CPSC 455 / MATH 452 Chaos and Dynamical Systems

Problem Set 2, due Wednesday, February 2

Directions: In order to receive full credit for correct answers it is necessary to follow instructions, show your clear organized work, and/or provide a well-reasoned explanation. Proper mathematical notation is expected.

Please note that unless indicated otherwise, Parts I & II should be completed by all students.

Part I. Exercises for Chapter 4: Graphical Analysis

1. Consider the map $F(x) = x^2 - 1.1$.

(a) Find the fixed points of F

(b) Use part (a) to find the a 2-cycle for F.

2. Perform a complete orbit analysis for:

(a) $F(x) = x \sin x$

(b) $F(x) = 1/x$.

(c) $F(x) = e^x$.

(d) $F(x) = x^2 + 1$.

3. (BONUS PTS) Recall that Newton's Method associated with a function F is $N(x) = x - \frac{F(x)}{F'(x)}$. Prove that p is a fixed point of N if and only if p is a zero of F.

Part II. Computer Exercise

If p is an attracting fixed point, then the set of all initial conditions x_0 with the property that the sequence $\{F^n(x_0)\}$ converges to p is called the **basin of attraction** of p. It can be shown that every fixed point of Newton's Method is attracting. Consider the map

$$F(x) = x(x^2 - 4).$$

C-1: Adapt your Newton's method code to find the basin of attraction of each fixed point of F.

C-2: What does the structure of the basin of attraction imply for Newton's Method.

C-3: Describe the set of initial values in the domain of F for which Newton's Method will fail. Explain graphically why it fails in these cases.