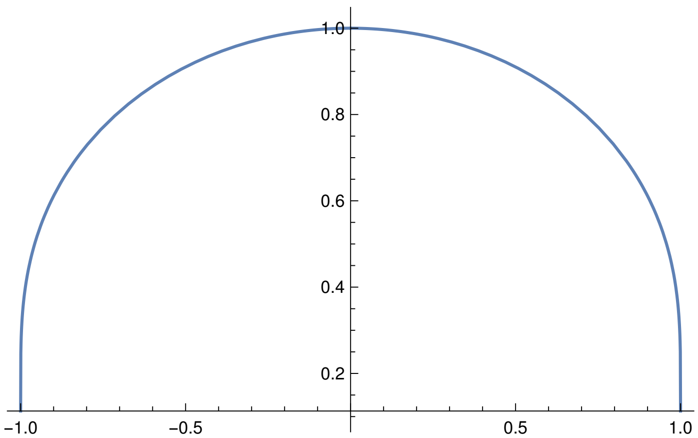
$$\frac{\sqrt{a^2 - b^2}}{\sin^{(-1)}\left(\frac{\sqrt{a^2-b^2}}{a}\right)}$$

Which as a function of the eccentricity becomes the following plot

In[27]:=
```
Plot[e / Log[(1+e)/Sqrt[1-e]], {e, -1, 1}]
```

$$\text{Plot}\left[\frac{e}{\text{Log}\left[\frac{(1+e)}{\sqrt{1-e}}\right]}, \{e, -1, 1\}\right]$$

Out[27]=



This is as far as I go in this problem but the plots are interesting. Capacitance as a function of eccentricity where the capacitance is highest in both cases at zero eccentricity. Implications are interesting.