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Math 87 HW 10
Due 12/9/2022

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1 Visualizing Lotka-Volterra

1. We can attempt to intuitively understand the following equations...

$$\frac{\partial r}{\partial t} = r(\alpha - \beta f)$$
$$\frac{\partial f}{\partial t} = f(\delta r - \gamma)$$

We consider f to be the population of the foxes, and r to be the population of the rabbits. Then $\frac{\partial r}{\partial t}$ is the rate of change of the rabbit population and same for the foxes.

We can see that the rabit population depends linearly on the current rabit population times some constant α. This makes sense, as we would expect the rabbit population to increase with the total rabbit population due to breeding.

It also has a negative term associated with the number of foxes as well as the number of rabbits. This is due to the foxes eating the rabbits.

Similarly we have a positive term dependent on the number of foxes and rabbits in the fox population equation. This is because the more rabbits the foxes eat, the higher their expected population growth. There is equally a negative population term associated with the starvation of a fox.

Thus we can see why this model might be reasonable for a small scale idealized system.

2. We want to write a function in python that handles the above equation and returns the result in an array (vector) or ideally a numpy array. I wrote the following code.

```
# initial conditions
params1  = [1., 0.1, 1.5, .75]
params2  = np.random.rand(4)
pops     = [10, 5]
dt       = 1000
minT     = 0
maxT     = 15

a,b,c,d      = params1
t            = np.linspace(minT, maxT, dt)

## we have to create our function rather specifically so we can use the odeint function
## we have a dummy varibale t, and then all our intial conditions
def gradient(pop_vec, t):
    R = pop_vec[0]
    F = pop_vec[1]
    return np.array([R * (a - b*F),
                     F * (d*R - c)])

```

Listing 1: Gradient Function

I decided to keep the initial conditions as global constants as I continue using them later in the project. Passing them into the function then is of little use, and complicates the `scipy` integration step by required an extra `args` tuple.

When we evaluate the gradient for a given (r, f) value pair we are characterizing the rate of change of our state space. In general the gradient characterizes the direction of the rate of fastest increase. In this case, though, it is characterizing the rate of our population change for both foxes and rabbits.

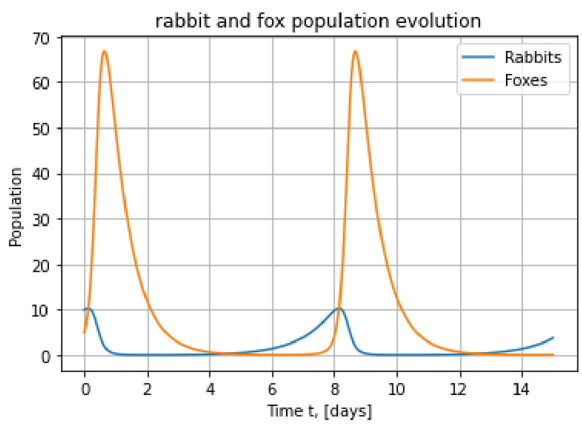
3. Integrating the equations from some starting time t to a later time t'. The values of these integrals for either function would likely indicate the total population size at a given time step Δt. I believe that will be more apparent when we do the next part of the problem.

4. We can go ahead and execute and plot the integrals we used above utilizing `scipy.integrate.odeint`. The code I wrote looks like the following.

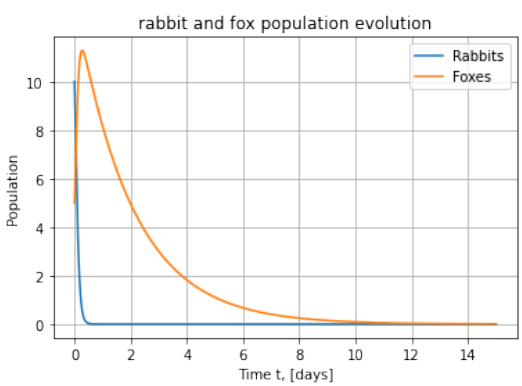
```
        sol = integrate.odeint(gradient, pops, t)
        rabbits, foxes = sol.T

```

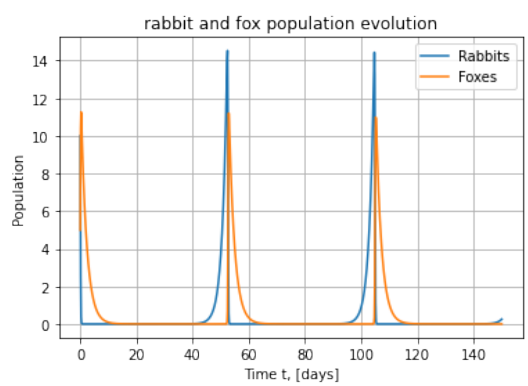
Listing 2: scipy integrate

This was pretty straightforward, and you can find the `pops` array in my initial conditions in Listing 1. I take the transpose here as, otherwise, the data isn't easily plottable in 2D. For the initial conditions I gave above (ie params1) we get a nice periodic plot.



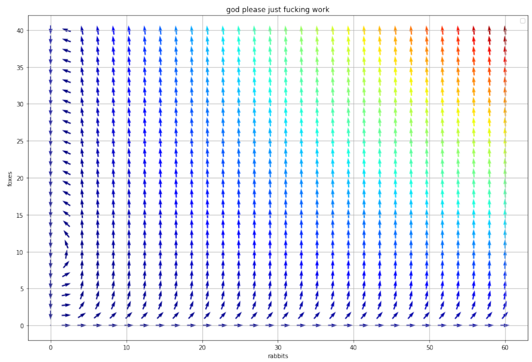
However, if we utilize some alternate initial conditions (params2 which) we get (more often than not) something that is not periodic within 15 days.

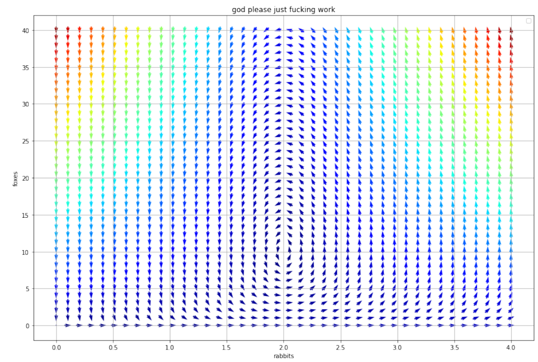




Above I've put the associated integral plots, one showing a range of 15 and the other of 150 to show that, almost no matter what, this is a periodic system.

5. Full disclosure, I had some trouble with this section because I was not familiar with `matplotlib.pyplot.quiver` or `numpy.meshgrid`. After much fiddling around, though, I was able to produce the following.





6. The two images above show my vector field graphs. On the right is the plot I was able to produce using the given inputs. I noticed, though, that there wasn't anything that would seemingly indicate a periodic nature in the vector field (loops). In fact, the vector field was surprisingly barren.

   I decided to try and crank up the resolution a bit, so I set my interpolation to 40 and, noticing the small swirll towards the origin, I cut my rabbits axis down to just 4. What results is the sort of swirling pattern I was expecting. Note, this is all for params1, params2 is not shown here.

   But yes, in general it is good to see the swirls indicating that the "stable" states are periodic (as seen in part 4) to some degree depending on the inital population conditions.

7. There are a few unrealistic assumptions made by the Lotka-Volterra model. One that I thought about is how it seemingly has some issues being solved analytically as you add more predators. If you were to try and model a full blown ecological system, you would have n coupled differential equations which I imagine trying to solve analytically would be a pain.

   As far as assumptions go, though, it assumes constant rates for hunting and death in different populations. It also assumes that, if necessary, a single predator could cosume rabbits indefinitely until dead or that it could consume rabbits every day. It also doesn't have any ability (in this formulation at least) to consider population caps.

   Overall, the model sacrifices some realism for simplicity, and is probably pretty okay for modelling small scale systems, or at the very least, fun toy models.

8. I believe a predator prey model with two predators, one of which being a super predator, would look like this...

$$\frac{\partial r}{\partial t} = \alpha r - \beta r f - \psi r h$$
$$\frac{\partial f}{\partial t} = -\gamma f + \delta r f - \omega f h$$
$$\frac{\partial h}{\partial t} = -\lambda h + \phi h r + \eta h f$$

   Where I have simply added "interference" terms with the himans in the mix. We have two positive terms associated with the human population, and one negative, similar to the fox in the previous system. We then have negative terms added to the fox and rabbit systems to show that humans are hunting both of them.