Concise representations of regular languages

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INRIA Lille, équipe Links

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— *Journées DeLTA – Bordeaux* —

mainly joint work with Giovanni Pighizzini and Luca Prigioniero, University of Milan

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- study of size of models recognizing languages

  e.g., number of transitions of an automaton

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This talk:

- focus on regular languages
- particular attention paid to 1-*limited automata*

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G is *self-embedding* if for some X, $X \overset{*}{\Rightarrow} \alpha X \beta$ with both α, β nonempty.

Otherwise, **G** is non-self-embedding.

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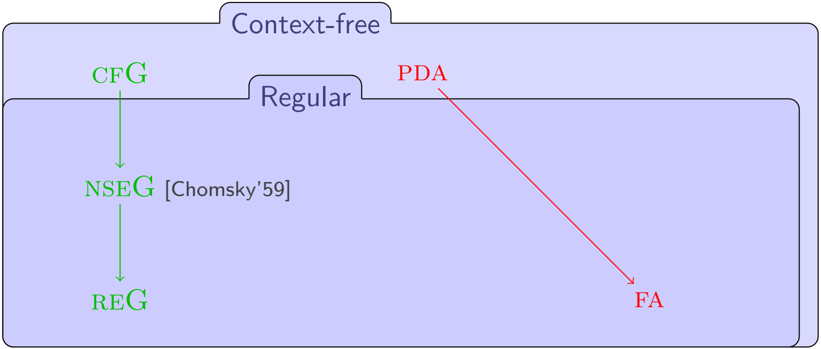

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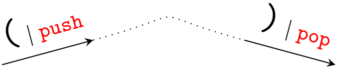

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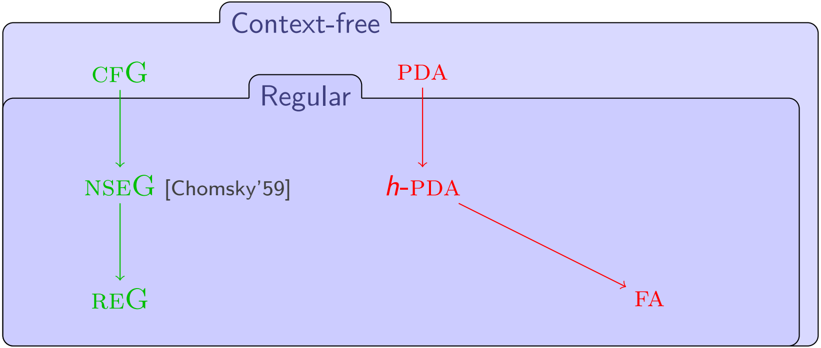

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Definition (h-PDA)

An *h-height* PDA is a PDA with stack size $\leq h \in \mathbb{N}$.

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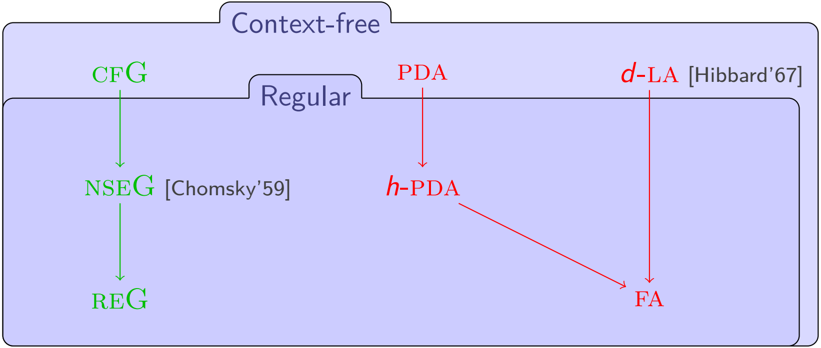

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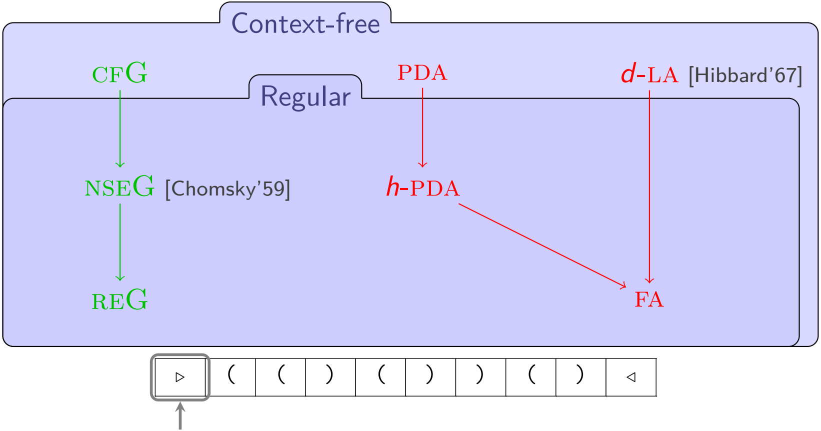

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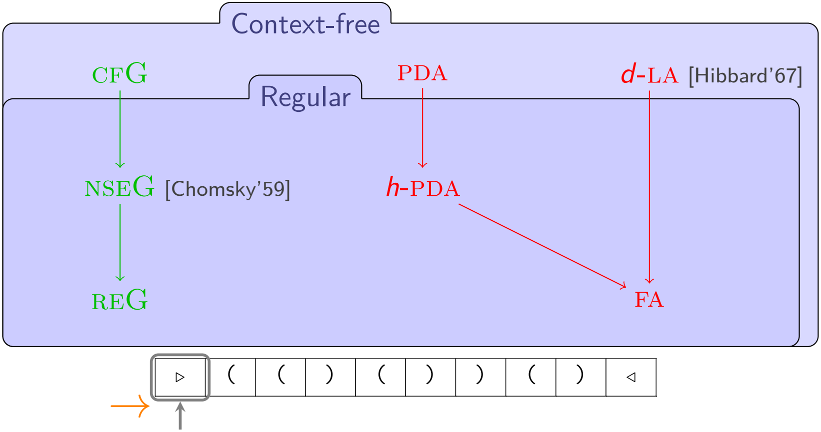

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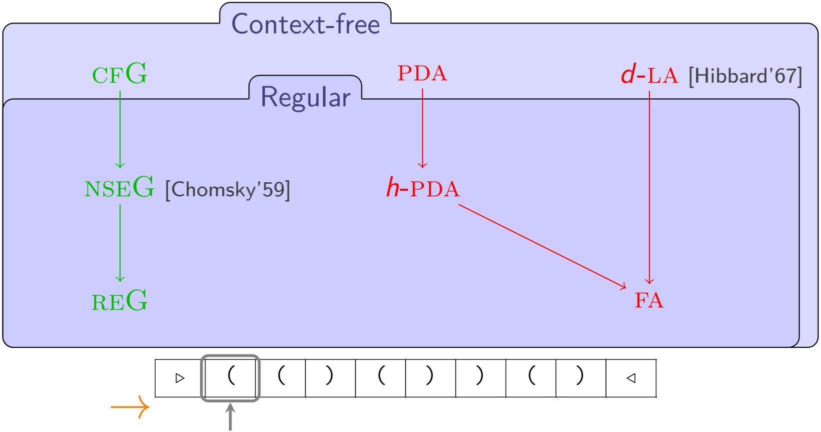

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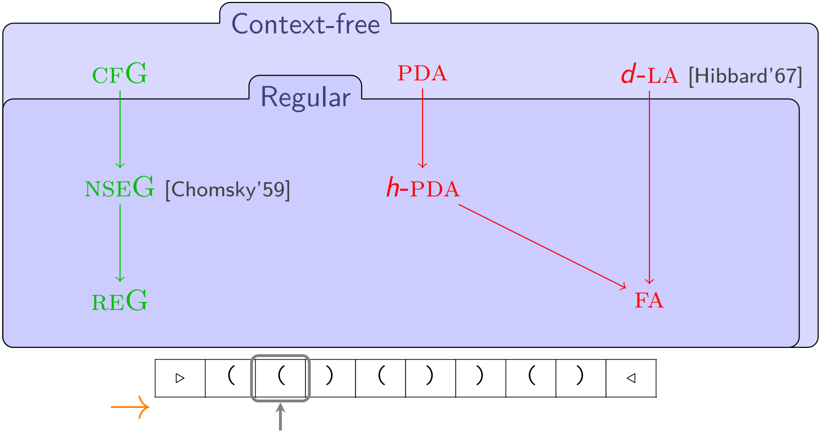

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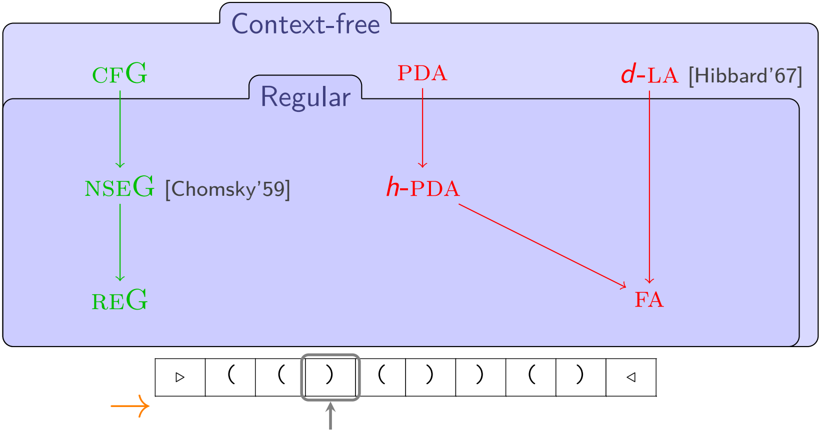

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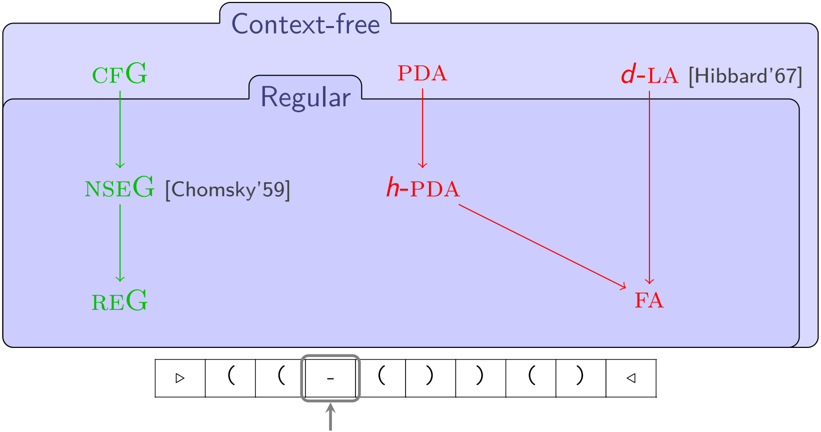

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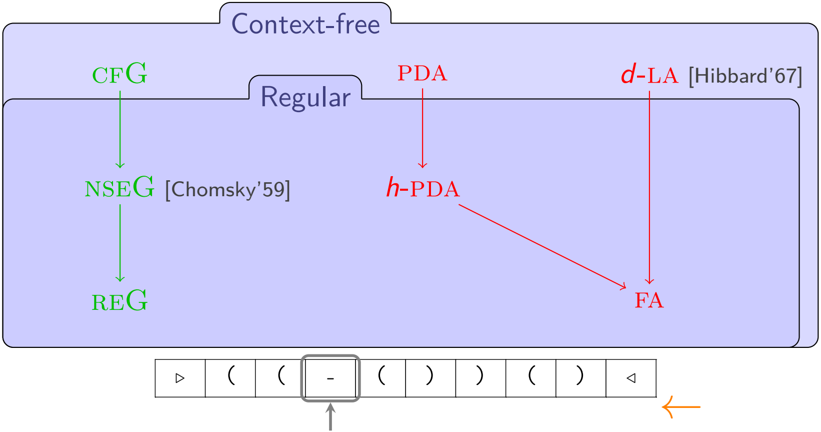

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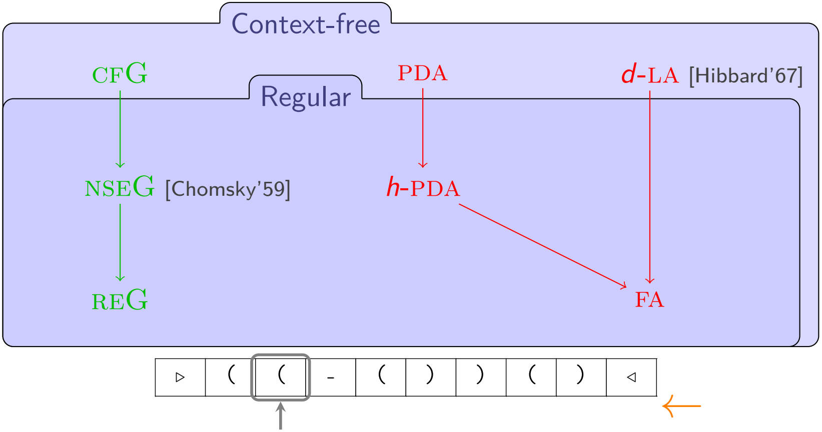

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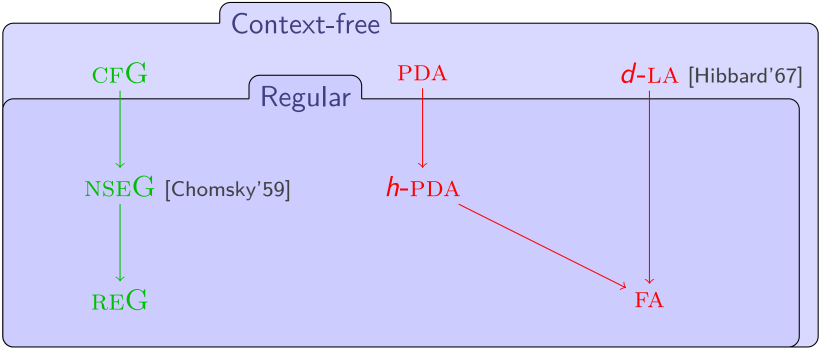

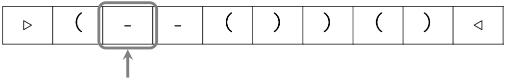

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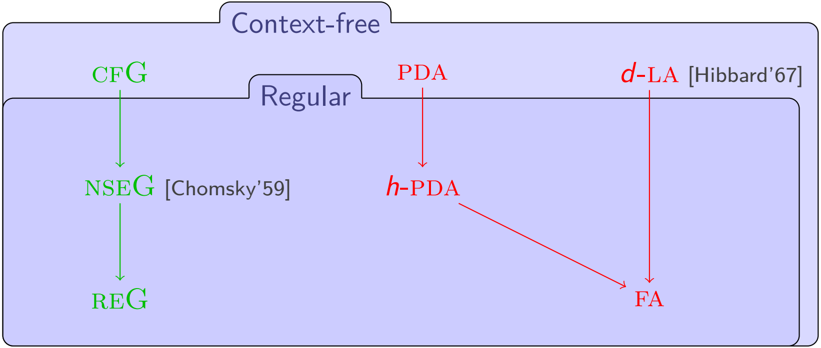

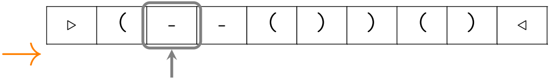

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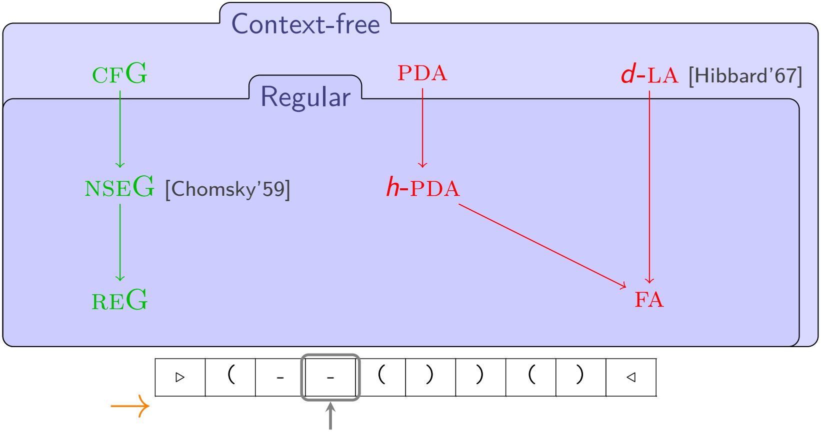

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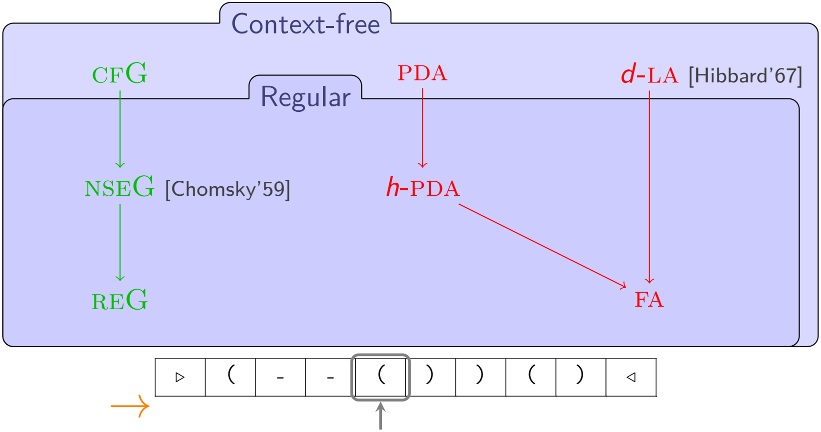

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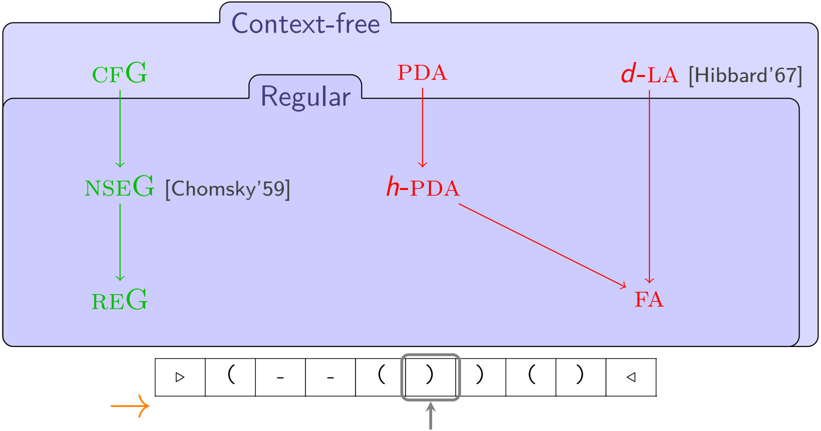

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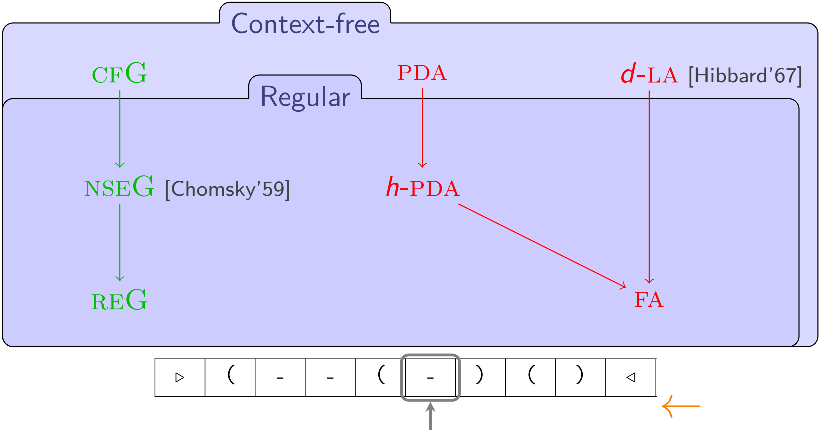

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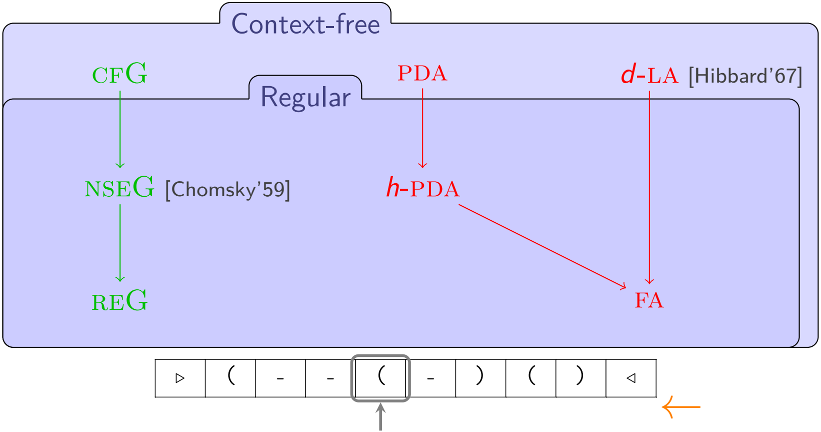

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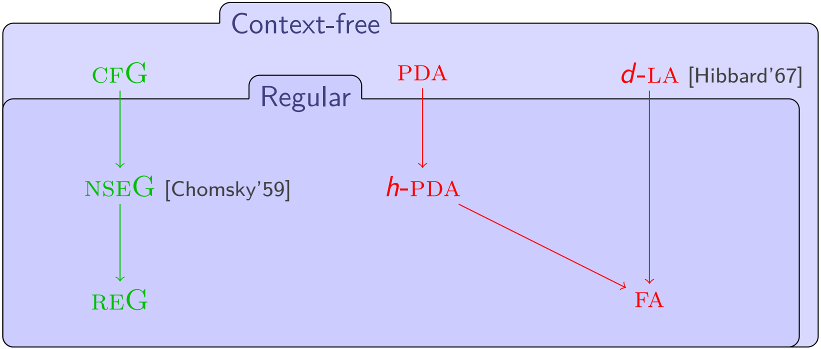

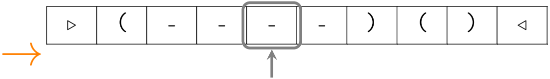

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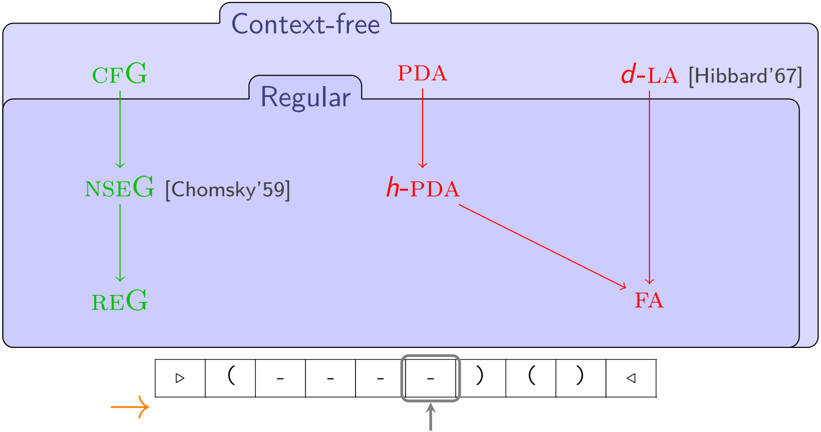

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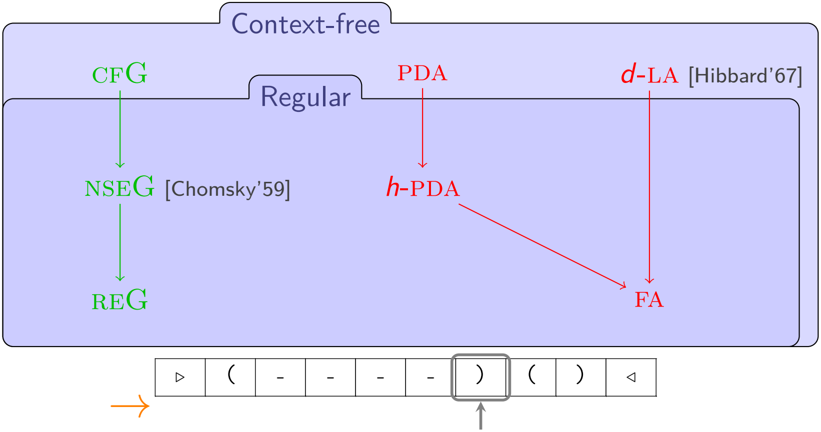

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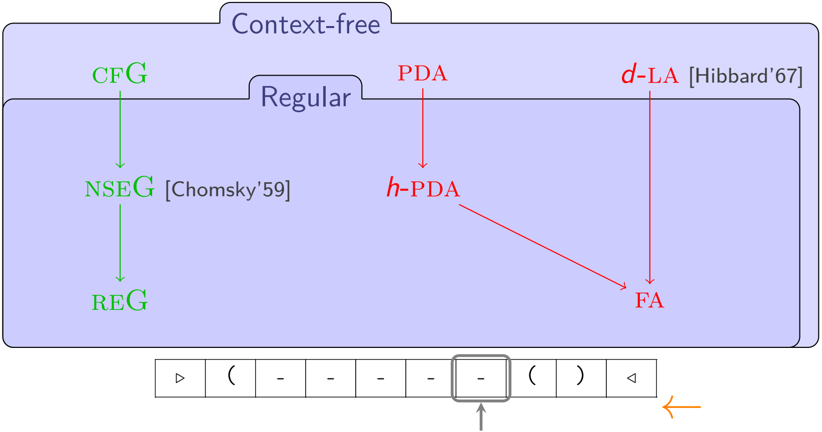

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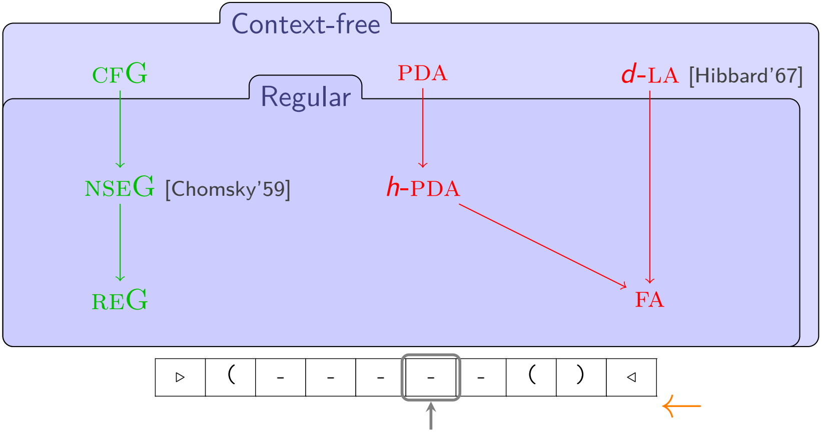

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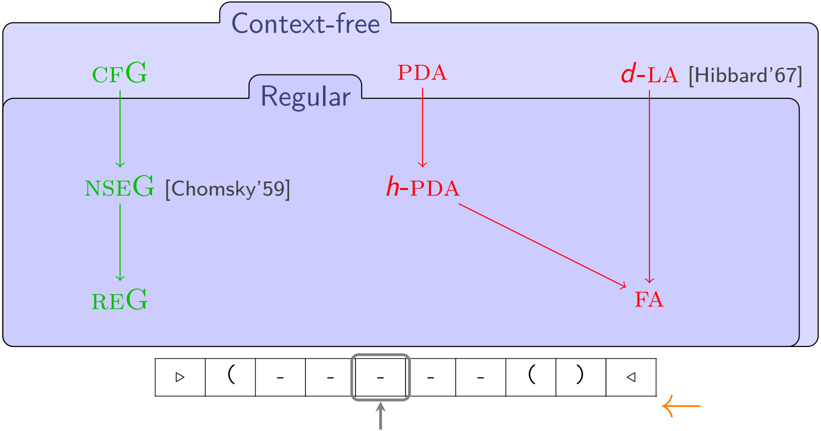

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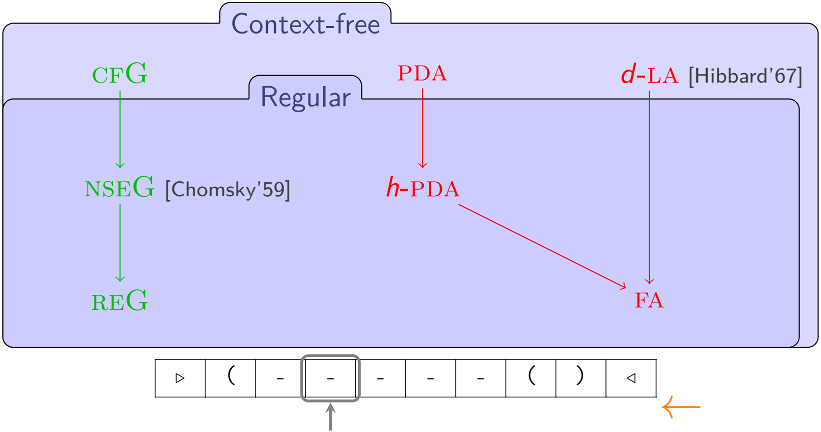

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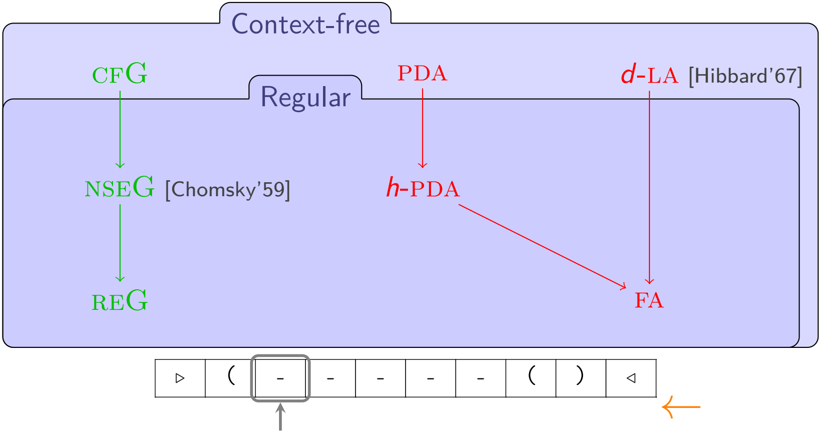

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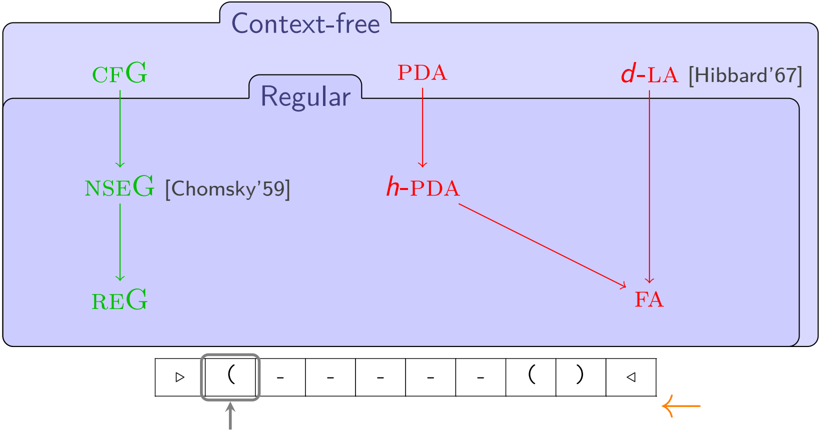

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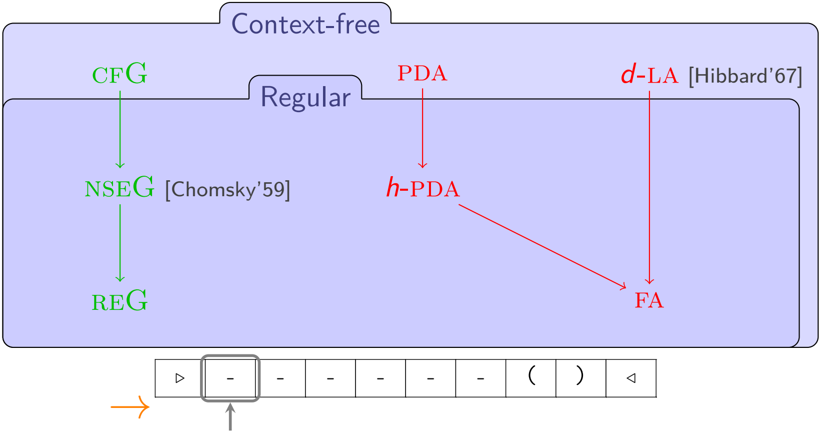

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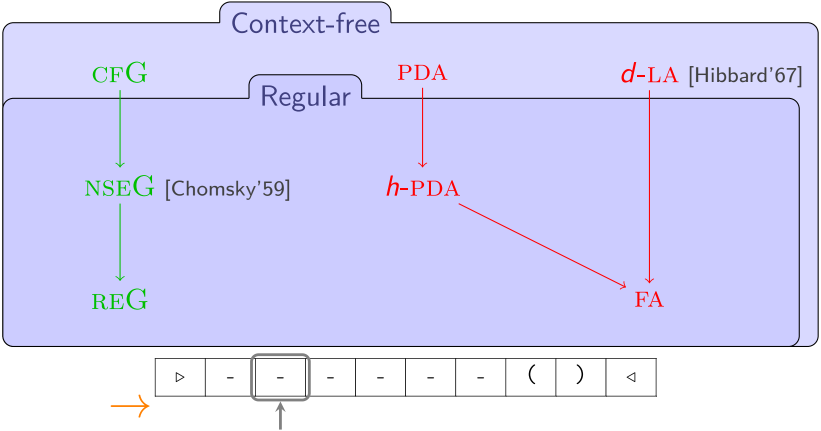

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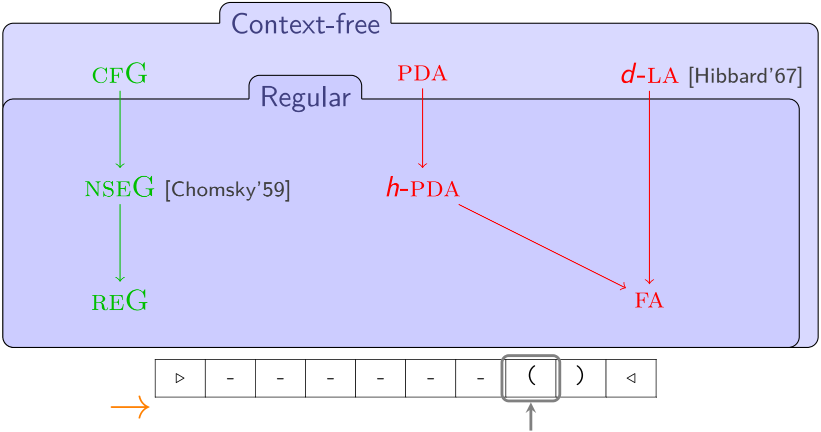

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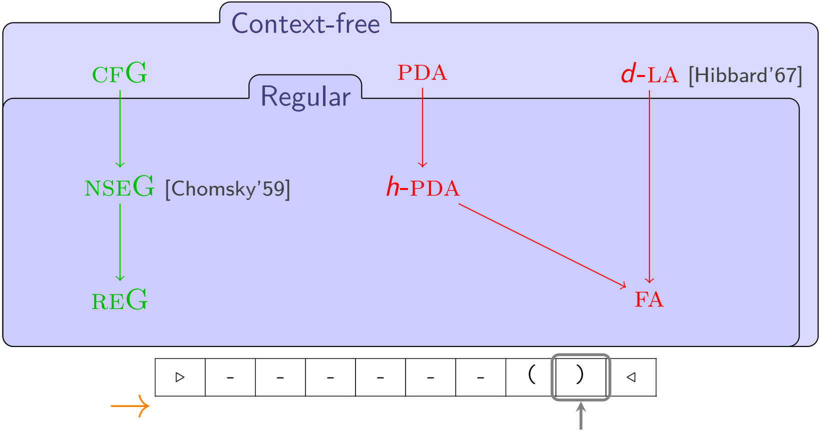

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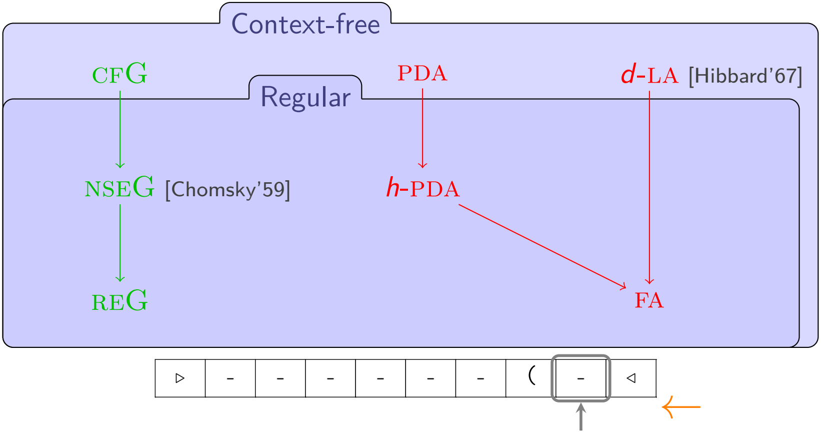

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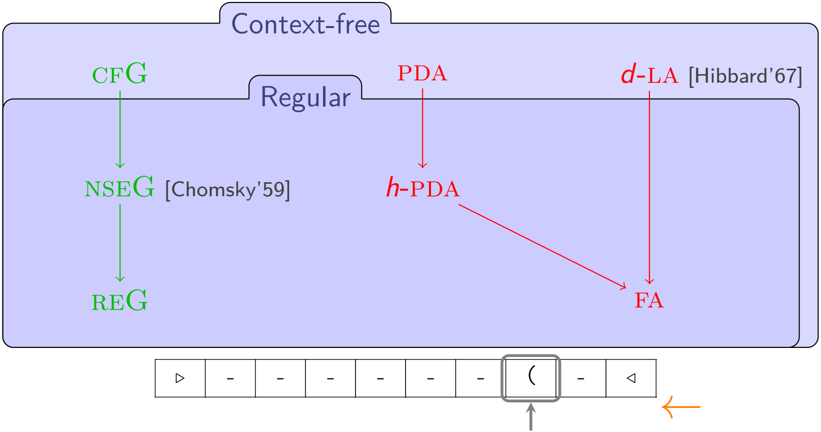

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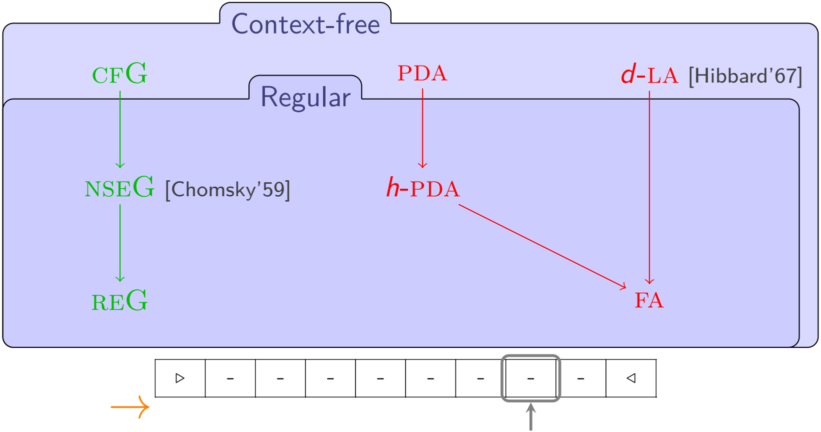

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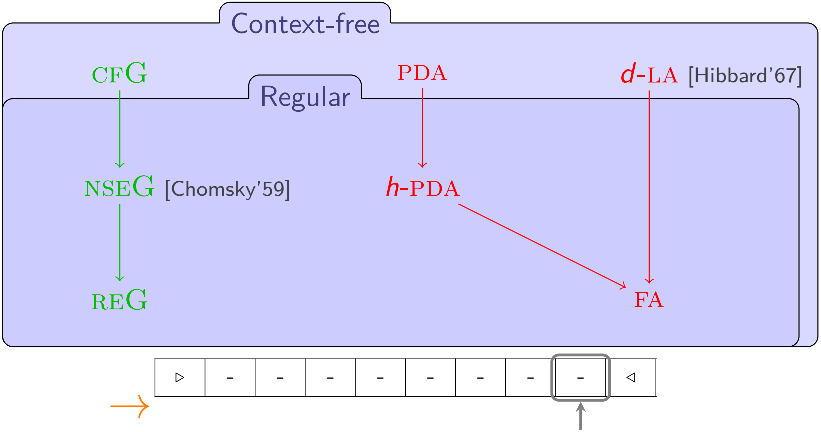

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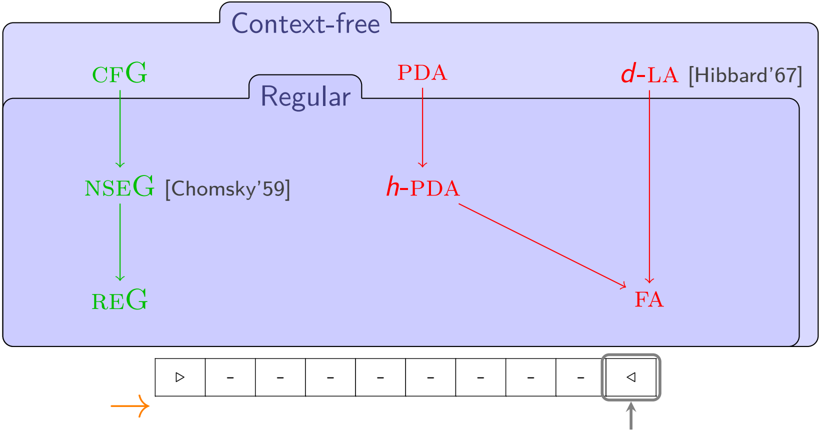

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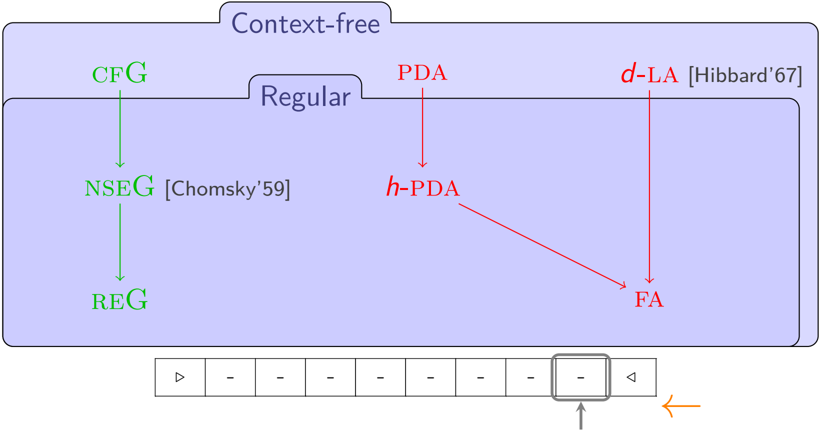

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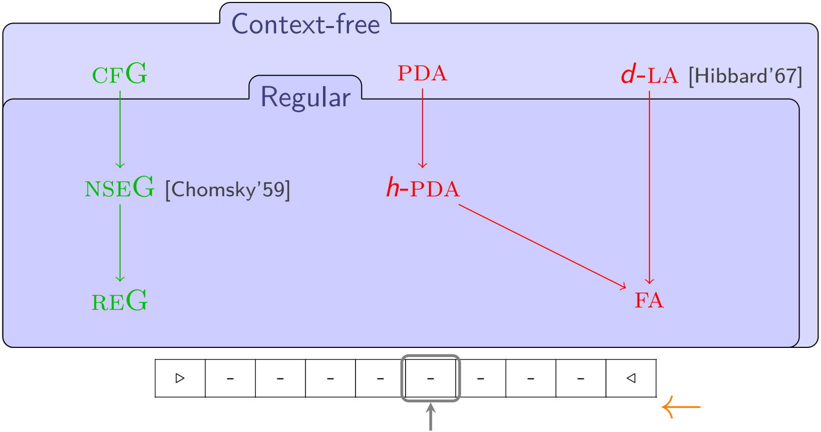

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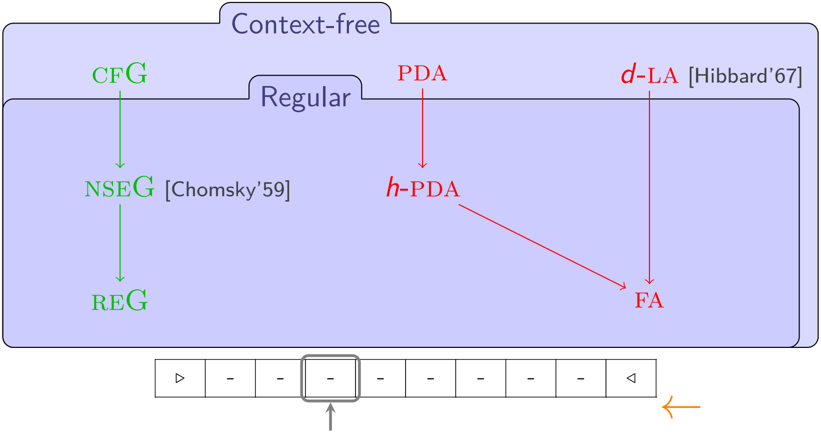

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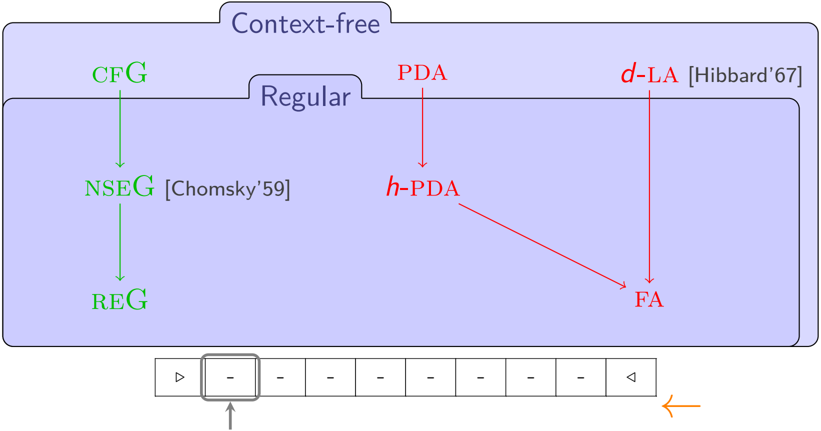

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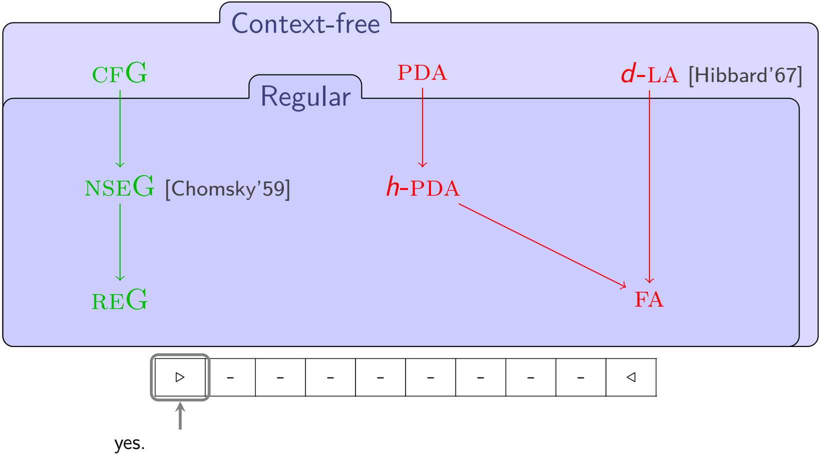

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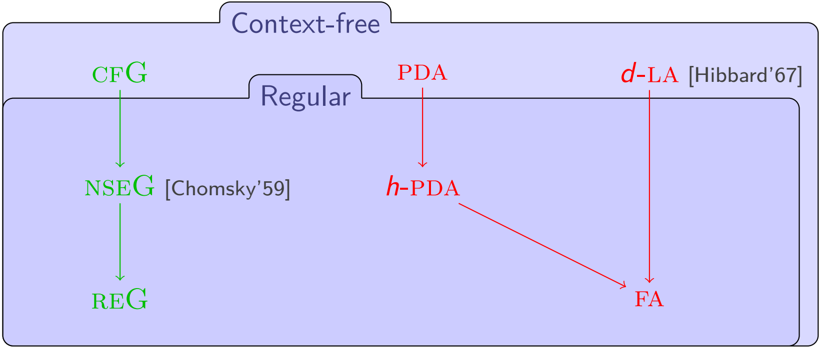

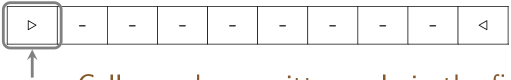

Cells can be rewritten only in the first 2 visits!

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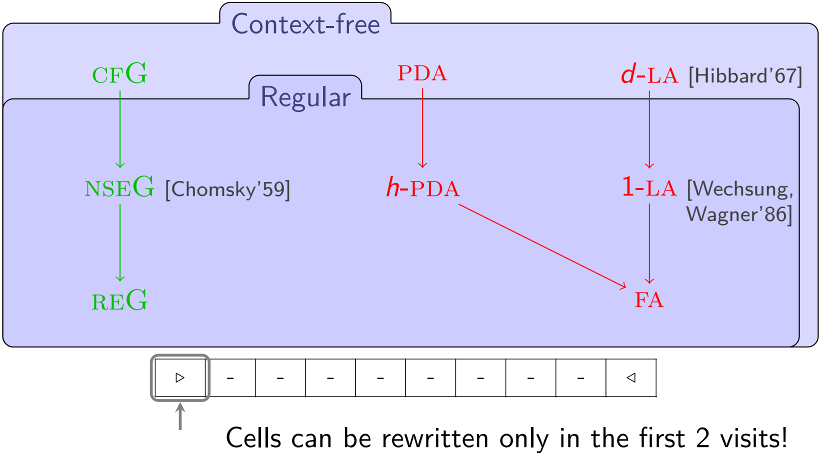

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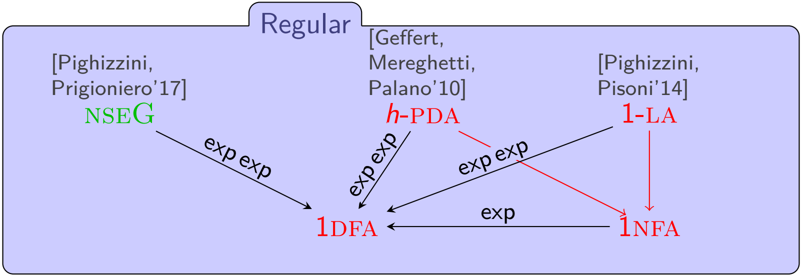
Concise representations of regular languages

Regular

NSEG h-PDA 1-LA

Concise representations of regular languages



Definition: Sizes of models:

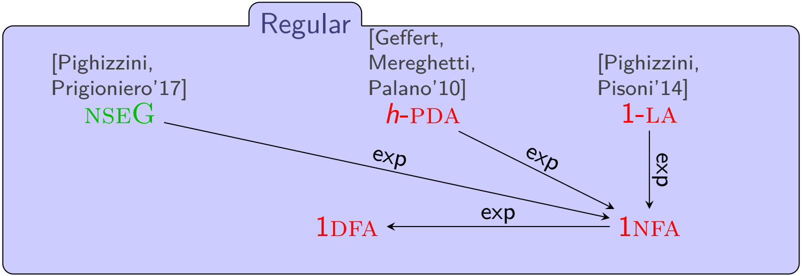
grammars
$$\sum_{X \to \alpha \in P} (2 + |\alpha|)$$

h-PDA
poly in $\#Q$, $\#\Delta$, h

1-LA
poly in $\#Q$, $\#\Gamma$

FA: poly in $\#Q$

Concise representations of regular languages



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Concise representations of regular languages

Regular

$$\begin{tikzcd} \textsc{nseG} \arrow[r, bend left, "?"] & h\text{-}\textsc{pda} \arrow[l, bend left, "?"] \arrow[r, bend left, "?"] & 1\text{-}\textsc{la} \arrow[l, bend left, "?"] \end{tikzcd}$$

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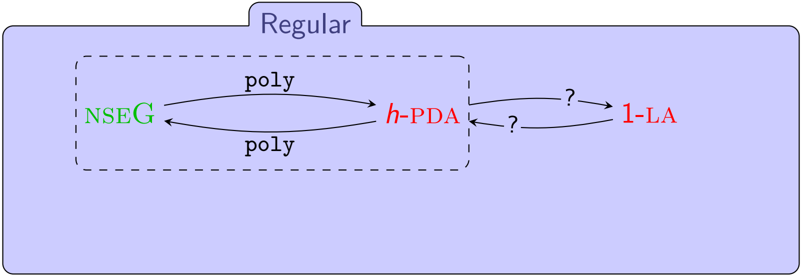

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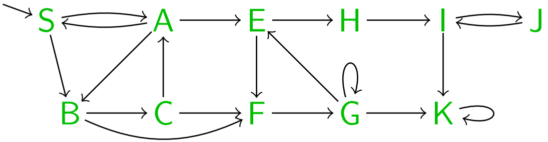
From NSEG to h-PDA and back

Production graph

There is an edge $X \to Y$ if there is a production $X \to \alpha Y \beta$



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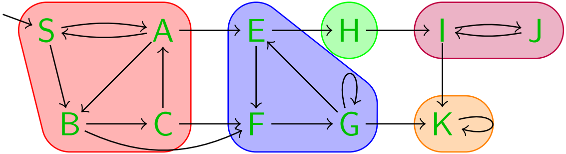

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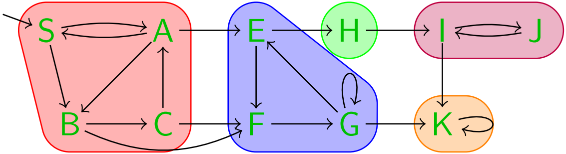

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[Anselmo, Giammarresi, Varricchio 2002]

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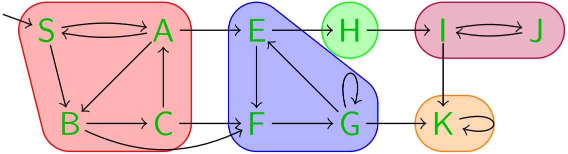

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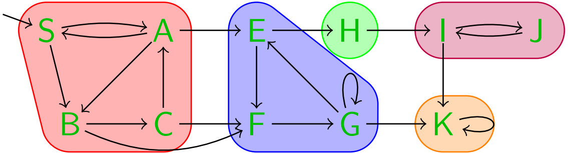

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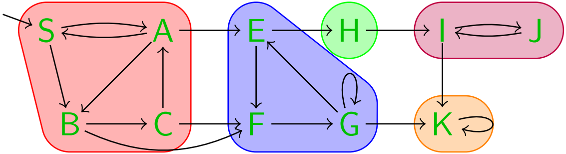

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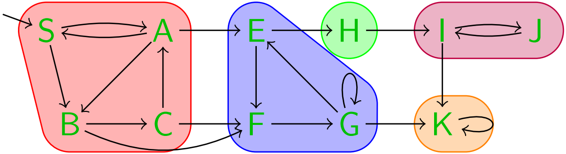

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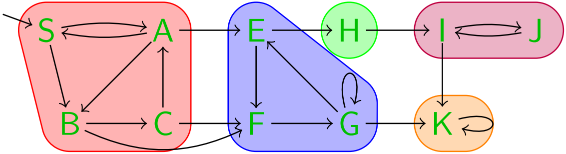

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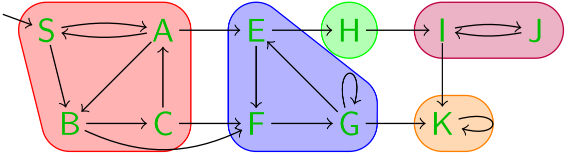

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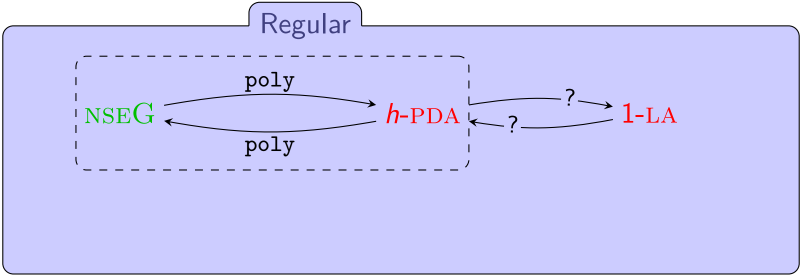

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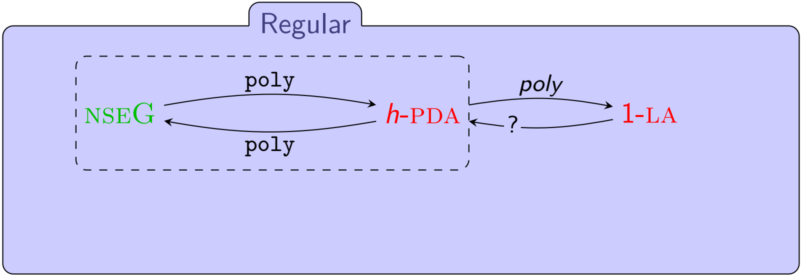

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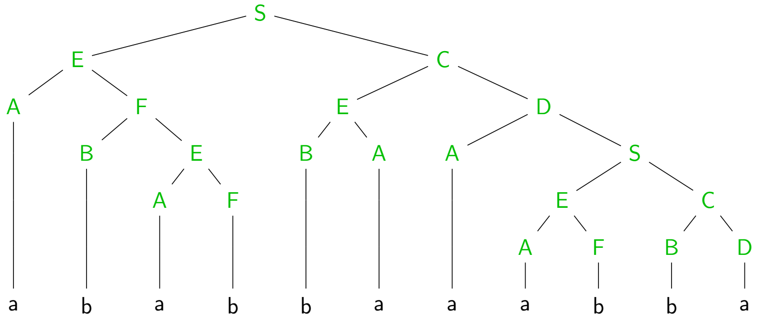
From NSEG to 1-LA

from NSEG in CNF with
$X \to YZ \implies Y > X$



▷	a	b	a	b	b	a	a	a	b	b	a	◁

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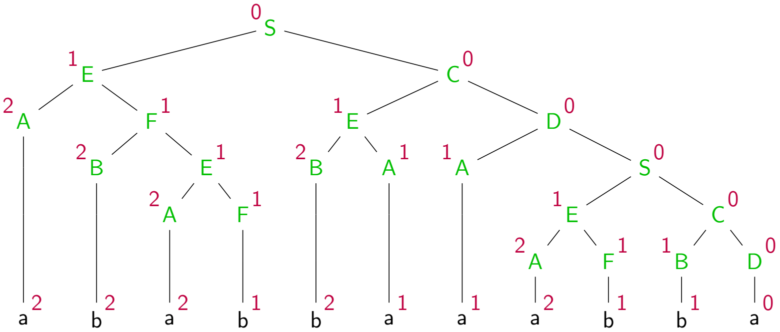

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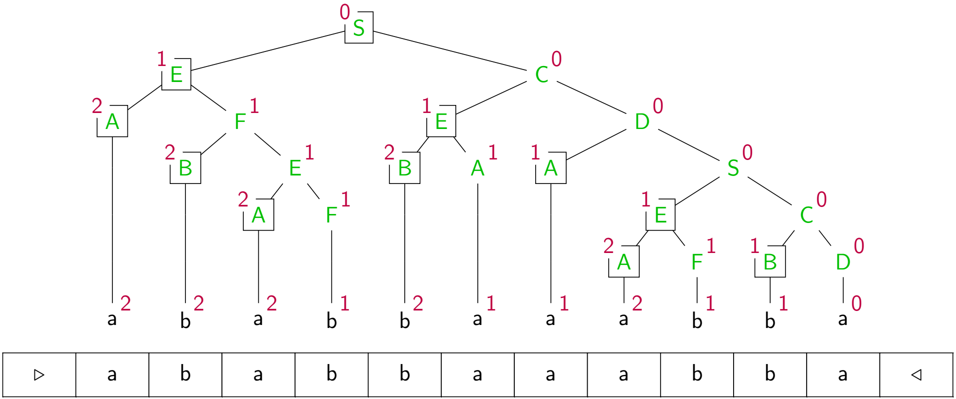

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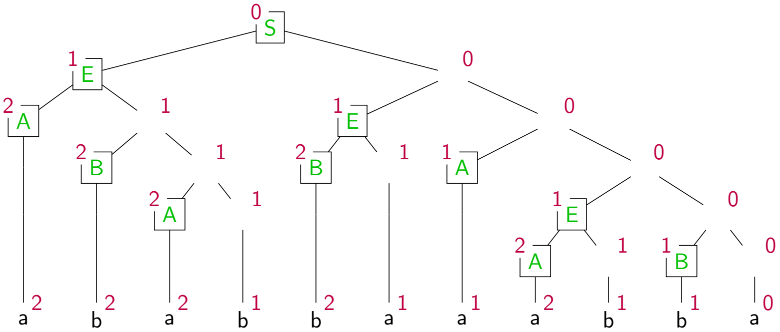

$\triangleright$	$a, A, 2$	$b, B, 2$	$a, A, 2$	$b, E, 1$	$b, B, 2$	$a, E, 1$	$a, A, 1$	$a, A, 2$	$a, E, 1$	$b, B, 1$	$a, S, 0$	$\triangleleft$

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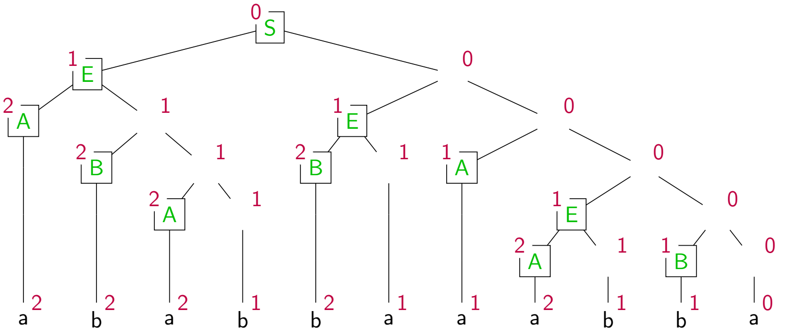

$\triangleright$	$a, A, 2$	$b, B, 2$	$a, A, 2$	$b, E, 1$	$b, B, 2$	$a, E, 1$	$a, A, 1$	$a, A, 2$	$a, E, 1$	$b, B, 1$	$\boxed{a, S, 0}$	$\triangleleft$

the root

From NSEG to 1-LA

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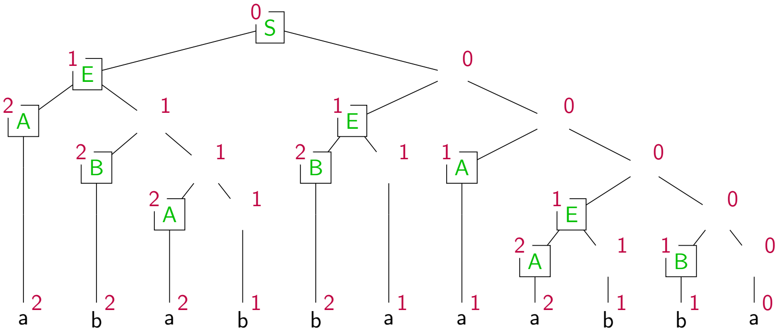

$\triangleright$	$a, A, 2$	$b, B, 2$	$a, A, 2$	$b, E, 1$	$b, B, 2$	$a, E, 1$	$a, A, 1$	$a, A, 2$	$a, E, 1$	$b, B, 1$	$\boxed{a, S, 0}$	$\triangleleft$

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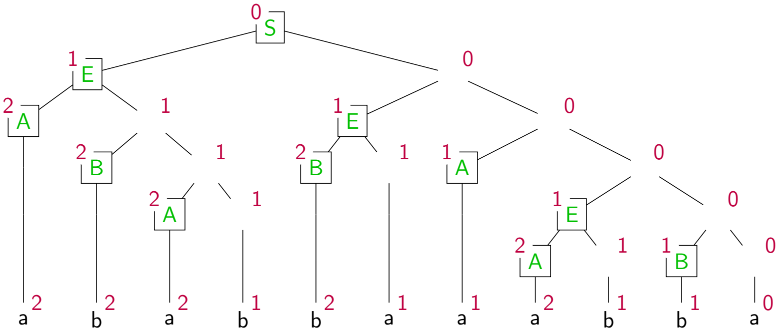

$\triangleright$	$a, A, 2$	$b, B, 2$	$a, A, 2$	$b, E, 1$	$b, B, 2$	$a, E, 1$	$a, A, 1$	$a, A, 2$	$a, E, 1$	$b, B, 1$	$a, S, 0$	$\triangleleft$

its left child

the root

From NSEG to 1-LA

from NSEG in CNF with
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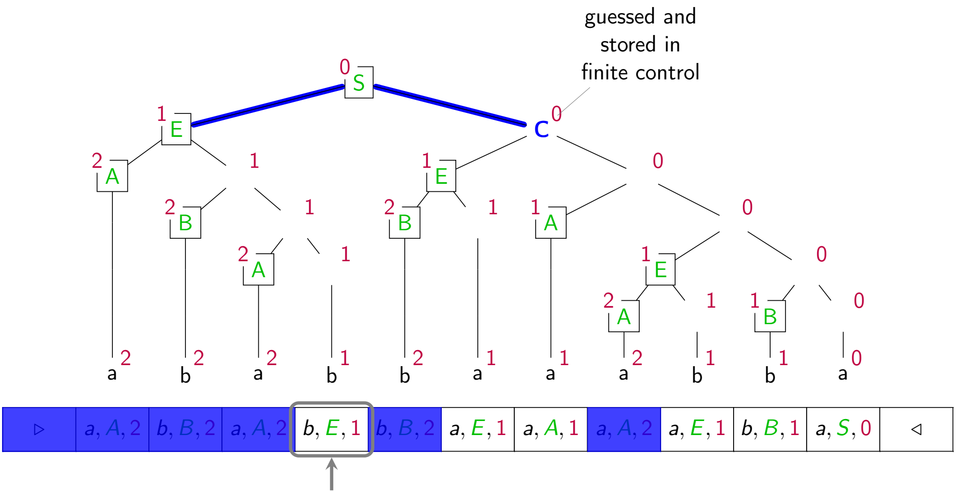

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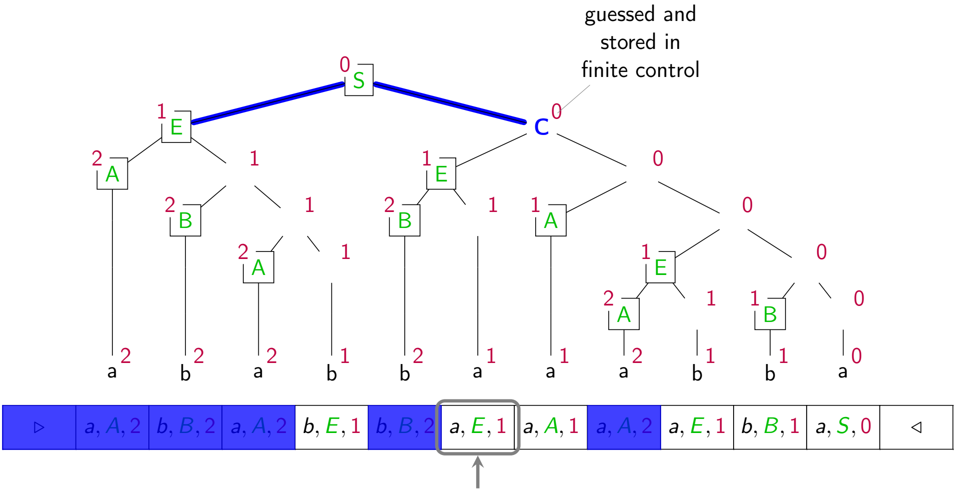

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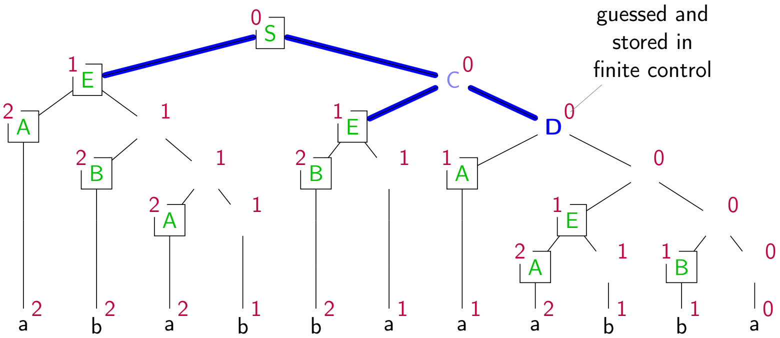

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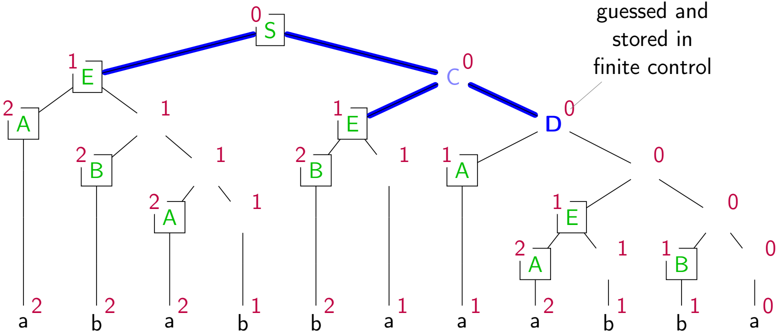

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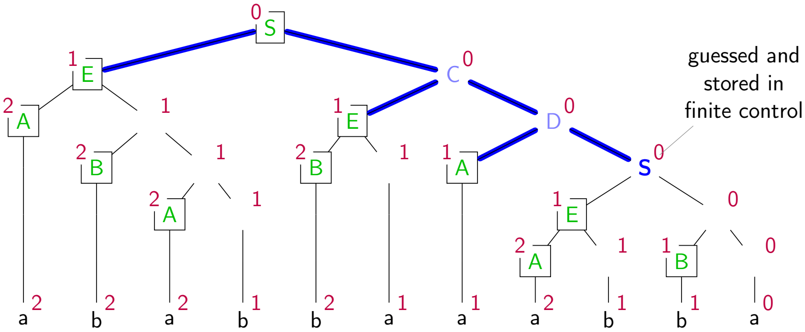

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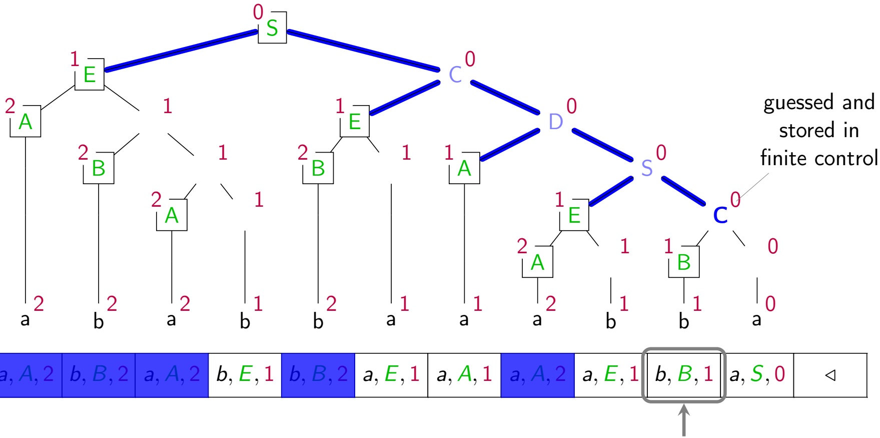

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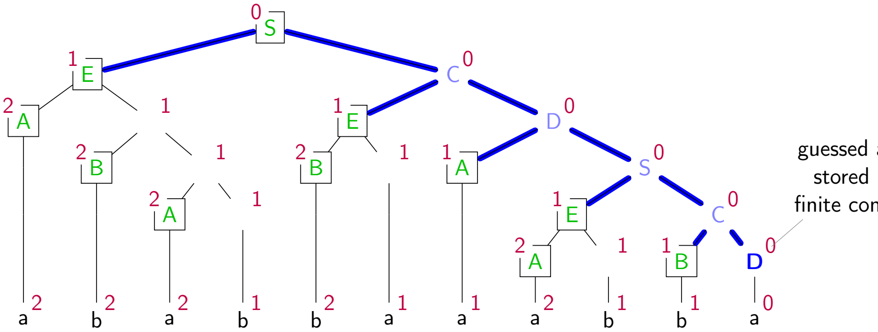

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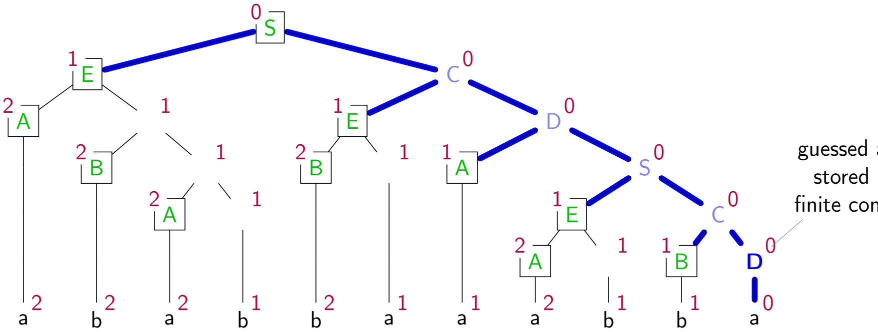

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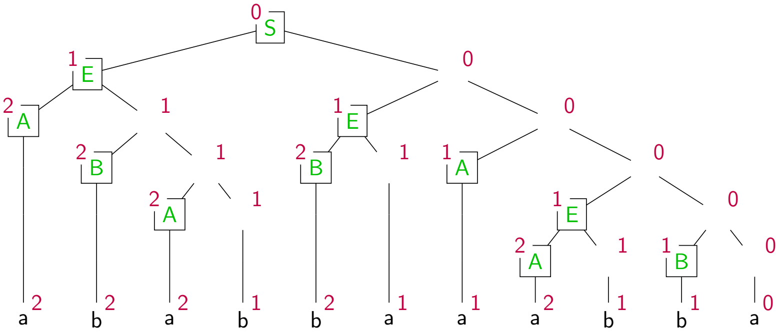

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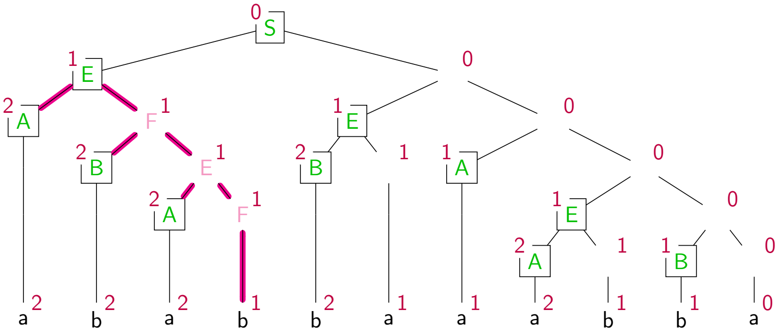

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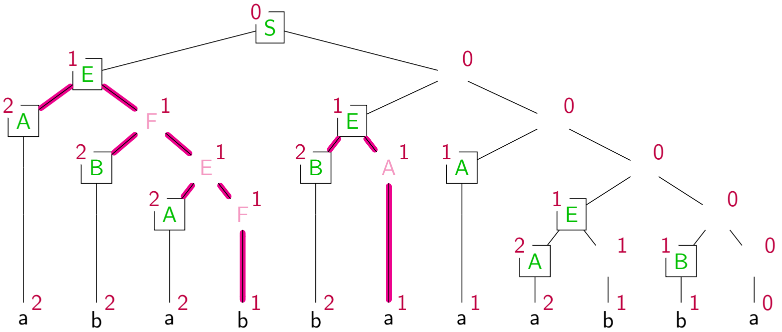

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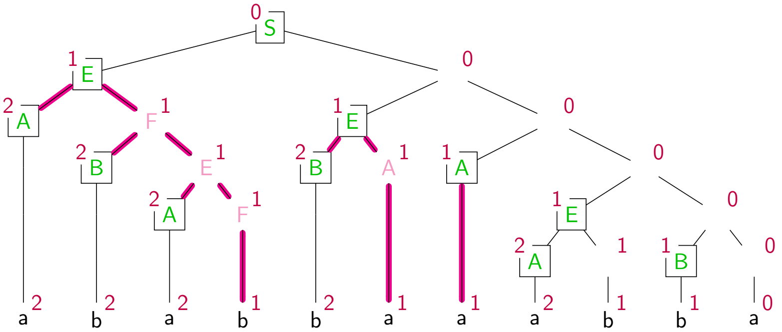

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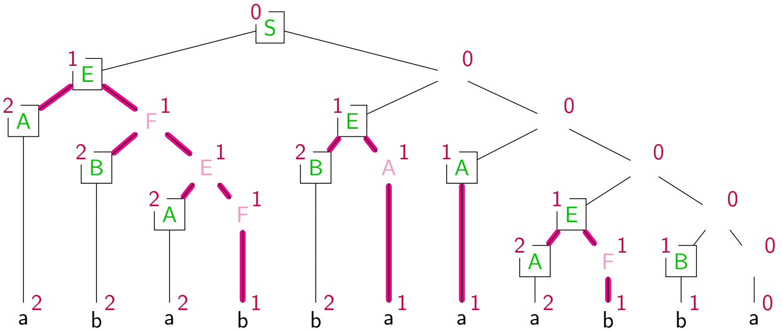

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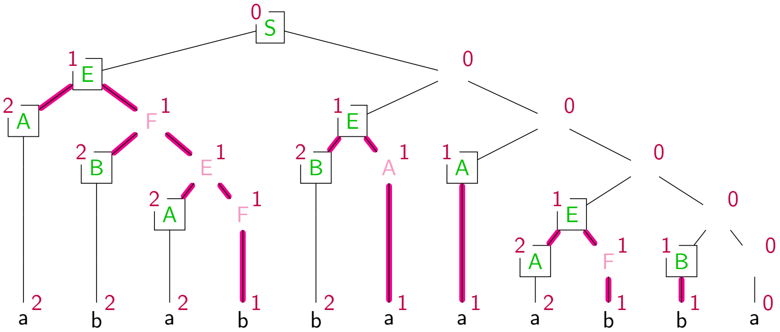

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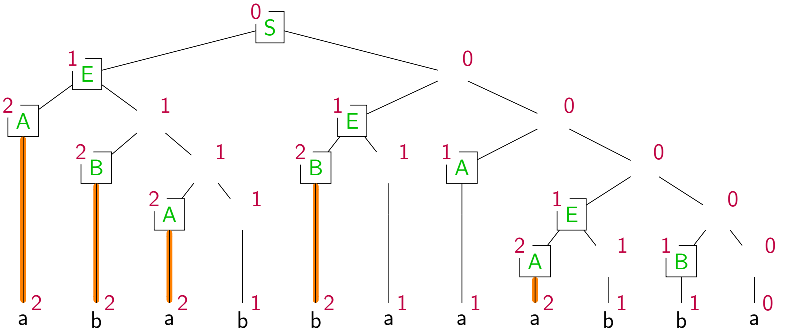

$\triangleright$	$\boxed{a, A, 2}$	$\boxed{b, B, 2}$	$\boxed{a, A, 2}$	$b, E, 1$	$\boxed{b, B, 2}$	$a, E, 1$	$a, A, 1$	$\boxed{a, A, 2}$	$a, E, 1$	$b, B, 1$	$a, S, 0$	$\triangleleft$

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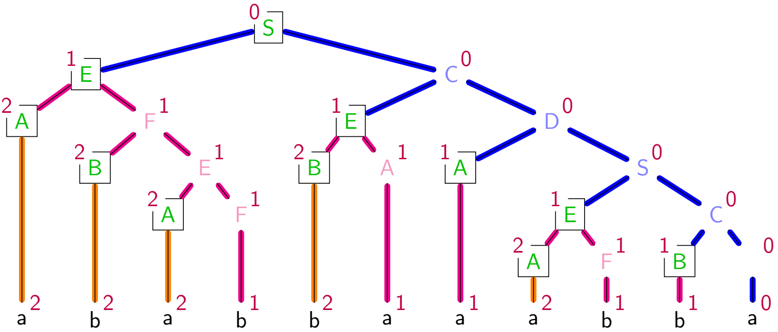

▷	$a, A, 2$	$b, B, 2$	$a, A, 2$	$b, E, 1$	$b, B, 2$	$a, E, 1$	$a, A, 1$	$a, A, 2$	$a, E, 1$	$b, B, 1$	$a, S, 0$	◁

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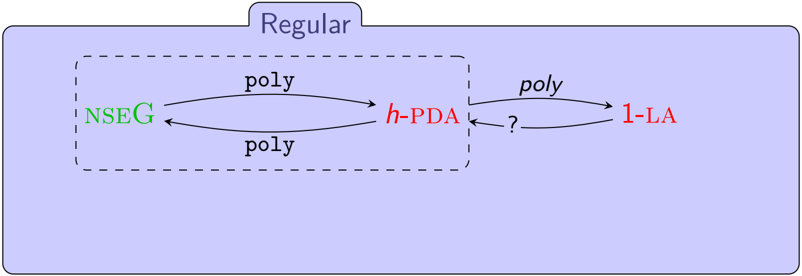

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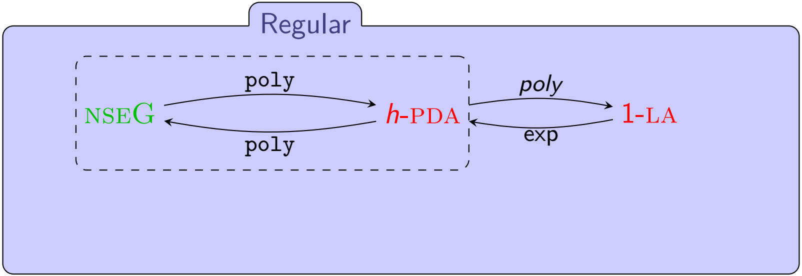

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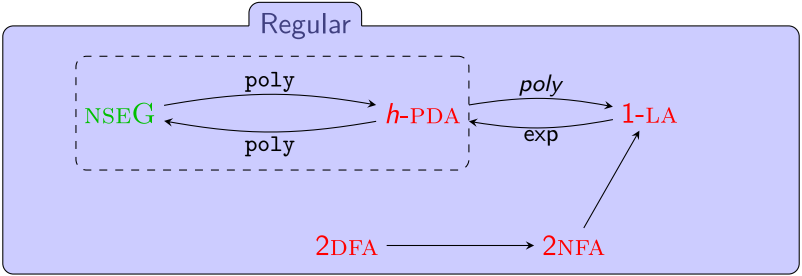

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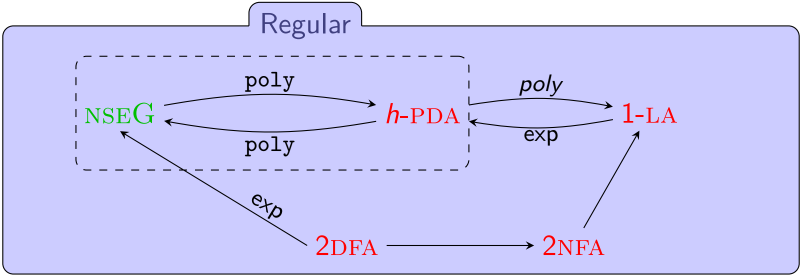

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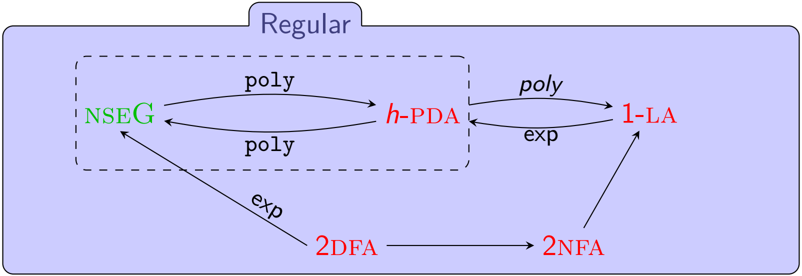

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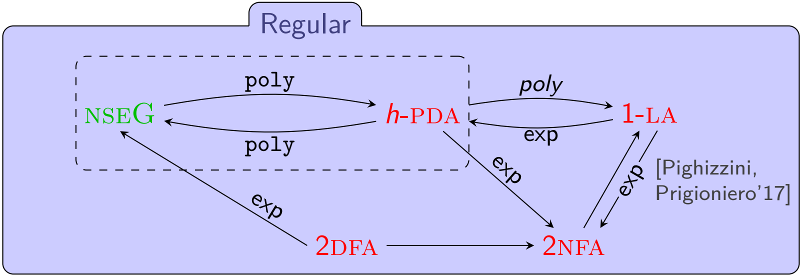

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Rewriting can be pushed to an initial phase in the resulting 1-LA.

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Thank you for your attention.