Internship proposal: The theory of universal graphs for games on graphs

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Abstract: This internship aims at developping the theory of universal graphs, in the context of constructing efficient algorithms for games on graphs.

Topics: games, automata, algorithms, graphs, logic

The algorithmic study of games on graphs is at the crossing of various fields in computer science: originating from automata theory and logic and motivated by program verification and controller synthesis, it develops and leverages tools from complexity theory, graph theory, and combinatorics.

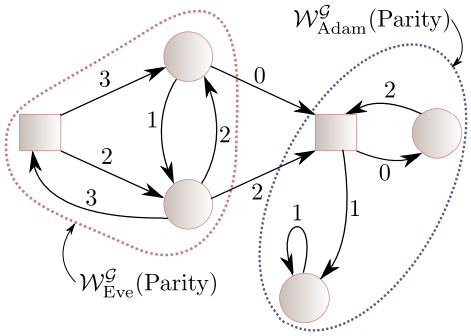

Figure 1: An example of a parity game. The labels on the edges are called priorities.

A central example is parity games, which are played in a directed graph with two types of vertices: circles controlled by Eve and squares controlled by Adam. A play consists in describing a path in the graph: when the current vertex is a circle Eve chooses the edge to follow, and when it is a square Adam chooses. The play stops when some vertex is visited for the second time, forming a loop. The parity objective specifies that Eve wins if the largest priority in the loop is even, and Adam wins otherwise.

We let $W_{\text{Eve}}(\mathcal{G})$ denote the set of vertices from which Eve has a winning strategy in the game $\mathcal{G}$, and $W_{\text{Adam}}(\mathcal{G})$ for Adam. In the example, $W_{\text{Eve}}(\mathcal{G})$ is circled in red and $W_{\text{Adam}}(\mathcal{G})$ in blue. For instance, a winning strategy for Eve consists of following the loop with priorities 1 and 2 between the top and bottom vertices, ensuring a loop with largest priority 2.

Goals of the internship

Universal graphs have been introduced recently by Colcombet and Fijalkow [CF19] to better understand the recent quasipolynomial time algorithms for parity games[1].

The goal of this internship is to develop a theory of universal graphs, and to investigate its uses for various questions beyond parity games. Different directions will be offered, and will be explored depending on the taste and interest of the intern.

- Develop a notion of universal graphs for stochastic games,
- Study the composition and combination of universal graphs, in particular for mean payoff games,
- Construct universal graphs for subclasses of graphs, such as nowhere dense graphs, planar graphs, or graphs of bounded tree width,
- Obtain upper and lower bounds on lazy progress measures.

The internship can naturally be continued into a PhD.

References

[CF19] Thomas Colcombet and Nathanaël Fijalkow. Universal graphs and good for games automata: New tools for infinite duration games. In *FoSSaCS*, pages 1–26, 2019.

[1] The interested reader could read more on the blog `https://games-automata-play.github.io/`.