

homework #1 modified

PHY 009I Spring 2022

- 1) For the standard Cartesian coordinates in two-dimensional flat space R^2 , a one-sphere S^1 of radius R centered at origin, $O = (0,0)$, is a circle, a set of points in two-dimensional space satisfying the equation:
- $$x^2 + y^2 = R^2$$

Introduce the polar coordinates on R^2 . A point P on can be described by the coordinates

$$\begin{aligned}x &= r' \cos \theta \\ y &= r' \sin \theta\end{aligned}$$

where θ the angle in $\hat{x} \times \hat{y}$ plane, measured from \hat{x} to \hat{y} .

Find the equivalent of Friedman-Walker expression for the square of the geometrical interval dl^2 between two points in S^1 . (Have a look at Max's thesis if you need guidance, or write me.) Find the circumference of S^1 or radius R by integration of the line element dl along the circle. Find the area of the disk bounded by a circle by integrating dA , the area element, in Cartesian and polar coordinates. One way to is consider the areas of thin shells (spherical slices) whose boundaries are two-spheres S^2 of radii varying from 0 to R .

- 2) For the standard Cartesian coordinates in three-dimensional flat space R^3 , a two-sphere S^2 of radius R and centered at origin, $O = (0,0,0)$, is a set of points in three-dimensional space satisfying the equation:
- $$x^2 + y^2 + z^2 = R^2$$

Introduce the spherical coordinates on S^2 . A point P on S^2 can be described by the coordinates

$$\begin{aligned}x &= r' \cos \theta \\ y &= r' \sin \theta\end{aligned}$$

where $r' = R \sin \phi$, θ the angle in $\hat{x} \times \hat{y}$ plane, measured from \hat{x} to \hat{y} , and ϕ the angle from \hat{z} to the vector \overrightarrow{OP} (you can, of course, define the angles in a different way, frequently the definitions of θ and ϕ are interchanged). Find the equivalent of Friedman-Walker expression for the square of geometrical interval dl^2 between two points in S^2 . (Have a look at Max's thesis if you need guidance, or write me.)

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solutions are due on March 9, 2022

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- 3) Find the area of a two-sphere S^2 of radius R by integrating dA , the area elements.
- 4) Find the volume of a 3D ball of radius R by integrating dV , the volume elements, over the volume enclosed by S^2 . with $r \in (0, R)$
- 5) Follow the same technique to find the volume of S^3 , which can be considered as a 3D hypersurface in a specially 4D manifold with a restriction

$$x^2 + y^2 + z^2 + d^2 = R^2$$

Follow Max's Senior Thesis to see how to introduce a 4D analogue of a spherical co-ordinate system, and so on.

- 6) Derive the Friedman-Walker metric for S^3 , basically following what is presented in Max's Senior Thesis
- 7) Find the volume of S^3 or radius R , you may use Mathematica to integrate the Friedman-Walker metric with curvature $k = 1$.
- 8) Check that the same integration for a space with $k = 0$ gives a familiar expression for a volume of a sphere in R^3