IEM 4013 Project

Team Name: Operation Equal Population

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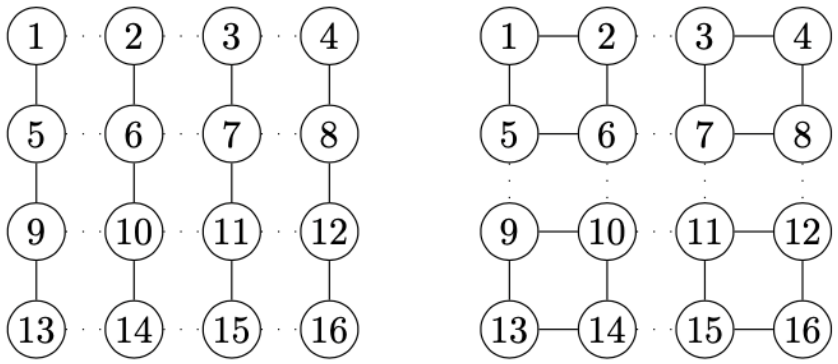
State: New Mexico

I. Executive Summary

The goal of this report is to find a new solution for redistricting in the state of New Mexico. The requirements are that all applicable laws and guidelines be followed, both federally and at the state level. Additionally, the team desired to keep the maximum population deviation between districts at less than 1%, and to avoid any sort of gerrymandering that might adversely affect certain groups of voters.

The project team used a purely mathematical system to determine the ideal districts, without taking the voting patterns of the state into account, so as to avoid bias. The mathematical method that was used to determine the most compact districts possible is called the Labeling Model. This method essentially minimizes the number of borders in between districts in order to ensure compactness. In the following example, the 16 dots each represent a single county, with lines drawn between counties that share a border. (ex. County 6 shares borders with counties 2, 5, 7, and 10) A cut edge is when the border of a district lines up with the border between counties, and thus, two neighboring counties are each in different districts.

Figure 1: Labeling Model Example



In Figure 1, the four districts create 12 cut edges, making that the least compact arrangement of the districts according to the Labeling Model. Figure 2, on the other hand, only creates 8 cut edges with the same number of districts, and thus is the most compact arrangement of the districts possible according to the labeling model, for this set of counties. This model was scaled up and used to determine the most compact arrangement of the counties of New Mexico into its 3 congressional districts.

II. Introduction

Redistricting is the process of redrawing political districts on the map in order to determine which constituents will vote together for the same set of candidates. This happens every ten years in the United States when the census is taken and the government gains data about the populations of certain areas. If one district has too many more people than the other districts in that state, then the state will then need to be redistricted. There are many possible goals when redistricting an area, and this process has been used frequently to gerrymander, or in other words, to draw the districts in such a way that certain candidates or political parties will have an advantage or disadvantage. Gerrymandering can also be used to give certain groups of voters more or less power. For example, the person who draws the district lines could separate a group of black voters into multiple districts to dilute their voting power. The goal of this report is the opposite: to draw the districts in such a way that elections will be as fair as possible. The project team endeavored to make the districts as even in population as possible, and as fair as possible for all groups of voters.

III. New Mexico Redistricting Criteria

There are several state and federal criteria that must be met when creating new congressional districts. These come from the U.S. Constitution, the Voting Rights Act of 1965, and the New Mexico State Legislative Council. All new plans proposed by the project team must follow all of these laws, and if any certain plan fails to adhere to them, it will need to be rejected.

United States Federal Requirements:

I. Apportionment Clause of Article I, Section 2, of the U.S. Constitution requires that all districts be as nearly equal in population as possible:

"Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons."

II. Section 2 of the Voting Rights Act of 1965 prohibits plans that either intentionally or unintentionally discriminate on the basis of race, which could dilute the minority vote:

"No voting qualification or prerequisite to voting, or standard, practice, or procedure shall be imposed or applied by any State or political subdivision to deny or abridge the right of any citizen of the United States to vote on account of race or color."

New Mexico State Requirements:

The following requirements were adopted by the New Mexico state legislative council for use in developing a congressional redistricting plan based upon the 2010 Census data:

1. Congressional districts shall be as equal in population as practicable.
2. The legislature shall use 2010 federal decennial census data generated by the United States bureau of the census.
3. Since the precinct is the basic building block of a voting district in New Mexico, proposed redistricting plans to be considered by the legislature shall not be comprised of districts that split precincts.
4. Plans must comport with the provisions of the Voting Rights Act of 1965, as amended, and federal constitutional standards. Plans that dilute a protected minority's voting strength are unacceptable. Race may be considered in developing redistricting plans but shall not be the predominant consideration. Traditional race-neutral districting principles (as reflected in paragraph seven) must not be subordinated to racial considerations.
5. All redistricting plans shall use only single-member districts.
6. Districts shall be drawn consistent with traditional districting principles. Districts shall be composed of contiguous precincts and shall be reasonably compact. To the extent feasible, districts shall be drawn in an attempt to preserve communities of interest and shall take into consideration political and geographic boundaries. In addition, and to the extent feasible, the legislature may seek to preserve the core of existing districts, and may consider the residence of incumbents.

IV. Current Districts

The state of New Mexico contains 3 congressional districts. The congressional districts are shown here:

Figure 2: Current New Mexico Congressional Districts

New Mexico meets the criteria of compactness, contiguity, preservation of communities of interest, and preservation of cores of prior districts. According to the 2010 Redistricting Deviation Table, the current population deviation for New Mexico is 0.0%. Using the numbers from the 2010 census, the population bounds would have to remain between 682,961 and 689,825 in order for it to retain a 1% population deviation. New Mexico actually retains its own redistricting information at nmlegis.gov. Other than that, the redistricting atlas and Dr. Buchanan's County data will probably be consulted.

V. OR Model

Sets: C is the set of counties in New Mexico

Indices: i is a county in New Mexico

j is a district in New Mexico

Parameters: P_i is the population in given county i

k is the amount of districts (k=3)

L = the district with the lowest population

U = the district with the highest population

Variables: x_{ij}= 1 if county i is assigned to district j (i ϵ C and j ϵ (1,2,3)); 0 otherwise

y_{ij} 1 if edge $e \epsilon E$ is cut; 0 otherwise

r_{ij}=a root of a county assigned to a specific district

f_{ij} =the flow of given a county assigned to a specific district

Minimize the total amount of cut edges	Min $\sum_{e\varepsilon E} y_e$	
Make edge $e = \{u, v\}$ cut when vertex u is assigned to district j	subject to: $x_{uj} - x_{vj} \leq y_e$	$\forall_e = \{u, v\}\epsilon E,\ \forall_j\epsilon\{1,2,\dots,k\}$
Make each vertex $i \epsilon assigned$ to a singular district	$\sum_{j=1}^{k} x_{ij}$=1	$\forall_i\epsilon V$
Make the population of each district between parameter L and U	L$\leq \sum_{\text{i}\ \epsilon\ C} p_i \leq$ U	$\forall_j\epsilon\{1,2,\dots,k\}$
Each county assigned to a specific district and every cut edge is either 0 or 1	$x_{ij}\epsilon\{0,1\}$ $y_e\epsilon\{0,1\}$	$\forall_i\epsilon V,\ \forall_j\epsilon\{1,2,\dots,k\}$ $\forall_e\epsilon E$
Make every district have a singular root	$\sum_{i\epsilon V} r_{ij} = 1$	$\forall_j\epsilon\{1,2,\dots,k\}$

Vertex $i \epsilon V$ must not root to any district j	$r_{ij} \leq x_{ij}$	$V_i \epsilon V, \forall_j \epsilon \{1,2,\dots,k\}$
Make vertex i consume flow when it is not a root	$\sum_{u \epsilon N(i)} (f_{ui} - f_{iu}) \geq 1 - M \sum_{j=1}^{k} r_{ij}$	$\forall_i \epsilon V$
Prevent flow across cut edges	$f_{ij} + f_{ji} \leq M(1 - y_e)$	$\forall_e = \{i, j\} \epsilon E$
Make the flow a nonnegative value	$f_{ij,}\, f_{ji} \geq 0$	$\forall_{\{i,j\}} \epsilon E$
Make each root either 0 or 1	$r_{ij} \epsilon \{0,1\}$	$\forall_i \epsilon V, \forall_j \epsilon \{1,2,\dots,k\}.$

Table 1: OR Model

Description of OR Model

For the integer program, the objective is to minimize the total amount of cut edges. The constraints for this program are 1) assigning each county to one particular district. 2) assign each vertex to a district. 3) Make each district have a population between L and U. 4) Assign each county to a specific district and every cut edge is either 0 or 1. 5) Assign each district to one root. 6) Assign vertex for a county to consume the flow when it is not assigned to a root. 7) Assign cut edges to not have any flow. 8) The value of flow must be positive. 9) Assign the value of a root to be 0 or 1.

VI. Problem Statement

The assigned problem is to develop a congressional districting plan that withstands legal scrutiny and satisfies state and federal laws. The objective function is to minimize the difference between the most populated district and the least populated district. The constraints used will make each county be fixed to one of the three districts, make the population of each district fall between the lowest and highest population of a Congressional District, and make a county either associate with a specific district or not (involve the full county population or not). Counties will be kept as a whole when accounting for the population of a county to make a Congressional District.

VII. Experiment Discussion

The optimization model was written, combined, and solved using the Gurobi application (version 9.1.1) Jupyter Notebook, and the coding language used was Anaconda. The model was run on a Dell G7 Gaming Laptop, which contains 16.0 GB of Ram and an Intel(R) Core(TM) i7-8750H CPU @ 2.20GHz as its processor. The program used is a cut edges program with geopandas and contiguity. Perimeter programs and moment of inertia programs were also tested, but the cut edges best represented the desired solution. In this program, the objective value is

listed as the number of cut edges, and in this case the number of cut edges was 17. The program solved the model in a negligible amount of time, but the finished model listed the solving time as 2.62 seconds. The model is solved to optimality.

IX. Plan and Map

Our proposed plan is as follows:

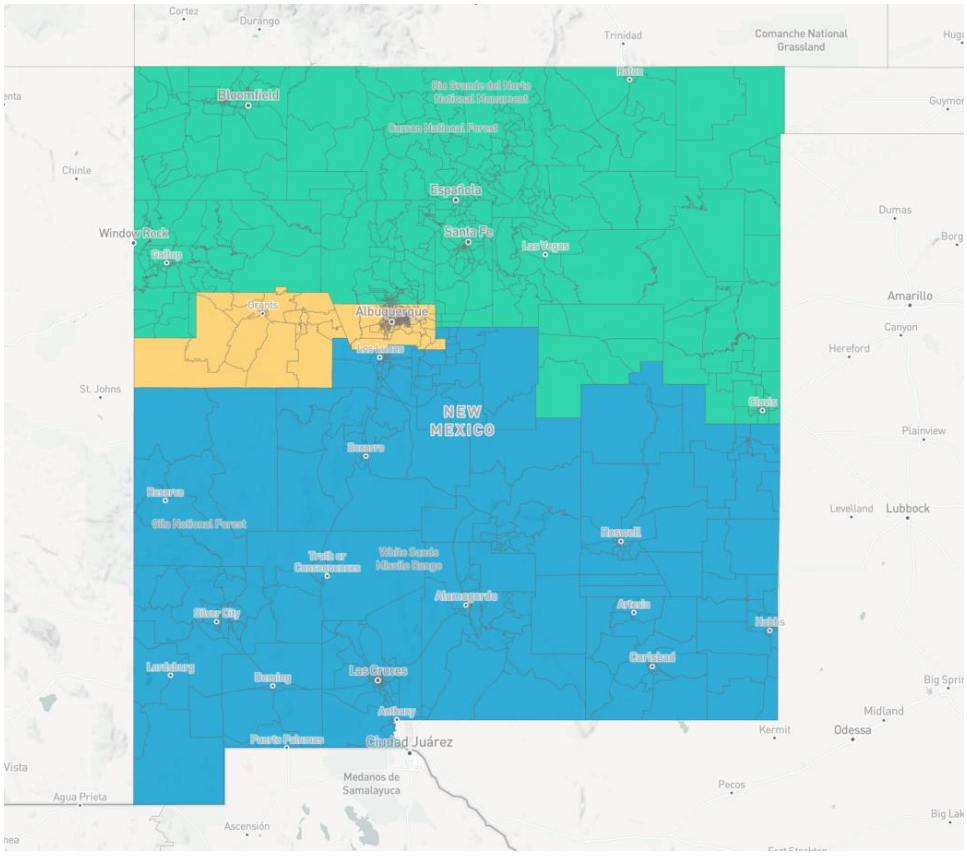
District 1 will consist of Cibola and Bernalillo counties.

District 2 will consist of Sierra, Lea, Torrance, Grant, Otero, Roosevelt, Hidalgo, Eddy, De Baca, Doña Ana, Chaves, Valencia, Catron, Socorro, Lincoln, and Luna counties.

District 3 will consist of Harding, Guadalupe, San Juan, Curry, Taos, Quay, Colfax, Los Alamos, Rio Arriba, San Miguel, Sandoval, Santa Fe, McKinley, Mora, and Union counties.

The populations of each district are 689,777, 685,630, and 683,772, respectively. The plan is shown here on a map of New Mexico with the yellow representing district 1, the blue district 2, and the green being district 3.

Figure 3: New Congressional Districts for New Mexico



X. Evaluation of Proposed Redistricting Plan

This plan conforms well to the criteria set by both the U.S. federal government as well as the state of New Mexico. The maximum population deviation between the most populated district (District 2) and the least populated district (District 1) is less than half a percent at 0.49%. This plan does not discriminate based upon race, nor does it separate any communities of interest or political subdivisions, instead only separating districts on the county level. The plan also creates districts that are quite compact, preserves the cores of the prior districts, and refrains from redistricting in such a way that the current incumbents would no longer reside in their districts.

The proposed solution is limited in many ways however. None of the project team's members reside in the state of New Mexico and thus local knowledge of the area and the communities living there is limited. The mathematical model used to draw the districts is also limited in that it does not consider subcommunities and voting patterns, which while making it unbiased in some sense, means that it could be possible for it to gerrymander the state unintentionally.

XI. Conclusion

By using a labeling model and a cut edges program, the optimal three districts have been created; with populations of 689,777, 685,630, and 683,772, respectively. The maximum population deviation remains under .5% on the upper and lower bounds, and the model conforms to the state and federal criteria. The project team recommends that the state of New Mexico adopt this districting plan to improve population deviation and the compactness of its districts.