An Analysis of Chronometric Cosmology and The CMB

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Abstract

The Cosmic Microwave Background Radiation is often taken as a cornerstone of evidence in contribution to the Standard Model of Cosmology. In the standard model, the CMB is taken to be the light last scattered from the hot and dense plasma permeating the early universe as it cooled approximately 300,000 years after the Big Bang. In this paper, however, the CMB is analyzed from an alternative cosmological model known as The Chronometric Cosmological model. This model considers the spatial component of the cosmos to be a 3-sphere in which light is permitted to take several circuits around the universe before potentially scattering. It is through the phenomenon of light taking circuits about the universe that one can derive the same notion of the CMB in this model. Thus, this paper endeavors to utilize this theoretical framework to derive first order approximations of the average temperature of the CMB as well as it's power spectrum in this model and determine whether or not the model is falsified in light of modern astrophysical data.

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1 Introduction

In 1927, Georges Lemaître presented a paper proposing a solution to Einstein's field equations in which the universe was expanding. Unbeknownst to Lemaître, however, this solution had already been derived several years prior by Alexander Friedmann, but notably, Friedmann did not place much weight on his derivation. During the Early 1920s, Friedmann produced many unique solutions to Einsteins field equations, and was largely concerned with the range of possible solutions as opposed to which of the bunch could be considered the most physical. Lemaître, however, attacked the problem of the expanding cosmos in order to attempt to understand how an expanding universe would impact physical theories [**?**]. Over the course of a century the SCM would change form slightly, adapting to more and more astrophysical data, however it still remains a titan in the field of cosmology whose predictive capabilities still have no true rival.

The SCM posits that the universe was once an incredibly dense sea of energy, too dense in fact for the formation of familiar matter to take place. This dense sea of energy eventually underwent a period of rapid inflation thus beginning the universe as we know it. The exponentially expanding cosmic soup of energy would eventually slow it's rate of inflation allowing the matter within it to cool substantially causing more familiar states of matter to form. Over the span of billions of years, this matter would become stars, galaxies, quasars, and all the other wonderful celestial bodies humanity has been familiarizing ourselves with over the last several millennium.

This theory would gain much traction after certain astrophysical phenomena were discovered. The redshift was the first of these phenomena, and it was it's detection that set the SCM in motion, setting a precedent for field equation solutions which involved an expanding universe.

In the Big Bang model, the cosmological redshift is attributed to the expansion of the universe and is generally a product of the light travelling through a space as the space expands. Heuristically this can be thought of as though the wavelength of the light is being stretched as the space it is traversing expands with it. It would be Hubble in 1929 who would first characterize the redshift relation, as he noticed distant galaxies appeared to be more redshifted than those nearer. This solidified Hubble's Law in the history of physics, which is a linear redshift relation given by

$$v = H_0 D \tag{1}$$

for D the proper distance, v the recessional velocity, and H_0 the Hubble parameter.

The second phenomena that has served a critical role in the formulation of the SCM is the Cosmic Microwave Background (CMB). The CMB in a completely phenomenological context is a faint glow of microwave frequency light permeating the universe. It has a spectrum which is very nearly a black body spectrum of temperature ~ 2.7 K. However, the CMB exhibits fluctuations in temperature (fractional deviations from the average $\Delta T/\langle T\rangle$) on the order of 10^{-5} K [**?**].

In the SCM, the CMB is explained in the context of the Big Bang and the years nearly following the first moments of the universe. Principally it is considered to be the last scattering of light in the early universe as the hot ionized matter in the universe gradually cooled. Eventually, it would cool enough to deionize allowing it to become transparent to radiation. Thus light would scatter freely to form the CMB which we see in microwave telescopes today [**?**].

The CMB, along with the cosmological redshift, make up two of the most physically important phenomena in modern cosmology, and the derivation and explanation of them is critical for any theory which wishes to exert explanatory power in the field. Few theoretical frameworks exist which can aptly do so, however few is far from none, and if a potential model proposes an explanation of the universe which is able to reproduce these crucial phenomena, it is certainly worthy of earnest investigation.

1.1 Organization of this thesis

This thesis centers around one such alternate cosmological model and in particular it's ability to offer a sufficient explanation for the CMB in a universe far outside the current mainstream of modern cosmology. The model itself is known as the Chronometric Cosmological Model (CCM) and is class of models which is of positive (spherical) curvature. Further, it is a completely static model and thus has no conception of the expansion or contraction of the universe or any notion of a beginning to the universe. It is considered infinite temporally.

This thesis will first discuss the Big Bang model and some of it's pertinent mathematical properties, then move into the Chronometric Model. The Chronometric Model's mathematical properties will be discussed without explicitly deriving them (an exercise best left to someone of a more purely mathematical inclination). The discussion will then turn to deriving certain important equations in dealing this this universe, namely the "Area" and "Volume" of the cosmos in this model, and a method of acquiring averages over this universe. It will then be possible, using averaging techniques, to make certain determinations about the CMB in the CCM, which will be discussed in section 3.

Section 4 will then concern itself with the power spectrum of the CMB and how possible calculations of this spectrum are impacted based on various parameters. Finally, we will concluding with a general discussion of the limits and implications of the contents of this thesis in the scope of modern cosmology, along with further points of research to be explored.

A pertinent question might be related to why this thesis will not be discussing aspects of the cosmological redshift. The reason for this is due to a preexisting (and rather complete) discussion of the redshift having already been conducted by a research colleague Maxwell Kaye in [**?**] and, of course, in Segal [**?**]

2 The Models

In science theories are formulated to explain phenomena and are constantly tested through empirical observation and experimentation. Robust theories that can successfully explain and predict the observed phenomena are generally not subject to modification or reformulation, while theories that fail to do so are subject to revision or replacement. Thus, a natural question to arise when discussing alternate cosmological models is: *is there anything wrong with the current model*?

The answer, truthfully, is a rather vague. As stated previously, the SCM has proven to be a reliable and robust framework for understanding the universe. It has successfully predicted and explained a wide spread of phenomena. This is not to say, however, that the model does not have any unresolved issues. Over the course of the last century, many such issues have been resolved, but as telescopes continue probing deeper and deeper into universe with progressively more advanced means of measurement, they may yet find more unsolved mysteries.

One such example which has become pertinent quite recently is relating to large structure formation in the early universe. The James Webb Space Telescope (JWST) in a recent paper has discovered a set of galaxies of uncharacteristically large mass given their redshift. The paper titled *A population of red candidate massive galaxies ∼600 Myr after the Big Bang* by Labbè et al. reported...

> Galaxies with stellar masses as high as $\sim 10^{11}$ solar masses have been identified out to redshifts z $\sim$ 6, approximately one billion years after the Big Bang... Here we make use of the 1-5 μm coverage of the JWST early release observations to search for intrinsically red galaxies in the first $\approx$ 750 million years of cosmic history. In the survey area, we find six candidate massive galaxies (stellar mass $> 10^{10}$ solar masses) at $7.4 \leq z \leq 9.1$, 500–700 Myr after the Big Bang, including one galaxy with a possible stellar mass of $\sim 10^{11}$ solar masses. If verified with spectroscopy, the stellar mass density in massive galaxies would be much higher than anticipated from previous studies based on rest frame ultraviolet-selected samples.

In all, the paper goes on to describe possible explanations for such a discrepancy at such a large redshift, but in general we can see this as a clear example of how possible mysteries can arise in modern cosmology [**?**].

Another example of unresolved issues in the SCM can be found in the cosmological redshift. For much of the model's existence, the cosmological redshift has existed as means by which light seemingly loses energy by no apparent mechanism. The wavelength of the light is simply "stretched" causing it's energy to vanish into the cosmos. This problem would eventually be dubbed the *cosmic sink* by Hermann Bondi, and it is a problem which led to dissatisfaction amongst some physicists. This eventually became the starting point for Irving Segal who was troubled by this apparent violation of the conservation of energy. In hopes of restoring this grand scale energy conservation, Segal developed the Chronometric Cosmology[1].

Segal had considered closely the work of Oswald Veblen on group deformations and particle classification of conformal space, and used these ideas formulated by him to begin his considerations of a conformal spacetime [**?**]. These considerations were motivated in part by the formulation of quantum mechanics and special relativity. Special relativity displacing Galilean relativity which in turn became a sort of limiting case of special relativity. In the same way, classical mechanics had started to be considered as a limiting case of a more fundamental theory; quantum mechanics. Perhaps, then, the ideas put forth by Minkowski could be considered a limiting case of a larger, more accurate formulation of cosmology.

2.1 Development of the Chronometric Cosmos

Segal's development of the chronometric cosmology starts with an axiomatic framework through which one can develop any generalized cosmos. The axioms given in [**?**] are as follows.

1. The Cosmos is a four-dimensional manifold[2]

2. The Cosmos is endowed with a notion of causality

3. The Cosmos admits stationary observers

4. Space is homogeneous and isotropic

5. Any given timelike direction at a point p is tangential to the forward direction of some admissible observer

[1]This development was primarily conducted via his 1976 book *Mathematical Cosmology and Extragalactic Astronomy* [**?**], however much of the mathematical framework regarding the CMB is found in other papers such as [**?**] and [**?**]

6. Two different observers at the same point see the Cosmos in causally compatible ways, i.e., the transformation between their respective maps of the Cosmos should be causality-preserving

While the exact nature of these axioms is outside the scope of this thesis[3], they outline the mathematical properties of the framework utilized by Segal, and they are based on common notions and intuitions about the universe in general [?]. In the context of this paper, a few of the axioms stand out in particular.

Axiom 2 indicates that at each point in the four-dimensional manifold, there exists a notion of causality. Formally, this means that at every point in space there is a nontrivial closed convex cone tangent to that point in space. This is considered to be the functional description of a light cone in the axiomatic description set forth by Segal. Further, axiom 3 admits stationary observers which also implies the cosmos has a global notion of causality and that there can be no close time-like paths (hence $\mathbb{R} \times \mathbb{S}^3$, $\mathbb{R}$ being the temporal component) [?]. These axioms, then, clearly derive notions which are already quite familiar to the field of cosmology, just in a more general way.

In the same way, the cosmos being a four-dimensional manifold is certainly not a new concept, but it, along with the other axioms, limits the space of possible world manifolds by a considerable degree. In fact, Segal shows that, given the axioms above, there are only two admissible world manifolds [?].

2.1.1 Admissible Manifolds

Throughout the course of [?], Segal shows that the only two admissible world manifolds are as follows.

$$M = \mathbb{R} \times \mathbb{R}^3 \qquad \bar{M} = \mathbb{R} \times \mathbb{S}^3$$

Where here we refer to M as the standard Minkowski space and $\bar{M}$ as the universal cosmos. M is then taken to be isomorphic to $\mathbb{R}^4$ and $\bar{M}$ is isomorphic to $\mathbb{R} \times \mathbb{S}^3$. While these distinctions become important for large scale astrophysical phenomenon, Segal found that for smaller scale interactions $\bar{M}$ is functionally indistinguishable from M. This is essentially the same effect as a tangent plane on the surface of a sphere being nearly identical with the surface of the sphere

[2] One must go out of their way to note the exclusion of the singularity from this definition, however the general principle applied is that if you were to remove singularities from consideration you would be left with a four-dimensional manifold from which you could then consider the adjunction of singularities [?].

[3] Again a more complete analysis is found in [?] and [?]

in a local approximation.

2.1.2 The Metric

Briefly, now, we return to the work of Lemaître and Friedmann. They initially (separately) derived the expanding solution of Einstein's field equations in the 1920s, and later this work would be picked up by Robertson and Walker, thus birthing the Friedmann-Lemaître-Robertson-Walker metric which would go on to be a critical component understanding the general characteristics of any cosmological model.

In general, a metric determines the distance between any two points in a space. It is essentially a distance function in it's most basic form, and the FLRW metric, in it's most general form, is as follows

$$dl^2 = R^2 \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2) \right) \tag{2}$$

Where R is a scaling factor which in general for the SCM is taken to be a function of time to account for the expansion of the universe. The parameter k is constant which characterizes the curvature of the space which the metric describes. It can take on three possible values:

- $k = +1 \quad \rightarrow$ closed/spherical space.
- $k = 0 \quad \rightarrow$ euclidean/flat space.
- $k = -1 \quad \rightarrow$ open/hyperbolic space.

Thus the standard Minkowski spacetime M has the constant k set to zero, converting equation (2) into the following.

$$dl^2 = R^2 \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta)d\phi^2 \right) \tag{3}$$

This can be identified with the general volume component in spherical coordinates of typical Euclidean space (merely scaled by the R factor. In fact, if we take the more general form of the metric, the metric tensor, and we utilize the property of the metric tensor which relates the square root of the determinant of the metric tensor to the volume component in the space described by the metric, we will see exactly how they relate.

The metric tensor (more formally the Riemannian metric tensor) is given as follows.

$$g = R^2 \begin{pmatrix} \frac{1}{1-kr^2} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2(\theta) \end{pmatrix}$$

And the general volume is then...

$$V(r) = \int \sqrt{det(g)}\ d\Omega$$

Which for Minkwoski space ($k = 0$) becomes

$$V(r) = R^2 \int_0^r r^2\ dr \int_0^{\pi/2} \sin(\theta)\ d\theta \int_0^{2\pi}\ d\phi$$

Where we are integrating for some volume enclosed by Rr at some arbitrary time t_0 so we will take $R = R_0$ at t_0 resulting in the following.

$$V(r) = \frac{4}{3}\pi(R_0 r)^3 \tag{4}$$

Which is in fact exactly what we would expect for the volume enclosed by some region of M. We can conduct an identical calculation to determine what this volume will be for $\bar{M}$.

$$\begin{aligned} V(r) &= R^2 \int_0^r \frac{r^2}{1-r^2}\ dr \int_0^{\pi/2} \sin(\theta)\ d\theta \int_0^{2\pi}\ d\phi \\ &= 2\pi R^3 \left[\ \sin^{-1}(r) - r\sqrt{1-r^2}\ \right] \end{aligned}$$

This however is a rather awkward form for a handful of reasons. One is that it is offering the volume enclosed by a certain region of $\bar{M}$ in terms of the total radius of the space ($\mathbb{S}^3$) and the *coordinate* r. The coordinate r in this case is not a particularly physical concept as it describes a *local* distance as opposed to the *manifold* distance. Pulling a lower dimensional example, we take an ant living on the two dimensional surface of $\mathbb{S}^2$. The coordinate r then would be akin to taking the taking the tangent plane at the current position of the ant and moving it from $\mathbb{S}^2$ onto that tangent plane, then letting it walk across that surface. It would eventually be walking some distance "above" the surface of it's original space. As put by Max Kaye in [?] "...imagine a sheet of glass balanced perfectly on the north pole of a sphere. The distance from the north pole to any other point p on the sphere, measured by r, would be the distance along the sheet of glass an ant must walk such that it is standing directly above the point p"

[3]Explicitly we should be calling this the hypersurface and consequently the hypersurface area and hypervolume. However, for the sake of readability and writing slightly more suc-

Another reason for which one may find this expression awkward is the arcsin which, granted, is an argument of beauty, however it is not without validity. The universe being described by these equations is highly unfamiliar, thus we desire to put it in a more appreciable form such that an intuition about the space can be formed.

The simplification we desire is found by utilizing the manifold distance as opposed to r. We know that the relationship between the metric distance l and the coordinate r comes from general radial paths. This means we take $d\theta = d\phi = 0$ in (2) for $k = 1$ which gives us the following.

$$l = R \int_0^r \frac{dr}{\sqrt{1 - r^2}}\, dr = R \sin^{-1}(r) \tag{5}$$

Thus

$$r = \sin(\frac{l}{R}) \tag{6}$$

Here the quantity l/R is a dimensionless quantity which is typically how distance is described in $\bar{M}$ and is exactly the manifold distance mentioned above. We now give the distance between objects on the surface of this universe, not as an ant on a sheet of glass, but as an ant crawling across the surface, which is a far more natural and physical coordinate.

Here it is also worth noting that, while in general R is a scaling factor and may depend on time, in the CCM R is taken to be static and exactly the radius of the universe.

Substituting equation (6) back into the general volume expression, and calling l/R as ρ, results in the following equation.

$$V(\rho) = 2\pi R^3 \left[\rho - \cos(\rho)\sin(\rho)\right] \tag{7}$$

Where $\rho \in (0, \pi)$.

Interestingly, if we expand this equation out in terms of small ρ, we see the following.

$$V(\rho) \approx \frac{4\pi}{3} R^3 \rho^3 \left[1 - \frac{\rho^2}{5} + \cdots\right]$$

Which shows us that ρ must be close to 1 in order for deviations from the geometry of M can be noticed.

cinctly, we will generally refer to the hypersurface area/hypersurface and hypervolume as surface area/surface and volume respectively

A unique result of having an expression such as this, is that it allows us to make averages over the whole of $\bar{M}$ as one typically would in M. This can be done by the useful relationship between the surface area of n-spheres and their volumes, which dictates that the surface area is exactly the derivative of the volume. Using this will allow us to a) easily derive the following expression for the surface area of $\mathbb{S}^3$ and b) normalize the surface area (again surface area being the actual volume of $\bar{M}$ which forms spacetime in the CCM) to allow for easy averaging over functions of ρ. $A(\rho)$ is then...

$$A(\rho) = 4\pi R^3 \sin^2(\rho) \tag{8}$$

The discussion of how to utilize these two expressions to take averages over the cosmos will be discussed and further utilized in section 3.1.3

3 The Cosmic Microwave Background

3.1 Average Temperature of CMB

The average temperature of the CMB is a critical aspect of the CMB as a phenomenon in the universe. This, along with the power spectrum characterize the primary features of the CMB, and thus, if the chronometric model is able to reproduce these aspects of the CMB, we can say that the model is, if nothing else, not falsified in light of modern astrophysical data. In order to determine these attributes from first principle, however, it is important to first get the bigger picture of how the CMB is accounted for within this model.

Light in the chronometric cosmos is broadly separated into two categories. The first we call pristine light, and the second we call residual. We call the light that has taken fewer than one half-cycle about the cosmos pristine and the light which has passed the $\rho = \pi$ manifold distance residual. The CMB then is concerned primarily with the residual light in the universe.

Given the relative sparsity of matter in the universe, light of this category would be able to take many circuits about the universe. The infinite time for this high-dispersion radiation to accumulate would directly imply that it is qualitatively distributed in accordance to Planck's law, and thus is in fact a black body spectrum. This fact is shown more formally in [**?**], but here will be taken as a fact.

It is worth noting before continuing that the origin of this light is unimportant for general considerations. However for a more specific analysis we can easily consider the source of the pristine and residual light in the universe to be the galaxies and other luminous matter in the universe.

3.1.1 Methodology

The methodology, then, for determining the average temperature of the CMB in this model will be to utilize the residual light in the cosmos. The residual light will be considered explicitly as the light "left over" after traversing multiple half circuits of the cosmos, and is thus the light which has *not* gone extinct. The residual light is considered to be of particular importance as, given the curvature of $\bar{M}$, it is allowed to continue traversing the universe becoming of higher and higher dispersion, directly contributing to the CMB.

This analysis will be quite general and is only intended to act as a first order approximation to the average temperature to the CMB. As such, a handful of approximations must be clarified. We will consider all galaxies to be *approximately* the same in radius (d), luminosity (L), and have an explicit number count (N). Each galaxy is approximated as a perfect absorbing sphere as well as an emitter of light. Using these factors we will calculate on average the amounts of pristine and residual light in the universe, and what of that reaches the Earth which will give us qualitative information regarding the CMB in this model. We will then utilize that to determine a range of values for the average temperature of the CMB.

3.1.2 Extinction Law

For a general order of magnitude estimate of the total radiative flux of the CMB which arrives at the Earth in the $\mathbb{S}^3$ cosmos, we first need to consider extinction of light as light takes half-circuits about the closed universe.

In general, we know that that the rate of change of photons (N_p) traversing $\bar{M}$ will be negative and proportional to the number of half-circuits (n) in accordance with general stochastic absorption of light. Using this simple relationship we can construct the following.

$$\frac{dN_p}{dn} = -\alpha n \longrightarrow N_p = N_{p_0} e^{-\alpha n}$$

For N_{p_0} the amount of light which has not yet taken a half circuit about the universe at some arbitrary time t_0, which formally makes it the pristine light P. This allows us to consider the total amount of residual light N_{p_0}.

$$N_p = N_{p_0} e^{-\alpha n}$$

With the identification of N_{p_0} as the pristine light P, we can then arrive at the following expression for the residual light in the universe.

$$N_p = Pe^{-\alpha n} \tag{9}$$

This expression is of course per half-circuit n. For the purposes of determining the extinction of light over any n circuits the summation to infinity must be taken. For the purposes of this analysis, n will be taken to start at 1 as this must be an examination of the residual light implying at least one half-circuit has been taken.

$$P \sum_{n=1}^{\infty} e^{-\alpha n} = P \left[e^{-\alpha} + e^{-2\alpha} + \cdots \right] = \frac{P}{1 - e^{-\alpha}}$$

In order to further simplify this expression an approximation via Taylor Series expansion is utilized. Given the Taylor series expansion for $e^{-\alpha}$, the following is known.

$$e^{-\alpha} = 1 - \alpha + O[\alpha]^2$$

From here all greater order terms will be taken as negligible as, for general considerations α is considerably less than one[4] [?]. Removing the higher order terms in this approximation allows for a massive simplification in the given expression leaving us with the following.

$$N_p \approx P\alpha^{-1} \tag{10}$$

Before continuing on to determine what exactly these parameters are in the context of the CCM, we should stop briefly to consider the physical meaning of this expression.

In typical considerations of the standard flat Minkowski spacetime M, as light radiates away from emitters, it is free to traverse the infinite plane of the cosmos until it is scattered. Thus, the average amount of light will always decrease irrespective of continuous emission of light from other sources in the cosmos. However, in the case of $\bar{M}$, light is permitted to continue taking circuits about the universe while the emitters continue radiating. Because P is constant the residual light will contribute to the net luminous flux over many cycles as an enhancement factor of sorts. This is precisely what allows the CMB to take form in the CCM. In this way, we can consider that from the theoretical framework of Segal's model, the CMB is not merely retrofitted, but it is, in fact, *predicted.*

The next steps, then, are of course to consider how we can determine the approximate values for the pristine light and the absorption coefficient α from the principles already discussed in the CCM.

3.1.3 Absorption Factor

To start, we consider how one can calculate the absorption factor as it relates to the residual light in the cosmos. As stated before, we consider this light to be the light which has *not* gone extinct. In this case, then, we need only consider the amount of light that arrives at the Earth, as this is the only light which we can factor into the chronometric model's explanation of the CMB. In order to calculate this, then, we can shift the question to be how much light is absorbed by galaxies in the universe en route to the Earth?

[4]This will be shown more clearly in 3.1.3

We consider the general volume of $\mathbb{S}^3$ as a function of the manifold distance ρ.

$$V(\rho) = 2\pi R^3 \left[\rho - \cos(\rho)\sin(\rho)\right]$$

We can then characterize the surface area as follows based on the analysis conducted in 2.1.2.

$$A(\rho) = 4\pi R^3 \sin^2(\rho) \tag{11}$$

As discussed previously, we can utilize these expressions to develop a general expression for the normalized surface area over $\mathbb{S}^3$. We derive the following normalization factor over the integral from 0 to π (ie the entire surface).

$$Q = \int_0^\pi A(\rho)\, d\rho = \int_0^\pi 4\pi R^3 \sin^2(\rho)\, d\rho = 2\pi^2 R^3 \tag{12}$$

From here we can find the average value of any function of ρ by utilizing this normalization factor in the following way.

$$F(\rho) = \int_0^\pi \frac{A(\rho)}{Q}\, f(\rho)\, d\rho \tag{13}$$

Where the integral goes from 0 to π for the sake of averaging over the entire surface (ie cosmos).

With this framework we can now turn our attention to what exactly it is we wish to be finding the average value of. It has been noted that we are interested in the amount of light occluded by our approximated galaxy objects at any given point in the cosmos (for posterity we consider Earth to be our point of relevance). Thus we are interested purely in the percentage of the sky taken up by such objects, and therefore we want to know the percentage of solid angle taken up by N galaxies.

We define the solid angle as the field of view obstructed by an object at some distance. In the consideration of celestial bodies, we can simply take the solid angle to be given as follows.

$$\Omega = 2\pi\left(1 - \cos(\theta)\right) = 4\pi \sin^2\left(\frac{\theta}{2}\right)$$

We now consider that for any general consideration of a distribution of galaxies, we can consider them to be quite far away. Thus, utilizing small angle approximations, we can arrive at the following.

$$\Omega = \pi\theta^2 \tag{14}$$

Or the area of the central cross section of our sphere typically called the great circle of our sphere. In this approximation, θ is characterizing the radius of of the great circle of the sphere. Thus θ is formally d in the small angle approximation.

For considerations of the ratio of the sky obstructed by the absorbing spheres, we must carefully consider the geometry of the situation. If it were the case that we were in typical flat Minkowski space M, then we would take no issue and simply take our ratio with respect to the total surface area of the sphere which we project or solid angle onto. However, given that, in $\bar{M}$ an object at $\rho = \frac{\pi}{4}$ and an object at $\rho = \frac{3\pi}{4}$ will take up the same Ω, we must utilize the surface area component of $\mathbb{S}^3$. Thus, our equation for the ratio of solid angle obstructed by a single spherical absorber of radius d becomes the following.

$$\frac{\Omega}{\Omega_{total}} = \frac{\pi d^2}{4\pi R^3 \sin^2(\rho)}$$

Thus for a general case of N such bodies we arrive at...

$$\frac{\Omega}{\Omega_{total}} = \frac{N\pi d^2}{4\pi R^3 \sin^2(\rho)}$$

Now we can consider the average based on our formulation of equation (5) substituting $\frac{\Omega}{\Omega_{total}}$ for $f(\rho)$.

$$F_\Omega(\rho) = \int_0^\pi \frac{A(\rho)}{Q} \frac{\Omega}{\Omega_{total}}\, d\rho = \int_0^\pi \frac{Nd^2}{2\pi R^3}\, d\rho$$

Thus...

$$F_\Omega(\rho) = \frac{Nd^2}{2R^3} \tag{15}$$

This value for the average absorption of light is in fact exactly identified as the absorption factor α in (3). We note that it is obvious given the inverse cubic relationship to the radius of the $\mathbb{S}^3$ cosmos that α must be less than 1.

3.1.4 Pristine Light Calculation

The consideration of the pristine light in the cosmos will follow very similarly to the former as the calculation is yet another average over the cosmos. Now, however, it is not about their properties as absorbers, but instead their properties as emitters.

As stated, we consider each galaxy to have identical properties for the sake of a first (or zeroth) order calculation. As such we take each galaxy to have uniform luminosity L, and define their flux as follows.

$$\phi = \frac{L}{4\pi R^3 \sin^2(\rho)} \tag{16}$$

Thus, our average becomes the following.

$$F_\phi = \int_0^\pi \frac{A(\rho)}{Q} \frac{NL}{4\pi R^3 \sin^2(\rho)}\, d\rho = \frac{NL}{2\pi R^3} \tag{17}$$

This is then our most general expression for the average pristine light in $\bar{M}$.

While it is rather useful to have an expression for any "typical point" in space, we here on Earth find ourselves in a rather peculiar situation. Namely, we happen to be located approximately half-way from the center of what seems to be a perfectly average spiral galaxy[5]. Thus, one could easily formulate the luminosity L in terms of the flux from the Milky Way within the solar neighborhood. Thus we can take L to be exactly the following.

$$L = 4\pi\phi_e(\frac{d}{2})^2 \tag{18}$$

Where ϕ_e is the aforementioned flux of the Milky Way received by the Earth. Notably we utilize the standard spherical surface area formula for this expression as the curvature of the universe is imperceptible on the scales of an individual galaxy and thus considerations of its larger topology are unnecessary.

If we substitute (19) into (18), it results in the following.

$$F_\phi(\rho) = \phi_e \frac{Nd^2}{2R^3} \tag{19}$$

This, then, is the average pristine light P in the cosmos as identified in (10).

3.1.5 Average Temperature of CMB

The tools for calculating the average temperature of the CMB have now been derived. The discussion of the average pristine and residual light in the cosmos has been conducted and the relationship between the two has given us an explicit way to calculate the total flux of light in the cosmos. Given expression (3), we can now substitute the calculated values of P and α to see that the net flux of

[5]This may or may not be the case exactly, and this will be discussed at greater length in 3.2.5

the CMB arriving at the Earth is simply ϕ_e.

$$P\alpha^{-1} = \phi_e \frac{Nd^2}{2R^3}\left[\frac{Nd^2}{2R^3}\right]^{-1} = \phi_e \tag{20}$$

The uniqueness of this result cannot be overstated. Due to the form of the calculations conducted we see that the terms relating to the radius of the $\mathbb{S}^3$ manifold, the number count of galaxies, and the radius of the average galaxy, all drop out of our expression and leave us with ϕ_e. Of course, this varies under considerations of our assumptions. For instance, our position in the Milky Way could be closer to $\frac{3}{4}d$ which would alter our solution accordingly. For the purposes of a zeroth order calculation, however, we will consider $\frac{d}{2}$ to suffice.

One factor which must be considered, however, is how the redshift will change this expression. Certainly, given that the pristine light will be coming in from some distance, the redshift must be considered. To do this we can find the redshift average over space given that the character of the redshift factor in $\bar{M}$ is $(1+z)^{-1}$ and $z = \tan^2(\rho/2)$ [?].

$$F_z(\rho) = \int_0^\pi \frac{A(\rho)}{Q}\left(1+\tan^2\left(\frac{\rho}{2}\right)\right)^{-1} d\rho = \frac{1}{2} \tag{21}$$

Thus, the total flux is reduced by a factor of 2 due to the redshift in $\bar{M}$ resulting in (21) becoming $P\alpha^{-1} = \phi_e/2$.

The CMB in the CCM is taken to be a perfect black body spectrum. As such, we can utilize the Stefan-Boltzmann equation to determine the temperature of the radiative flux density of a black body spectrum.

$$J = \sigma T^4 \longrightarrow T = \left[\frac{\phi_e}{2\sigma}\right]^{\frac{1}{4}} \tag{22}$$

For J, the radiative flux density which here we take to be ϕ_e, σ the Stefan-Boltzmann constant ($5.6710^{-8}\frac{W}{m^2K^4}$), and T the temperature[6].

3.1.6 Analysis of Average Temperature

Given the unique relationships derived above, it is now the hope to put them to the test of actual astrophysical data.

Equations (20) and (21) together show that the expected average temperature

[6]It should be noted that the more general expression of the Stefan-Boltzmann relation includes the factor of ϵ or emissivity. However, the emissivity of a perfect black body is simply 1.

of the CMB power spectrum in $\bar{M}$ is given by $T = [\phi_e/2\sigma]^{1/4}$. In order, then, to calculate the value of the temperature, data regarding the flux from the Milky Way in the solar neighborhood is needed.

Several probes have made attempts at calculating this value, and to varying degrees of accuracy and through various methods. In *The surface brightness of the Galaxy at the solar neighbourhood* by Melchior et al., several estimates from various data sets and papers were summarized in discussed, along with a new synthetic estimation method proposed by the authors [**?**]. However, the discussion of the flux is broken down into various frequency bands which are not able to be simply summed over. There is often considerable overlap between frequency bands, and determination of where these overlaps are and how much overlap exactly exists is a task which requires more precision than is perhaps required for such an approximate calculation. The frequency bands also do not cover the entire spectrum of light being emitted in most cases, thus it may not offer the most complete perspective. Because of these reasons, it will be convenient to consider this calculation from the perspective of the absolute luminosity L.

Given equation (18), for any general emitting body (which we are sufficiently nearby to such that the curvature of $\bar{M}$ does not affect the geometry of the situation) we know the following.

$$\phi = \frac{L}{4\pi d^2}$$

Thus we can simply calculate the flux of the Milky Way based on the luminosity of the Milky Way. This values is not necessarily more easily obtained in the literature, however it is relatively more studied relative to magnitude relations. Utilizing the luminosity relationship in this way also means that we must have some idea of how far the solar neighborhood is from the center of the galaxy. Typically this is taken to be the distance from Sgr A*, and it typically taken to be ~ 8.3 kpc, or $\sim 27,000$ light years [**?**] [**?**]. For the sake of attaining the correct units for the Stefan-Boltzmann constant (W/m^2), this will be converted into a rather large quantity of meters; $\sim 2.6 \times 10^{20}$m.

The luminosity itself is a rather variable parameter, and is quite tricky to determine for the Milky Way due to obstructions such as dust and the general disc of the galaxy. However, the luminosity is typically considered to be on the order of $\sim 1.1 \times 10^{10}\ L_{\odot}$, or $\sim 4.2 \times 10^{36}$ Watts [**?**].

For these given values of L and d, utilizing equations (20) and (21) result in an average temperature value of...

$$T \approx 2.54 \text{ K}$$

While this value varies somewhat from the expected value of 2.7 K, it is quite surprisingly close.

It is worth noting the approximate nature of this result and how easily it falls out of basic approximations. We have not done anything more than relating averages of values of the pristine and residual light, then utilizing modern astrophysical data to discern an approximate value of the average temperature. The mere fact that this calculation is within the same order of magnitude is surprising to some degree.

The figures used here are also quite approximate. As stated previously, accurate figures for the flux, luminosity, or even general magnitude of the Milky Way is quite nontrivial. Thus it may be more accurate to utilize other galaxies as the "average" galaxy of some average luminosity L and radius d. However, given the point of our detection of the CMB being the Earth, it seems fitting to utilize our home galaxy.

Perhaps even more surprising is the uniqueness of equation (20), as it simplifies what seems to be a problem relating difficult to discern values of the universe, such as the number count of galaxies and the radius of $\mathbb{S}^3$ into one relating to nothing other than the flux of the Milky Way galaxy or perhaps more generally, the flux of some average galaxy in the cosmos.

3.2 CMB Power Spectrum

Considerations of the average temperature of the CMB is a critical step for developing a functioning model within the CCM, however it is merely part of the story. The CMB has a well understood and experimentally determined power spectrum as well as the average temperature. Thus, if there is any hope of the CCM being a world model capable of describing even the most basic phenomena within the universe, it must be able to reproduce at least some of the major characteristics of the CMB power spectrum.

In general terms, the CMB power characterizes the temperature fluctuations in the CMB across different angular scales. These angular scales themselves are characterized by a variable l called the multipole moment. The temperature anisotropies are often decomposed into a sum of spherical harmonics, which are functions that describe the variation of a scalar field over the surface of a sphere. The spherical harmonics can be expressed in terms of the multipole moments l and m, where l"is a positive integer and m is an integer that satisfies $-l \leq m \leq l$.

The multipole moments l correspond to the angular scale of the temperature fluctuations in the CMB radiation. Specifically, larger values of l correspond to smaller angular scales, and vice versa. The lowest values of l correspond to the largest angular scales, which are typically referred to as the "quadrupole" and "octupole" moments. The higher values of l correspond to smaller angular scales and more detailed structure in the temperature anisotropies.

The CMB power spectrum is then calculated by squaring the amplitude of each multipole moment and averaging over all possible orientations (m values). The resulting power spectrum tells us how much power is present in each multipole moment and therefore how the temperature fluctuations are distributed across different angular scales.

3.2.1 The Known Spectrum

As stated previously, the CMB has a very well known power spectrum (see Fig 1) and is a critical tool for physicists to understand the cosmos. In general, the explanation given for the CMB in the SCM allows astrophysicists to probe various multipole moment regimes which are then interpreted depending on the relative power at the given scale. In the CCM, however, the interpretation is notable different as the source of the spectrum shifts from the last scattering of light in a hot and dense early universe, to that of the net average flux of the residual light in the universe as explained in 3.1.3.

Given this explanation of the CMB in the CCM, we anticipate that the power spectrum will be dependent on the large scale structure formation of the cosmos, as we take the emitting and absorbing objects in $\bar{M}$ to be galaxies which have a known hierarchical structure. There are namely three large scale structures which we will concern ourselves with.

- Superclusters - a cluster of a cluster of galaxies
- Clusters - a cluster of several galaxies
- Galaxies - The galaxies themselves

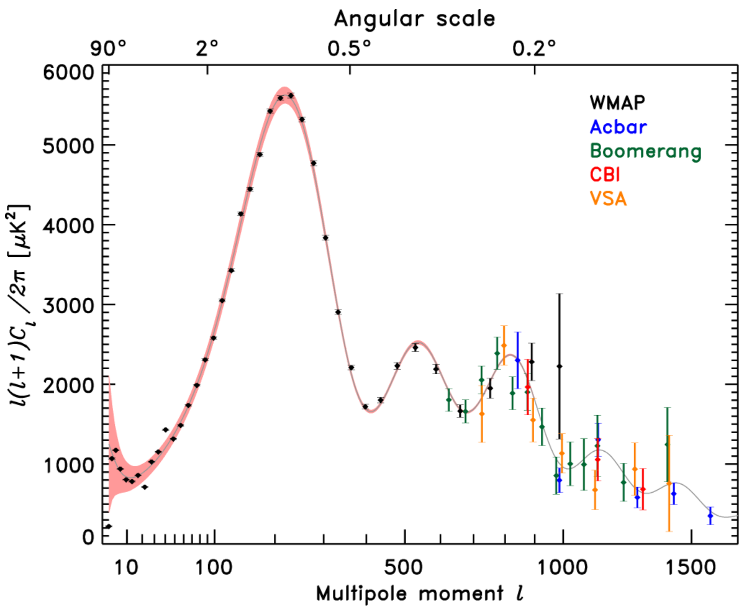

Figure 1: Figure showing the power spectrum from [?]. Data from the Wilkinson Microwave Anisotropy Probe (WMAP) with high-l values coming from various other collaborations shown in diagram., figure originally sourced from [?]. The plot is multipole moment against the historically utilized quantity TT (temperature-temperature correlation).

Having only these structures, however, is perhaps not the most meaningful way to consider how the power spectrum will form. Along with these, there is a larger underlying structure, which has to do with the average void distance between these superclusters of galaxies. It is through these large structures and the numerical relationships between them, that the CCM must reproduce the power spectrum of the CMB.

3.2.2 Methodology

The principal concept of reproducing the power spectrum of the CMB in the CCM relies heavily on being able to reproduce the large scale structures mentioned above. Thus approaching this problem via simulation was a natural conclusion. The aim then will be to *pack* super clusters of galaxies into an $\mathbb{S}^3$ surface area based on typical void size and number count. Doing so will rely on certain parameters which must be inputted manually and numerically (such as the radius of the $\mathbb{S}^3$ universe, super cluster number count, void size, etc.) to then attempt to recover the power spectrum by projecting these objects onto the sky using some standard astrophysics libraries.

Exact numbers for galaxies per cluster and cluster per supercluster are few and far between, but approximations exist online which gave a reasonable starting estimate for the simulations. Exact parameters utilized will be discussed further in 3.2.4.

3.2.3 Simulations

The simulation itself is broken into two parts. The first part happens in Mathematica and is the more computationally expensive part. This consists of packing the $\mathbb{S}^3$ volume with the super clusters of galaxies. This is done by first selecting a number of super clusters (which in most cases we will consider to be on the order of 10^7). From there, the super clusters are distributed throughout the $\mathbb{S}^3$ space according to a cubical packing[7](ie each void is modelled as a cube and the super clusters are placed on each vertex of the vacuum cube). This is done uniformly and allows for one to discretize the volume of the universe into layers or shells, each of which containing a certain number of super clusters. After this packing is done, the super clusters are broken down into a specific number of clusters, and further into galaxies, thus adding many more points into the simulation (see Figure 3).

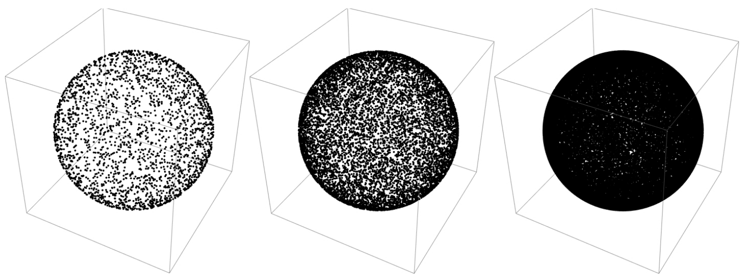

Figure 2: Figures showing layer slicing of Mathematica simulation of cubical packing of super clusters. Left shows Layer 2, middle shows Layer 3, right shows Layer 10 of projected slices onto celestial sphere.

From here, the galaxies are projected onto celestial sphere layer by layer (a general depiction of this layering process can be seen in Figure 2).

The output from this projection onto the celestial sphere is then fed into a python library called healpy which is a library built on HEALPix. The healpy library was developed to handle pixelated data on a sphere and HEALPix was developed precisely for efficient handling of data from the likes of WMAP and BOOMERANG. Thus, this software has plenty of tools for developing images of

[7]A cubical packing is at least a decent first guess. There are any number of different ways to pack the superclusters in any number of geometries, however for now only cubical packings will be considered. This will be discussed further in section 4

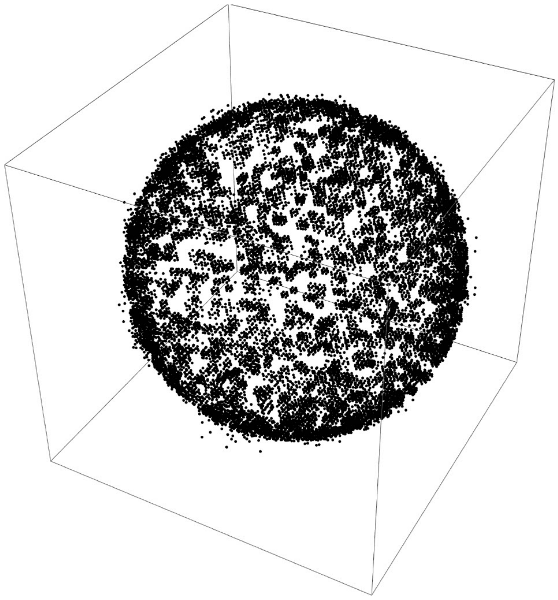

Figure 3: Replacement of super cluster points with clusters and galaxies for low n layer.

the sky and taking the pixelated data output from Mathematica and translating it into a power spectrum as well as a general temperature map of the sky.

3.2.4 Results

The results of running the simulation scheme mentioned above vary to some degree or another, largely depending on the ratio of the void size to the radius of the universe. However, certain sets of parameters produce power spectra that are of particular interest.

First, it is worth using healpy on the WMAP data set in order to determine what the CMB power spectrum looks like using this software. Doing so results in Figure 4.

The results of running the simulation and developing the power spectrum from the output of the Mathematica code can be seen in Figure 5. This plot was simulated utilizing a supercluster count of 10^6, with 9 clusters per supercluster, and 20 galaxies per cluster, totally out to 180 million galaxies in total. The void size if taken to be 10 Megaparsecs, which when utilizing a packing of this type, results in the radius of the $\mathbb{S}^3$ ball coming out to be 210 Megaparsecs.

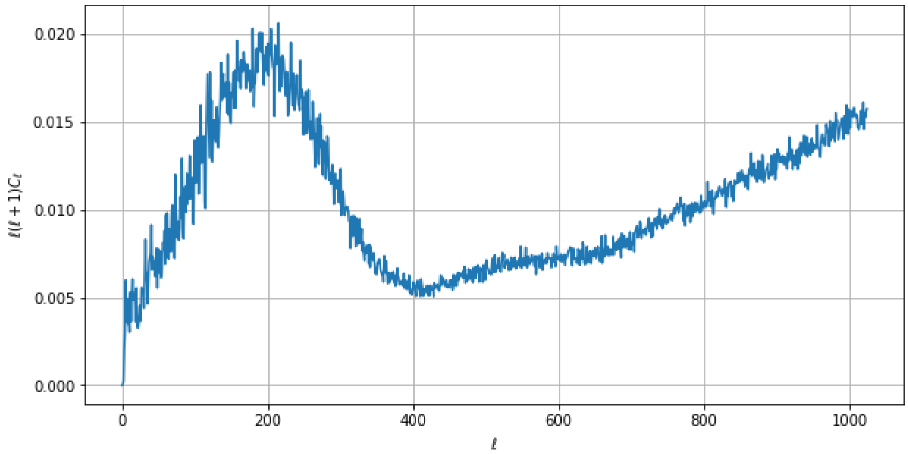

Figure 4: The CMB power spectrum produced in healpy directly from the WMAP data. Notably, the Planck data is not used in this thesis as it's resolution is quite high and comparing to a resolution that high would require significantly more computation time in the simulations where that resolution is generally not required for first order approximations.

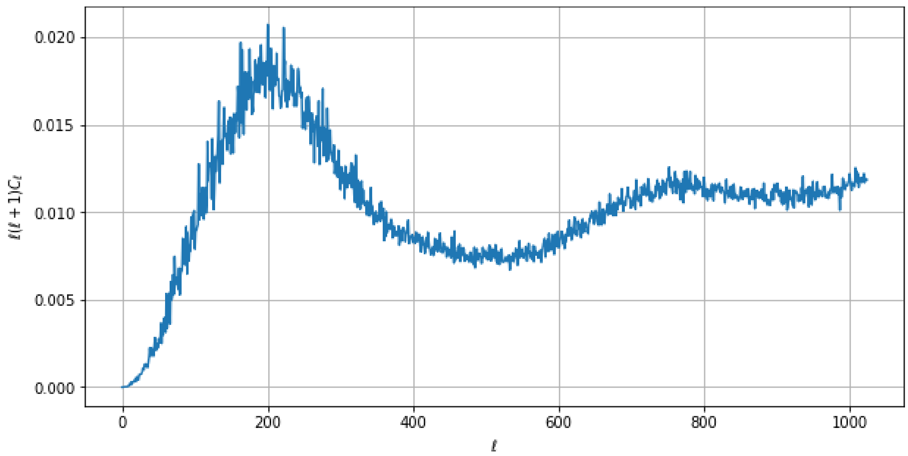

Figure 5: The CMB power spectrum produced in healpy directly from the Mathematica output utilizing cubic packing. We note the spectrum is characteristically varied, and notably starts from zero, likely due to there being no constraint on how close exactly the packed superclusters are allowed to be to some origin point (the celestial sphere).

3.2.5 Analysis

While a complete analysis of the average error between the two plots would certainly provide a rather complete and quantitative analysis of the Figures 4 and 5, the results seem to be rather self explanatory in a more qualitative way.

The simulation utilizing cubical packing has reproduced to a reasonably high degree of accuracy the first major feature of the CMB. That being the first peak in the power spectrum at $l \approx 200$. Beyond the $l \approx 400$ point, there are noticeable discrepancies, however the whole of this simulation is meant to provide merely a first order approximation to determine the feasibility of reproducing the CMB from first principles in the CCM.

An analysis comparing the two also does not entirely do justice to just how unique the result in Figure 5 is. It may seem as though, perhaps, any random scattering of the galaxies across the cosmos would likely produce a spectrum which may approximate a black body well enough construct this near black body spectrum. In order to show how this is not the case, we will do exactly this, and conduct the same analysis as done for Figure 5, however instead of cubically packing superclusters and then decomposing them into their constituent componenets thus creating large scale hierarchical structure, we will uniformly scatter 180 millions galaxies across the same cosmos. The result of this analysis is seen in Figure 6.

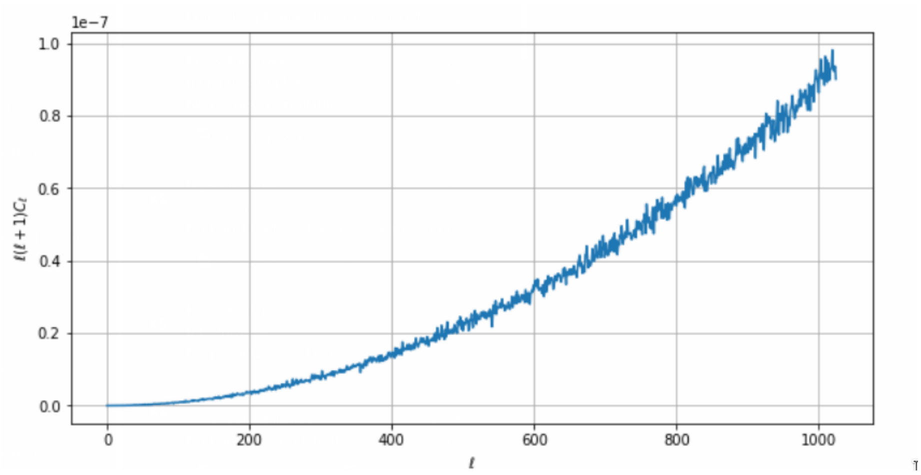

Figure 6: The CMB power spectrum produced in healpy directly from the Mathematica output utilizing completely random packing. We note that there is no apparent structure to the spectrum, and also that the scale of the vertical axis has changed considerably

Notably, the spectrum from the completely random scattering has no structure even remotely similar to a typical black body spectrum nor the CMB power spectrum. This highlights the fact that there is something unique about the packing of the clusters of galaxies in the cosmos and the detected power spectrum within the CCM. Further, the deviations noted between the simulated power spectrum and the true power spectrum past the $l \approx 400$ point could possibly be resolved by an alternate packing geometry, or various ratios of void size to cosmic radius.

As stated previously however, replicating the exact power spectrum is outside the bounds of this thesis. Doing so is likely an exercise in searching a wide parameter space for possible values which could reproduce various characteristics of the known power spectrum. However, it would appear that the CCM can, at the very least, reproduce the primary characteristics of the CMB, which is certainly a surprising result to fall out of such an approximate analysis.

4 Further Research

4.1 CMB Temperature

The calculation conducted throughout 3.1 provided a general overview of how the CMB functions within the chronometric model, and while the result of $T \approx 2.54$ K is promising in the context of reproducing the CMB in this model, it is not as complete an analysis as might be hoped.

There is the question of what exactly the "average" galaxy is in the context of the calculations in 3.1. The utilization of the Milky Way may or may not be entirely accurate, and, for instance, if one conducts the same analysis as found in 3.1.5, they will find the Andromeda galaxy (Messier 31) yields an average temperature of ~ 3.17 K. For general purposes we assumed the luminosities of the galaxies to be approximately equal, thus implying that there may be some mean distribution from which one can draw an average luminosity value. However, discerning these figures is not quite as straight forward. Astronomical measurements are subject to uncertainties coming from various sources, and in particular magnitude suffers from this. Thus, figures for these values vary largely between sources depending on the equipment and methods utilized. Further research is required for determining selection of this data so that a better understanding of the average temperature can be formed.

4.2 CMB Power Spectrum

The power spectrum simulation, while reproducing the primary characteristics of the experimentally observed power spectrum, is not exactly in line with the expected. While it would be perhaps foolish to assume that such an approximate simulation of the cosmos could produce an exact replica of the data, it would be ideal if the discrepancies following the $l = 400$ multipole moment could be brought into a smaller range. How exactly to do this, though, is a topic in searching a wide space of parameters in which there is not much direction to work from.

4.3 The Redshift

As stated in section 1.1, an analysis of the number count to redshift relation and magnitude to redshift relation has already been conducted to reasonable depth by Kaye in [?] utilizing modern astrophysical data. However, in light of new, high redshift data coming in from JWST as mentioned in [?], there will be a much larger pool of data to operate with than in the past, and at z values never before observed.

5 Conclusion

Throughout the course of this thesis we have explored a truly fantastical cosmological model. It is a world vastly unfamiliar to our intuitions as physicists and perhaps that is what drives it's appeal. However, the question of if this model is a mere toy in the sandbox of mathematics or a possible description of the universe we inhabit is yet unanswered.

While the world described in the Chronometric Model may feel overly simplisitc, appealing to the early Einstein universe as a static 3-sphere with no concept or care for a beginning or end, it is perhaps precisely in this simplicity which we find it's appeal. The Standard Cosmological model calls for dark matter and dark energy, a period of exponential inflation, and several free parameters in order to describe the cosmos at large, whereas the Chronometric Cosmos could not be more simplistic in its formulation and description of the observed universe. This disregard for the many parameters and place holders established by the Standard Model, along with a more rigorous mathematical underlying structure, only add to the model's inherit beauty. This is perhaps no more true than now, in a time where the Hubble parameter measurements are no longer overlapping in error, and as the fundamental concepts of early galaxy formation are beginning to come under question [**?**] [**?**].

While the beauty of the model is certainly notable, it remains critical to consider it from the perspective of what is and isn't physically possible. In Kaye [**?**] an analysis of the cosmological redshift (z-$N(z)$ and z-$M(z)$) was conducted in which the Chronometric Redshift relation was put to the test of modern data. Reasonable (and sometimes perhaps even great) agreement was shown between CCM predictions and data, thus showing that the Chronometric Model is not falsified by the redshift measurements at hand. In this thesis, the Chronometric Model was put to the test of reproducing yet another critical phenomena in modern cosmology; the CMB. It was shown that the two major characteristics of the CMB are in fact able to be reproduced in a first order approximation thus further solidifying Segal's theory as not yet falsified in light of modern data.

In physics, it is our job to probe the very nature of the universe, and in doing so gain some understanding of our surroundings much larger than ourselves. While the efficacy of Segal's theory as an overall description of the universe remains to be seen, there is certainly no doubt in it's ability to reproduce critical aspects of modern cosmological data from first order approximations, which is why the Chronometric model has been and continues to be an important line of consideration.

6 Acknowledgments

First and foremost I would like to acknowledge Krzysztof Sliwa for his continued guidance, patience, and mentorship, and to thank him for sharing this wonderful research experience with me.

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