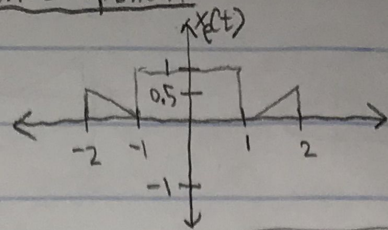
Prof. Kao
ECE102

Homework #1

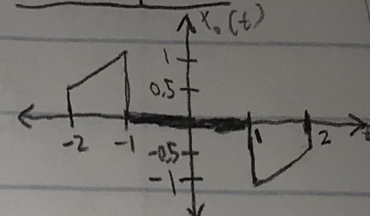
1. $x_e(t) = \frac{x(t)+x(-t)}{2}$

$x_o(t) = \frac{x(t)-x(-t)}{2}$

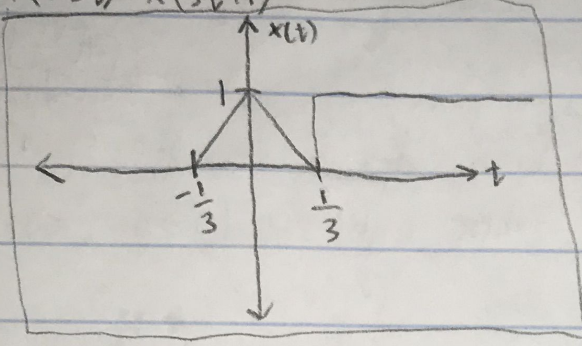
even component



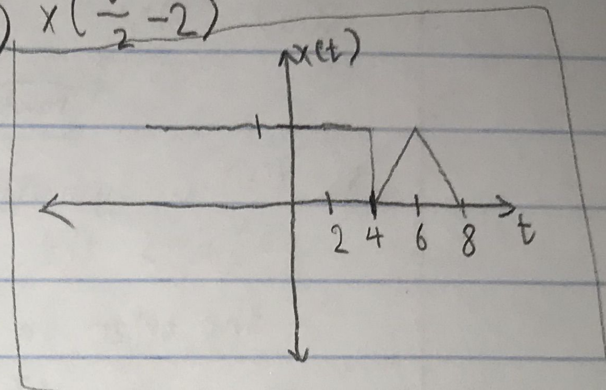
odd component



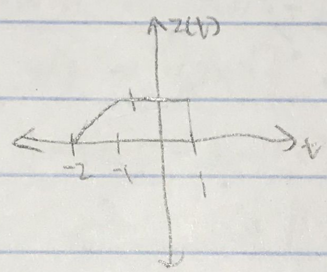
2. ai) $x(1-3t) = x(-3t+1)$



aii) $x(\frac{t}{2}-2)$



b) $\boxed{y(t) = z(-2t)}$



3 ai) $\boxed{\text{Periodic}}$ $T = \frac{2\pi}{w} = \frac{2\pi}{2} = \boxed{\pi}$ $f = \frac{w}{2\pi} = \boxed{\frac{1}{\pi} Hz}$

aii) $\boxed{\text{Periodic}}$ $T = \frac{2\pi}{\sqrt{2}\pi} = \boxed{\sqrt{2}}$ $f = \frac{w}{2\pi} = \frac{\sqrt{2}\pi}{2\pi} = \boxed{\frac{\sqrt{2}}{2} Hz}$

aiii) $\boxed{\text{Periodic}}$ $\sin^2(3\pi t+3) = \frac{1-\cos(6\pi t+6)}{2}$ $T = \frac{2\pi}{6\pi} = \boxed{\frac{1}{3}}$ $f = \frac{6\pi}{2\pi} = \boxed{3 Hz}$

aiv) $\boxed{\text{Not Periodic}}$

av) $\boxed{\text{Periodic}}$ $x_1(\pi t) + x_3(t) = \sin(2\pi t + \frac{\pi}{3}) + \frac{1}{2} - \frac{1}{2}\cos(6\pi t+6)$ $T = \frac{2\pi}{6\pi} = \boxed{\frac{1}{3}}$ $f = \frac{6\pi}{2\pi} = \boxed{3 Hz}$

avi) $\boxed{\text{Not Periodic}}$ avii) $\boxed{\text{Not periodic}}$

b) $\boxed{x(T_0) = x(0)}$

c) $x_e(t) = \frac{x(t)+x(-t)}{2}$ $x_o(t) = \frac{x(t)-x(-t)}{2}$ $\boxed{\text{Periodic}}$ because LCM of $x(t)$ and $x(-t)$'s w values is rational.

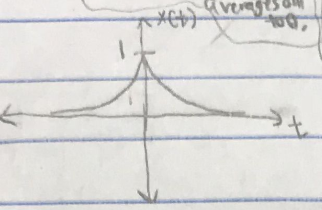
$$E=\int_{-T}^{T} e^{-2|t|}\,dt = -\frac{1}{2}\left[e^{-2|t|}\right]_{-T}^{T} = 0 \text{ as T approaches } \infty \text{ and } -T \text{ approaches } -\infty,$$

but it isn't actually 0, it just averages out too.

$$P=\frac{1}{2T}\cdot\int_{1}^{T} t^{-1}\,dt = \frac{1}{2T}\Big[\ln|x|\Big]_{1}^{T} = \infty - 0 = \infty$$

$$E=\int_{1}^{T} t^{-1}\,dt = \big[\ln|x|\big]_{1}^{T} = \infty$$

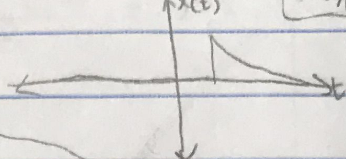
4. ai) $x(t)=e^{-|t|}$



$\boxed{\text{Energy Signal}}$

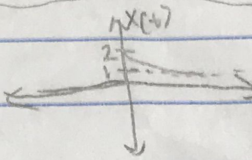
aii) $x(t)=\begin{cases}\frac{1}{\sqrt{t}}, & t\geq 1\\ 0, & \text{else}\end{cases}$

$\boxed{\text{Neither energy/power Signal}}$



aiii) $x(t)=\begin{cases}1+e^{-t}, & t\geq 0\\ 0, & \text{else}\end{cases}$

$\boxed{\text{Power Signal}}$



$$P=\frac{1}{2T}\int_{0}^{T}(1+e^{-t})^2\,dt = \frac{1}{2T}\int_{0}^{T}(e^{-2t}+2e^{-t}+1)\,dt$$
$$=\frac{1}{2T}\left[\frac{e^{-2t}}{-2}\right]_0^T+\frac{1}{2T}\left[-2e^{-t}\right]_0^T+\frac{1}{2T}\left[t\right]_0^T$$
$$=\frac{1}{2T}\left[\frac{e^{-2T}}{-2}+\frac{1}{2}\right]+\frac{1}{2T}\left[-2e^{-T}+2\right]+\frac{1}{2T}\left[T-0\right]=0+0+\frac{1}{2}=\boxed{\frac{1}{2}}$$

b) $x(t)$ is symmetric on both sides of the y-axis, so multiplying it by $y(t)$ adjusts the signal where one side ($t>0$, $t<0$) is lower/higher than the other on the y-axis.

The integral $\int_{-T}^{T} z(t)\,dt=0$ always because by definition, one side of an odd signal is "flipped" on the other side, where $z(t)=-z(-t)$. Therefore, $z(t)+z(-t)=z(t)-z(t)=0$ for all t, and they cancel out.

$$E=\int_{-\infty}^{\infty}|x(t)|^2\,dt \qquad E_{x_e}=\int_{-\infty}^{\infty}\left|\frac{x(t)+x(-t)}{2}\right|^2 dt \qquad E_{x_o}=\int_{-\infty}^{\infty}\left|\frac{x(t)-x(-t)}{2}\right|^2 dt$$

$$=\frac{1}{4}\int_{-\infty}^{\infty}\left((x(t))^2+\cancel{2x(t)x(-t)}+(x(-t))^2\right)dt \qquad =\frac{1}{4}\int_{-\infty}^{\infty}\left((x(t))^2-\cancel{2x(t)x(-t)}+(x(-t))^2\right)dt$$

$$E_{x_e}+E_{x_o}=\frac{1}{4}\int_{-\infty}^{\infty}(x(t))^2\,dt+\frac{1}{4}\int_{-\infty}^{\infty}(x(-t))^2\,dt+\frac{1}{4}\int_{-\infty}^{\infty}(x(t))^2\,dt+\frac{1}{4}\int_{-\infty}^{\infty}(x(-t))^2\,dt$$

$$=\frac{1}{2}\int_{-\infty}^{\infty}(x(t))^2\,dt+\frac{1}{2}\int_{-\infty}^{\infty}(x(-t))^2\,dt=\boxed{\int_{-\infty}^{\infty}(x(t))^2\,dt}$$

5. ai) $\frac{d}{d\theta}\sin\theta = \cos\theta$

$$\frac{d}{d\theta}e^{j\theta} = \frac{d}{d\theta}(\cos\theta + j\sin\theta)$$

$$\frac{d}{d\theta}e^{j\theta} = je^{j\theta} = -\sin\theta + j\cos\theta$$

$$\frac{d}{d\theta}(\cos\theta + j\sin\theta) = -\sin\theta + j\cos\theta$$

$$\frac{d}{d\theta}\cos\theta = -\sin\theta$$

$$\frac{d}{d\theta}j\sin\theta = j\cos\theta$$

$$\boxed{\frac{d}{d\theta}\sin\theta = \cos\theta}$$

aii) $\sin^2(\theta) = \frac{1}{2}(1-\cos(2\theta))$

$$(e^{j\theta})^2 = (\cos\theta + j\sin\theta)^2$$

$$e^{j2\theta} = (\cos\theta + j\sin\theta)^2$$

$$\cos 2\theta + j\sin 2\theta = \cos^2\theta + 2j\sin\theta\cos\theta - \sin^2\theta$$

$$Re = \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \cos^2\theta - \cos 2\theta$$

$$\sin^2\theta = (1-\sin^2\theta) - \cos 2\theta$$

$$\cancel{2\sin^2\theta = 1 - \cos 2\theta}$$

$$\boxed{\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)}$$

aiii) $e^{j\alpha} + e^{j\beta} = 2\cos\left(\frac{\alpha-\beta}{2}\right)e^{j\frac{\alpha+\beta}{2}}$

$$2\cos\left(\frac{\alpha-\beta}{2}\right)e^{j\frac{\alpha+\beta}{2}} = 2\cos\left(\frac{\alpha-\beta}{2}\right)\left(\cos\left(\frac{\alpha+\beta}{2}\right) + j\sin\left(\frac{\alpha+\beta}{2}\right)\right)$$

$$= 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right) + j2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$Re = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right) = \cos\alpha + \cos\beta \qquad Im = 2j\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = j\sin\alpha + j\sin\beta$$

$$= Re + Im = \cos\alpha + j\sin\alpha + \cos\beta + j\sin\beta = \boxed{e^{j\alpha} + e^{j\beta}}$$

bi) $x(t) = -(1+j)e^{j(1+2t)} = -e^{j(1+2t)} + je^{j(1+2t)} = -\cos(1+2t) - j\sin(1+2t) + j\cos(1+2t) - \sin(1+2t)$

$$\boxed{Re = -\cos(1+2t) - \sin(1+2t) \qquad Im = j(-\sin(1+2t) + \cos(1+2t))}$$

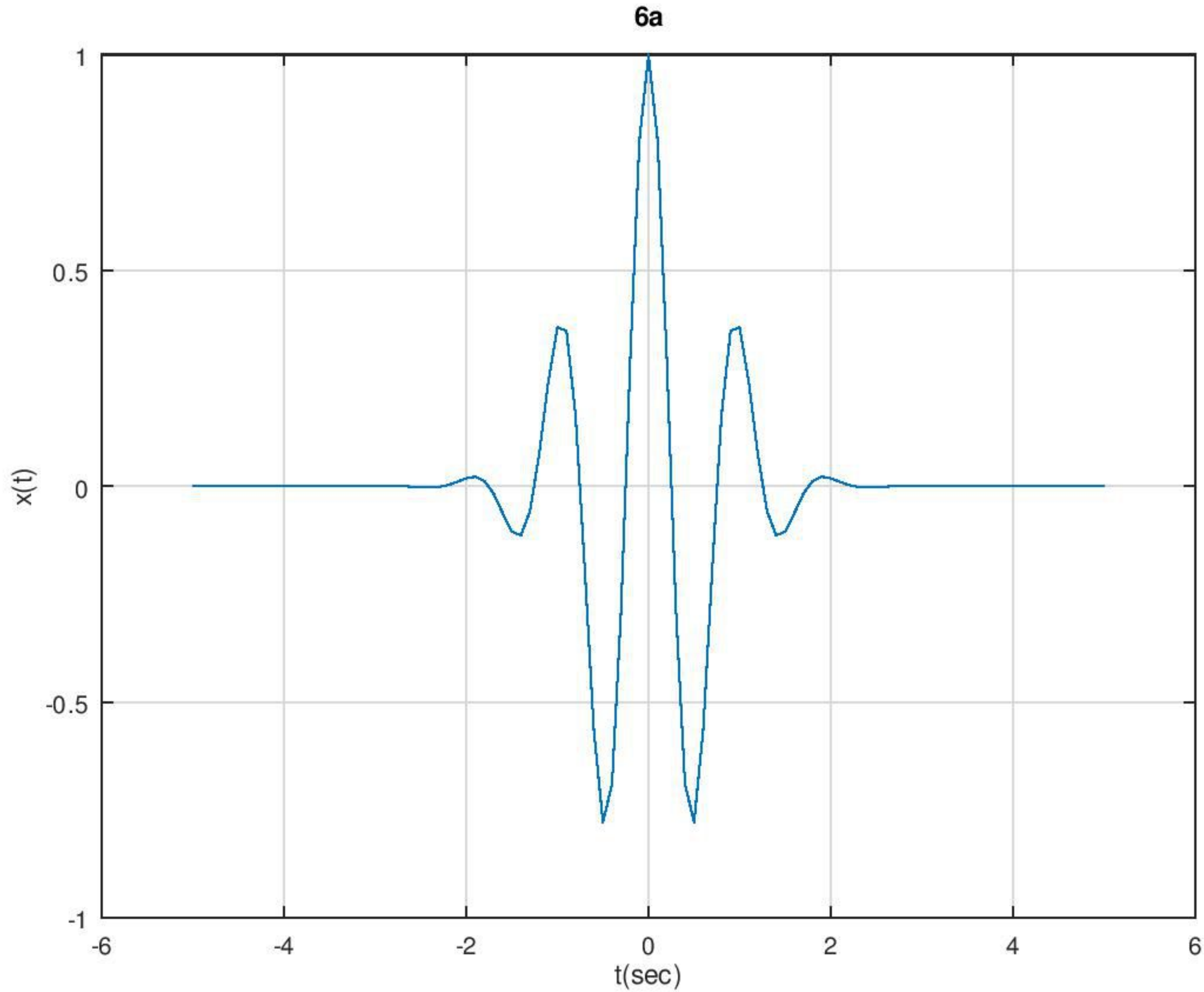
bii) Magnitude $= \sqrt{(-\cos(1+2t) - \sin(1+2t))^2 + (-\sin(1+2t) + \cos(1+2t))^2}$

$$= \sqrt{\cos^2(1+2t) + \sin^2(1+2t) + 2\cos(1+2t)\sin(1+2t) + \sin^2(1+2t) + \cos^2(1+2t) - 2\cos(1+2t)\sin(1+2t)}$$

$$= \boxed{\sqrt{2\cos^2(1+2t) + 2\sin^2(1+2t)}}$$

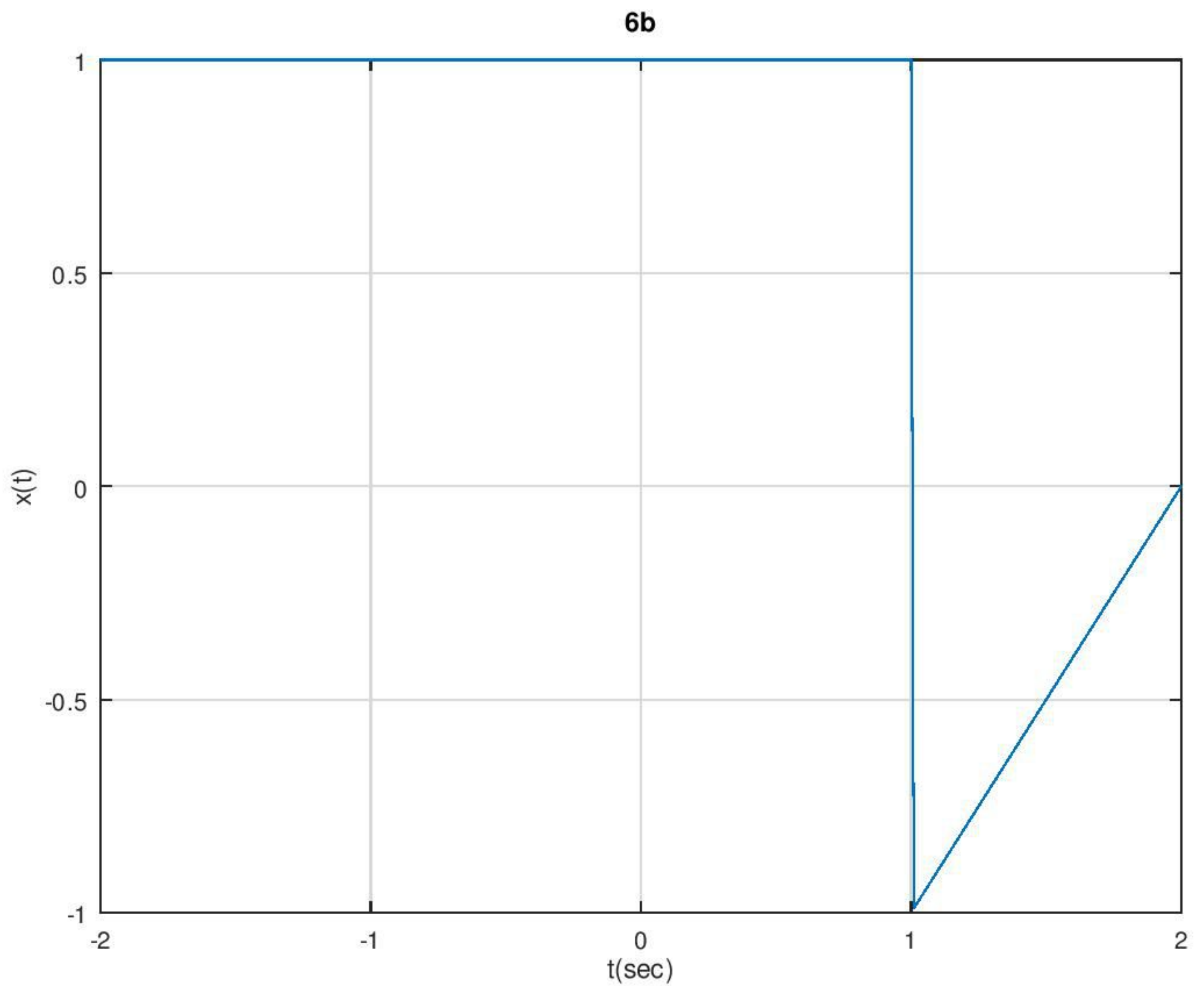
$$\text{Phase} = \boxed{\arctan\left(\frac{\cos(1+2t) - \sin(1+2t)}{\cancel{-\cos(1+2t) - \sin(1+2t)}}\right)}$$

```
# Octave 4.4.1, Wed Oct 10 01:54:08 2018 GMT <unknown@nathan-laptop>
t=-5:0.1:5; x=exp(-(t.^2)).*cos(2.*pi.*t); plot(t,x); grid on; title('6a'); xlabel('t(sec)'); ylabel('x(t)');
```



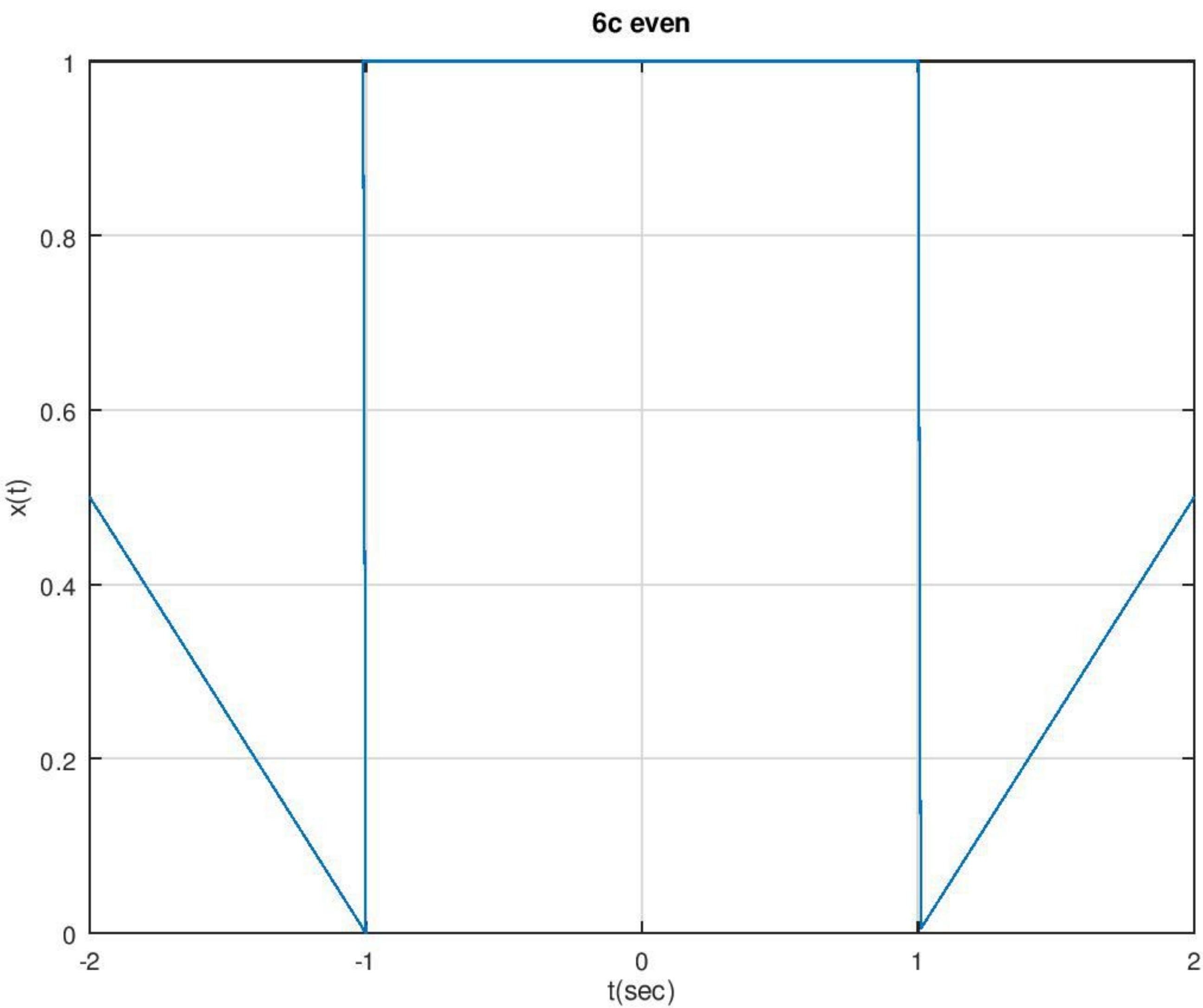
6a
x(t)
t(sec)
1
0.5
0
-0.5
-1
-6
-4
-2
0
2
4
6

```
t1=-2:0.01:1; t2=1.01:0.01:2; x1=ones(1,length(t1)); x2=-2+t2;
t=[t1 t2]; x=[x1 x2];
plot(t,x); grid on; title('6b'); xlabel('t(sec)'); ylabel('x(t)');
```

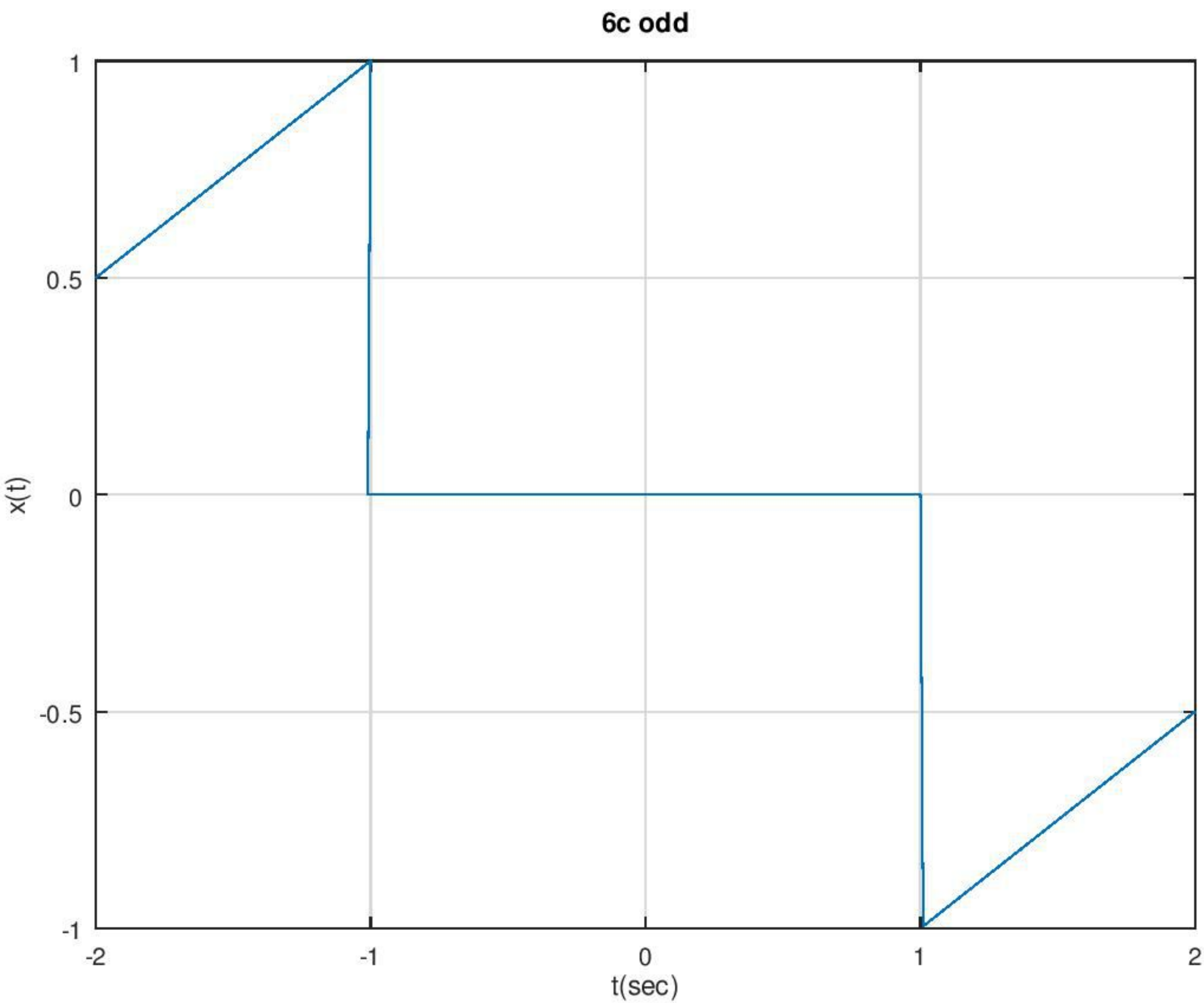


6b
1
0.5
0
-0.5
-1
x(t)
-2
-1
0
1
2
t(sec)

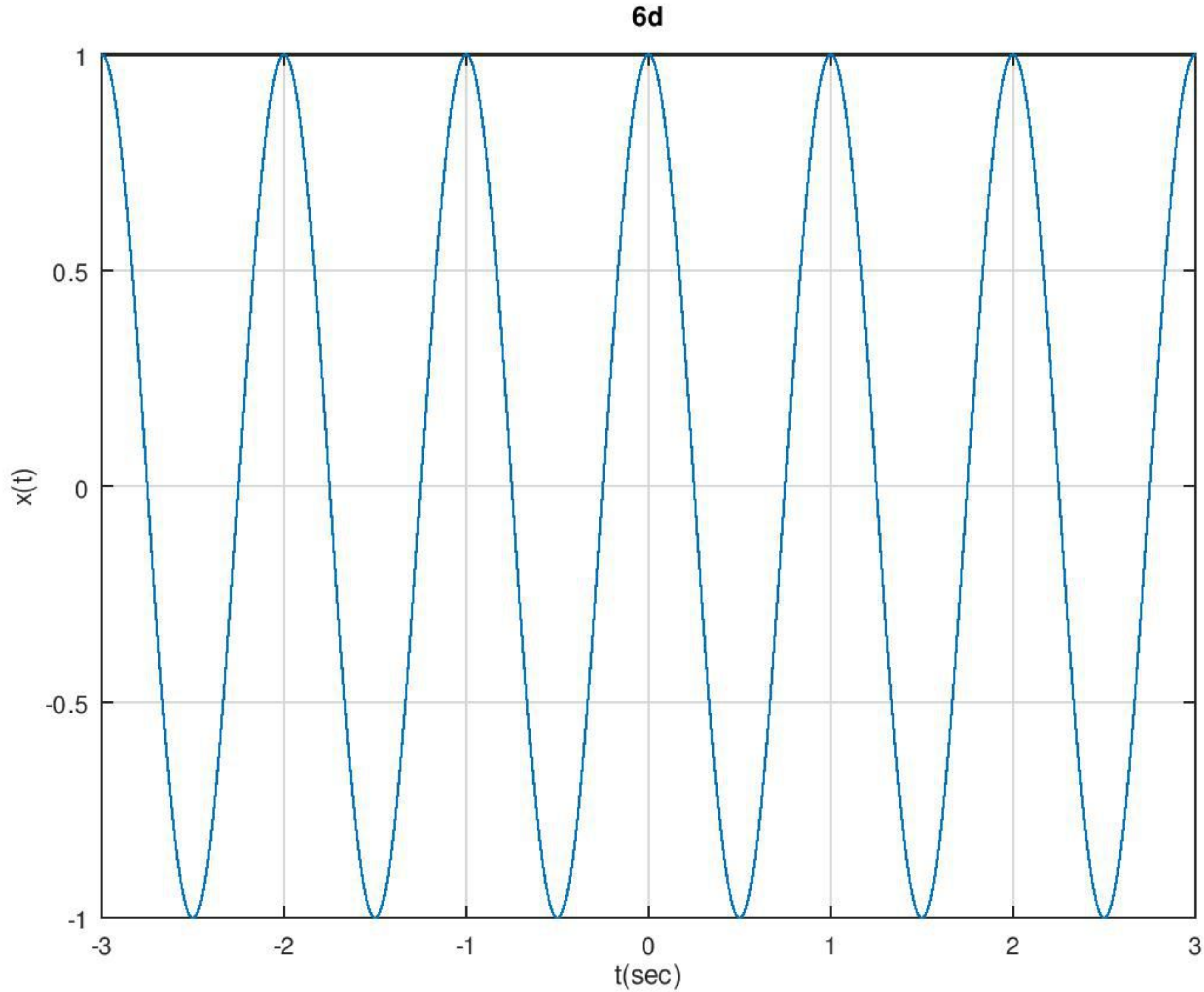
```
t1=-2:0.01:-1; t2= -1.01:0.01:1; t3=1.01:0.01:2;
x1=-0.5.*t1-0.5; x2=ones(1,length(t2)); x3=t3-1;
 plot(t,x); grid on; title('6c even'); xlabel('t(sec)'); ylabel('x(t)');
t=[t1 t2 t3]; x=[x1 x2 x3];  plot(t,x); grid on; title('6c even'); xlabel('t(sec)'); ylabel('x(t)');
x3=0.5.*t3-0.5;
x=[x1 x2 x3];
 plot(t,x); grid on; title('6c even'); xlabel('t(sec)'); ylabel('x(t)');
```


6c even
x(t)
t(sec)
1
0.8
0.6
0.4
0.2
0
-2
-1
0
1
2

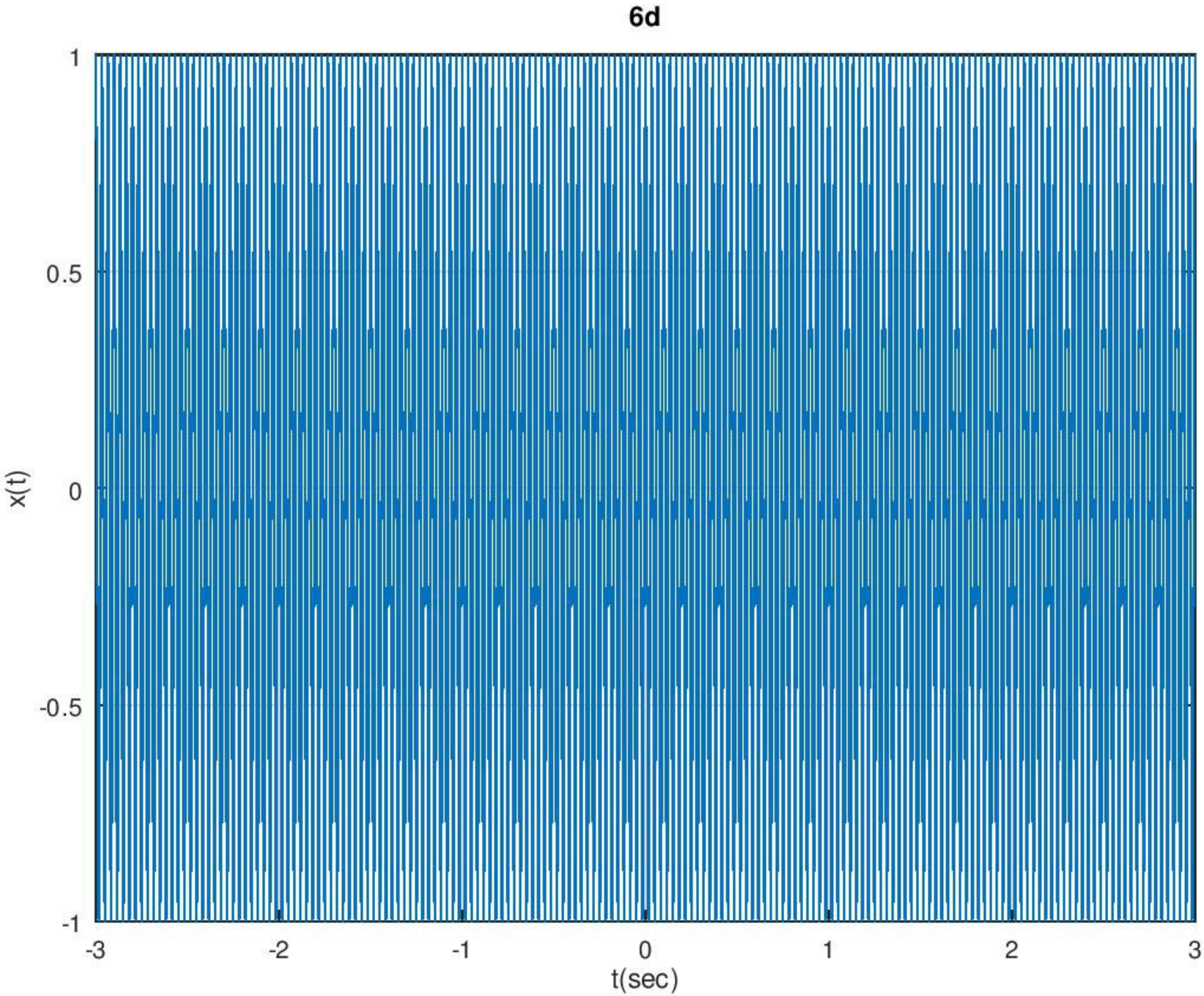
```
x1=0.5.*t1+1.5; x2=zeros(1,length(t2)); x3=0.5.*t3-1.5;
x=[x1 x2 x3];
plot(t,x); grid on; title('6c odd'); xlabel('t(sec)'); ylabel('x(t)');
```


6c odd
x(t)
t(sec)
1
0.5
0
-0.5
-1
-2
-1
0
1
2

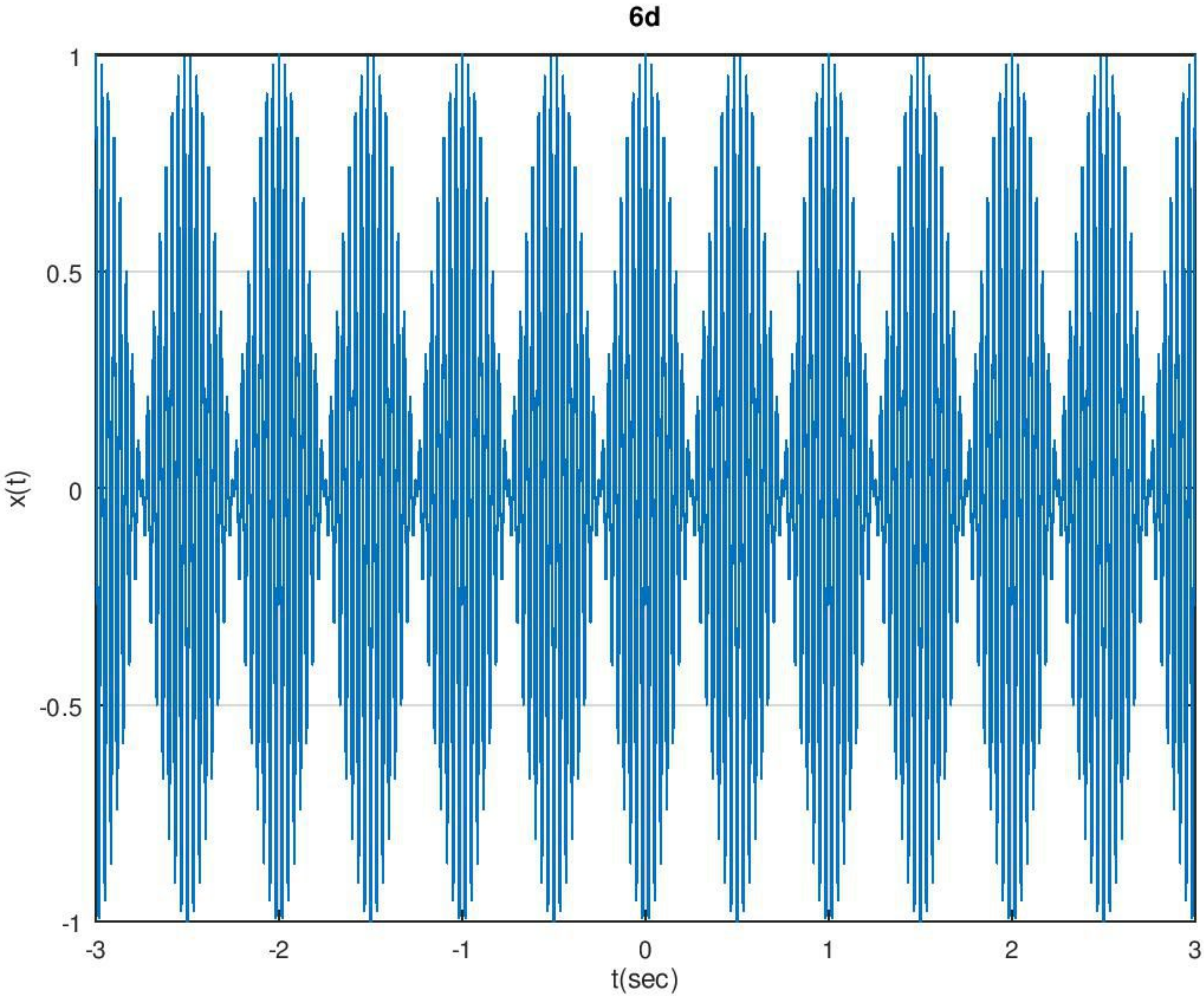
```
t=-3:0.001:3;
x1=cos(2.*pi.*t); x2=cos(60.*pi.*t);
 plot(t,x1); grid on; title('6d');  xlabel('t(sec)'); ylabel('x(t)');
```


6d
1
0.5
0
-0.5
-1
x(t)
-3
-2
-1
0
1
2
3
t(sec)

```
t=-3:0.001:3;
x1=cos(2.*pi.*t); x2=cos(60.*pi.*t);
 plot(t,x1); grid on; title('6d');  xlabel('t(sec)'); ylabel('x(t)');
 plot(t,x2); grid on; title('6d');  xlabel('t(sec)'); ylabel('x(t)');
```


6d
1
0.5
0
-0.5
-1
x(t)
-3
-2
-1
0
1
2
3
t(sec)

```
x3=x1.*x2;
 plot(t,x3); grid on; title('6d');  xlabel('t(sec)'); ylabel('x(t)');
exit
# Octave 4.4.1, Wed Oct 10 02:20:46 2018 GMT <unknown@nathan-laptop>
```


6d
1
0.5
0
-0.5
-1
x(t)
-3
-2
-1
0
1
2
3
t(sec)