homework #1 PHY 0091 Spring 2022

solutions are due on March 9, 2022

1) For the standard Cartesian coordinates in two-dimensional flat space R^2, a one-sphere S^1 of radius R centered at origin, $O = (0,0)$, is a circle, a set of points in two-dimensional space satisfying the equation:

$$x^2 + y^2 = R^2$$

Introduce the polar coordinates on R^2. A point P on can be described by the coordinates

$$x = r' cos\theta$$
$$y = r' sin\theta$$

where θ the angle in $\hat{x} \times \hat{y}$ plane, measured from $\hat{x}$ to $\hat{y}$. Find the equivalent of Friedman-Walker expression for the square of the geometrical interval dl^2 between two points in S^1. (Have a look at Max's thesis if you need guidance, or write me.) Find the circumference of S^1 or radius R by integration of the line element dl along the circle. Find the area of the disk bounded by a circle by integrating dA, the area element, in Cartesian and polar coordinates. One way to is consider the areas of thin shells (spherical slices) whose boundaries are two-spheres S^2 of radii varying from 0 to R.

2) For the standard Cartesian coordinates in three-dimensional flat space R^3, a two-sphere S^2 of radius R and centered at origin, $O = (0,0,0)$, is a set of points in three-dimensional space satisfying the equation:

$$x^2 + y^2 + z^2 = R^2$$

Introduce the spherical coordinates on S^2. A point P on S^2 can be described by the coordinates

$$x = r' cos\theta$$
$$y = r' sin\theta$$

where $r' = Rsin\phi$, θ the angle in $\hat{x} \times \hat{y}$ plane, measured from $\hat{x}$ to $\hat{y}$, and ϕ the angle from $\hat{z}$ to the vector $\overrightarrow{OP}$ (you can, of course, define the angles in a different way, frequently the definitions of θ and ϕ are interchanged). Find the equivalent of Friedman-Walker expression for the square of geometrical interval dl^2 between two points in S^2. (Have a look at Max's thesis if you need guidance, or write me.)

3) Find the area of a two-sphere S^2 of radius R by integrating dA, the area elements. One way to contract dA is to consider the areas of thin slices of S^2 whose boundaries are circles, one-spheres or S^1, of the radii varying from 0 to R and then to 0, while traveling along a great circle on S^2 from one antipode to another (or from one pole to another)

4) Find the volume of a ball of radius R by integrating dV, the volume elements, over the volume enclosed by S^2. One way to is to consider the volumes of thin shells (spherical slices) whose boundaries are two-spheres S^2 of radii varying from 0 to R.