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Problem Set 2 Due Feb. 3

CPSC 455 Chaos and Dynamical Systems

1. Consider the map $F(x) = x^2 - 1.1$

a) find fixed points

$x = x^2 - 1.1$

$x^2 - x - 1.1 = 0$

$\frac{1 \pm \sqrt{1^2 - 4(1 \times 1.1)}}{2}$ $x = \frac{1}{2} + \frac{\sqrt{5.4}}{2}$

$x = \frac{1}{2} - \frac{\sqrt{5.4}}{2}$

b) find 2-cycle ~~2-cycle between~~ $\frac{1-\sqrt{5.4}}{2} < x_0 < \frac{1+\sqrt{5.4}}{2}$

Graphs on next page

2. a orbit analysis $F(x) = x\sin(x)$

$\frac{\pi}{2} + 2m\pi = a_m \sin(a_m)$

$$\lim_{n\to\infty} F^n(x) = \begin{cases} 0 & \text{if } |x_0| < \frac{\pi}{2} \\ \frac{\pi}{2} + 2m\pi & \text{if } \frac{\pi}{2} + 2m\pi < x_0 < a_m \end{cases}$$

b. $F(x) = \frac{1}{x}$

Fixed point at 1

$$\lim_{n\to\infty} F^n(x) = \begin{cases} 1 & \text{if } |x_0| = 1 \\ \text{2-cycle between } \frac{1}{x_0} \text{ and } x_0 & \text{if } 0 < |x_0| < 1, \; 1 < |x_0| < \infty \\ x_0 \neq 0 & \end{cases}$$

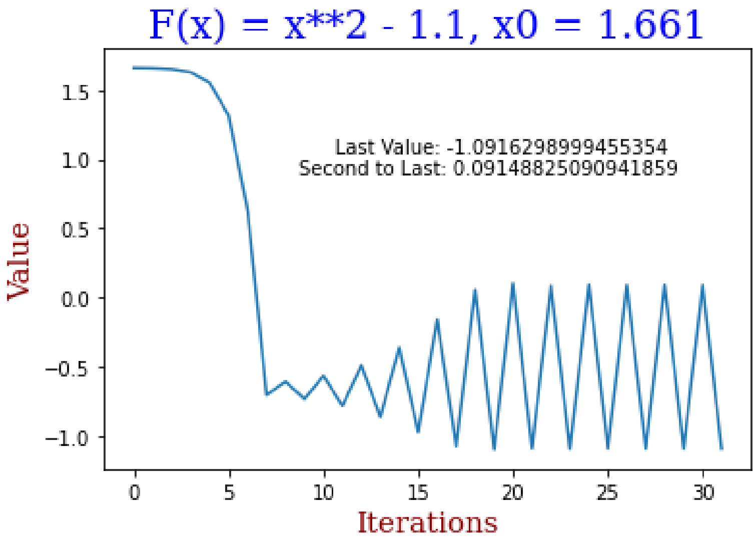
c. $F(x) = e^x$ $\lim_{n\to\infty} F^n(x) = \infty$ for any x_0

no fixed points

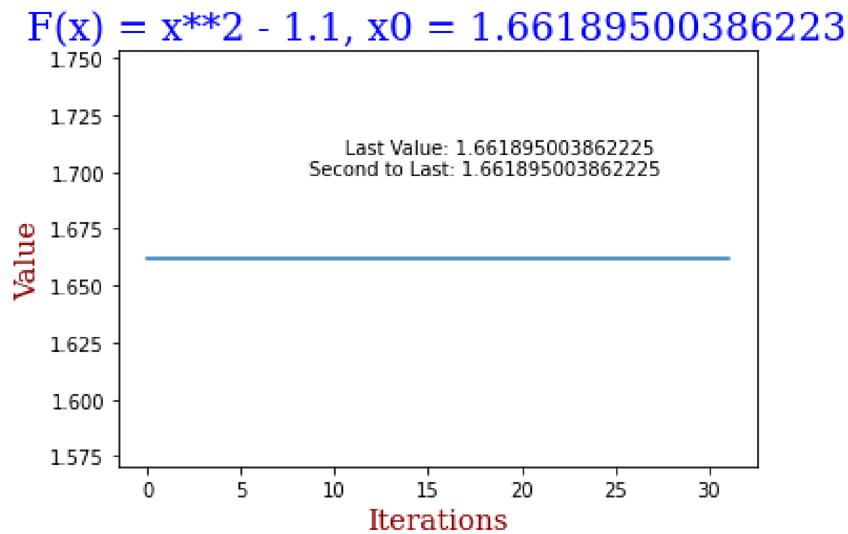
d. $F(x) = x^2 + 1$ $\lim_{n\to\infty} F^n(x) = \infty$ for any x_0

no fixed points

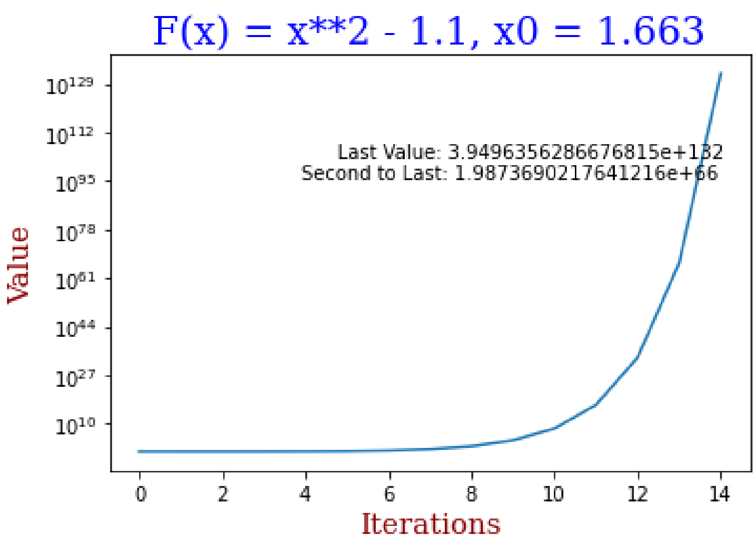
1. A, B



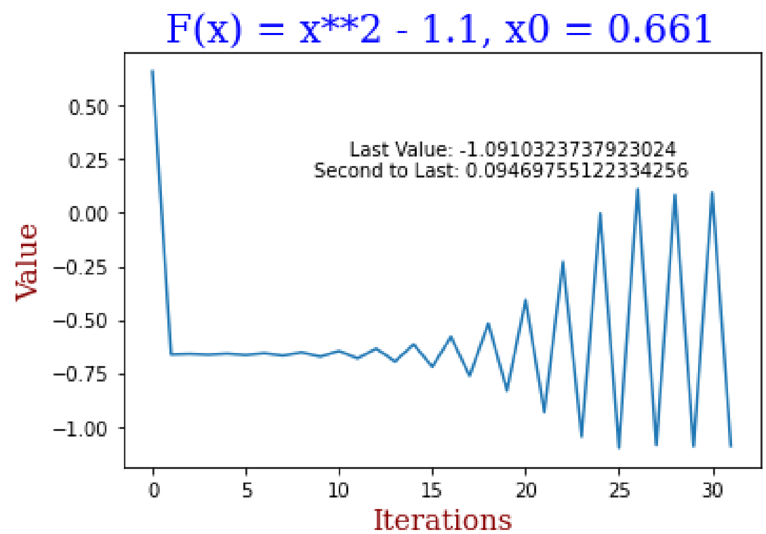
$\left|\frac{1}{2} - \frac{3\sqrt{0.6}}{2}\right| < x_0 < \frac{1}{2} + \frac{3\sqrt{0.6}}{2}$, 2-cycle



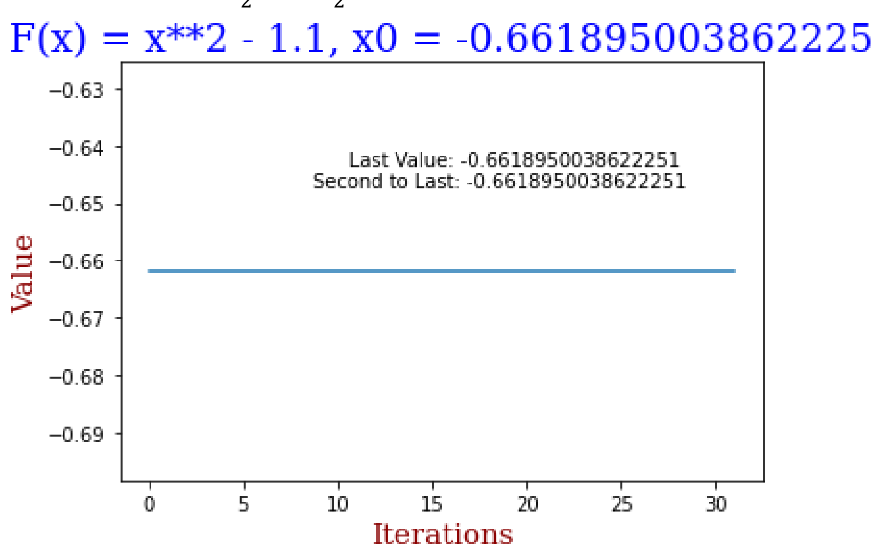
$x_0 = \frac{1}{2} + \frac{3\sqrt{0.6}}{2}$, fixed point



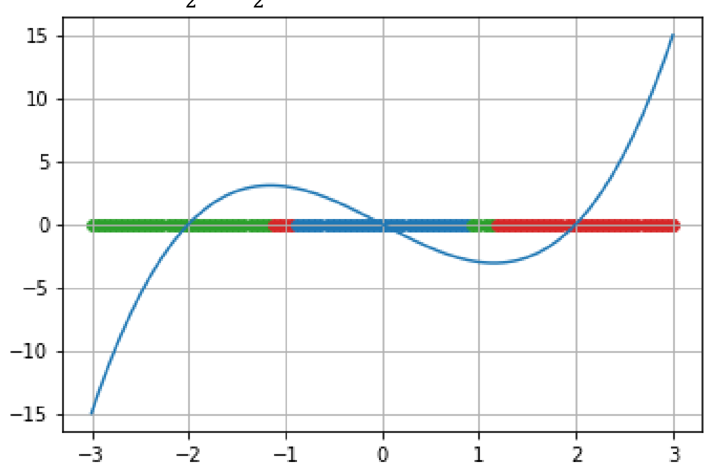
$X_0 > \frac{1}{2} + \frac{3\sqrt{0.6}}{2}$, -> ∞



$0 \le x_0 < \left|\frac{1}{2} - \frac{3\sqrt{0.6}}{2}\right|$, 2-cycle



$x_0 = \frac{1}{2} - \frac{3\sqrt{0.6}}{2}$, fixed point



Blue values converge to 0
Red values converge to 2
Green values converge to -2

C2: The structure of the basin of attraction implies that newton's method can be used to find the basin of attraction

C3: Newton's method will fail if the derivative of F(x), F'(x) is equal to 0. Because:

$$N(x) = x - \frac{F(x)}{F'(x)}$$

And there cannot be a zero in the denominator