Day 6: 3D Geometry

3D Geometry Formulas

Volume of cube = s^3

Volume of sphere = 4/3 pi*r^3

Volume of rectangular prism = lxwxh = bh

Volume of pyramid = bh/3

Volume of triangular prism = lxwxh/2 = bh/2

Volume of cone = pi*r^2*h/3 = bh/3

Volume of cylinder = bh = pi*r^2*h

3D Geometry Formulas

SA of cube = 6s^2

SA of sphere = 4* pi * r^2

SA of rectangular prism = 2(lxw + lxh + wxh)

SA of square pyramid = a^2 + 2al, l is the slant height

SA of cone = πr(r+ √(h^2+r^2))

Lateral surface area means surface area excluding the bases

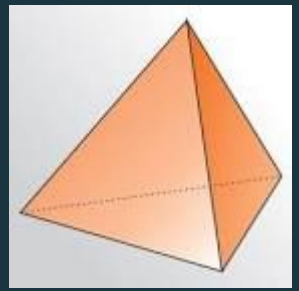
Distance between two points (a,b,c) and (d,e,f) is √((a-d)^2 + (b-e)^2 + (c-f)^2)

Distance between a point (m,n,q) and the plane Ax + By + Cz + D = 0 is
|Am + Bn + Cq + D|/(√(A^2 + B^2 + C^2))

Tetrahedrons

A tetrahedron is a 3D figure with 4 triangular faces. The volume of any tetrahedron is bh/3.

For regular tetrahedrons, the volume is $\sqrt{2}/12 * s^3$, where s is the length of any edge.



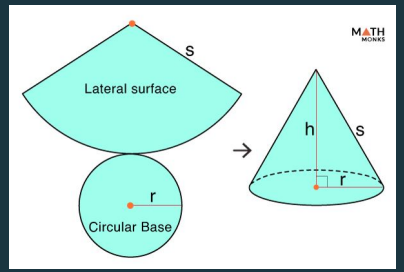
Unfoldings/Nets

A cone will unfold into a circle for the base and a sector of a circle for the lateral surface.

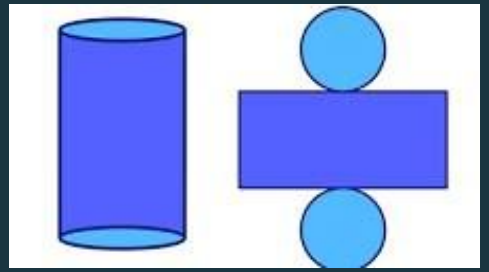
A cylinder will unfold into a rectangle with two circular bases.

A cube unfolds into a net with two square faces on one end, one square face in the middle and on each other end.

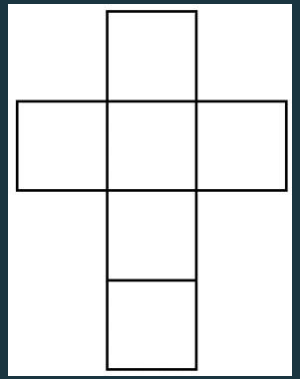
Unfoldings/Nets



Cone



Cylinder



Cube

Practice!

1. A regular hexagon with sides of length 6 has an isosceles triangle attached to each side. Each of these triangles has two sides of length 8. The isosceles triangles are folded to make a pyramid with the hexagon as the base of the pyramid. What is the volume of the pyramid?
2. A sphere with center O has radius 6 . A triangle with sides of length 15,15 , and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?
3. Eight spheres of radius 100 are placed on a flat surface so that each sphere is tangent to two others and their centers are the vertices of a regular octagon. A ninth sphere is placed on the flat surface so that it is tangent to each of the other eight spheres. The radius of this last sphere is a + b √ c, where a, b, and c are positive integers, and c is not divisible by the square of any prime. Find a + b + c.
4. Two right circular cones with vertices facing down contain the same amount of liquid. The radii of the tops of the liquid surfaces are 3 cm and 6 cm. Into each cone is dropped a spherical marble of radius 1 cm, which sinks to the bottom and is completely submerged without spilling any liquid. What is the ratio of the rise of the liquid level in the narrow cone to the rise of the liquid level in the wide cone?
5. A lampshade is made in the form of the lateral surface of the frustum of a right circular cone. The height of the frustum is 3√ 3 inches, its top diameter is 6 inches, and its bottom diameter is 12 inches. A bug is at the bottom of the lampshade and there is a glob of honey on the top edge of the lampshade at the spot farthest from the bug. The bug wants to crawl to the honey, but it must stay on the surface of the lampshade. What is the length in inches of its shortest path to the honey?
6. Let ABCD be a tetrahedron such that AB = CD = √ 41, AC = BD = √ 80, and BC = AD = √ 89. There exists a point I inside the tetrahedron such that the distances from I to each of the faces of the tetrahedron are all equal. This distance can be written in the form m √ n p , when m, n, and p are positive integers, m and p are relatively prime, and n is not divisible by the square of any prime. Find m + n + p. (this was problem 14 on the 2024 AIME I!)