CPSC 455 Chaos and dynamical systems
Problem Set 3, Due Wednesday, February 9

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Part 2.

1. (a) $A(x) = \arctan x$ $\arctan x = x$

$A'(x) = \dfrac{1}{x^2+1}$ $x = 0$

fixed point at $x = 0$

$A'(0) = \dfrac{1}{0+1} = 1$ $A'(0.1) = \dfrac{1}{1.01} < 1$

attracting fixed point $A'(-0.1) = \dfrac{1}{1.01} < 1$
at $x = 0$

(b)
$$T(x) = \begin{cases} 2x & \text{if } x \le \frac{1}{2} \\ 2-2x & \text{if } x > \frac{1}{2} \end{cases}$$

$2x = x$ $2-2x = x$
$x = 0$ $2 = 3x$
      $x = \frac{2}{3}$

$$T'(x) = \begin{cases} 2 & \text{if } x < \frac{1}{2} \\ -2 & \text{if } x > \frac{1}{2} \end{cases}$$

$|T'(0)| = 2$
$|T'(\frac{2}{3})| = |-2| = 2$

repelling fixed points at $x = 0$, $x = \frac{2}{3}$

2. $F(x) = -\frac{1}{2}x^3 - \frac{3}{2}x^2 + 1$ $-\frac{1}{2}x^3 - \frac{3}{2}x^2 + 1 = x$

$F'(x) = -\frac{3}{2}x^2 - 3x = -\frac{3}{2}x(x+2)$ $F^n(x_0) = 0, 1, -1, 0, 1, -1$
                         $x_0 = 0$

$F'(0) \times F'(1) \times F'(-1) = 0$ This orbit is attracting

3. $D(x) = \begin{cases} 2x & \text{if } 0 \le x < \frac{1}{2} \\ 2x-1 & \text{if } \frac{1}{2} \le x < 1 \end{cases}$ $D^n = D(D(x))$

$(D^n)'(0.2) = 4$
$(D^n)'(0.4) = 4$
$(D^n)'(0.6) = 4$ repelling
$(D^n)'(0.8) = 4$

4. $F(P) = \dfrac{x^3}{6} + x$ $F(P) = \dfrac{P^3}{6} + P$ $F(0) = 0$

                      $F'(P) = \dfrac{P^2}{2} + 1$ $F'(0) = 1$ ✓

$F''(0.1) = 0.1$ positive concavity $F''(0) = 0$ ✓

$F''(-0.1) = -0.1$ negative concavity $F''(P) = P$ $F'''(0) = 1$ $1 > 0$ ✓
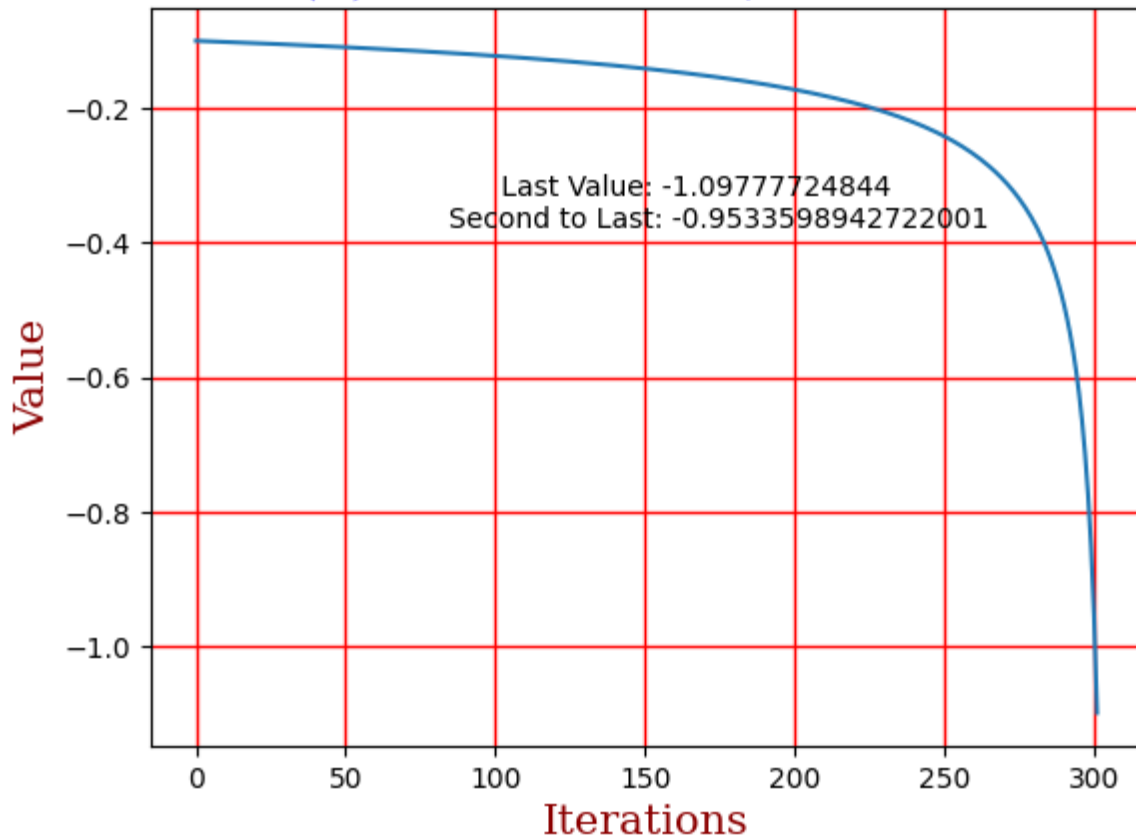
                         $F'''(P) = 1$

F(x) = x**3/6 + x, x0 = 0.1

Last Value: 1.09777724844
Second to Last: 0.9533598942722001

Value

Iterations

F(x) = x**3/6 + x, x0 = -0.1

Last Value: -1.09777724844
Second to Last: -0.9533598942722001

Value

Iterations
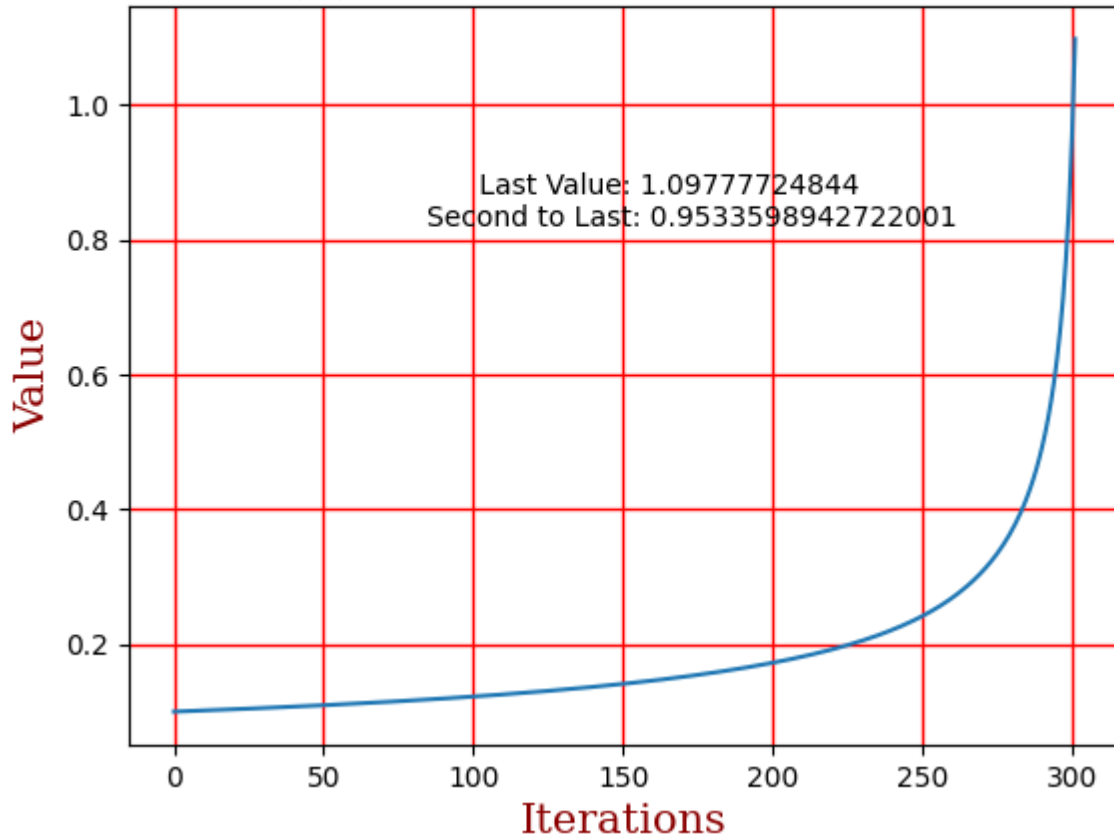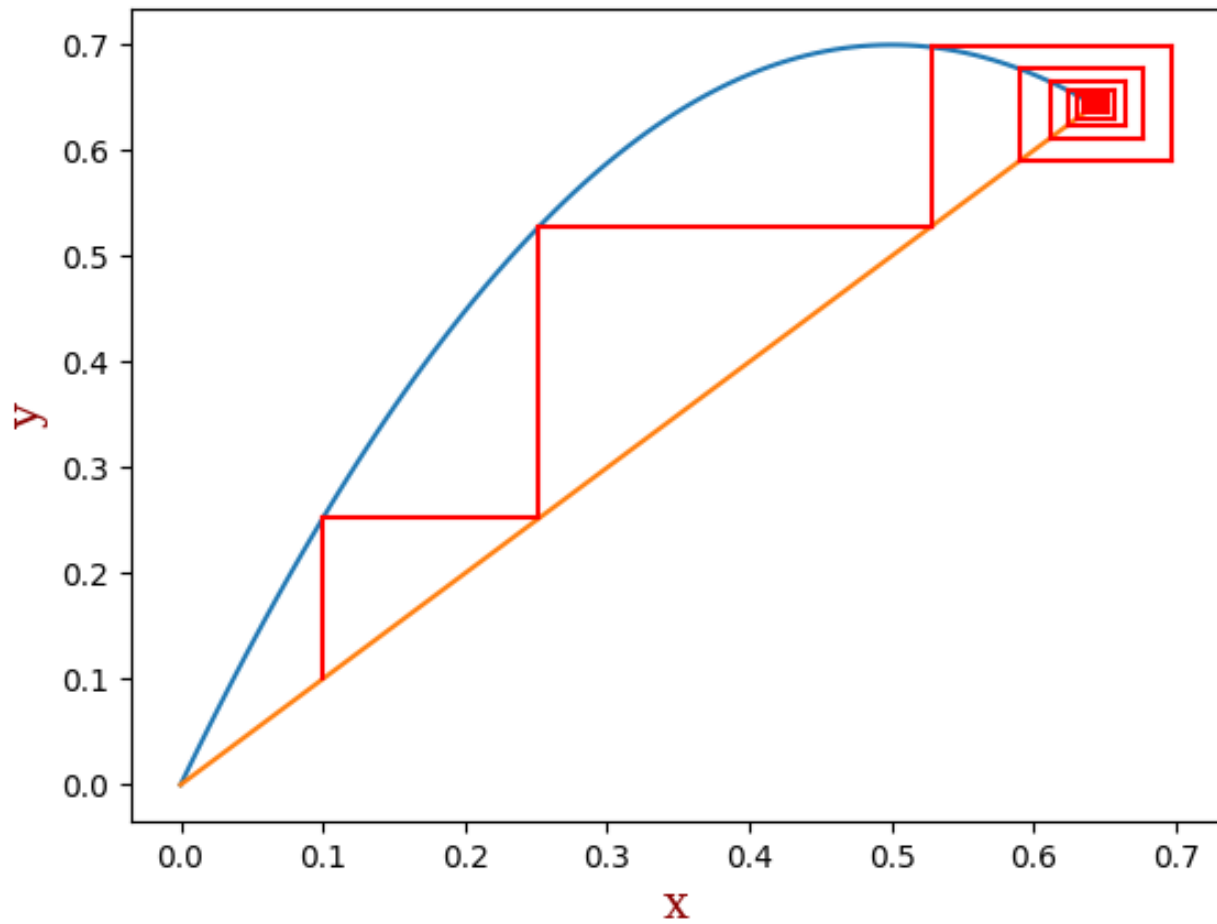
F(x) = 2.8*x*(1 - x), x0 = 0.1

F(x) = x**3/6 + x, x0 = 0.11
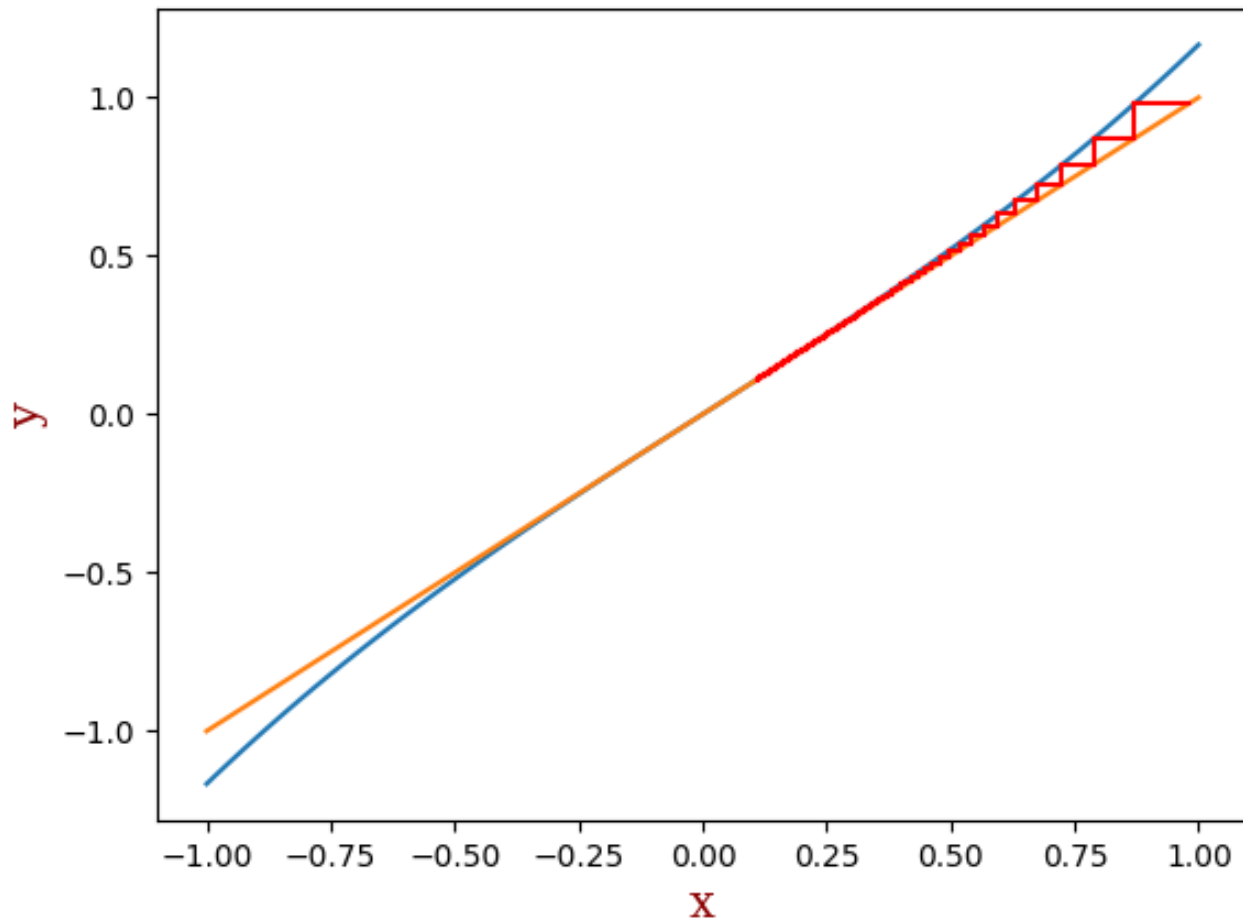
F(x) = atan(x), x0 = 1
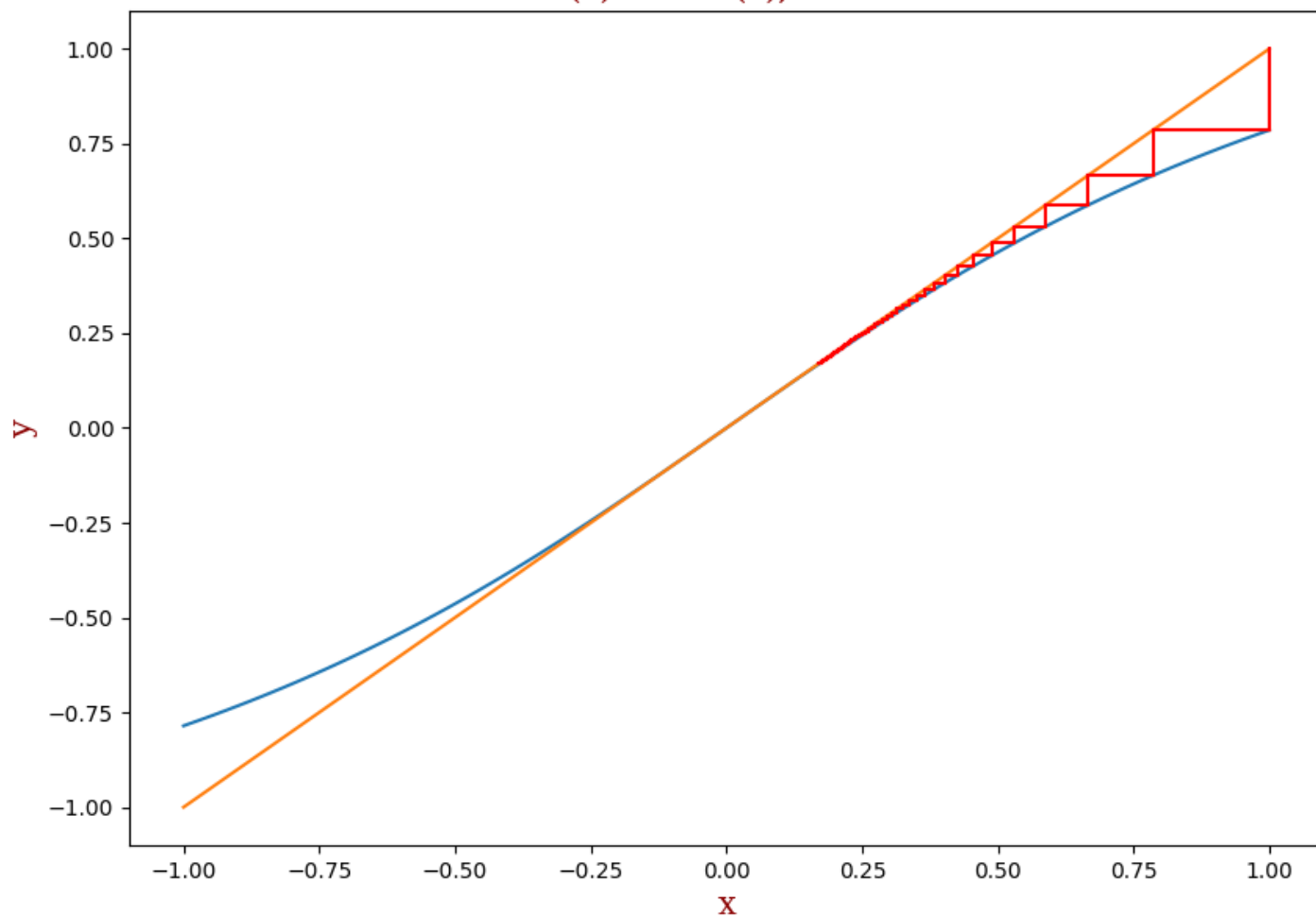
F(x) = Piecewise((2*x, (x >= 0) & (x < 0.5)), (2*x - 1, (x >= 0.5) & (x < 1)), (x, True)), x0 = 0.1
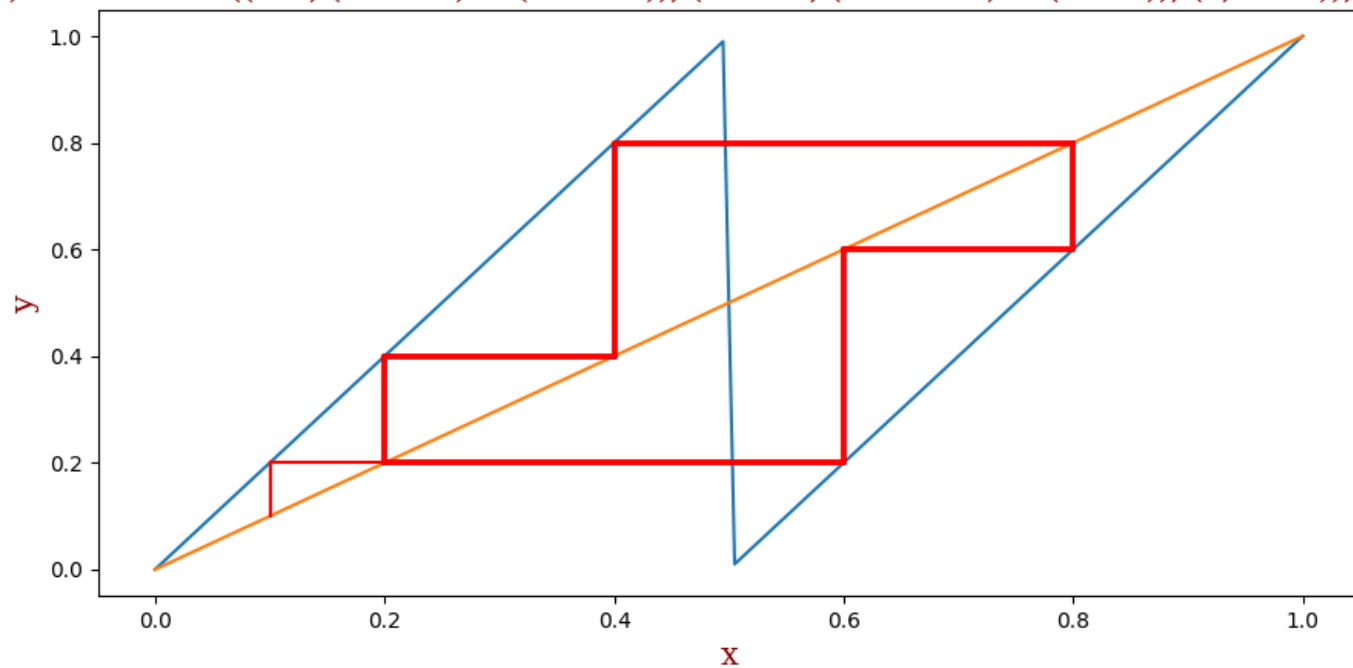
F(x) = 2.8*x*(1 - x), x0 = 0.9
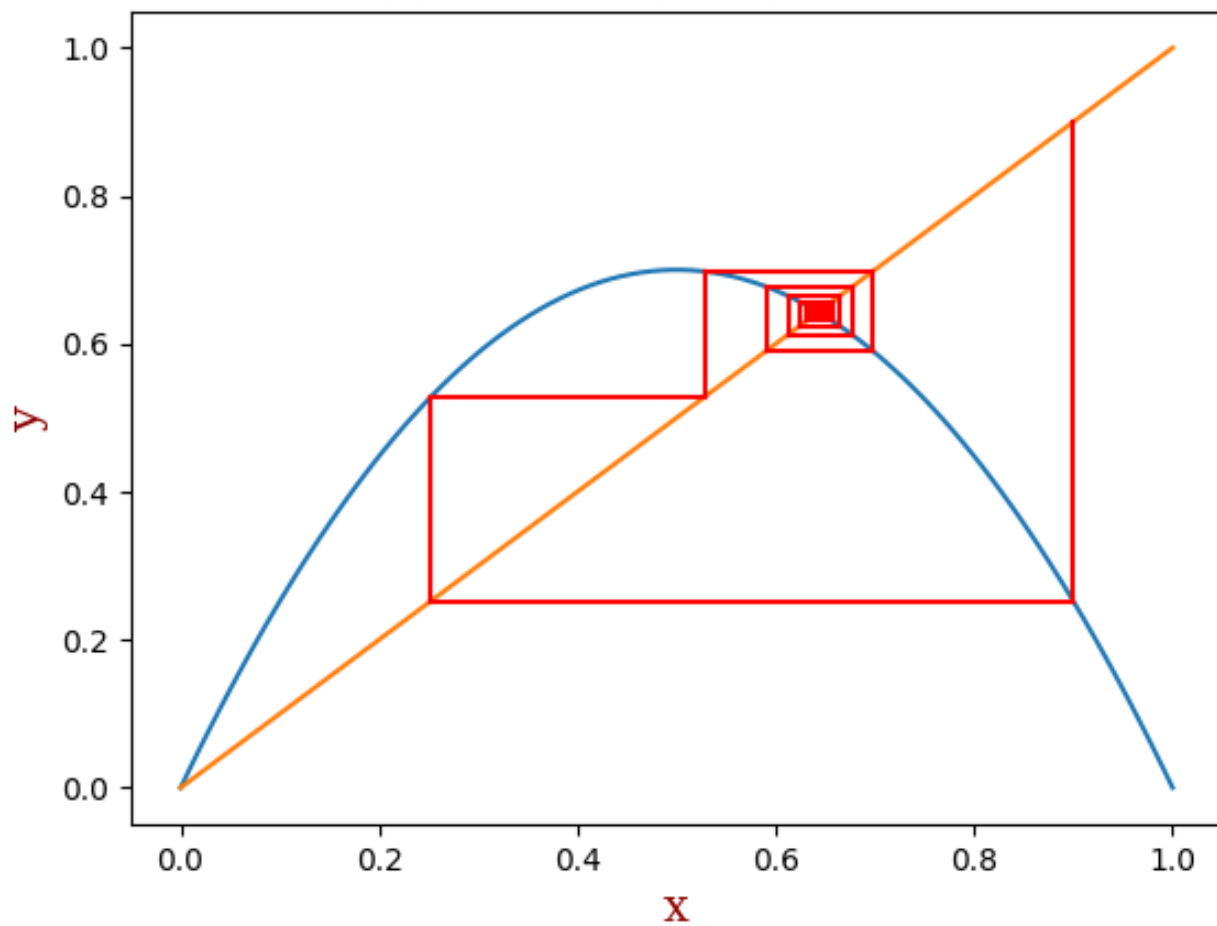
F(x) = x**3/6 + x, x0 = -1
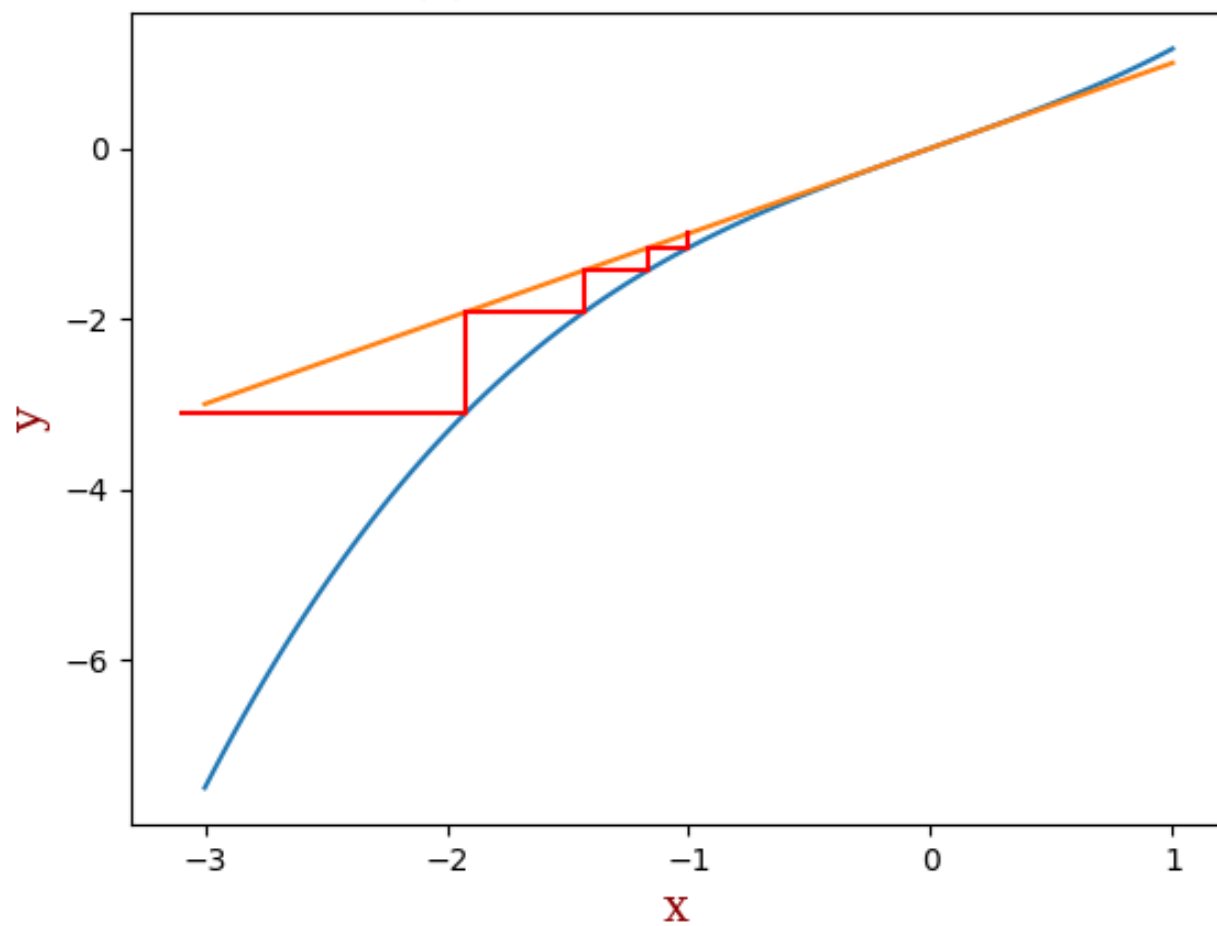
F(x) = atan(x), x0 = 2

F(x) = Piecewise((2*x, (x >= 0) & (x < 0.5)), (2*x - 1, (x >= 0.5) & (x < 1)), (x, True)), x0 = 0.111