Homework 3 (due Tuesday September 24)

Quiz 3 Thursday September 26.

Main Topics:

Solving nonlinear equations in one variable. Chapter 3 in textbook.

Main Objectives:

Introduce and implement popular methods to solve scalar nonlinear equations.

Main Tools:

Basic calculus and previous course results.

1. Review questions
   a) Is the bisection method (i) efficient (ii) robust (iii) does it require a minimal amount of additional knowledge (iv) require f to satisfy only minimum smoothness properties (v) generalize easily to several functions in several variables?
   b) Answer similar questions for the Newton and secant methods.
   c) State at least two advantages and two disadvantages of Newton's method.
   d) What are order of convergence and rate of convergence, and how do they relate?
   e) In what situation does Newton's method converge only linearly?
   f) Explain the role that roundoff errors play in the convergence of solvers for nonlinear equations, and explain their relationship with convergence errors.

2. Write a MATLAB routine bisect to find the root of the function $f(x) = \sqrt{x} - 1.1$ starting from the interval $[0,2]$, with $atol = 1.e-8$.
   a) How many iterations are required? Does the iteration count match the expectations, based on our convergence analysis?
   b) What is the resulting absolute error? Could this absolute error be predicted by our convergence analysis?

3. Consider the function $g(x) = x^2 + \frac{3}{16}$
   a) This function has two fixed points. What are they?
   b) Consider the fixed point iteration $x_{k+1} = g(x_k)$ for this g. For which of the points you have found in part a) can you be sure that the iterations will converge to that fixed point? Briefly justify your answer. You may assume that the initial guess is sufficiently close to the fixed point.
   c) Write a MATLAB routine to implement this fixed point method for this particular function g.

4. Write MATLAB routines for problem 2 that implement Newton's and fixed point method, and compare results and stablish rate of convergence of each of the 3 methods. Plot the function and show in this plot the progress of each of the methods.

5. Two of the four zeros of $x^4 + 2x^3 - 7x^2 + 3$ are positive. Find them by Newton's method, correct to two significant figures.

6. Find the root of the equation $2x(1 - x^2 + x)\ln x = x^2 - 1$ in the interval $[0,1]$ by Newton's method using double precision. Make a table that shows the number of correct digits in each step.

7. Find the root of the equation $\frac{1}{2}x^2 + x + 1 - e^x = 0$ by Newton's method, starting with $x_0 = 1$, and account for the slow convergence.

8. For problem 4, use and get familiar with the features of a build in MATLAB function to get the root or roots. Compare the accuracy and execution times. Comment on the results obtained.

9. Suppose that we want to find all the roots of a polynomial of degree 20. Explain how you might try to do this and comment on the possible issues that arise. You might experiment numerically with an actual polynomial and present your results. Note. There is not just a unique answer to this question.