V

Discussion

1. General conclusions

Substantially all potentially relevant published systematic data on extragalactic objects have played a part in the foregoing parallel tests of the chronometric and expansion hypotheses. The satisfactory, and for the most part strikingly good, agreement of the chronometric predictions with the raw observations is in clear contrast with the only rarely really good agreement of the direct expansion-theoretic predictions with corrected observations. When consideration is confined to samples that are complete out to specified limiting magnitudes, the comparison is still more one-sided in favor of the chronometric theory; there is no such sample, either of galaxies or quasars, which is at all well fitted by the unembellished expansion theory. Furthermore there is no present indication that more refined studies are at all likely to alter the basic fact that the chronometric theory, with essentially no free parameter, provides a much better overall fit to extragalactic data than do straightforward general relativistic models with the free parameters q_0 and Λ. Indeed the trend of recent work has largely been in the opposite direction.

This is not to say that the expansion theory has been disproved. Its generally idealistic, nonoperational nature is readily compounded by the introduction of ad hoc mechanisms—superclustering for nearby galaxies, number-luminosity evolution for quasars, and other features reminiscent of epicycles—which may serve to render moot its apparent disagreement with the observations. But saving the theory in this way largely elminates its predictive power, and by virtue of the effective increase in the number of adjustable parameters, renders it scientifically highly uneconomical in comparison with the chronometric theory.

In any event, the remarkable degree of observational confirmation of the chronometric theory naturally raises the question of whether there is any sound scientific reason not to employ this hypothesis in theoretical astrophysics in preference to the expansion hypotheses. The expansion theory has for many years enjoyed the status of a preferred theory, with its concomitant influence on both the direction of observational research and its quantitative results. Inevitably questionable observations tend to be resolved in conformity with an established theory, while conversely observation in apparent conformity with the theory tend to be regarded as relatively unexceptionable. This general feature of experimental science is particularly important in an area in which facilities have been extremely limited, in which observations are not readily repeated by independent observers, and in which there is inherently little control or capacity for more intensive examination of the objects under study. The highly limited telescope time suitable for extragalactic work, and the intrinsic restriction to the information obtainable from the observation of their electromagnetic emission, imply that astrophysics falls into this category to the *n*th degree. The inability of the expansion theory to make useful fundamental observable predictions, despite its dominance over the past 40 years, is in striking contrast with the capacity of the chronometric theory to predict accurately a broad variety of theory-independent relations derived from observations published prior to its existence. This suggests that the chronometric theory is, at the least, likely to be relatively useful as a framework for the organization and study of observations on extragalactic objects.

Figure 30 is an illustration of the coherence which the use of the chronometric theory can introduce into the study of the nature of different types of extragalactic objects. The generally highly satisfactory fit of the theory to the samples on which Figure 30 is based, as well as a number of other samples, is shown in Figures 31 and 32. No substantial or otherwise cogent published samples of galaxies or quasars are unrepresented in these figures, except that due to Colla *et al.* (1975), which was published too late to be included in these figures, but which as earlier noted has the same qualitative implications as the large or otherwise statistically cogent galaxy samples.

It has to be admitted that the square redshift–distance law predicted for low-redshift objects is in striking variance with a generation of instruction in cosmological astrophysics, and at first glance appears to be contradicted by Sandage's observations on brightest cluster galaxies. While referring to

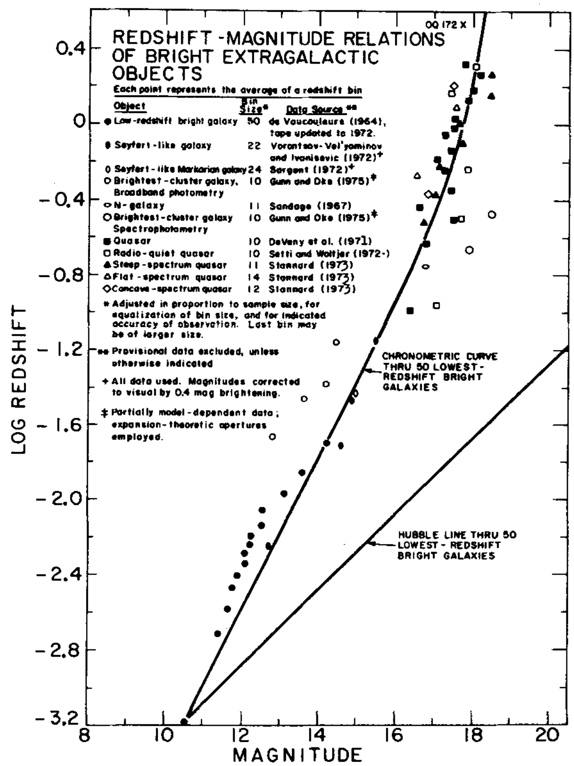

***Figure 30** Redshift–magnitude relations of bright extragalactic objects.*
Sources: de Vaucouleurs and de Vaucouleurs (1964), Gunn and Oke (1975), DeVeny *et al.* (1971), Sargent (1972), Vorontsov-Vel'yaminov and Ivanisevic (1974), Sandage (1967), Setti and Woltjer (1973), and Stannard (1973). Thanks are due the authors of the last two cited sources for communicating the data on which their graphs and other reduced results were based.

Chapter IV for a detailed analysis of the latter point, a decent regard for the natural prejudices and conservative proclivities of those brought up scientifically on the expanding universe seems to require an attempt to explain how so fundamentally misleading an apparent observational result could become so firmly imbedded in astrophysical thinking. There are sociological and biographical matters here which while probably quite interesting are beyond the scope of this book, and of the author's competence, and require additional information which is not readily available. But some nontrivial illumination is derivable from material in the scientific literature.

The boldness of Hubble's first paper (1929), which was in all probability influenced in part by theoretical considerations, as emphasized by Hetherington (1971), was one factor. The small sample of galaxies studied in this

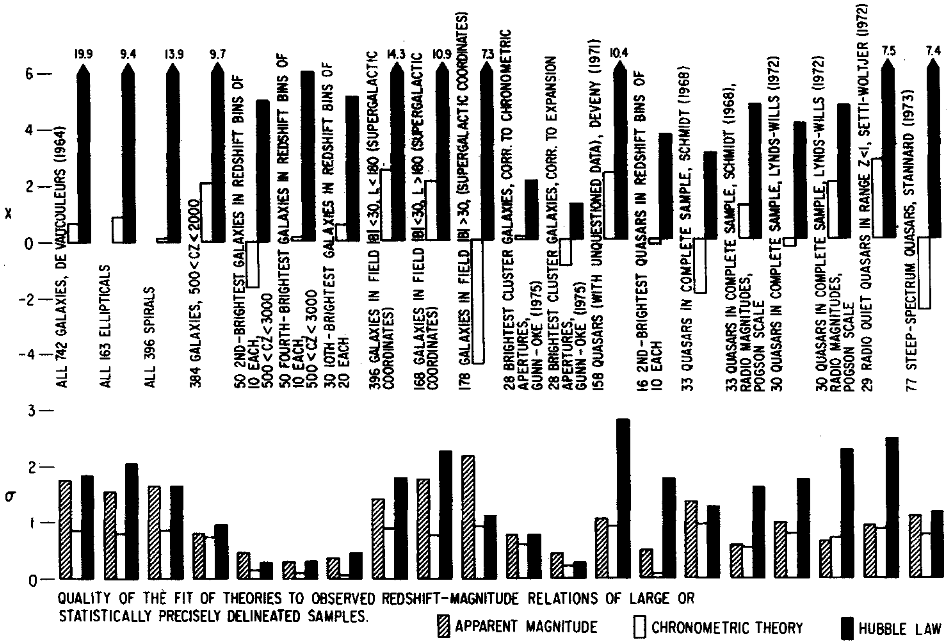

Figure 31 The quality of the fit of theories to observed redshift-magnitude relations of large or statistically precisely delineated samples.

The quantity X shown is the negative of the normalized reduction in variance brought about by the theory and is asymptotically normally distributed with zero mean and unit variance, for a fair sample and a correct theory. The variances σ^2 are in the apparent magnitude and in the residuals from the chronometric and Hubble law predictions. (In each case, the zero points of the predictions are adjusted to the mean luminosity of each sample.) The tendency toward order-of-magnitude equality of the variances in apparent magnitude and in residuals from the Hubble law for galaxy samples (or others at $z \lesssim 0.5$) is predicted by the chronometric theory, as is the larger variance in these residuals for larger-redshift samples. The subsamples specified by supergalactic coordinates were defined ex post facto (by G. de Vaucouleurs) in accordance with a theory indicating different redshift-distance exponents in the respective regions of the sky; the X values are correspondingly equivocal, but favor the square law over the linear one even in the region $|B| > 30$ hypothetically maximally favorable to the linear law. See also Tables 11 and 12. Data sources, in addition to those for Figure 30: Lynds and Wills (1972) and Schmidt (1968).

paper had an observed m–z relation which is distinctly better fitted by a square redshift–distance law than by a linear one. Nevertheless Hubble described the law as "roughly linear," on the explicit basis of uncertain and surely rough estimates of distance to only 10 of the galaxies; and the implicit basis of the prior theoretical prediction of a roughly linear law by a suitable development of general relativity. Later the estimates of distances to galaxies was refined by a study of the "brightest stars" in low-redshift galaxies, leading

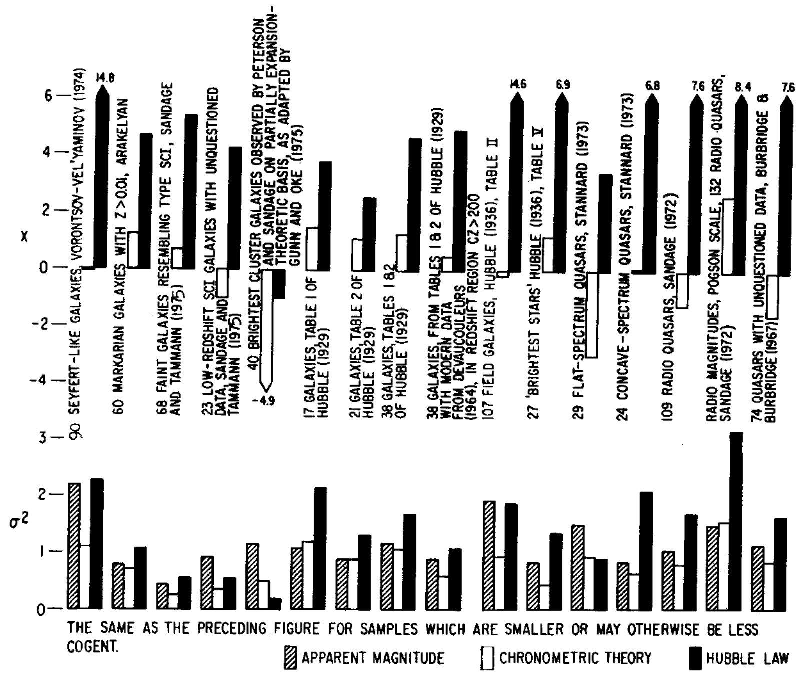

Figure 32 *The same as Figure 31 for samples which are smaller or may otherwise be less cogent than those in Figure 31.*

Data sources, in addition to those cited in Figures 30 and 31: Arakelyan *et al.* (1972), Sandage and Tammann (1975), Hubble (1929, 1936), and Burbridge and Burbridge (1967).

to Hubble's conclusion that these were suitable distance indicators. But the observed *m–z* relation of these "stars" is again in distinctly better agreement with a square redshift–distance law than with a linear one; and again, there is no indication in Hubble's work of this fact, or that he was at all cognizant of it.

In collaboration with Humason, Hubble (1931, 1936b) made additional observations, more than 100 field galaxies being included in their sample published in 1936. Again, this sample is in much better agreement with a square than a linear law, but mention or cognizance of this is not in evidence. Of course, the lack of agreement with a linear law could always be ascribed to an extreme breadth of the luminosity function for the galaxies observed; but such conceivable agreement with a linear law is very different from a positive indication for it. Moreover, if one attempts to suppress or limit the effect of the breadth of the luminosity function by utilizing only the brightest galaxies in bins ordered by redshift, the results are equally favorable to the square law vis-à-vis the linear law. But in these papers appeared for the first time the class of galaxies which were later to form the foundation for the linear redshift–distance law which had been proposed by Hubble in 1929. It consisted of bright cluster galaxies, and was a relatively much smaller sample than those previously considered by Hubble; however, its m–z relation was in extremely close agreement with the linear law prediction. The papers included no definite indication of how the ten clusters in this sample were chosen, and with the passage of time and the deaths of the authors, it appears that this may never be known.

It was difficult to make similar observations on other telescopes. Over the next two decades Hubble's original program was developed by Humason, Mayall, and Sandage, culminating in their classic paper giving redshifts and magnitudes for a large number of field and cluster galaxies. Again, the field galaxies formed a large sample whose m–z relation was in considerably better agreement with the prediction of a square rather than a linear law; and the brightest cluster galaxies formed a relatively small sample in somewhat better agreement with a linear law. This sample of bright cluster galaxies was intensively studied and extended by Sandage over the next decade and a half, during which time preliminary results were reported by him, generally in graphical form, and fairly widely accepted as definite proof for the linear redshift–distance law. The superiority, indeed unicity, of the 200-in. Mt. Palomar facility, and the decades of intensive study initiated by Hubble, made difficult the performance of comparable work elsewhere. With the publication of numerical observational results by Sandage in 1972, the basic case for the linear law seemed to be finally documented by the observed m–z relation for the sample of 41 brightest cluster galaxies which he treated. But as detailed earlier, analysis of the $N(z)$ relation and the apparent Hubble core radii for galaxies in the sample naturally raises a question of apparent selection effects, which cannot be dispelled on the basis of published information. The very satisfactory agreement of the phenomenological m–z relation for other types of galaxies with the prediction from a square law naturally reinforces the apparent anomaly of the Sandage sample.

The main moral is perhaps the importance of taking effective cognizance of the distinction between model-building on the one hand, and hypothesis-testing on the other. Another is the need for the exploration of foundational observable relations by several independent groups of observers. Finally, the great difficulty and expense of the observations do not supersede the need for utilizing the best available robust statistical analysis, but on the contrary, strongly enhance the marginal utility of the extraction of all relevant information from the samples. This need is closely related to that for statistically unexceptionable sampling procedures, involving notably the designation of explicit objective criteria for the inclusion or exclusion of objects from the sample.

In work published since the original manuscript of this book was completed, Rust (1974) has given a precise study of data for 36 supernovae, including particularly time-delay estimates. Here it can only be commented that the objects treated by Rust appear to be among the best "standard candles" available, but that the sample is too small for definitive analysis, and that the time-delay effect is theoretically less clear-cut than those treated in Chapter IV (cf. the discussion of observed versus theoretical times in Chapter III). The peak magnitude–redshift relation for all non-blueshifted objects (with or without removal of unrepresentatively large redshift objects) is in quite satisfactory agreement with the chronometric prediction, but poor agreement with the expansion prediction. The $N(<z)$ relation in the redshift region $cz < 1000$, which is unlikely to be greatly affected by an observational magnitude cutoff, is also in closer agreement with the chronometric prediction, the observational estimate of $\partial \log N / \partial \log z$ at $z = 0$ being fairly close to the chronometric value, $\frac{3}{2}$.

2. Theoretical aspects

Although the present theory in principle alters only physical kinematics, and so is vastly more limited than any complete dynamical theory, the alteration is of a fundamental character, which may in consequence cause some reflection by specialists. It remains to reconsider, if necessary, a variety of developments in astrophysical theory in terms of the chronometric theory. Most fundamentally, the questions arise: (a) what is the relation to the theory of Friedmann models; more broadly, how does it relate to general relativity, or to gravitation as a purely physical process? (b) what observable consequences, if any, does it have as regards elementary particle phenomena? A brief discussion of these and some related questions follows.

a. Slow expansion?

In principle, there is no difficulty in combining the chronometric redshift theory with some degree of expansion in accordance with a closed Friedmann model. As long as the rate of expansion is kept sufficiently low, the excellent agreement of the chronometric theory with observation is sufficient to ensure a statistically acceptable fit of the combined theories in standard cosmological tests. Moreover, the basic features of the Friedmann theory and its correlation with cosmology, apart from the redshift itself, would be retained.

Such a mixed theory cannot be excluded on a purely statistical basis, and would permit conventional ideas concerning the evolution and age of galaxies to persist in the combined theory without essential change.

However, the remarkable quality of the fit of the theory to observation is impaired noticeably as the rate of expansion increases. This is particularly the case for some of the key nonstatistical matters: the problem of the energy output of quasars, the apparent near cutoff in quasars circa $z = 2.5$, and the apparent existence of superlight (or near superlight) velocities. In addition, it would generally diminish the economy and predictive power of the theory. It would therefore seem more interesting and promising to seek to reexamine conventional ideas on the age, evolution, and colors of galaxies, in the light of the pure chronometric theory, than to develop a combined theory at this time. There is no apparent reason to anticipate any greater difficulties in so doing than exist already at present.

As noted by Segal (1972), and discussed in Chapter III, the chronometric theory can be regarded as defining (and is largely defined by) a virtual motion of the canonical local Lorentz frame at each point of the Cosmos with respect to the same frame at any other point. This canonical frame is that tangential to the universal (chronometric) frame at the point in question, and has natural units specified by setting $\hbar = c = R$. Thus a virtual Doppler redshift is implicit in the chronometric theory. However, the virtual point of view has no physical advantages, but only the theoretical one of possibly facilitating the correlation with the formalism of general relativity. The canonical local Lorentz frames at different points differ in scale as well as by a conventional Lorentz transformation. This variation in scale can be removed only by making the radius of the universe time-dependent, since the distance scale is fixed chronometrically by setting $R = 1$. Having made this change in scale, one has a pure Doppler relation between the correspondingly rescaled local Lorentz frames, and a formally expanding universe. The velocity of this initial recession varies approximately as the square of the distance for small distances, but for larger distances is not the Doppler velocity in formal correspondence with the chronometric redshift.

b. Gravitation

In mathematical respects, there is no significant difference between the chronometric and the Minkowski models from the standpoint of general relativity as a local theory of gravitation. The chronometric model is conformally flat, indeed the physically relevant local correspondence between local chronometric space–time and Minkowski space–time is given in explicit analytic form in Chapter III.

The increasing but still not yet totally definitive validation of general relativity as a local theory of gravitation does not imply its physical applicability in the form of the theory of Friedmann or similar models. As a local theory of gravitation, it is one in a hierarchy of local dynamical theories, none taking precedence over the others; as a cosmological theory, it has a more fundamental and different status from the others. Being inherently lacking in temporal homogeneity, the conservation of energy in our earlier group-theoretic sense is violated, and indeed there does not yet exist a broadly accepted and fully viable definition of energy in classical (unquantized) general relativity. The time itself is defined in terms of the formalism, rather than observable physical processes. In these respects it differs greatly from elementary particle and quantum field theories, in which there are formally well-defined positive definite energies, and in which the time may be characterized uniquely, apart from choice of scale, by the constraint of temporal invariance, and observed directly in terms of a theoretically precise frequency standard.

It would seem distinctly metaphysical to extrapolate the mathematical formalism of general relativity from a theory valid on the galactic scale as one of a hierarchy of theories of different interactions, to a theory on which the dynamics of the entire universe must be based, and to which the clocks of elementary particle processes must conform. Probably still less justified physically is the application of general relativistic hydrodynamics to extragalactic questions such as the mass density and the stability of the entire Cosmos. The approximation of the distribution of galaxies by a fluid is quite uncontrolled and open-ended; at best, conclusions drawn in this way are merely suggestive. The astrophysically fundamental fact that much, if not most, of the mass of the universe is in the form of the discrete bound states called galaxies, is completely lost sight of in the process of this approximation, and may represent a more crucial physical point of departure than the study of overall mass density of the Cosmos.

Admittedly, as a nondynamical theory, the chronometric model is incapable of predicting the average density of the universe. But this separation between kinematics and dynamics is quite possibly the way it should be. The circumstance that there is basically no such separation in global general relativity, while philosophically striking and unique, can be regarded as a major source of the ambiguities in the elucidation of its precise physical meaning. The clear-cut separation between kinematics and dynamics in elementary particle theory has made for empirical lucidity of the theory, and has on the whole been very satisfying. Indeed, a strong current trend in general relativity has been toward its recasting in terms analogous to those employed in the theory of elementary particles and their associated quantum fields. Work by Faddeev (1971) on the correlation of general relativistic and quantum concepts, leading in particular to a possible appropriate notion of energy, represents fundamental progress in this direction. The chronometric model could serve equally well with Minkowski space in Faddeev's work; its use in place of Minkowski space would actually lead to simplifications, in that the delicate question of the appropriate boundary conditions at infinity in space is superseded by the closure of space.

Likewise adaptable to the chronometric framework is the important foundational work of Lichnerowicz (1961) on the quantization of general relativity.

As pointed out to us by C. C. Lin, the mass density given by the standard closed Friedmann model with fixed radius is, with the radius of the universe given by the chronometric theory of the order of 10^{-27} gm cm^{-3}. This is quite high, but perhaps not unacceptably so, particularly in the light of comparable difficulties with missing mass in conventional theory. It remains to be explored to what extent observational estimates of the mass density of the universe may be affected by employment of the chronometric rather than the expansion model as the theoretical substructure.

In a more theoretical vein, it is interesting to note possibilities for correlating the chronometric model with general relativistic local gravitational theory through the scalar field provided by the presently unspecified scale of the conformally flat metric involved in the model together with the vector that vanishes in the special relativistic limit, as in Section 9 of Chapter III. In this connection, mention should be made of Weyl's conformally oriented theory (1921) and of his projective tensor, having the feature that it vanishes if and only if the metric is conformally flat. It is interesting that in the very natural form of the theory presented by Veblen (1933) there intervene both a scalar and a vector field, such as are provided locally by the chronometric theory in the $R \to \infty$ limit. Moreover, the mere modification of a Minkowskian metric by the introduction of these two fields in the indicated fashion is sufficient to imply all the observational consequences of general relativity (cf., e.g., Hawking and Ellis, 1973). Finally, the frequently-conjectured elimination of singularities in general relativity by the introduction of quantum considerations seems closer in that the strict form of the theory, for finite R, involves the five hermitian operators whose approximately scalar form for large R gives rise to the indicated scalar and vector fields.

In summary, the chronometric theory, although based on a physically entirely different redshift mechanism from expansion or gravitation, is in both observational and mathematical respects otherwise compatible with much of general relativity, including all of its local ($\sim$ galactic) features.

c. *Elementary particles*

As indicated in Chapter III, the fundamental local dynamical variables of the chronometric theory, energy, momenta, etc., differ from those of special relativistic field theory by terms of order R^{-1}, or less, where R is the radius of the universe in conventional laboratory units. These dynamical variables are here regarded as generators of local symmetries. The difference is therefore too small to affect observationally any known elementary particle processes, apart from possible selection rules and classification features, assuming the state spaces to be the same in both cases. For particles of zero mass, the wave functions are locally essentially unchanged, but for massive particles the chronotheoretic structure remains to be developed. The existence of mass is not compatible with the transformation of elementary particles under the full conformal group; current ideas of broken symmetry and the like indicate, however, that a group of related massive elementary particles may well arise from restriction of this group (more precisely, of its universal covering group) to an appropriate mass-conserving subgroup.

The most obvious choice for this subgroup is that leaving fixed an observer's infinity and local distance scale; this is the conventional inhomogeneous Lorentz group. The square of the mass is then represented by the image under the relevant representation of the D'Alembertian $\Box$. When combined with conventional representations for internal symmetries or quantum numbers, it would leave the theory of elementary particles and their local interactions basically unchanged. A physically more natural choice for the subgroup is however $O(2, 3)$. This also facilitates mass splitting, which is forbidden for the Lorentz subgroup by the O'Raifeartaigh theorem. The role of $\Box$ would be taken over by the Casimir operator for $O(2, 3)$, which differs essentially from $\Box$ only by terms of order R^{-1}.

d. The mechanism of energy production in galactic nuclei and quasars

Obviously any hypothesis regarding this matter can be validated only in a quite indirect fashion. From the chronometric standpoint there is, however, an extremely simple and natural supposition regarding the nature of the mechanism: it consists basically of the transformation of the excess of the unienergy $i^{-1}(\partial/\partial\tau)$ over the special relativistic energy $i^{-1}(\partial/\partial t)$, i.e., of the new form of energy corresponding to the difference between the two times involved in the theory (earlier shown to be positive) into elementary particle processes. As earlier noted, the excess unienergy appears with redshifting, and is then diffused in space in a fashion which causes no observable local particle production. The amount of energy involved is, however, quite substantial, and over very large distances and times should be responsible for significant interactions. The formation, dissolution, and early intensive development of galaxies would appear on the one hand to involve interactions of this nature, and on the other to be the most likely known physical process not clearly explicable by elementary particle combined with gravitational interactions.

If the conversion of the unienergy excess over the special relativistic energy into local particle processes is indeed a significant feature of galactic development, it should be one of the main mechanisms by means of which the unienergy excess energy density in the universe is kept stationary, as it would be natural to assume it is in the chronometric theory. White dwarfs and other elderly, metal-rich objects may provide fuel which can be ignited with sufficient unienergy excess to yield elementary particles and hydrogen. Such "burning" of moribund objects in galaxies could provide very large amounts of energy in quite small regions, and plausibly take place on a short time scale, as in the case of supernovae, giving rise to variability such as is observed in active galactic nuclei. At the same time it would serve to maintain the population of moribund objects at an approximately stationary level.

Highly speculative as these considerations are, they are perhaps less so than those treating the origin and early dynamics of the universe, which involve much larger regions of space and much longer reaches of time. The group-theoretic nature of the chronometric hypothesis implies a variety of conservation laws which are relatively stringent, physically meaningful, and to a considerable extent serve to define the theory. It is therefore operationally much more subject to definitive validation than is the "big-bang" theory and similar hypotheses.

In any event, because of the temporal homogeneity of the universal cosmos, there must be processes that convert the superrelativistic into the relativistic energy. This is effected in part by free propagation, which while leaving the total energy unchanged alters its partition between the delocalized superrelativistic energy and the microscopically observable relativistic energy. However, the energetics of the microwave background indicates that other processes must be more important.

Consider, for example, what may be the basic cycle involving the bulk of matter and radiation in the universe: gas + microwave background + dust + local radiation + old stars → galaxies → gas + microwave background + dust + local radiation + old stars. The local radiation density over regions of the order of 10 parseconds is naturally subject to large random fluctuations in the course of cosmic time, especially if enhanced by cosmic turbulence along the lines of von Weizsäcker (1951). These fluctuations should eventually reach the flash point required to "ignite" the other ingredients of a generic galaxy core. Of course, this does not exclude other possible mechanisms, such as collisions involving cores of old galaxies or old stars within the cores, which collisions necessarily take place because of the infinitude of time and the finiteness of space. A particularly interesting feature of the chronometric outlook for the evolution of galaxies is the availability of ample energy from the superrelativistic radiation for making up the mass loss from galaxies with active nuclei. This mechanism would both explain the otherwise persistently puzzling large mass loss, of the order of one solar mass per year (cf. Oort, 1974), and provide for additional large-scale conversion of the superrelativistic radiation into conventional forms. In particular, it would lower the estimated cosmic background radiation temperature from the correct order-of-magnitude but slightly high figure earlier derived.

e. Intergalactic matter and the microwave background

The precise equilibrium attained by radiation in space, following possibly many complete circuits in space, will naturally depend on the extent and character of intergalactic matter which may be present. The existence of such matter proposed by Holmberg (1958) and indicated by some later studies (Takase, 1972, among others) is difficult to substantiate directly. A small rate of extinction, of the order of $\lesssim 10^{-6}$ mag/kpc if $H \sim 80$ at 10 Mpc is consistent with the redshift–magnitude relations for quasars and galaxies, in the chronometric theory. Because of the smaller size of the chronometric than the expansion-theoretic universe, relatively little dust would be required to produce this rate of extinction, but on the other hand relatively little would be necessary to play a significant role in the possible thermalization of intergalactic radiation. The very general analysis by Purcell (1969) (based on the Kramers–Kronig relation) indicates a density of matter $> 10^{-33}$ g cm^{-3}, assuming transmission characteristics not grossly dissimilar from those of interstellar dust and the indicated extinction rate, but there are no other known restrictions on the dust. Compare in this connection, the mechanism for production of the background radiation proposed by Layzer and Hively (1973).

A detailed analysis of how the spectrum of emitted radiation is transformed depends also on the absorption and emission characteristics of galaxies and intergalactic matter, as well as on having globally more precise wave functions for the emitted radiation than are afforded by simple plane wave. The chronometric theory predicts quite directly the existence of very nearly isotropic and highly energetic blackbody radiation diffused homogeneously throughout the universe. The temperature of this radiation is a dynamical quantity which the theory can only correlate with other dynamical quantities. This is effectively the case also with the primeval fireball concept. Indeed, the latter development involves more parameters than the chronometric prediction, and these parameters must be quite specially chosen in order to lead to an observationally correct prediction (cf. the careful account by Weinberg, 1972). On the whole, the observed cosmic background radiation is predicted by the chronometric theory in at least as definitive a fashion as the primeval fireball complex of hypotheses.

f. The Friedmann model with $q_0 = \infty$

To take q_0 as ∞ would exacerbate the missing mass problem in general relativistic cosmology, and otherwise appear at first glance not to be observationally sustainable. It has however been observed by J. F. Nicoll that the value $q_0 = \infty$ is in many ways the best-fitting of the Friedmann models to the general run of cosmological data, on galaxies and quasars. Its redshift–distance law is identical with that of the chronometric theory, and it thereby gives an equally good account of the $N(< z)$ relations for galaxies and quasars. Its m–z and N–S relations are however different by virtue of the "number effect" due to the recession. This results in a distinctly poorer fit to the quasar m–z relation than the chronometric relation. In the case of the N–S relation, there is the same difficulty as in other Friedmann models, that without evolution the values of the index $-\partial \log N/\partial \log S$ fall below the Euclidean value 1.5, in disparity with the radio source observations earlier described. From a purely theoretical viewpoint, the $q_0 = \infty$ case is one of the most interesting Friedmann models by virtue of its exceptional symmetry.

3. Further observational work

In view of the historical observational basis for the expansion theory in the data on low-redshift galaxies, the question arises of the existence of statistically rigorously appropriate data which are actually favorable to, and not merely perhaps marginally consistent with, this model. Its relation to the cosmic microwave background, coincidence of order of magnitudes of time scales, and the apparent helium abundance are dependent on stringent dynamical assumptions supplementary to the expansion hypothesis itself. Consequently, these relations only indicate the possibility, and not an objective or definitive likelihood, that the model is correct. The only clear possibility for the vindication of the predictive cogency of the expansion hypothesis appears to lie in systematic galaxy observations in randomized fields out to a limiting magnitude of ~ 15. But it is evident that what is to be anticipated on the basis of the present work is rather a *reductio ad absurdum* of the model.

In any event, the observations on each galaxy should include magnitudes over a range of apertures, in order to form an appropriate basis for testing several theories. The sample should either be complete in a designated field to specific limits, or constitute a randomized subsample from such a complete list. Radio observations might additionally be quite useful in this connection. Although there is no special reason to doubt that the results would be generally similar to those earlier obtained, such observations would then lead to more precise estimates of the radius of the universe, the number density and size of bright galaxies, and other important cosmological parameters.

The fact that increased accuracy and numbers of observations, over the years, has not appreciably altered the phenomenological situation as regards the *m–z–N* relations of galaxies, except strongly to confirm earlier indications appears equally true of quasars. The *m–z* and *N–z* relations of the ~ 70 quasars known approximately eight years ago are similar to those for the $\gtrsim 200$ quasars known today. For example, for the 74 quasars with unquestioned data listed by Burbridge and Burbridge (1967), the respective dispersions in apparent magnitude, deviation from the Hubble line, and in chronometric absolute magnitude are: (a) 1.09, (b) 1.29, (c) 0.92, which is qualitatively similar to the results from the later DeVeny and other samples, in the reduction and increase in dispersion associated, respectively, with the chronometric and expansion theories. As was to be expected, the average magnitudes are fainter for the later sample, but only by slight amounts: (a)

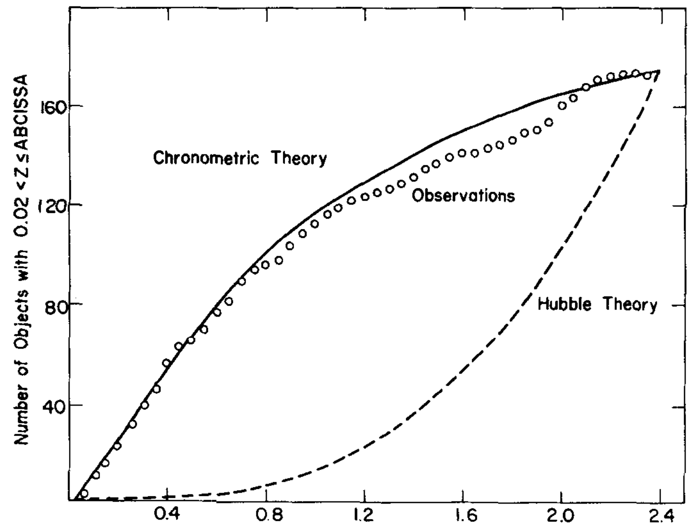

Figure 33 *The redshift distribution of the quasarlike objects with redshifts published as of* 1969.

All quasars, N and Seyfert galaxies listed by Burbridge and Burbridge (1969) for the redshift range $0.02 < z < 2.40$ were included. The redshift distribution is similar to that for the later list published by DeVeny *et al.* (1971) and the earlier list of Burbridge (1967), except that the inclusion of N and Seyfert galaxies removes the apparent deficiency in the number of quasars at redshifts ≤ 0.3. The *m–z* relations are also similar, and there is no apparent reason to expect that additional quasar observations over the next few years will materially affect their overall *m–z–N* relation.

0.07, (b) 0.32, (c) 0.15. The same is true of the $N(z)$ relation, which is qualitatively similar on the basis of today's observations to that of several years ago. Figure 33 shows in fact the excellent fit of the direct chronometric prediction to the redshift distribution of quasarlike objects listed by Burbridge and Burbridge (1969). Here "quasarlike" means that the object is listed either as a QSO, a Seyfert, or an N galaxy. As earlier indicated, inclusion of the latter removes an apparent if statistically not significant deficiency of quasars at redshifts < 0.3 from the chronometric outlook. Whether these objects are included or not, the observational relation is generally similar to that shown in Figure 25, based on the DeVeny sample. From the expansion-theoretic standpoint, quasars are murky, variable objects, among the least likely to provide an observational foundation for the theory. But larger samples of quasars which are complete out to fainter limits, or random subsamples of such, could lend additional confirmation to the chronometric theory; and in any event would help to clarify the nature of quasars, even if, as is to be anticipated, their basic redshift–luminosity–number relation is not substantially altered.