TOWARD A GAUSSIAN MIXTURE MODEL FOR CHANCE CONSTRAINED ORBITAL TRANSFER OPTIMAL CONTROL

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MOTIVATION

- Spacecraft often must undergo orbital transfers
- Optimal control is used to determine the solution
- Control solution depends on the state
- State uncertainty is guaranteed
- Can represent the state uncertainty with a Gaussian
- Works for two body problem, run into problems with 3-body

OVERVIEW OF PROJECT

01

SURVEY OF PRIOR WORK

Survey of chance constrained optimal control for optimal control

02

DERIVATION

Derivation of necessary conditions for gaussian mixture model

03

UNCONSTRAINED ALGORITHM DEVELOPMENT

Development of Algorithm and testing

04

GAUSSIAN MIXTURE MODEL

Future work

BUILDING OFF CURRENT RESEARCH

- Oguri et al. give a solution for the constrained optimal control problem, solved with indirect approach
  - Derive the necessary conditions with a Gaussian model of the state
- Choose to build off this research with a Gaussian mixture model, to capture higher order moments of the state uncertainty.

DERIVING THE NECESSARY CONDITIONS

- The first half of the semester was devoted to deriving the necessary conditions for the Gaussian Mixture model.
- Stochastic problem to deterministic problem
- New state representation with pdf
- Path constraint is computed with unscented transform
- Leads to optimal control problem
- Augment state with K*n dimensions, for each component in Gaussian mixture
- Augment Covariance with K*n x K*n dimensions

$$p(t,x) = \sum_{i=1}^{K} w_i p_{g_i}$$

$$p_{g_i} = \mathcal{N}(x(t)|\mu_i, P_i)$$

$$Pr[h(x,t) \leq 0] \geq \alpha$$

$$[\bar{h}_i, P_{h,i}] = UT(\mu_i, P_i, h(x,t))$$

$$\bar{h} = \sum_{i=1}^{K} w_i \bar{h}_i$$

$$P_h = \sum_{i=1}^{K} w_i P_{h,i}$$

DETERMINISTIC OPTIMAL CONTROL PROBLEM

$$J = \int_{t_0}^{t_f} L dt$$

$$\dot{\mu} = f(\mu, m, u, t), \dot{m} = -\Gamma \dot{m}_m ax$$

$$\dot{P} = AP + PA^T + GG^T$$

$$\psi_s \leq 0$$

$$\psi_0(\mu_i, t)|_{(t_0)} = 0$$

$$\psi_f(\mu_i, t)|_{(t_f)} = 0$$

Where $\mu \in R^{Kn_x}$ and $P \in R^{Kn_x \times Kn_x}$

Time optimal transfer, L = 1

IMPLEMENTATION OF UNCONSTRAINED CASE, GAUSSIAN MODEL

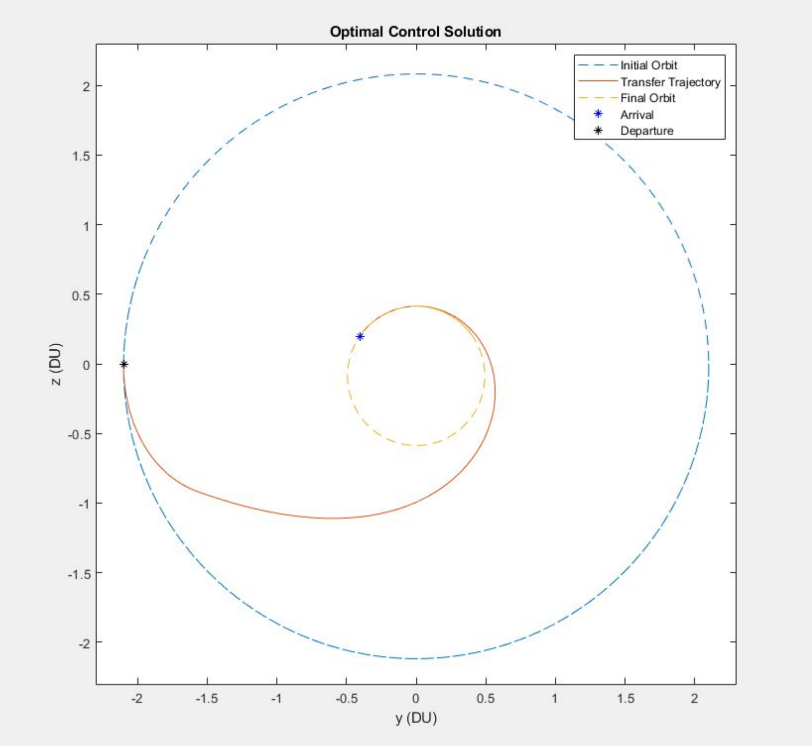
- Solver was implemented using MATLAB
- The 50 coupled ODEs were propagated with ODE45
- The TPBVP was solved with fsolve
- Used same test case as Oguri et. al.:
- Spacecraft Maneuver orbiting an asteroid

semi major axis: 2 km $\longrightarrow$ 0.5 km

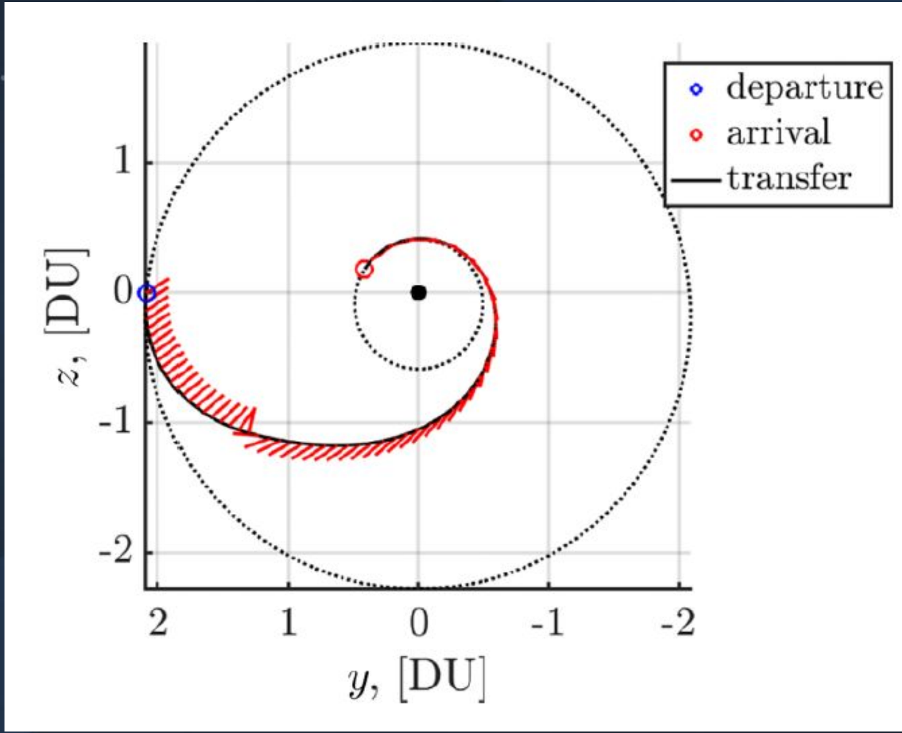
eccentricity: 0.173 $\longrightarrow$ 0.085

true anomaly: free

SOLUTION



Optimal Control Solution
Initial Orbit
Transfer Trajectory
Final Orbit
Arrival
Departure
z (DU)
y (DU)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2



departure
arrival
transfer
z, [DU]
y, [DU]
1
0
-1
-2
2
1
0
-1
-2

FUTURE WORK

- Finalize writeup for development of the Gaussian Mixture necessary conditions
- Now that unconstrained indirect solver is working, add constraint equations to necessary condition propagation which uses the Unscented Transform
- Implement the Gaussian Mixture model on the constrained indirect solver.

THANK YOU FOR YOUR TIME

Any questions?