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Observation Synthesis for Games with Imperfect Information

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Joint work with Paulin Fournier

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Games with Imperfect Information

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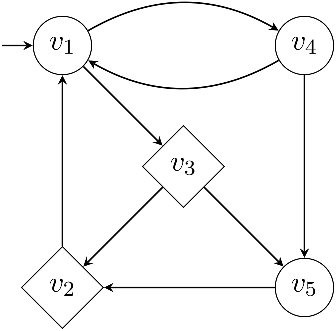

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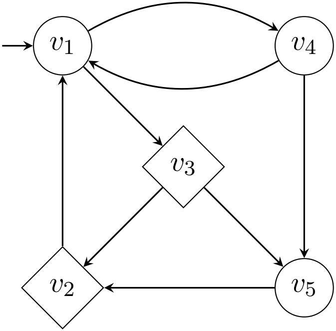

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  $V = \{v_1, v_2, v_3, v_4, v_5\}$

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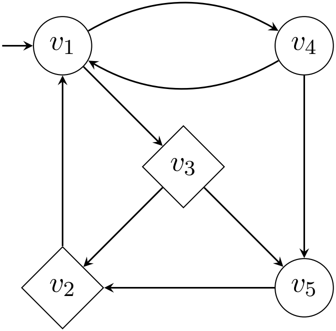

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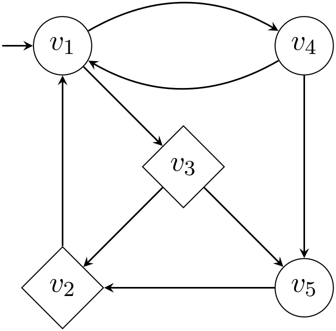

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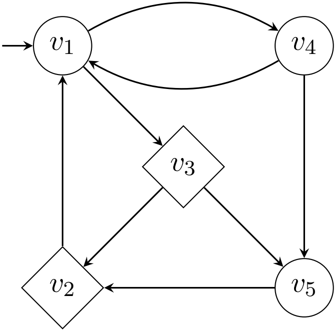

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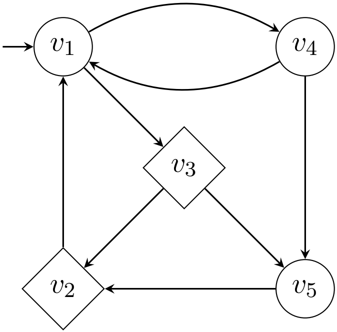

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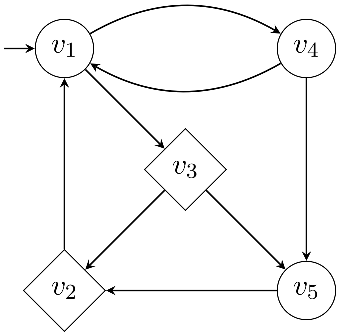
Indistinguishable histories (for Player 1):

Games with Imperfect Information



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Indistinguishable histories (for Player 1):

Equivalence relation $R \subseteq V^* \times V^*$

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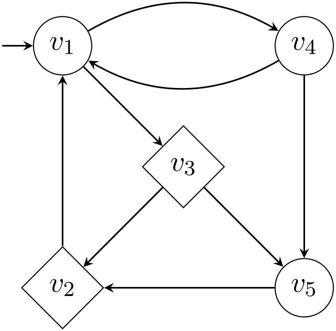

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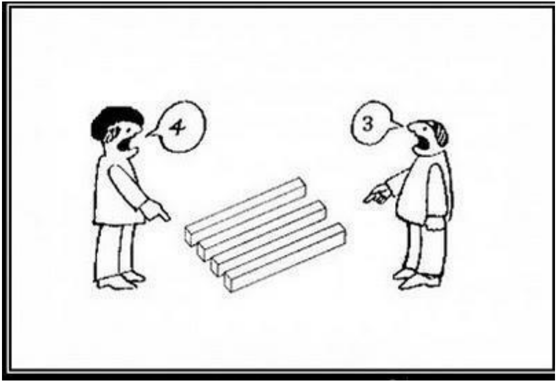
Equivalence relation $R \subseteq V^* \times V^*$
Given as a transducer

Observation functions



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Remark

$\ker f := \{(u,v) \in (V^*)^2 \mid f(u) = f(v)\} = f^{-1} \circ f$
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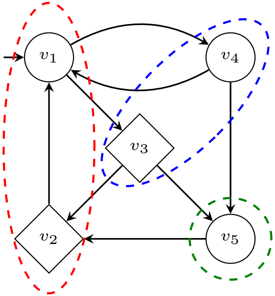
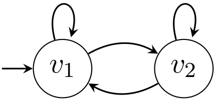
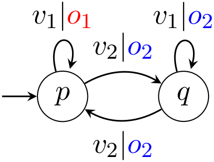
Ex:



Arena:



Observation function:



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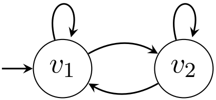

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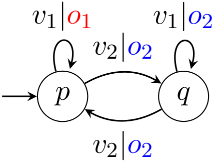

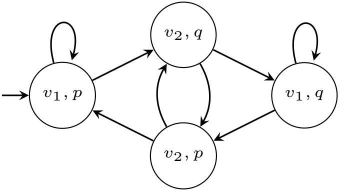

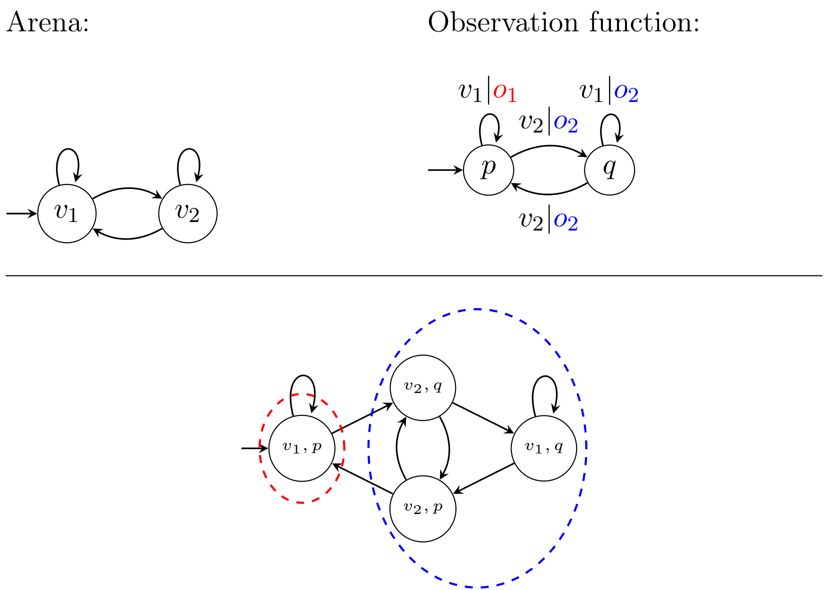
Arena:
Observation function:
v_1
v_2
$v_1|o_1$
$v_1|o_2$
$v_2|o_2$
$v_2|o_2$
p
q
v_1, p
v_2, q
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$R \setminus f$	Functional	Sequential	Mealy
Arbitrary	?		X
Deterministic	Yes		X
Letter-to-letter	Yes		

Mealy observation function

Arena V, indistinguishability relation R.
Is there f given by a *sequential* and *letter-to-letter* transducer, such that $\ker f = R$?

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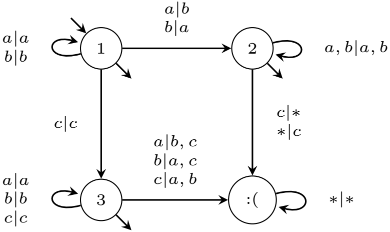
- 3) R satisfies condition 3)

Co-example



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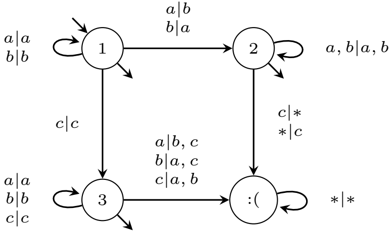

Syntactic congruence of R

uSv if $\forall w\ uwRvw$

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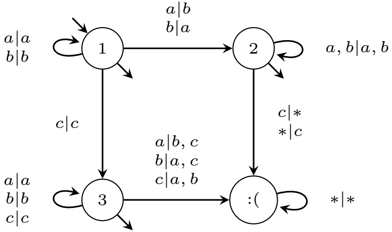

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Condition 3):

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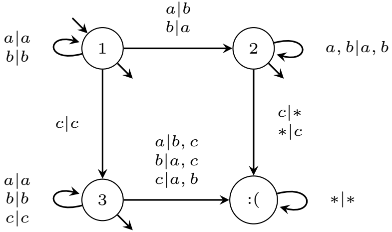

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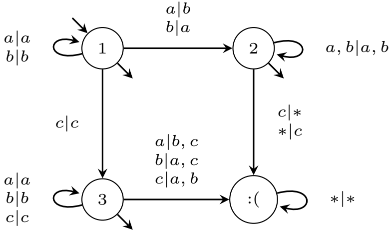

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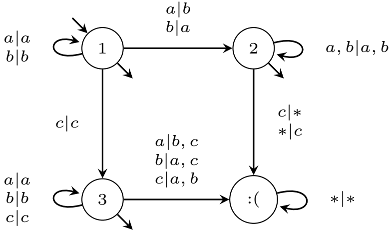

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First result:

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- 1)-3) are sufficient

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  (hard to show)

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Second result

- 1)-2) are sufficient (same kind of proof)
- 2) is undecidable (thanks Bruno Guillon)

Sum up *etc*

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Deterministic	Yes	U	X
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Qs

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Qs

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Qs

- Find interesting classes of observation functions (*i.e.* deterministic, increasing)
- Solve games!

Thanks !