Question 1

Holding Costs

Holding costs are costs incurred when a firm stores inventory in a warehouse. A firm's holding costs can come from storage space, labor, and insurance.

For a company producing hobby electric aircraft, the holding costs can appear from:

- **Storage cost**: Costs associated with setting up, building, and/or renting warehouse. This cost may also include labour costs associated with the maintenance and security of the warehouse. For example, to store electronic components to build the aircraft, a specific warehouse with a controlled environment (controlled humidity, temperature, no dust, etc.) is required and this can incur a lot of significant cost.
- **Depreciation**: Some items turn obsolete when stored for too long. These costs contribute to a firm's holding cost and is also known as depreciation. For example, if the aircraft company stores a certain electronic chip for too long (say, 5 years, hypothetically), the chip may become outdated for the most recent type of aircraft, making it unusable.
- **Insurance**: As a form of risk management, companies enrol for insurance schemes. In the event of theft, weather damage or vandalism, the company can claim insurance benefits from the insurer, enabling them to minimize risk in the long term at the cost of insurance fees. These costs contribute to holding costs since they are necessary in storing inventory. For example, the aircraft company may need to store various expensive components such as semiconductors and other electronic chips. These chips are highly valued and thieves may try to steal them. Thus, a certain level of protection and risk management is required, appearing in the form of insurance.

To compute the holding cost, companies typically sum up all contributing costs listed above (and other cost which may arise from inventory maintenance) and divide it with their total inventory value. This gives the rate of holding cost.

$$\text{Holding Cost (\%)} = \frac{\text{Inventory Holding Sum}}{\text{Total Inventory Value}} \times 100$$

Smoothing Costs

Smoothing costs are costs associated with the adjustment of workforce levels to cater for a certain production level deemed necessary by the company. Smoothing costs arise from two major components: hiring costs and firing costs. To hire a new employee, the company needs to post a job listing, process a bunch of job applications, and screen potential candidates through technical and behavioural interviews. In this case, the aircraft company may need to have an HR department overseeing this recruitment process, and some cost would definitely be incurred in the process.

On the contrary, firing workers can also incur certain costs. In certain countries, a company cannot simply fire a worker without good reason. Furthermore, they may be required to provide some amount of support to the worker. This support may appear in various forms (depending on the law), and may include retrenchment compensation package and aid in looking for a new job. As such, firing a worker can be an expensive process and the aircraft company must account for this before committing any fires.

Backordering Costs

Backordering costs are costs associated with the company's inability to satisfy customer demand. When the aircraft company fails to satisfy customer demand a certain period of time, two things can happen:

(1) the customer chooses to wait but feels disappointed, causing a loss of goodwill (which can turn into financial losses in the long term), or (2) the customer chooses to engage with a different company who can serve them faster. Either way, when backordering happens, the company suffers from losses which can be quantified in terms of money.

The backordering cost can be seen as the "cost of consequences." It can be calculated by considering sales lost, additional fees incurred due to the need for expediting production and delivery services, and even the permanent lost of a customer. In reality, backordering costs are tricky to compute but play a pivotal role in a company's supply chain management.

Planning Horizon

According to (Swamidass, 2000), a planning horizon is the length of time (i.e., the number of weeks or months) into the future for which plans are made. In the context of supply chain and demand planning, it represents the period over which forecasts are made and decisions regarding inventory, production, and capacity are determined. The planning horizon is crucial for ensuring that the right products are available at the right time and place to meet customer demand.

As production manager of the aircraft company, selecting the planning horizon involves a thorough understanding of the company's supply chain, including the lead times and the cumulative impact of various decisions. For instance, if it takes 1 month to acquire certain parts to build the company's aircrafts, then the planning horizon must not span less than 1 month. As production manager, I would design a planning horizon that is long enough such that the company can wholistically plan its manufacturing process.

Planning Period

The planning period is the time within the planning horizon whereby plans are actually made. This is the period where the company makes decisions regarding production and inventory management. Selecting the right planning period is critical for effective production and inventory management as it determines how frequently plans are reviewed and updated, impacting the agility and responsiveness of the company's production process. In the case of a hobby aircraft company, the planning period would probably be shorter than other volatile products such as fashion items (e.g. ZARA, which focuses on fast-changing fashion trends) because aircrafts develop over a longer time period. Similar to the planning horizon, determining an appropriate planning period requires a production manager to understand the nature of the company's business.

Question 2

(a)

In a constant work force plan, the number of workers remain constant from the first to last period of the planning. Since no backlogging is allowed, cumulative production must never be less than the cumulative demand at any given period. With this in mind, we can implement a constant work force plan as follows.

A	B	C	D	E	F	G
Month	Work days	Demand	B × k	Cumulative Demand	Cumulative Units/Worker	[E / F]
1	26	850	8.01	850	8.008	107
2	24	1260	7.39	2110	15.4	138
3	20	510	6.16	2620	21.56	122
4	18	980	5.54	3600	27.104	133
5	22	770	6.78	4370	33.88	129
6	23	850	7.08	5220	40.964	128
7	14	1050	4.31	6270	45.276	139
8	21	1550	6.47	7820	51.744	152
9	23	1350	7.08	9170	58.828	156
10	24	1000	7.39	10170	66.22	154
11	21	970	6.47	11140	72.688	154
12	13	680	4	11820	76.692	155

H
Min Workers
156

I	J	K	L
Monthly Production (H × D)	Cumulative Production	Inventory	Is Cumulative Demand Satisfied?
1249.2480	1249.2480	399.2480	TRUE
1153.1520	2402.4000	292.4000	TRUE
960.9600	3363.3600	743.3600	TRUE
864.8640	4228.2240	628.2240	TRUE
1057.0560	5285.2800	915.2800	TRUE
1105.1040	6390.3840	1170.3840	TRUE
672.6720	7063.0560	793.0560	TRUE
1009.0080	8072.0640	252.0640	TRUE
1105.1040	9177.1680	7.1680	TRUE
1153.1520	10330.3200	160.3200	TRUE
1009.0080	11339.3280	199.3280	TRUE
624.6240	11963.9520	143.9520	TRUE
	Total =	5704.7840	

Here, note that the demand in month 1 is adjusted to 850 from 1050 considering the company's 200 unit inventory prior to month 1.

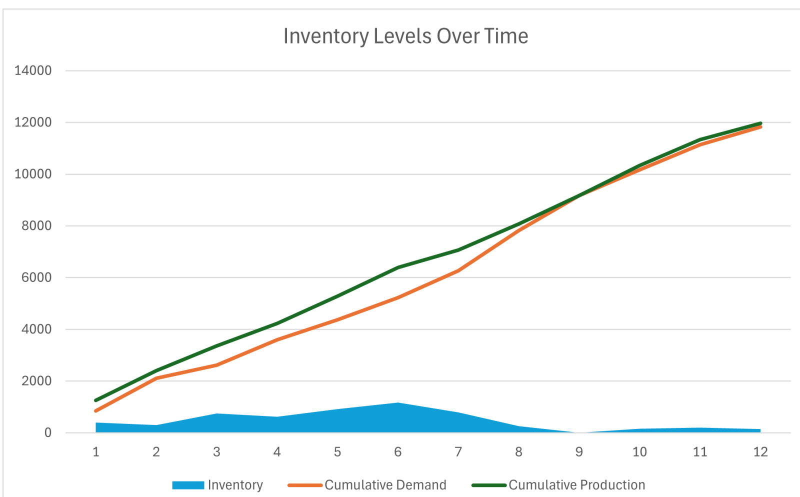

Figure 1: Constant Work Force Plan, No Backlogging

In this plan, there are three contributors to cost being considered: the cost of hire ($c_H = \$100$), the cost of fire ($c_F = \$200$), and the cost of inventory holding ($c_I = \$0.1$/unit·month). With this, our plan's total cost can be computed by the following equation:

$$C = \sum_{t=1}^{12} \left(100 \cdot H_t + 200 \cdot F_t + 0.1 \cdot I_t \right)$$

where H_t, F_t, and I_t refers to the number of hires, fires, and inventory in period t respectively. Assuming the company starts with 0 workers prior to the first month, then the plan incurs a total cost of:

$$C = 100 \cdot (156) + 0.1 \cdot (5704.784)$$
$$\boxed{C = \$16170.48}$$

(b)

Zero Inventory Plan

In a Zero Inventory Plan, the primary goal is to minimize inventory at the end of each period as close as possible to 0. This is done at the cost of hiring and firing people rapidly (where a certain cost would also be incurred for every fire and every hire). Similar to (a), no backlogging is allowed and thus cumulative production must always satisfy cumulative demand. There are two ways to implement the Zero Inventory Plan. The first way is to let some amount of inventory accumulate over time.

A	B	C	D	E	F	G	H	I	J	K	L
Month	Workdays	Demand	B × k	Worker Level [C / D]	Fires	Hires	Monthly Production (k×B×E)	Cumulative Production	Cumulative Demand	Inventory	Is Cumulative Demand Satisfied?
1	26	850	8.0080	107	0	107	856.8560	856.8560	850.0000	6.8560	TRUE
2	24	1260	7.3920	171	0	64	1264.0320	2120.8880	2110.0000	10.8880	TRUE
3	20	510	6.1600	83	88	0	511.2800	2632.1680	2620.0000	12.1680	TRUE
4	18	980	5.5440	177	0	94	981.2880	3613.4560	3600.0000	13.4560	TRUE
5	22	770	6.7760	114	63	0	772.4640	4385.9200	4370.0000	15.9200	TRUE
6	23	850	7.0840	120	0	6	850.0800	5236.0000	5220.0000	16.0000	TRUE
7	14	1050	4.3120	244	0	124	1052.1280	6288.1280	6270.0000	18.1280	TRUE
8	21	1550	6.4680	240	4	0	1552.3200	7840.4480	7820.0000	20.4480	TRUE
9	23	1350	7.0840	191	49	0	1353.0440	9193.4920	9170.0000	23.4920	TRUE
10	24	1000	7.3920	136	55	0	1005.3120	10198.8040	10170.0000	28.8040	TRUE
11	21	970	6.4680	150	0	14	970.2000	11169.0040	11140.0000	29.0040	TRUE
12	13	680	4.0040	170	0	20	680.6800	11849.6840	11820.0000	29.6840	TRUE
					259	429				224.8480	

Here, note that the demand in month 1 is adjusted to 850 from 1050 considering the company's 200 unit inventory prior to month 1.

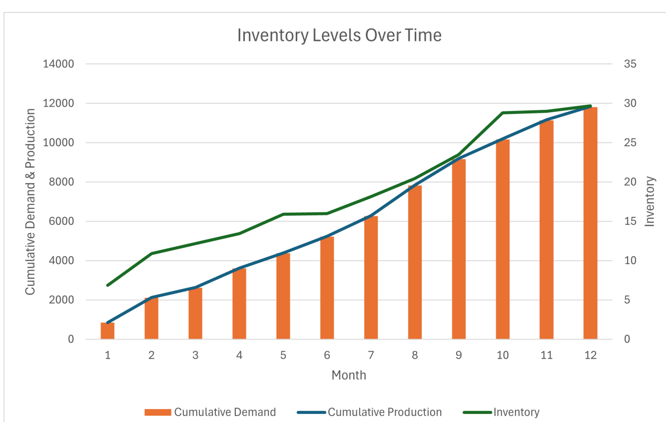

Figure 2: Zero Inventory Plan, No Backlogging, No Demand Adjustment

Using the cost function, this plan would incur a total cost of:

$$C = \sum_{t=1}^{12} \Big(100 \cdot H_t + 200 \cdot F_t + 0.1 \cdot I_t\Big)$$
$$= 100 \cdot (429) + 200 \cdot (259) + 0.1 \cdot (224.848)$$
$$\boxed{C = \$94722.48}$$

Zero Inventory Plan with Monthly Demand Adjustment

Meanwhile, a second way to implement the plan is by adjusting the net demand every period by using any remaining inventory from the previous period. The net demand thus needs to be calculated, and in particular by subtracting the original demand by the past period's inventory.

A	B	C	D	D	F	G	H	I	J	K	L	M	N
Month	Workdays	Demand	Net Demand	B × k	Worker Level [C / D]	Fires	Hires	n(workers)	Monthly Production	Cumulative Production	Cumulative Demand	Inventory	Is Cumulative Demand Satisfied?
1	26	850	850.00	8.008	107	0	107	107	856.8560	856.8560	850.0000	6.8560	TRUE
2	24	1260	1253.14	7.392	170	0	63	170	1256.6400	2113.4960	2103.1440	3.4960	TRUE
3	20	510	506.50	6.16	83	87	0	83	511.2800	2624.7760	2609.6480	4.7760	TRUE
4	18	980	975.22	5.544	176	0	93	176	975.7440	3600.5200	3584.8720	0.5200	TRUE
5	22	770	769.48	6.776	114	62	0	114	772.4640	4372.9840	4354.3520	2.9840	TRUE
6	23	850	847.02	7.084	120	0	6	120	850.0800	5223.0640	5201.3680	3.0640	TRUE
7	14	1050	1046.94	4.312	243	0	123	243	1047.8160	6270.8800	6248.3040	0.8800	TRUE
8	21	1550	1549.12	6.468	240	3	0	240	1552.3200	7823.2000	7797.4240	3.2000	TRUE
9	23	1350	1346.80	7.084	191	49	0	191	1353.0440	9176.2440	9144.2240	6.2440	TRUE
10	24	1000	993.76	7.392	135	56	0	135	997.9200	10174.1640	10137.9800	4.1640	TRUE
11	21	970	965.84	6.468	150	0	15	150	970.2000	11144.3640	11103.8160	4.3640	TRUE
12	13	680	675.64	4.004	169	0	19	169	676.6760	11821.0400	11779.4520	1.0400	TRUE
						257	426					41.588	

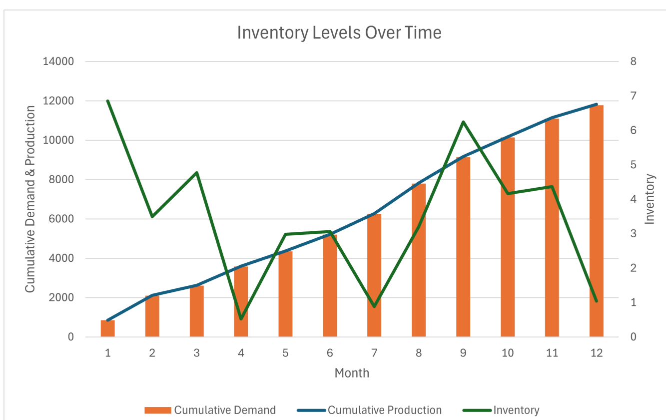

Figure 3: Zero Inventory Plan, No Backlogging, Demand Adjustment

Using the cost function, this plan would incur a total cost of:

$$C = \sum_{t=1}^{12} \Big(100 \cdot H_t + 200 \cdot F_t + 0.1 \cdot I_t\Big)$$
$$= 100 \cdot (426) + 200 \cdot (257) + 0.1 \cdot (41.588)$$
$$\boxed{C = \$94004.16}$$

Evidently, the plan incurs less of a cost since excess inventory is utilised rather than hoarded indefinitely.

(c)

Linear Programming

To solve the problem by adjusting workforce and inventory levels, we can implement the following linear program.

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{T} \left(c_H \cdot H_t + c_F \cdot F_t + c_I \cdot I_t \right) & \\
\text{s.t.} \quad & W_t = W_{t-1} + H_t - Ft & \forall t = 1, \dots, T \\
& P_t = K \cdot n_t \cdot W_t & \forall t = 1, \dots, T \\
& I_t = P_t - (D_t - I_{t-1}) & \forall t = 1, \dots, T \\
& W_0 = 0 & \\
& I_0 = 200 & \\
& H_t, F_t, I_t, W_t, P_t, D_t \geq 0 & \forall t = 1, \dots, T
\end{aligned}
$$

To solve the linear program, I used the GLPK solver in Julia. The data was prepared in a CSV file and processed into a dataframe in Julia. The program terminated almost instantly.

	A	B	C
1	t	n	D
2	1	26	1050
3	2	24	1260
4	3	20	510
5	4	18	980
6	5	22	770
7	6	23	850
8	7	14	1050
9	8	21	1550
10	9	23	1350
11	10	24	1000
12	11	21	970
13	12	13	680

Figure 4: Homework 2 Q2c data.csv

```julia
using JuMP, GLPK, DataFrames , CSV , XLSX
df = DataFrame(CSV.File("Homework 2 Q2c data.csv"))
display(df)
n = df[:, :n]
D = df[:, :D]

m = Model(GLPK.Optimizer)
T = 12
c_I = 0.10
c_H = 100
c_F = 200
K = 0.308

@variable(m, H[1:T] >= 0)
@variable(m, F[1:T] >= 0)
```

```
@variable(m, I[0:T] >= 0)
@variable(m, W[0:T] >= 0)
@variable(m, P[1:T] >= 0)

@objective(m, Min, sum(c_H*H[t] + c_F*F[t] + c_I*I[t] for t in 1:T))

for t in 1:T
    @constraint(m, W[t] == W[t-1] + H[t] - F[t]);
    @constraint(m, I[t] == P[t] - D[t] + I[t-1]);
    @constraint(m, P[t] == K * n[t] * W[t]);
end
@constraint(m, I[0] == 200);
@constraint(m, W[0] == 0);

optimize!(m)
```

```
* Solver : GLPK

* Status
  Result count       : 1
  Termination status : OPTIMAL
  Message from the solver:
  "Solution is optimal"

* Candidate solution (result #1)
  Primal status      : FEASIBLE_POINT
  Dual status        : FEASIBLE_POINT
  Objective value    : 1.61520e+04
  Objective bound    : -Inf
  Dual objective value : 1.61520e+04

* Work counters
  Solve time (sec)   : 0.00000e+00

Objective Value: 16151.97762970014
```

Figure 5: Solution Summary

To analyze the results, I plotted the optimal solution found by the program in a similar format as in part (b). This plan incurs a total cost of \$16151.98 as reported in Julia.

A	B	C	D	D	F	G	H	I	J	K	L	M	N
Month	Workdays	Demand	Net Demand	B × k	Worker Level [C / D]	Fires	Hires	n(workers)	Monthly Production	Cumulative Production	Cumulative Demand	Inventory	Is Cumulative Demand Satisfied?
1	26	850	850.00	8.0080	107	0	155.8782	155.87815	1248.27225	1248.27225	850	398.272251	TRUE
2	24	1260	861.73	7.3920	117	-2.8E-14	0	155.87815	1152.25131	2400.52356	1711.72775	290.52356	TRUE
3	20	510	219.48	6.1600	36	0	0	155.87815	960.209424	3360.73298	1931.20419	740.732984	TRUE
4	18	980	239.27	5.5440	44	-1.8E-14	0	155.87815	864.188482	4224.92147	2170.4712	624.921466	TRUE
5	22	770	145.08	6.7760	22	0	0	155.87815	1056.23037	5281.15183	2315.54974	911.151832	TRUE
6	23	850	-61.15	7.0840	-8	0	0	155.87815	1104.24084	6385.39267	2254.39791	1165.39267	TRUE
7	14	1050	-115.39	4.3120	-26	0	0	155.87815	672.146597	7057.53927	2139.00524	787.539267	TRUE
8	21	1550	762.46	6.4680	118	0	0	155.87815	1008.2199	8065.75916	2901.46597	245.759162	TRUE
9	23	1350	1104.24	7.0840	156	0	0	155.87815	1104.24084	9170	4005.70681	-1.0004E-11	TRUE
10	24	1000	1000.00	7.3920	136	-2.8E-14	0	155.87815	1152.25131	10322.2513	5005.70681	152.251309	TRUE
11	21	970	817.75	6.4680	127	0	0	155.87815	1008.2199	11330.4712	5823.4555	190.471204	TRUE
12	13	680	489.53	4.0040	123	0	0	155.87815	624.136126	11954.6073	6312.98429	134.60733	TRUE
						-7.5E-14	155.8782					5641.62304	

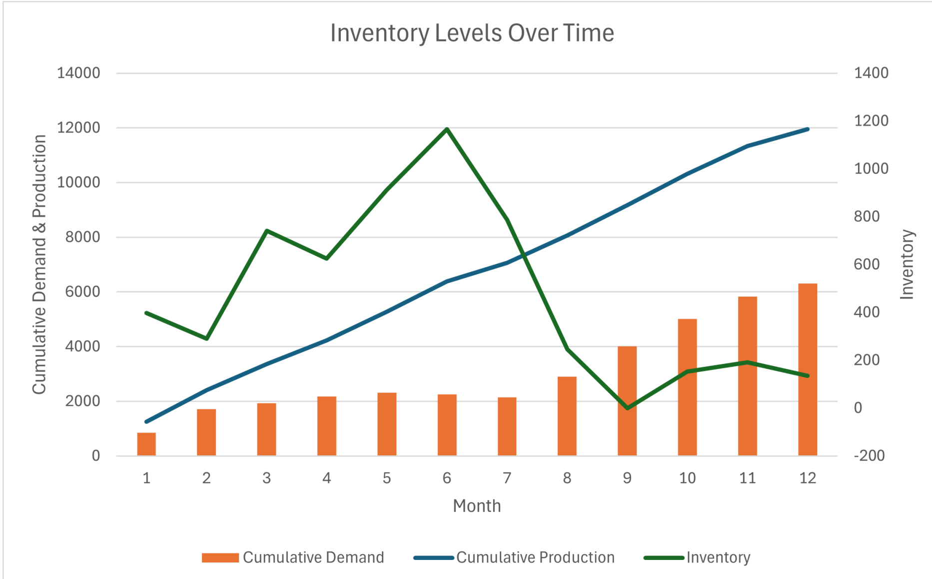

Figure 6: Inventory Plan Using Linear Programming

Integer Linear Programming

Now, we add integer constraints towards the worker level.

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{T} \left(c_H \cdot H_t + c_F \cdot F_t + c_I \cdot I_t \right) \\
\text{s.t.} \quad & W_t = W_{t-1} + H_t - F_t && \forall t = 1, \dots, T \\
& P_t = K \cdot n_t \cdot W_t && \forall t = 1, \dots, T \\
& I_t = P_t - (D_t - I_{t-1}) && \forall t = 1, \dots, T \\
& W_0 = 0 \\
& I_0 = 200 \\
& H_t, F_t, I_t, W_t, P_t, D_t \geq 0 && \forall t = 1, \dots, T \\
& H_t, F_t, W_t \in \mathbb{Z} && \forall t = 1, \dots, T
\end{aligned}
$$

```
m = Model(GLPK.Optimizer)
T = 12
c_I = 0.10
c_H = 100
c_F = 200
K = 0.308

@variable(m, H[1:T] >= 0, Int)
@variable(m, F[1:T] >= 0, Int)
@variable(m, I[0:T] >= 0)
@variable(m, W[0:T] >= 0, Int)
@variable(m, P[1:T] >= 0)

@objective(m, Min, sum(c_H*H[t] + c_F*F[t] + c_I*I[t] for t in 1:T))

for t in 1:T
```

```julia
    @constraint(m, W[t] == W[t-1] + H[t] - F[t]);
    @constraint(m, I[t] == P[t] - D[t] + I[t-1]);
    @constraint(m, P[t] == K * n[t] * W[t]);
end
@constraint(m, I[0] == 200);
@constraint(m, W[0] == 0);

optimize!(m)

println(solution_summary(m))
println("Objective Value: ", objective_value(m))
```

```
* Solver : GLPK

* Status
  Result count       : 1
  Termination status : OPTIMAL
  Message from the solver:
  "Solution is optimal"

* Candidate solution (result #1)
  Primal status      : FEASIBLE_POINT
  Dual status        : NO_SOLUTION
  Objective value    : 1.61705e+04
  Objective bound    : 1.61705e+04
  Relative gap       : 4.48889e-04

* Work counters
  Solve time (sec)   : 0.00000e+00

Objective Value: 16170.478400000033
```

Figure 7: Solution Summary

To analyze the results, I plotted the optimal solution found by the program in a similar format as in part (b). Similar to the LP, the plan incurs a cost of $16170.48.

A	B	C	D	E	F	G	H	I	J	K	L	M	N
Month	Workdays	Demand	Net Demand	B × k	Worker Level [C / D]	Fires	Hires	n(workers)	Monthly Production	Cumulative Production	Cumulative Demand	Inventory	Is Cumulative Demand Satisfied?
1	26	850	850	8.008	107	0	156	156	1249.248	1249.2480	850.0000	399.2480	TRUE
2	24	1260	860.752	7.392	117	0	0	156	1153.152	2402.4000	1710.7520	292.4000	TRUE
3	20	510	217.6	6.16	36	0	0	156	960.96	3363.3600	1928.3520	743.3600	TRUE
4	18	980	236.64	5.544	43	0	0	156	864.864	4228.2240	2164.9920	628.2240	TRUE
5	22	770	141.776	6.776	21	0	0	156	1057.056	5285.2800	2306.7680	915.2800	TRUE
6	23	850	-65.28	7.084	-9	0	0	156	1105.104	6390.3840	2241.4880	1170.3840	TRUE
7	14	1050	-120.38	4.312	-27	0	0	156	672.672	7063.0560	2121.1040	793.0560	TRUE
8	21	1550	756.944	6.468	118	0	0	156	1009.008	8072.0640	2878.0480	252.0640	TRUE
9	23	1350	1097.94	7.084	155	0	0	156	1105.104	9177.1680	3975.9840	7.1680	TRUE
10	24	1000	992.832	7.392	135	0	0	156	1153.152	10330.3200	4968.8160	160.3200	TRUE
11	21	970	809.68	6.468	126	0	0	156	1009.008	11339.3280	5778.4960	199.3280	TRUE
12	13	680	480.672	4.004	121	0	0	156	624.624	11963.9520	6259.1680	143.9520	TRUE
						0	156					5704.7840	

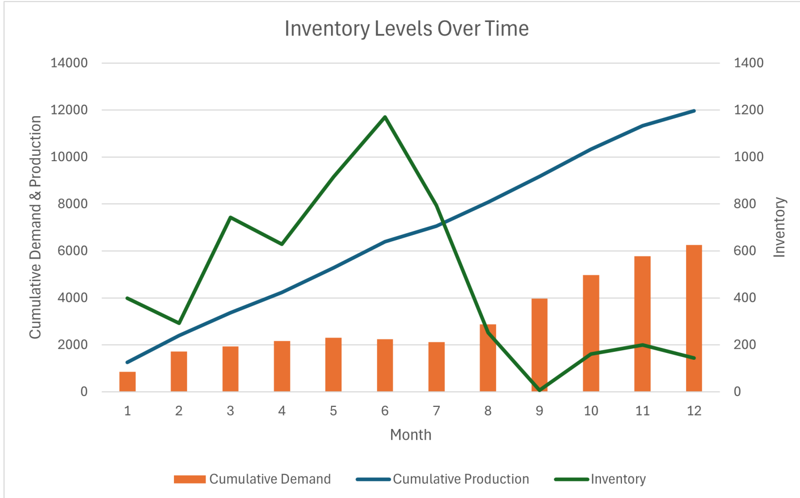

Figure 8: Inventory Plan Using Integer Linear Programming

Question 3

(a)

On average, Chewy's cost of production per unit involves two components: the order setup cost per unit and the holding cost per unit. This total cost can be expressed using the equation:

$$\begin{aligned} G(Q) &= \frac{K + cQ}{T} + \frac{hQ}{2} \\ &= \frac{K\lambda}{Q} + c\lambda + 0.1cQ \quad (\text{since } T = Q/\lambda \text{ and } h = 0.2c) \end{aligned}$$

where $K = \$45$ is the order setup cost, T is the time between order placements, $\lambda = 280$ kg/year is the demand rate, $c = \$2.40$/kg is the cost of muesli ingredients, and $h = \$0.2c$/kg is the holding cost. To find the optimal ordering quantity, we simply take the derivative of $G(Q)$ and set it to 0.

$$\frac{dG(Q)}{dQ} = -\frac{K\lambda}{Q^2} + 0.1c = 0$$

This gives the economic ordering quantity Q^*:

$$\begin{aligned} Q^* &= \sqrt{\frac{10K\lambda}{c}} \\ &= \sqrt{\frac{10(45)(280)}{2.40}} \\ &= 229.1287847 \text{ kg} \end{aligned}$$

$$\boxed{Q^* = 229.129 \text{ kg}}$$

Note that Q^* is minimum due to the fact that $\frac{d^2G(Q)}{dQ^2} = \frac{2K\lambda}{Q^3} > 0$ for all $Q > 0$

(b)

The time between order placements is simply given by $T = Q/\lambda$ and thus:

$$T = \frac{Q}{\lambda} = \frac{229.1287847}{280} = 0.818317088$$

$$\boxed{T = 0.818317088 \text{ years}}$$

(c)

Let $n = 1/T$ be the number of periods in a year. The annual holding cost H and the annual setup cost S is given by:

$$\begin{aligned} H &= \frac{hQ}{2} \\ &= \frac{(0.20 \cdot 2.40)(229.1287847)}{(2)} \\ &= \$54.990908328/\text{year} \end{aligned}$$

$$\boxed{H = \$54.991/\text{year}}$$

$$\begin{aligned} S &= Kn \\ &= \frac{45}{0.818317088} \\ &= \$54.9909084/\text{year} \end{aligned}$$

$$\boxed{S = \$54.991/\text{year}}$$

(d)

Given $\tau = 3$ weeks $= 21$ days, the reorder point is simply given by:

$$R = \lambda\tau = \frac{280 \text{ kg}}{\text{year}} \cdot \frac{1 \text{ year}}{365 \text{ days}} \cdot 21 \text{ days} = 16.10958904 \text{ kg}$$

$$\boxed{R = 16.110 \text{ kg}}$$

Question 4

(a)

Similar to question 3, the average cost of production per unit involves the order setup cost and the holding cost. The total cost can be expressed by the equation:

$$G(Q) = \frac{K}{T} + \frac{hH}{2}$$

where $K = \$45$ is the order setup cost, T is the time between order placements, $h = \$0.2c/\text{kg}$ is the holding cost and H is the maximum inventory level. H can be computed by dividing a period into uptime (T_1) and downtime (T_2). The uptime period is the time taken for production whereas the downtime period is the time left between the end of the uptime period and the next production period. By analyzing the geometry of the graph as seen in figure 9, we see that

$$H = (P - \lambda) \cdot T_1$$

where $\lambda = \$280$ kg/year is the demand rate for the cereal and $P = 1120$ kg/year is the production rate. Since $T_1 = Q/P$ by definition, we get:

$$\begin{aligned} H &= Q - \lambda T_1 \\ &= Q - \lambda Q/P \\ &= Q(1 - \lambda/P) \end{aligned}$$

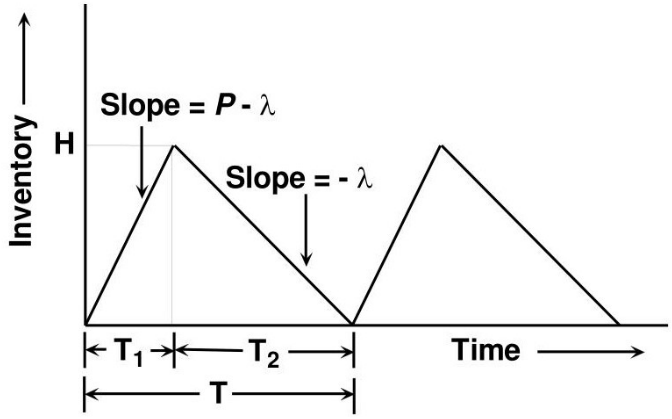

Figure 9: Taken from Slide Serve

Thus our equation for $G(Q)$ becomes:

$$\begin{aligned} G(Q) &= \frac{K}{T} + \frac{0.2cQ(1-\lambda/P)}{2} \\ &= \frac{K\lambda}{Q} + 0.1cQ(1-\lambda/P) \quad \text{(since } T = Q/\lambda\text{)} \end{aligned}$$

To find the optimal ordering quantity Q^*, we find the derivative of $G(Q)$ and set it to 0.

$$\frac{dG(Q)}{dQ} = -\frac{K\lambda}{Q^2} + 0.1c(1-\lambda/P) = 0$$

This gives:

$$Q^* = \sqrt{\frac{10K\lambda P}{c(P-\lambda)}} = \sqrt{\frac{10(45)(280)(1120)}{(2.4)(1120-280)}} = 264.575131106$$

$$\boxed{Q^* = 264.575 \text{ kg}}$$

Q^* is minimal due to the fact that $\frac{d^2G(Q)}{dQ^2} = \frac{K\lambda}{Q^3} > 0$ for all $Q > 0$.

(b)

The time between order placement is simply given by:

$$T = \frac{Q}{\lambda} = \frac{264.575131106}{280}$$

$$\boxed{T = 0.944911182521 \text{ years}}$$

(c)

As derived in (a), the maximum inventory level is given by:

$$H = Q(1-\lambda/P) = 264.575131106 \cdot (1 - 280/1120) = 198.431348329$$

$$\boxed{H = 198.431 \text{ kg}}$$

Question 5

In this discount scheme, the total purchasing cost of acquiring Q units (disregarding order setup cost) is given by:

$$C(Q) = \left\{ \begin{array}{lr} \$350Q, & \text{if } 0 \leq Q \leq 25 \\ \\ \$8750 + \$315(Q-25), & \text{if } 26 \leq Q \leq 50 \\ \\ \$16,625 + \$285(Q-50), & \text{if } 51 \leq Q \end{array} \right\}$$

such that we have the following per unit purchasing cost.

$$C(Q)/Q = \left\{ \begin{array}{lr} \$350, & \text{if } 0 \leq Q \leq 25 \\ \\ \$875/Q + \$315, & \text{if } 26 \leq Q \leq 50 \\ \\ \$2375/Q + \$285, & \text{if } 51 \leq Q \end{array} \right\}$$

The average annual cost is given by:

$$\begin{aligned} G(Q) &= \frac{K\lambda}{Q} + \lambda[C(Q)/Q] + \frac{hQ}{2} \\ &= \frac{K\lambda}{Q} + \lambda[C(Q)/Q] + 0.18[C(Q)/Q]\frac{Q}{2} \end{aligned}$$

where K is the order setup cost, λ is the demand rate, and h is the holding cost per unit as given by the equation $h = 0.18[C(Q)/Q]$. To find the most optimal ordering quantity, we can compute Q^* under each pricing scheme and compare $G(Q)$ accordingly. Under the first scheme, we have:

$$\begin{aligned} G_1(Q) &= \frac{K\lambda}{Q} + \lambda(350) + 0.18(350)\frac{Q}{2} \\ G_1'(Q) &= -\frac{K\lambda}{Q^2} + 31.5 = 0 \\ &\hookrightarrow Q_1^* = \sqrt{\frac{K\lambda}{31.5}} = 11.5470053838 \\ G_1''(Q) &= \frac{2K\lambda}{Q^3} > 0 \text{ for all } Q > 0 \\ &\hookrightarrow Q_1^* \text{ is minimum} \end{aligned}$$

Under the second scheme:

$$\begin{aligned} G_2(Q) &= \frac{K\lambda}{Q} + \lambda(875/Q + 315) + 0.18(875/Q + 315)\frac{Q}{2} \\ &= \lambda(K + 875)\frac{1}{Q} + 28.35Q + 315\lambda + 78.75 \\ G_2'(Q) &= -\lambda(K + 875)\frac{1}{Q^2} + 28.35 = 0 \\ &\hookrightarrow Q_2^* = \sqrt{\frac{(K + 875)\lambda}{28.35}} = 66.8515953622 \\ G_2''(Q) &= \frac{2\lambda(K + 875)}{Q^3} > 0 \text{ for all } Q > 0 \\ &\hookrightarrow Q_2^* \text{ is minimum} \end{aligned}$$

Finally, under the third scheme:

$$\begin{aligned}
G_3(Q) &= \frac{K\lambda}{Q} + \lambda(2375/Q + 285) + 0.18(2375/Q + 285)\frac{Q}{2} \\
&= \lambda(K + 2375)\frac{1}{Q} + 25.65Q + 285\lambda + 213.75 \\
G_3'(Q) &= -\lambda(K + 2375)\frac{1}{Q^2} + 25.65 = 0 \\
&\hookrightarrow Q_3^* = \sqrt{\frac{(K + 2375)\lambda}{25.65}} = 114.571836212 \\
G_3''(Q) &= \frac{2\lambda(K + 2375)}{Q^2} > 0 \text{ for all } Q > 0 \\
&\hookrightarrow Q_3^* \text{ is minimum}
\end{aligned}$$

This gives us several possible economic ordering quantities:

$$Q_1^* = 11.5470053838 \qquad Q_2^* = 66.8515953622 \qquad Q_1^* = 114.571836212$$

However, as these values are non-integer, they are not immediately realizable (the company sells microcontrollers, which are discrete objects). Furthermore, Q_2^* is not realizable because it is out of bounds ($G_2(Q)$ is only valid for $26 \leq Q \leq 50$). To find the EOQ, therefore, we must test the ceiling and floor of the Q^* values we have found. Since $C(Q)/Q$ is a monotonic function, the optimal Q^* for each scheme must lie either in the ceiling or floor of the Q^* we have computed earlier. This means we need to test the following Q^* values.

$$\begin{aligned}
Q_{1,1}^* &= 11 & Q_{2,1}^* &= 26 & Q_{3,1}^* &= 114 \\
Q_{1,2}^* &= 12 & Q_{2,1}^* &= 50 & Q_{3,2}^* &= 115
\end{aligned}$$

In the case of Q_2^*, the upper and lower bounds of the interval for $G_2(Q)$ is tested. Since $G_2(Q)$ is a monotonic function and Q_2^* as found earlier does not lie within $G_2(Q)$' s interval, the minimum must lie in either of the function's cornerpoints. Therefore, given $K = 30$ and $\lambda = 140$, we have:

$$\begin{aligned}
G_1(Q) &= \frac{K\lambda}{Q} + \lambda(350) + 0.18(350)\frac{Q}{2} \\
G_1(Q_{1,1}^*) &= G_1(11) = 49728.3181818 \\
G_1(Q_{1,2}^*) &= G_1(12) = 49728 \\
G_2(Q) &= \frac{K\lambda}{Q} + \lambda(875/Q + 315) + 0.18(875/Q + 315)\frac{Q}{2} \\
G_2(Q_{2,1}^*) &= G_2(26) = 49788.9269231 \\
G_2(Q_{2,2}^*) &= G_2(50) = 48130.25 \\
G_3(Q) &= \frac{K\lambda}{Q} + \lambda(2375/Q + 285) + 0.18(2375/Q + 285)\frac{Q}{2} \\
G_3(Q_{3,1}^*) &= G_3(114) = 45991.3587719 \\
G_3(Q_{3,2}^*) &= G_3(115) = 45991.326087
\end{aligned}$$

It is now evident that the most optimal ordering quantity would be $Q_{3,2}^* = 115$ with an average annual cost of \$45991.33. Indeed, this conclusion is verified when a graph is plotted using a graphic calculator.

After all this hard work, we can proudly claim that we have saved \$0.0326849 by selecting $Q_{3,2}^*$ rather than $Q_{3,1}^*$.

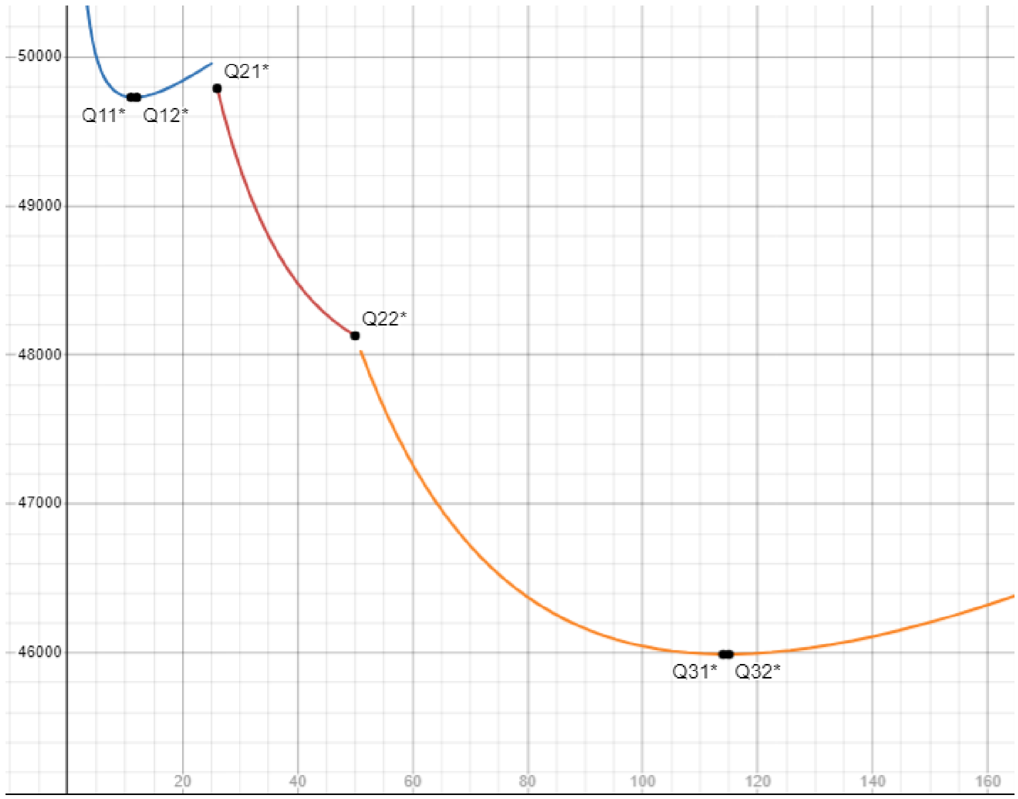

Figure 10: Continuous Q values. https://www.desmos.com/calculator/ylz2s22der

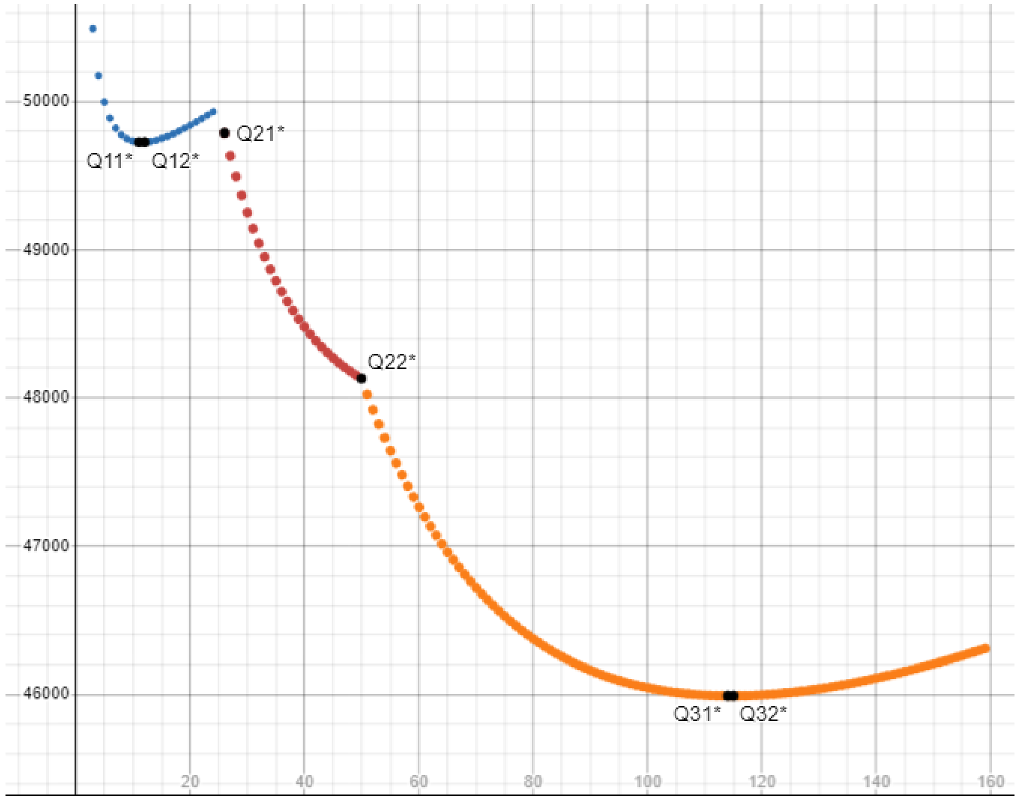

Figure 11: Discrete Q values. https://www.desmos.com/calculator/x5x0aqiczd