SENG 474: Assignment 2 Report

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Part 1: Loading the Data

In order to use sci-kit learn's implementations of logistic regression and support vector machines, we must load the data into a digestible form. For this, we use a pandas data frame. Note: if you are trying to run my code, make sure that the fashionmnist repo is in the folder above the folder containing the code.

```
def read_data():
    X_train, y_train = mnist_reader.load_mnist('../fashionmnist/data/fashion', kind='train
    X_test, y_test = mnist_reader.load_mnist('../fashionmnist/data/fashion', kind='t10k')

    #rescale the pixels to be between 0 and 1
    X_train = X_train / 255
    X_test = X_test / 255

    #normalize feature vectors to have euclidean norm 1
    X_train = X_train / np.linalg.norm(X_train, axis=1).reshape(-1, 1)
    X_test = X_test / np.linalg.norm(X_test, axis=1).reshape(-1, 1)

    train_df = pd.DataFrame(np.concatenate((X_train, y_train.reshape(-1, 1)), axis=1))
    test_df = pd.DataFrame(np.concatenate((X_test, y_test.reshape(-1, 1)), axis=1))

    #print(train_df.shape)
   # print(test_df.shape)

    #print(train_df.head())
    return train_df, test_df

train_df, test_df = read_data()
```

```
'''For this assignment we're only concerned with classes 0 and 6, so we'll drop the rest o

def filter_data(df):
    df = df[df[784].isin([0, 6])]
    #print(df.shape)
    for index, row in df.iterrows():
        if row[784] == 6:
            row[784] = 1
    shuffled_df = df.sample(frac=1)
    return shuffled_df
```

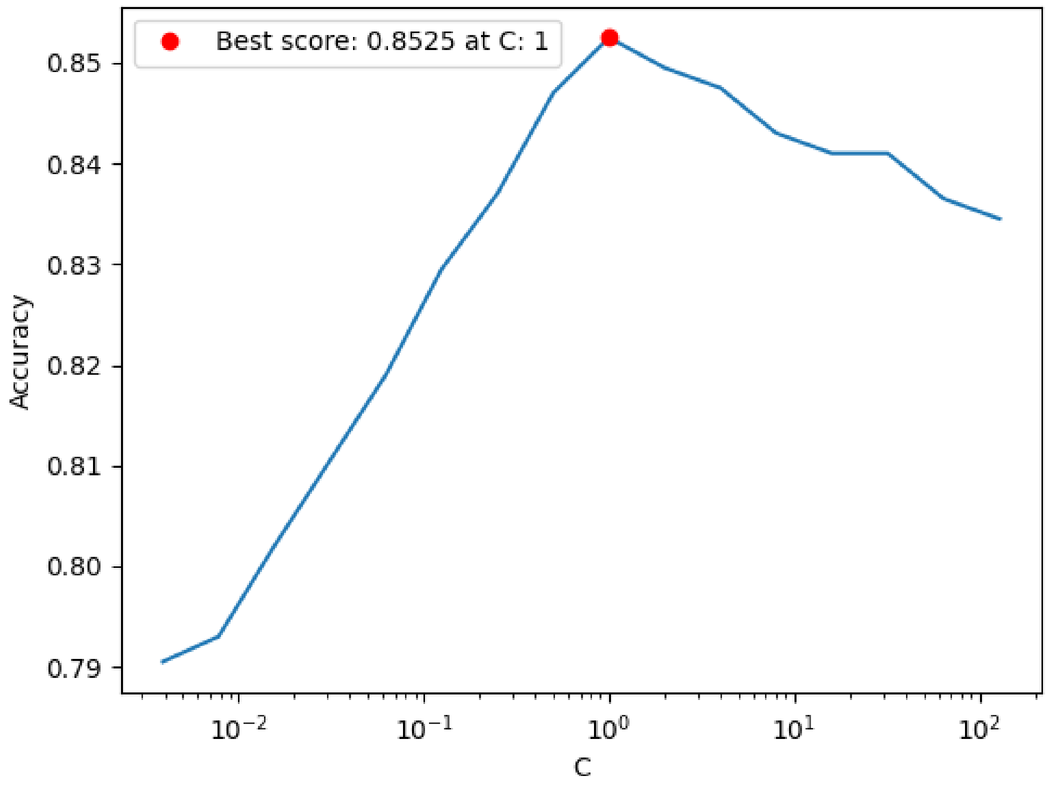
The read_data function grabs all the information from the fashionmnist data set and puts it into a pandas data frame. The filter_data function takes a data frame and filters it to only contain the classes we are interested in. In this case, we are only interested in classes 0 and 6, so we drop all the other classes. We also shuffle the data frame so that the data is not ordered by class.

Part 2: Logistic Regression

We use sci-kit learn's implementation of logistic regression to classify the data. We vary the regularization parameter C, according to a logarithmic scale, and plot the accuracy of the model on the training and test data.

```
def plot_accuracy_vs_C(X_train, y_train, X_test, y_test, C_values):
    '''A function to plot the accuracy of the model as a function of C.'''
    accuracies = []
    best_accuracy = 0
    best_C = 0
    for C in C_values:
        model = lm.LogisticRegression(C=C, multi_class="multinomial", solver="lbfgs", pena
        model.fit(X_train, y_train)
        accuracies.append(model.score(X_test, y_test))
        if model.score(X_test, y_test) > best_accuracy:
            best_accuracy = model.score(X_test, y_test)
            best_C = C
    plt.plot(C_values, accuracies)
    plt.plot(best_C, best_accuracy, 'ro', label='Best score: '
        + "{:.4f}".format(best_accuracy) + ' at C: ' + str(best_C))
    plt.xlabel("C")
    plt.xscale("log", base=2)
    plt.ylabel("Accuracy")
    plt.legend(loc="best")
    plt.savefig("logistic_accuracy_vs_C.png")

C_values = [2**i for i in range(-8, 8)]
plot_accuracy_vs_C(X_train, y_train, X_test, y_test, C_values)
```

Running the above code produces the following plot:



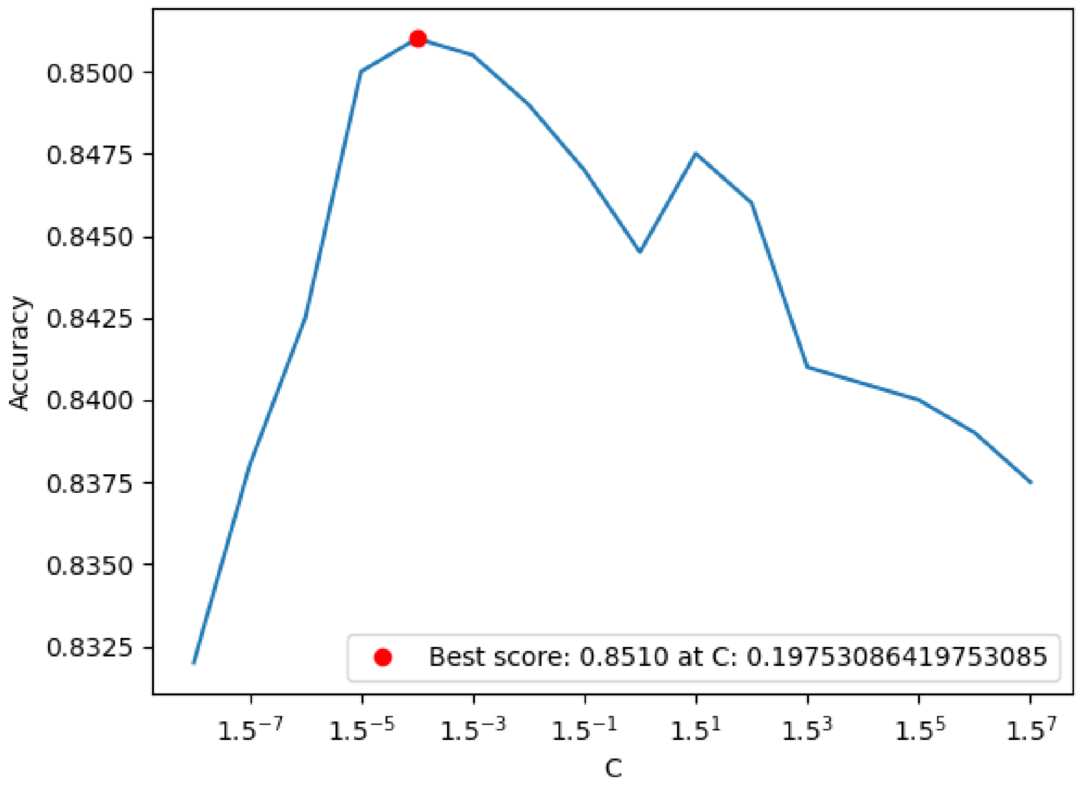
We can see that the best accuracy is achieved at C = 1, and the accuracy is 0.8525.

Part 3: Linear SVM

We use sci-kit learn's implementation of linear SVM to classify the data. We vary the regularization parameter C, according to a logarithmic scale, and plot the accuracy of the model on the training and test data. After a few experiments with different bases for the logarithmic scale, I found that base 1.5 was able to give the best performance.

```python
def plot_accuracy_vs_C(X_train, y_train, X_test, y_test, C_values):
    accuracies = []
    best_accuracy = 0
    best_C = 0
    for C in C_values:
        model = svm.LinearSVC(C=C, max_iter=10000)
        model.fit(X_train, y_train)
        accuracies.append(model.score(X_test, y_test))
        if model.score(X_test, y_test) > best_accuracy:
            best_accuracy = model.score(X_test, y_test)
            best_C = C
    plt.plot(C_values, accuracies)
    plt.plot(best_C, best_accuracy, 'ro', label='Best score: '
        + "{:.4f}".format(best_accuracy) + ' at C: ' + str(best_C))
    plt.xlabel("C")
    plt.xscale("log", base=1.5)
    plt.ylabel("Accuracy")
    plt.legend(loc="best")
    plt.savefig("lin_svm_accuracy_vs_C.png")

C_values = [1.5**i for i in range(-8, 8)]
plot_accuracy_vs_C(X_train, y_train, X_test, y_test, C_values)
```

Running the above code produces the following plot:



Our best accuracy is achieved at $C \approx 0.2$, and the accuracy is 0.8510. Slightly worse than the logistic regression model, but very close.

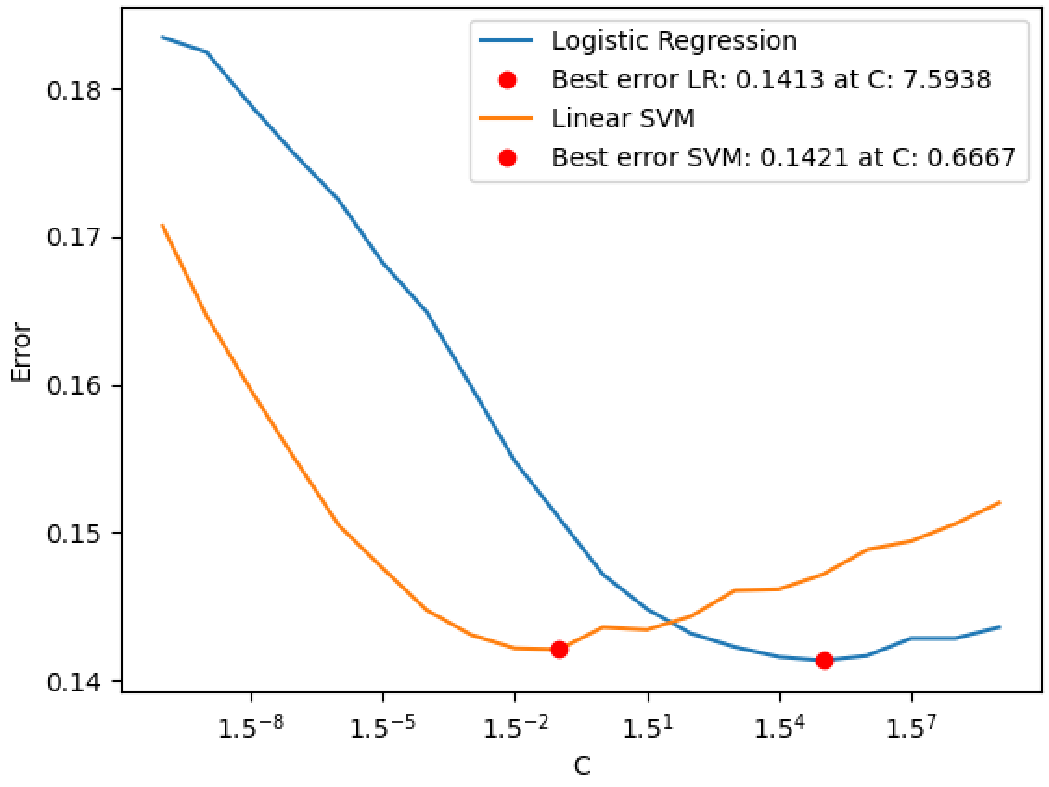
Part 4: K Fold Cross Validation

We implement K fold cross validation from scratch, and use it to find the best regularization value C for both the logistic regression and linear SVM models. We use a logarithmic scale for C, with base 1.5. We use 5 folds for the cross validation.

First, we partition the data into 5 folds. Then we write a function to train the model on each possible selection of validation fold, and return the average accuracy of the model on the validation folds. Then we write a function to find the best C value by iterating over a range of C values, and finding the C value that gives the best average accuracy.

```
def k_fold_separation(test_data, k=5):
    '''This function takes in a dataframe and returns a list of dataframes, each of which
    test_data_length = len(test_data)
    fold_length = test_data_length // k
    folds = []
    for i in range(k):
        folds.append(test_data.iloc[i*fold_length:(i+1)*fold_length])
    return folds

def k_fold_cross_validation(folds, model):
    '''this function takes in the folds and a SKL model, fits the model on each possible c
    scores = []
    for i in range(len(folds)):
        validation_set = folds[i]
        training_set = pd.concat(folds[:i] + folds[i+1:])
        model.fit(training_set.iloc[:, :-1], training_set.iloc[:, -1])
        scores.append(model.score(validation_set.iloc[:, :-1], validation_set.iloc[:, -1])
    return np.mean(scores)


def plot_error_logistic_regression_varying_c(c_values):
    '''This function creates an SKL logistic regression model and plots the error for vary
    folds = k_fold_separation(train_df)
    errors = []
    best_error = 1
    best_c = 1
    for c in c_values:
        lr = lm.LogisticRegression(C=c)
        errors.append(1 - k_fold_cross_validation(folds, lr))
        if 1 - k_fold_cross_validation(folds, lr) < best_error:
            best_error = 1 - k_fold_cross_validation(folds, lr)
            best_c = c
    plt.plot(c_values, errors, label='Logistic Regression')
    plt.xscale("log", base=1.5)
    plt.xlabel("C")
    plt.ylabel("Error")
    plt.plot(best_c, best_error, 'ro', label='Best error LR: '
        + "{:.4f}".format(best_error) + ' at C: ' + "{:.4f}".format(best_c))

def plot_error_linear_svm_varying_c(c_values):
    '''This function creates an SKL linear SVM model and plots the error for varying value
    folds = k_fold_separation(train_df)
    errors = []
    best_error = 1
    best_c = 1
    for c in c_values:
        linsvm = svm.LinearSVC(C=c)
        errors.append(1 - k_fold_cross_validation(folds, linsvm))
        if 1 - k_fold_cross_validation(folds, linsvm) < best_error:
            best_error = 1 - k_fold_cross_validation(folds, linsvm)
            best_c = c
    plt.plot(c_values, errors, label='Linear SVM')
    plt.plot(best_c, best_error, 'ro', label='Best error SVM: '
        + "{:.4f}".format(best_error) + ' at C: ' + "{:.4f}".format(best_c))
    plt.legend(loc='best')
    plt.savefig("kfcv_linear_svm_error_vs_C.png")
```

Running all of the above code produces the following plot:



According to this, the optimal value of C for the logistic regression model is $C \approx 7.5$, and the optimal value of C for the linear SVM model is $C \approx 0.67$. So now we can finally train our models on the entire training set, and evaluate them on the test set, using these optimal values of C.

```
train_df, test_df = read_data()

train_df = filter_data(train_df)
test_df = filter_data(test_df)

X_train = train_df.iloc[:, :-1]
y_train = train_df.iloc[:, -1]

X_test = test_df.iloc[:, :-1]
y_test = test_df.iloc[:, -1]

linsvm = svm.LinearSVC(C=0.6667)

logreg = lm.LogisticRegression(C=7.5938)

linsvm.fit(X_train, y_train)

logreg.fit(X_train, y_train)

print("Linear SVM accuracy: ", linsvm.score(X_test, y_test))

print("Logistic Regression accuracy: ", logreg.score(X_test, y_test))
```

Which produces the following output:

```
Linear SVM accuracy:  0.847
Logistic Regression accuracy:  0.8475
```

This shows that when optimally parameterized, the linear SVM and logistic regression models perform very similarly on the test set. The logistic regression model is slightly better, with an accuracy of 0.8475, compared to 0.847 for the linear SVM model, but this could easily be noise.

We can calculate 95% confidence intervals for the accuracy of the models, using the following formula:

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where $\hat{p}$ is the accuracy of the model, and n is the number of samples in the test set. We can calculate the accuracy of the models as follows:

This comes out to 0.847 ± 0.0158 for the linear SVM model, and 0.8475 ± 0.0158 for the logistic regression model. This shows that the difference between the two models is not significant, and that there is significant overlap between the 95% confidence intervals of the two models.

Part 5: Nonlinear SVM

Going beyond linear classifiers, we introduce a kernel SVM model, which is a nonlinear classifier. We use the Gaussian kernel, which is defined as follows:

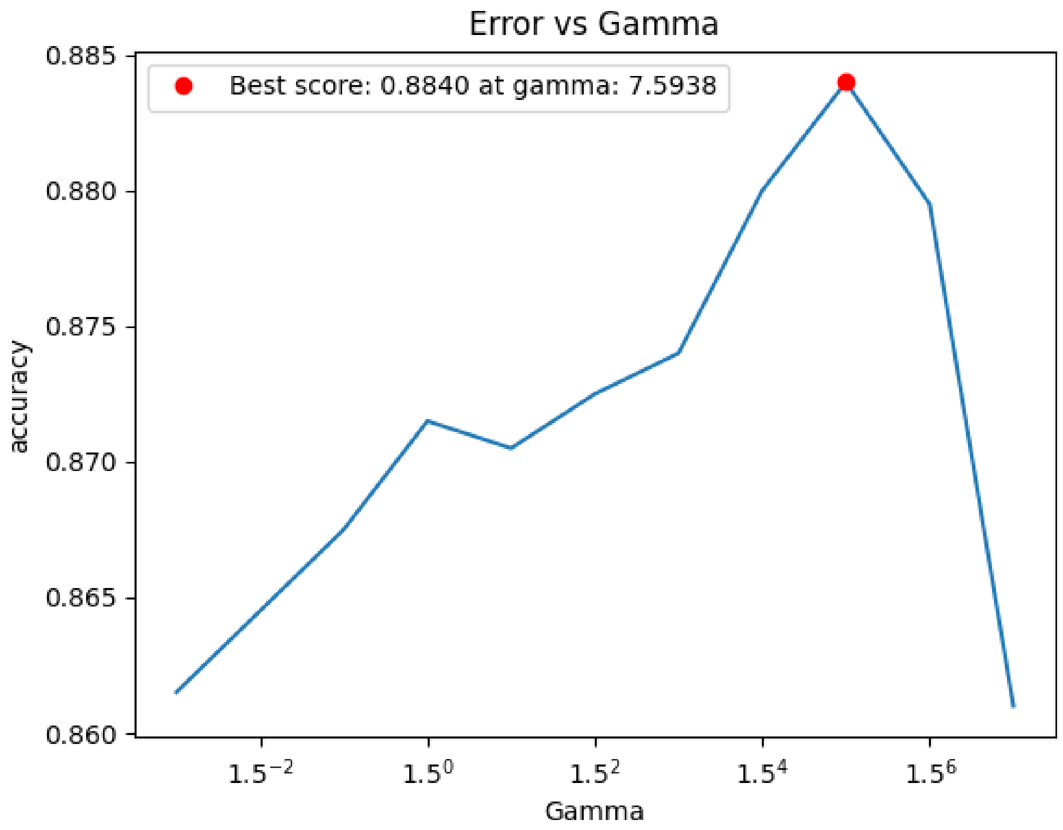
$$K_u(d(x_u, x)) = e^{\frac{1}{2\sigma_u^2} d^2(x_u, x)}$$

We re use our previous implementation of the K fold cross validation, and use it to find the best values of the regularization parameter C and the kernel parameter σ. We use a logarithmic scale for C, with base 1.5, and a logarithmic scale for σ, with base 2. We use 5 folds for the cross validation.

```
train_df = filter_data(train_df)
test_df = filter_data(test_df)

folds = k_fold_separation(train_df)


def plot_error_gaussian_svm_varying_gamme_and_c(gamma_values):
    ''' This function creates an SKL gaussian SVM model and plots the error for varying va
    For each gamma, we will use k fold cross validation to find an optimal C_gamma value.
    To train on the entire training set and evaluate on the test set. We will then plot th
    accuracies = []
    best_acc = 0
    best_gamma = 0
    c_gammas = []
    for gamma in gamma_values:
        start_time = time.perf_counter()
        print('gamma: ', gamma)
        cur_gamma_c_values = [1.5**i for i in range(-5, 8)]
        best_c = 0
        best_c_score = 0
        for c in cur_gamma_c_values:
            print('c: ', c)
```

```
                gs = svm.SVC(C=c, gamma=gamma)
                cur_c_score = k_fold_cross_validation(folds, gs)
                if cur_c_score > best_c_score:
                    best_c_score = cur_c_score
                    best_c = c
            c_gammas.append(best_c)
            print(best_c)
            gs = svm.SVC(C=best_c, gamma=gamma)
            gs.fit(train_df.iloc[:, :-1], train_df.iloc[:, -1])
            cur_gamma_score = gs.score(test_df.iloc[:, :-1], test_df.iloc[:, -1])
            accuracies.append(cur_gamma_score)
            print(time.perf_counter() - start_time)
            if cur_gamma_score > best_acc:
                best_acc = cur_gamma_score
                best_gamma = gamma

        plt.plot(gamma_values, accuracies)
        plt.xlabel('Gamma')
        plt.xscale('log', base=1.5)
        plt.ylabel('accuracy')
        plt.title('Error vs Gamma')
        plt.plot(best_gamma, best_acc, 'ro', label='Best score: '
            + "{:.4f}".format(best_acc) + ' at gamma: ' + "{:.4f}".format(best_gamma))
        plt.legend(loc='best')
        plt.savefig('error_vs_gamma.png')

    gamma_values = [(1.5**i) for i in range(-3, 8)]
    plot_error_gaussian_svm_varying_gamme_and_c(gamma_values)
```

First we choose a list of γ values to test, and then for each γ we test a range of C_γ values. We then use k fold cross validation to find the optimal C_γ value for each γ, and then train a model on the entire training set using the optimal C_γ value, and evaluate it on the test set. We then plot the test set accuracy for each γ value. The code for this is shown above. The plot produced by this code is shown below:



According to this, the optimal value of γ is $\gamma \approx 7.5$, and this achieves an accuracy of 0.884 on the test set, which is better than the linear SVM and logistic regression models, which both achieve about 0.85 accuracy - this means that the Gaussian kernel SVM makes about 20% fewer errors than the linear classifiers! It is possible that an even stronger nonlinear classifier could be achieved with a more extensive search for the optimal γ value, but this would take a long time to run, running the above code took quite some a while on my laptop and had to be left running overnight.