homework #1 modified

PHY 0091 Spring 2022

1) For the standard Cartesian coordinates in two-dimensional flat space R^2, a one-sphere S^1 of radius R centered at origin, $O = (0,0)$, is a circle, a set of points in two-dimensional space satisfying the equation:

$$x^2 + y^2 = R^2$$

Introduce the polar coordinates on R^2. A point P on can be described by the coordinates

$$x = r' cos\theta$$
$$y = r' sin\theta$$

where θ the angle in $\hat{x} \times \hat{y}$ plane, measured from $\hat{x}$ to $\hat{y}$.

Find the equivalent of Friedman-Walker expression for the square of the geometrical interval dl^2 between two points in S^1. (Have a look at Max's thesis if you need guidance, or write me.) Find the circumference of S^1 or radius R by integration of the line element dl along the circle. Find the area of the disk bounded by a circle by integrating dA, the area element, in Cartesian and polar coordinates. One way to is consider the areas of thin shells (spherical slices) whose boundaries are two-spheres S^2 of radii varying from 0 to R.

2) For the standard Cartesian coordinates in three-dimensional flat space R^3, a two-sphere S^2 of radius R and centered at origin, $O = (0,0,0)$, is a set of points in three-dimensional space satisfying the equation:

$$x^2 + y^2 + z^2 = R^2$$

Introduce the spherical coordinates on S^2. A point P on S^2 can be described by the coordinates

$$x = r' cos\theta$$
$$y = r' sin\theta$$

where $r' = R sin\phi$, θ the angle in $\hat{x} \times \hat{y}$ plane, measured from $\hat{x}$ to $\hat{y}$, and ϕ the angle from $\hat{z}$ to the vector $\overrightarrow{OP}$ (you can, of course, define the angles in a different way, frequently the definitions of θ and ϕ are interchanged). Find the equivalent of Friedman-Walker expression for the square of geometrical interval dl^2 between two points in S^2. (Have a look at Max's thesis if you need guidance, or write me.)

homework #1 Summer Scholars 2022

solutions are due in June 2022

3) Find the area of a two-sphere S^2 of radius R by integrating dA, the area elements.

4) Find the volume of a 3D ball of radius R by integrating dV, the volume elements, over the volume enclosed by S^2. with $r \in (0,R)$)

5) Follow the same technique to find the volume of S^3, which can be considered as a 3D hypersurface in a specially 4D manifold with a restriction

$$x^2 + y^2 + z^2 + d^2 = R^2$$

Follow Max's Senior Thesis to see how to introduce a 4D analogue of a spherical co-ordinate system, and so on.

6) Derive the Friedman-Walker metric for S^3, basically following what is presented in Max's Senior Thesis

7) Find the volume of S^3 or radius R, you may use Mathematica to integrate the Friedman-Walker metric with curvature $k = 1$.

8) Check that the same integration for a space with $k = 0$ gives a familiar expression for a volume of a sphere in R^3

9) Check that your expression found in 6) agrees with equation 8. in Sandage's paper (there is a typo in that formula, the factor in from of the integral sign should be $2R^3$, not $2\pi R^3$)

10) Integrate the volume element in S^3 obtained with the F-W metric and check that you obtain equation 9. in Sandage's paper

11) Introduce a new variable $\rho = l/R$, where l is the manifold distance (distance traveled by light along a geodesic in S^3. Show that with this change of variables in 9. one obtains (modulo a constant involving constants $2,\ \pi, R$) the formula for $V(\rho)$ on page 97 in Segal's book