Electromagnetic Form Factors of Nucleons

Nathan Apfel | Prof. Jerry Miller

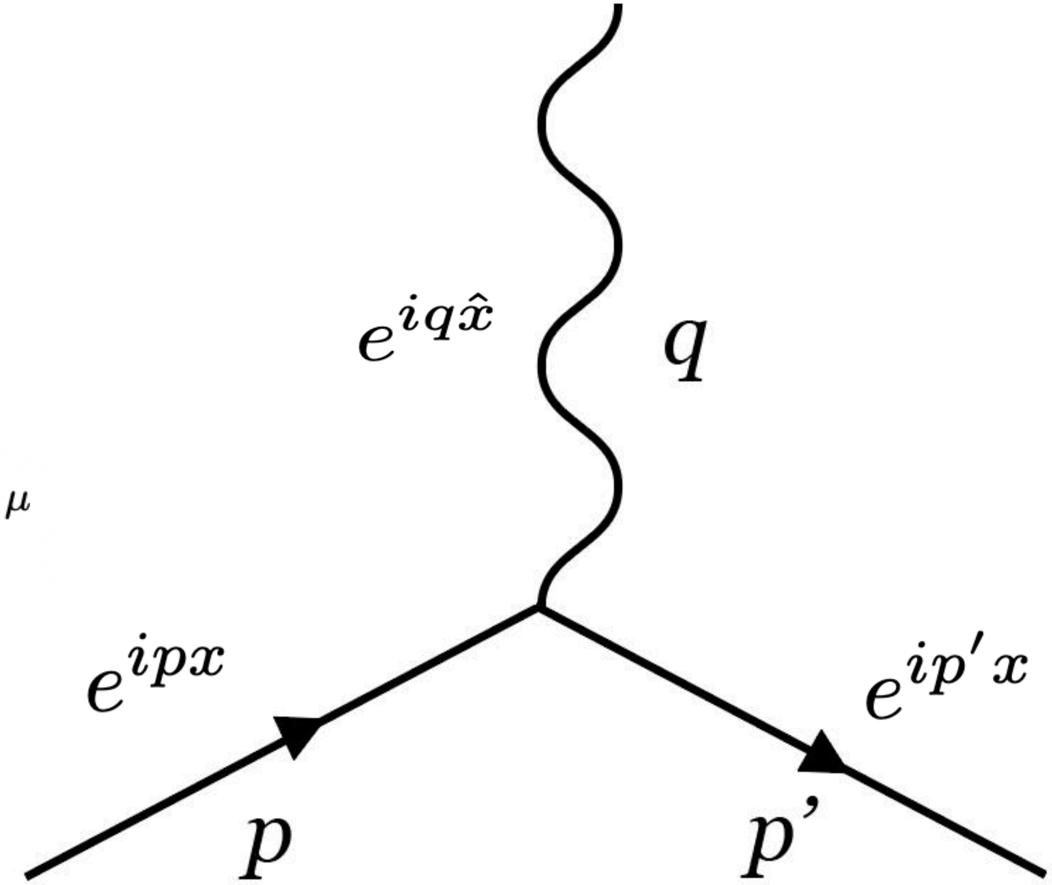
2. What is a form factor?

Basically

$$T_{fi} = \langle f|\, \hat{V}\, |i\rangle \qquad \hat{V} = \hat{j}^\mu A_\mu(\hat{x}) \quad *$$

$$T_{fi} = \int dx\, \underbrace{\psi'^\dagger(x)\hat{j}^\mu\psi(x)}_{j^\mu_{fi}(x)}\, A_\mu(x)$$

$$\hat{j}^\mu = e\left(\overleftarrow{\hat{p}} + \overrightarrow{\hat{p}}\right)^\mu$$

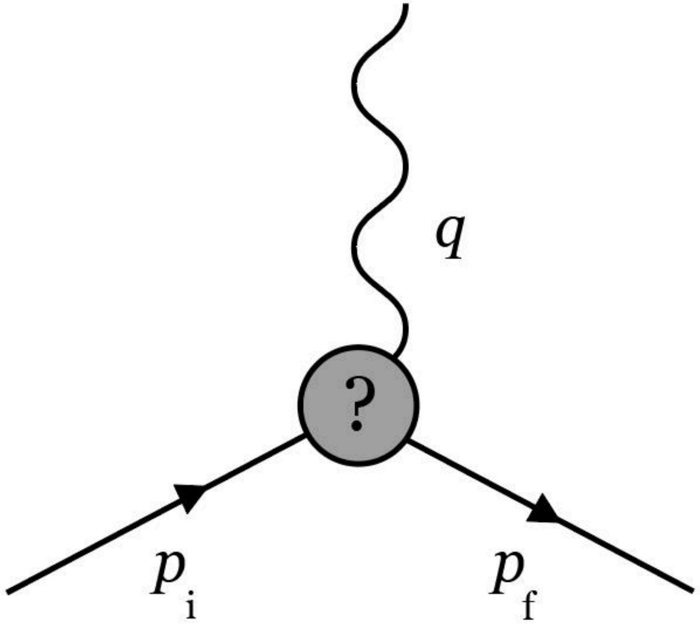


- Plane waves:

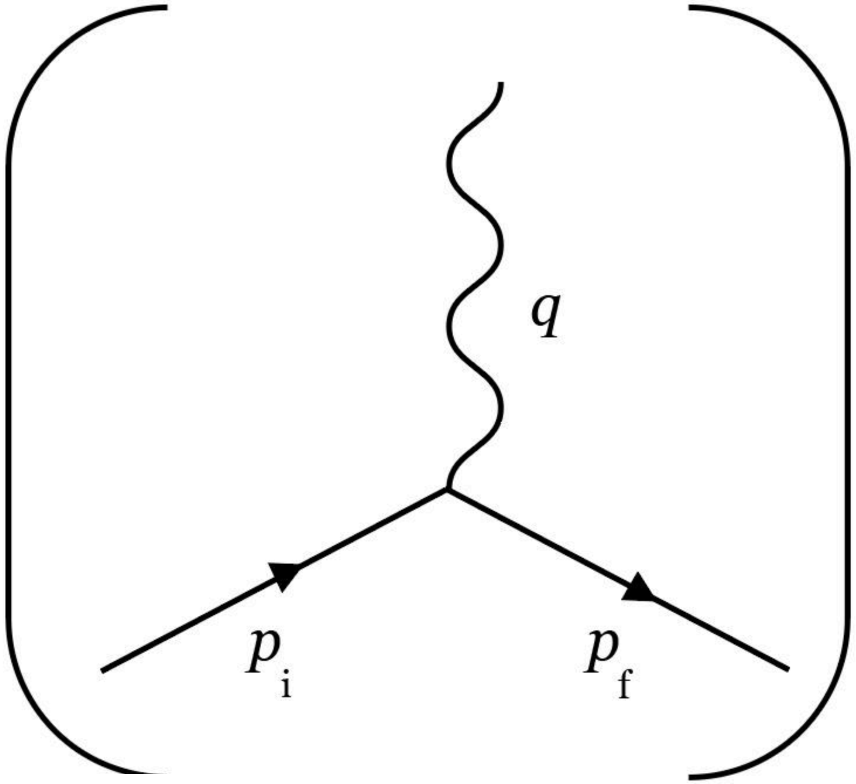
$$T_{p\to p'} = \epsilon_\mu\, j^\mu_{p\to p'}(0)\, \frac{1}{(2\pi)^4}\delta((p+q)-p')$$

*This is a bit of an abuse of notation, but for conceptual purposes it’s close enough

What is a form factor?



$$= F(q) \times$$



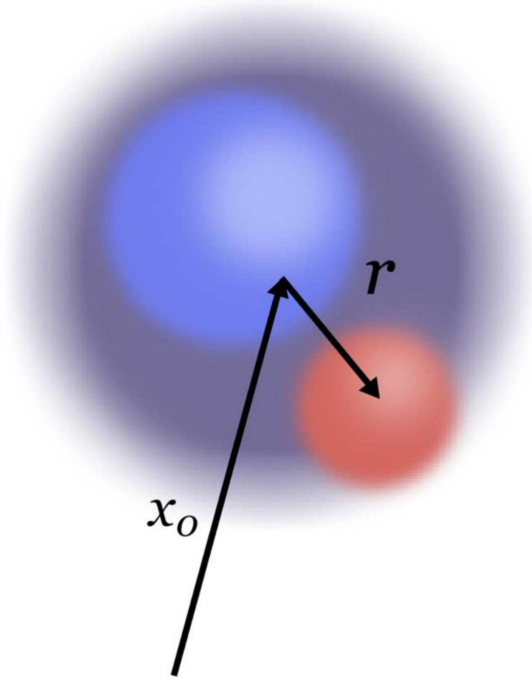
$$j_{fi}^{\mu\,\text{composite}} = F(q) \times j_{fi}^{\mu\,\text{point}}$$

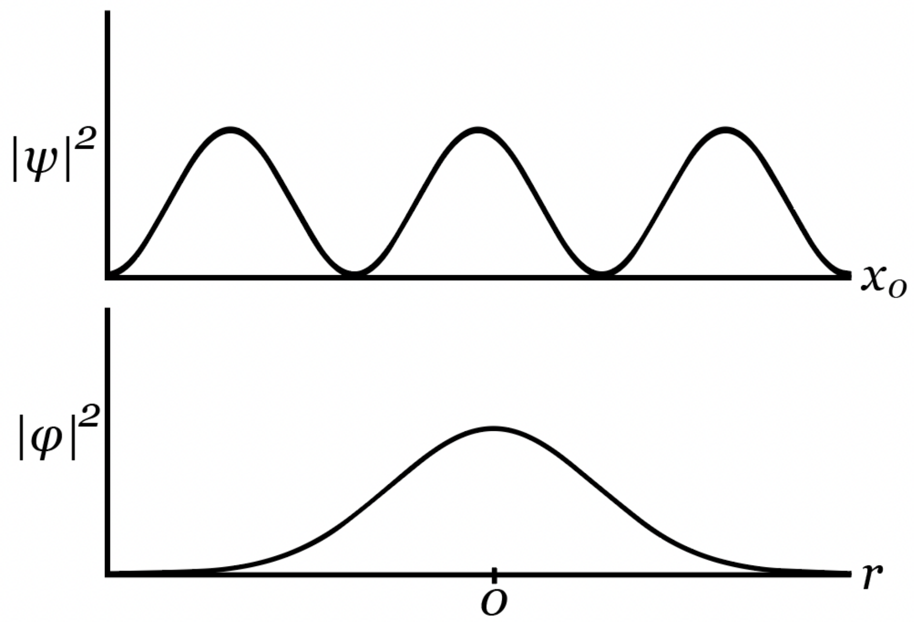
What's a form factor, physically?

$$x = x_0 + r$$

$$\Psi(x_0, r) = \psi(x_0)\phi(r)$$

$$\Psi'(x_0, r) = \psi'(x_0)\phi'(r)$$

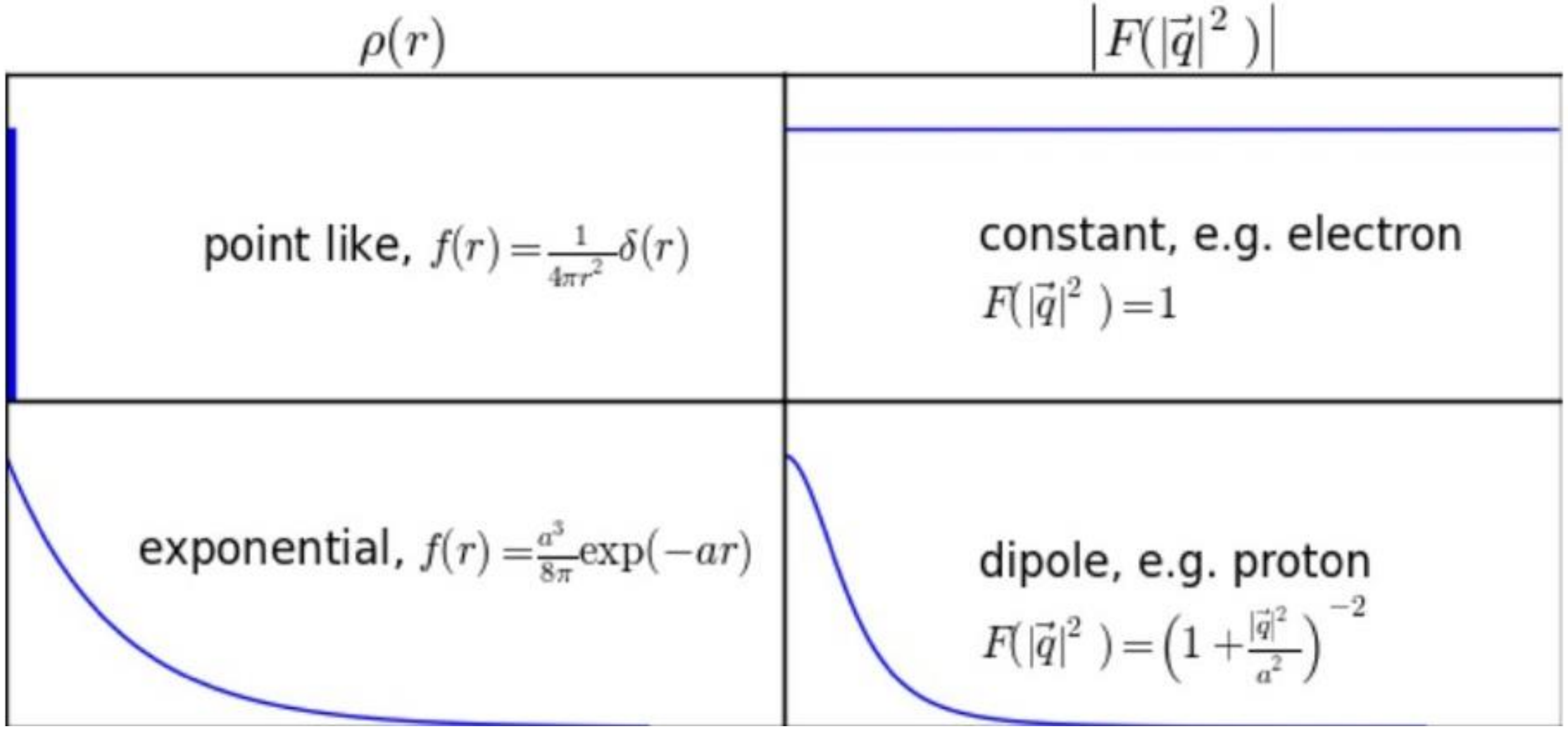




$$\underbrace{\langle f|\, e^{iq\hat{x}}\, |i\rangle}_{T_{fi}^{\text{composite}}} = \underbrace{\int dx_0\, \psi'^{\dagger}(x_0) e^{iqx_0} \psi(x_0)}_{T_{fi}^{\text{point}}} \underbrace{\int dr\; \phi'^{\dagger}(r) e^{iqr} \phi(r)}_{F(q)}$$

$$F(q) = \int dr\ \phi'^{\dagger}(r) e^{iqr} \phi(r) \quad \sim \int dr \phi^{\dagger} \phi e^{iqr}$$

A Fourier transform of the density! *



3. Adding spin

Form factors with spin

- Klein-Gordon field (without spin):

$$\psi = \psi(x)$$

$$\hat{j}^\mu = e\left(\overleftarrow{\hat{p}} + \overrightarrow{\hat{p}}\right)^\mu$$

- Dirac field (with spin):

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}$$

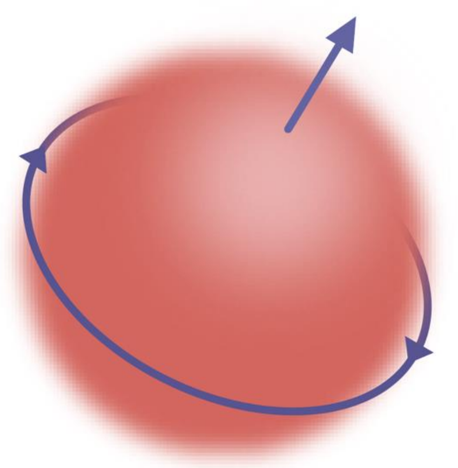
$$\psi = u_s \psi(x)$$

$$\hat{j}^\mu = e\gamma^\mu$$

$\sqrt{p^0 + m}$

$\frac{[illegible]}{\sqrt{p^0+m}}$

Form factors with spin



- Klein-Gordon field (without spin):

$$T_{P\to P'} = \langle P'|\,\hat{V}\,|P\rangle$$

- Dirac field (with spin):

$$T_{\uparrow\to\uparrow} = \langle P',\uparrow|\,\hat{V}\,|P,\uparrow\rangle \quad ;S\rangle$$

$$T_{\downarrow\to\uparrow} = \langle P',\uparrow|\,\hat{V}\,|P,\downarrow\rangle$$

Form factors with spin

- Klein-Gordon field (without spin):

$$j^{\mu}_{P\to P'} = e(P+P')e^{-i(P'-P)x}F(q)$$

- Dirac field (with spin):

$$j^{\mu}_{s\to s'} = \bar{u}_{s'}(P')\left(\gamma^{\mu}F_1(q) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M}F_2(q)\right)u_s(P)e^{-i(P'-P)x}$$

$$\sigma^{\mu\nu} = \frac{i}{2}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}), \quad \bar{u}'_s(P') = u'_s(P')^{\dagger}\gamma^0$$

Nonrelativistic approximation

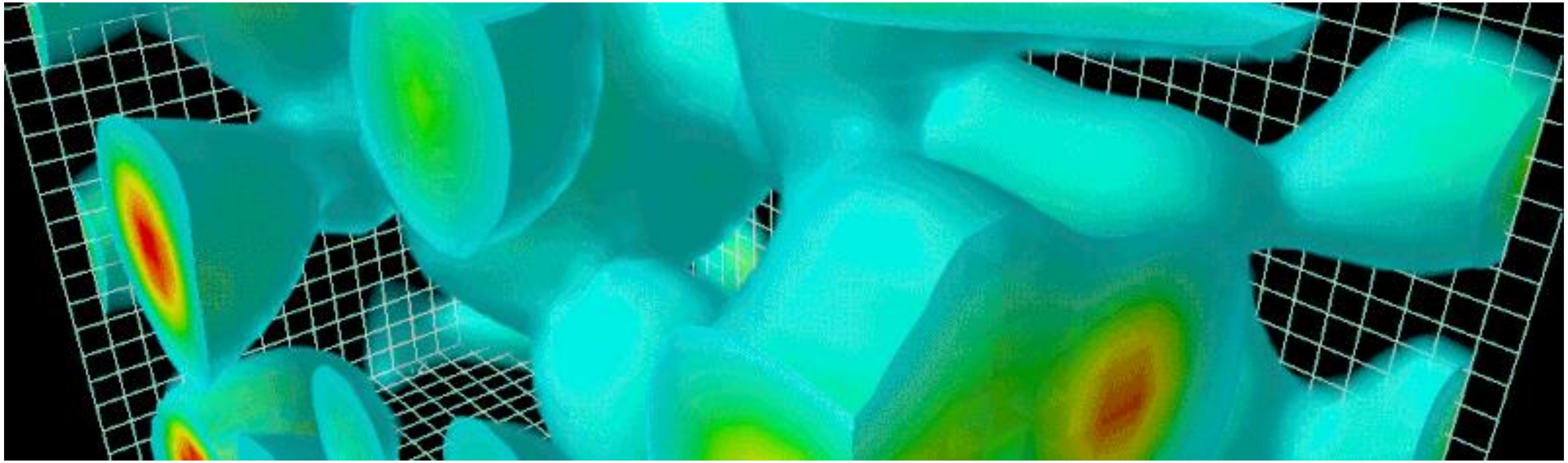
$$T_{fi} \sim 2M \, \langle P', S'| \left(eV(\hat{x}) \, F_1 - \mathbf{B}(\hat{x}) \cdot \overbrace{\frac{e\hat{\boldsymbol{\sigma}}}{2M} \, \underbrace{(F_1 + F_2)}_{g/2}}^{\hat{\boldsymbol{\mu}}} \right) |P, S\rangle$$

$$F_1(0) \equiv 1, \quad F_2(0) = \frac{g-2}{2}$$

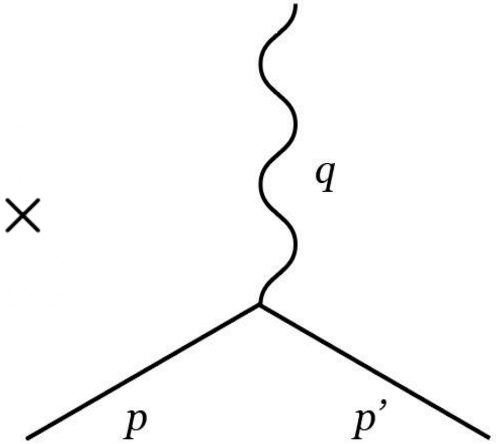
$$a_{\mathrm{e}} = 0.001\,159\,652\,181\,643(764)$$

$$a_{\mathrm{e}} = 0.001\,159\,652\,180\,59(13)$$

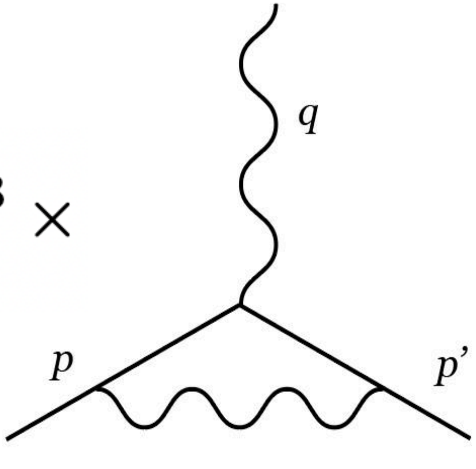
4. Neutron form factors



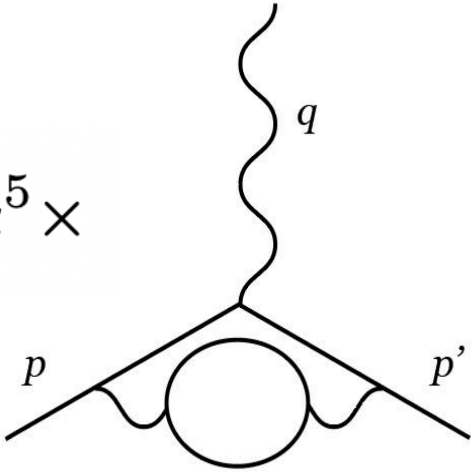
$\approx \alpha \times$



$+\alpha^3 \times$



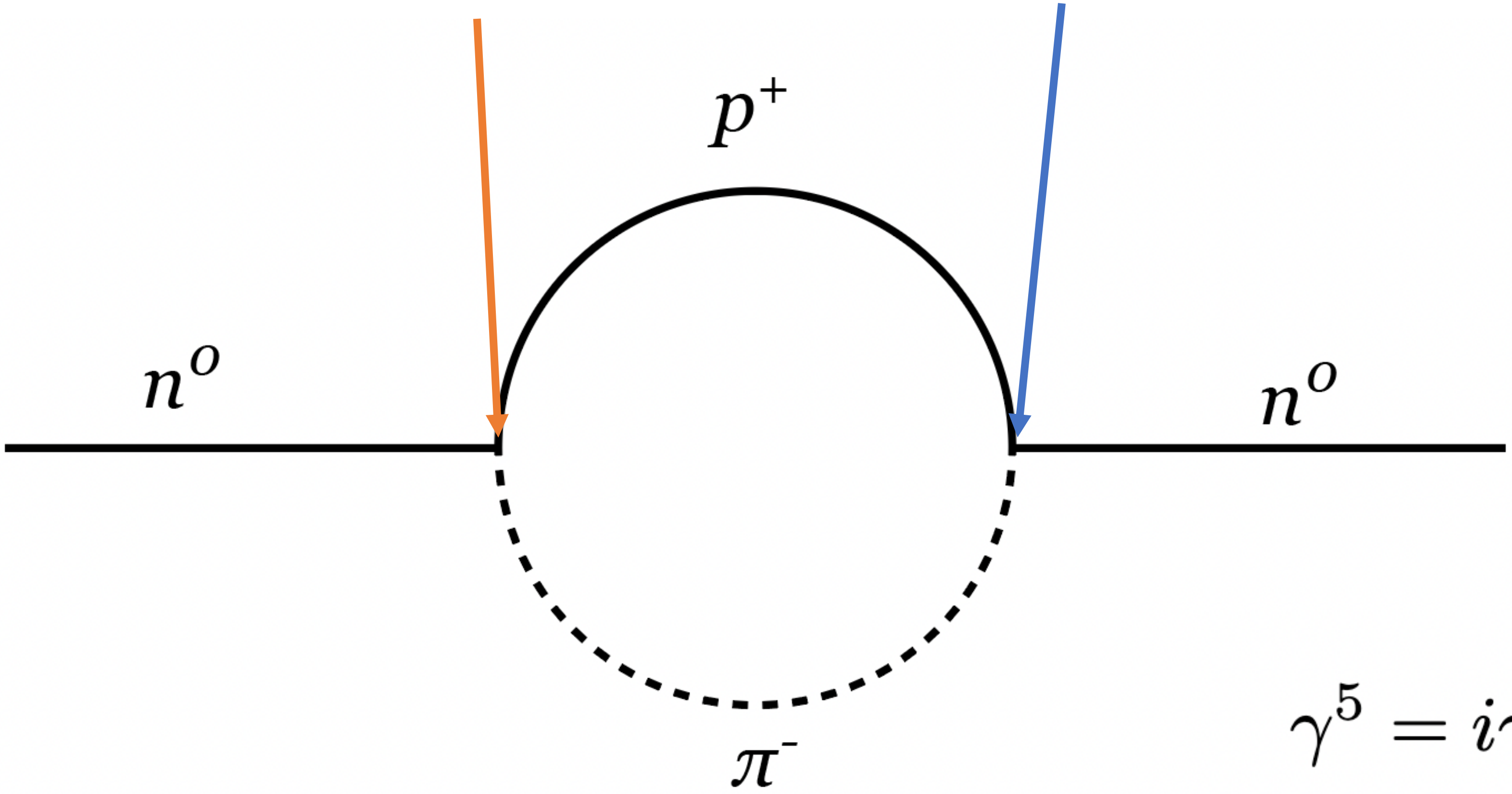
$+\alpha^5 \times$



$\alpha_e \approx 1/137 \qquad \alpha_s > 1$

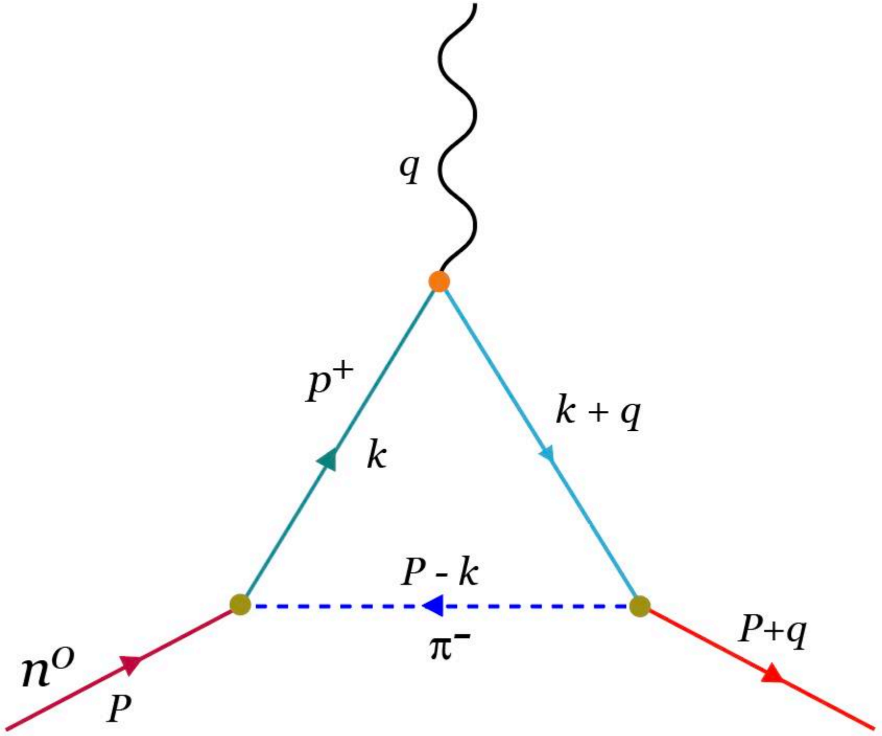
The model: $n^0 \to p^+\pi^- \to n^0$

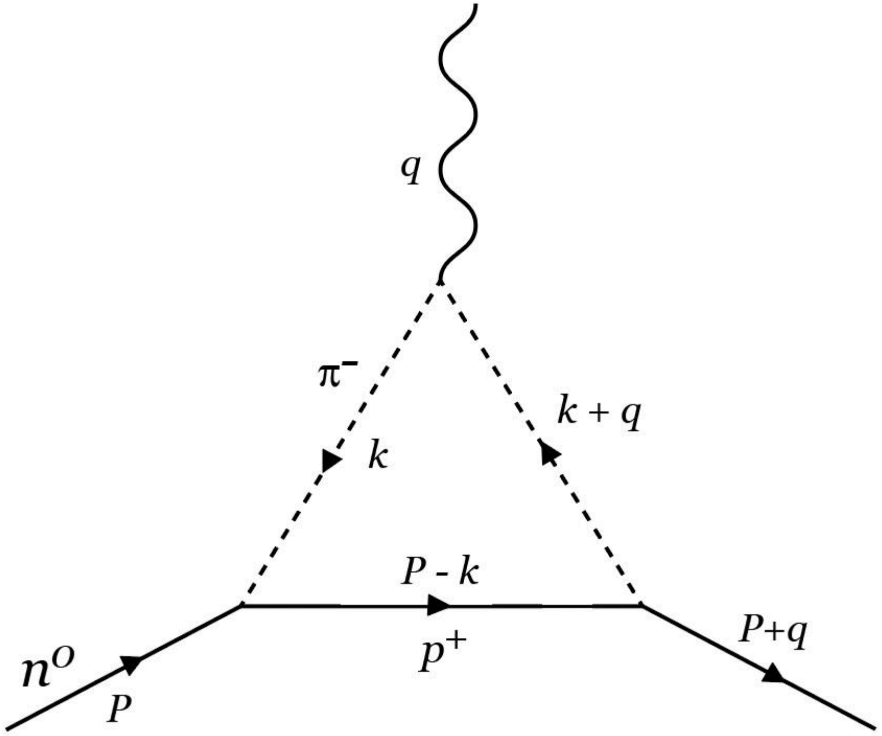
$$\mathcal{L}_{\text{int}} = ig\left(\bar{\psi}_p\gamma^5\psi_n\phi_-^* + \bar{\psi}_n\gamma^5\psi_p\phi_-\right) + \ldots$$



$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

Feynman diagrams





$$
\begin{aligned}
j^{\mu}_{P\to P+q} \;=\; & -\bar{U}'\gamma_5\, g^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{(P-k)^2 - m_1^2 + i\epsilon} \frac{i(\not{k} + \not{q} + m_2)}{(k+q)^2 - m_2^2 + i\epsilon} \gamma^{\mu} \frac{i(\not{k} + m_2)}{k^2 - m_2^2 + i\epsilon}\, \gamma_5 U \\
& + \bar{U}'\gamma_5\, g^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_1^2 + i\epsilon} (2k^{\mu} + q^{\mu}) \frac{i}{(k+q)^2 - m_1^2 + i\epsilon} \frac{i\;(\not{P} - \not{k} + m_2)}{(P-k)^2 - m_2^2 + i\epsilon}\, \gamma_5 U
\end{aligned}
$$

Pauli-Villars Regularization

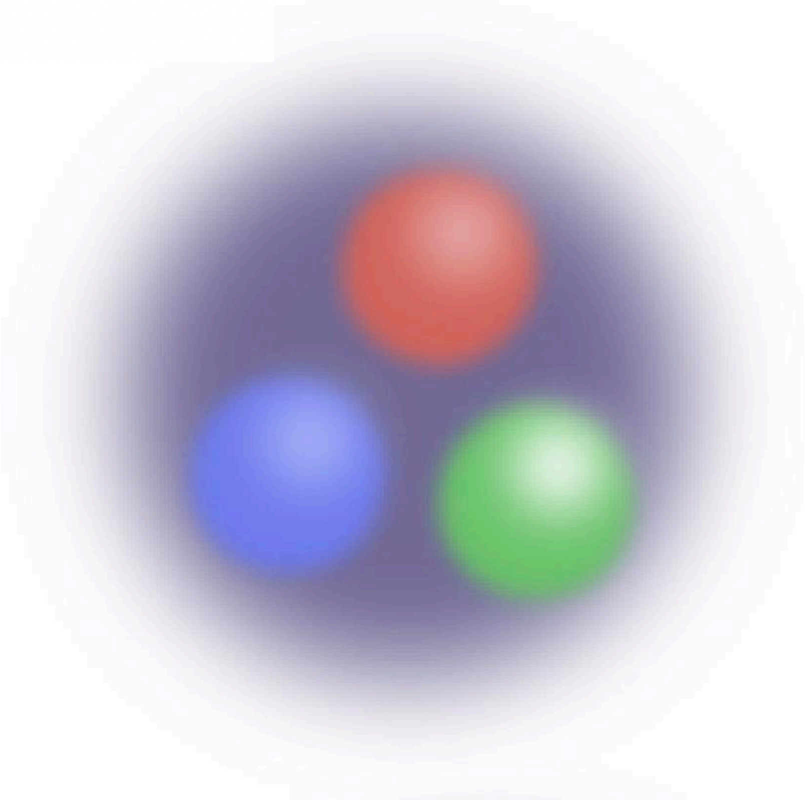
$$j^{\mu}_{P\to P+q} = \infty ??$$

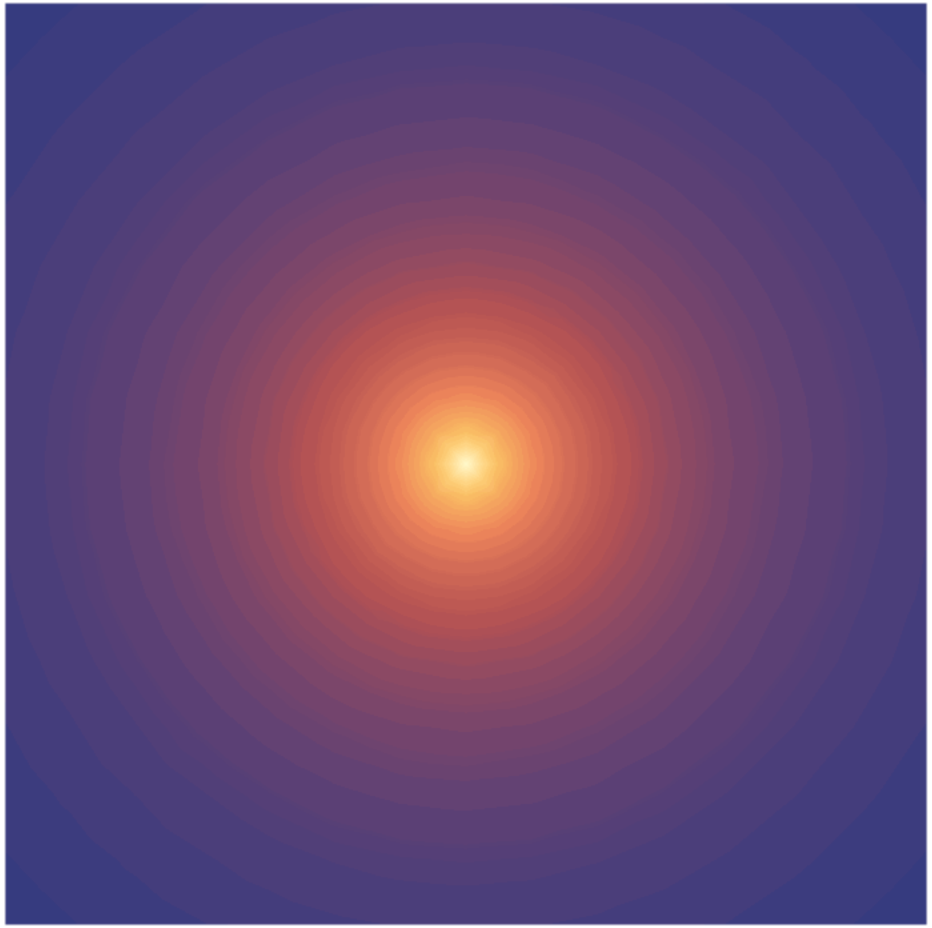
Renormalization!

$$\frac{1}{k^2 - m^2} \longrightarrow \frac{1}{k^2 - m^2} - \frac{1}{k^2 - \Lambda^2}$$

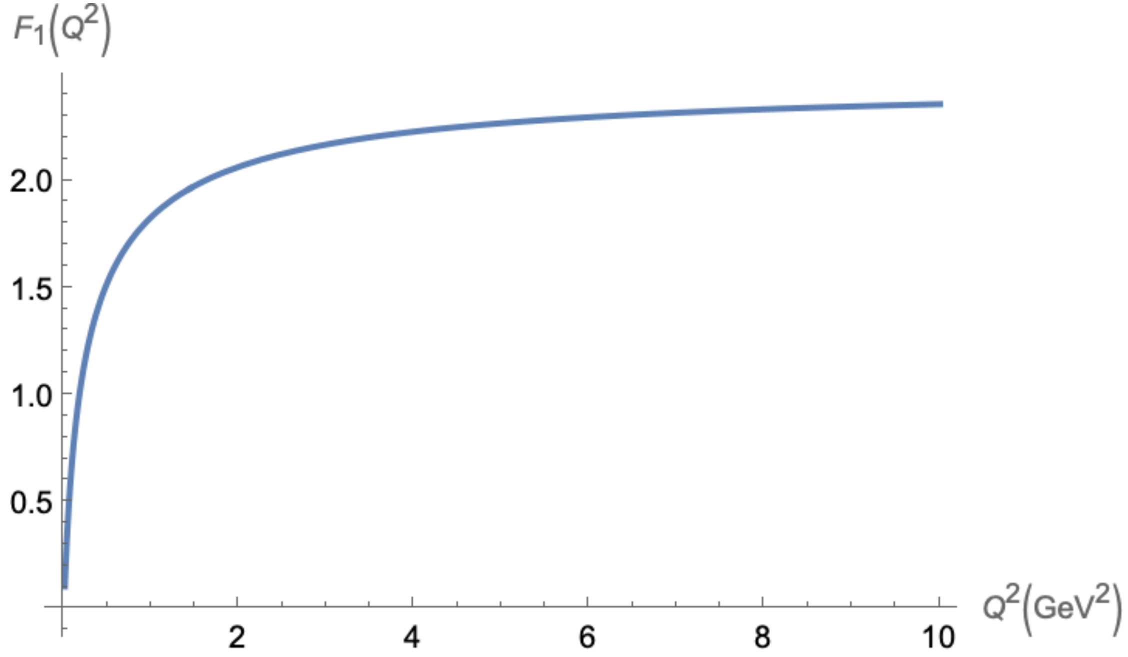
$$\frac{}{- m_1^2 + i\epsilon}(2k^\mu + q^\mu)\frac{}{(k -}$$

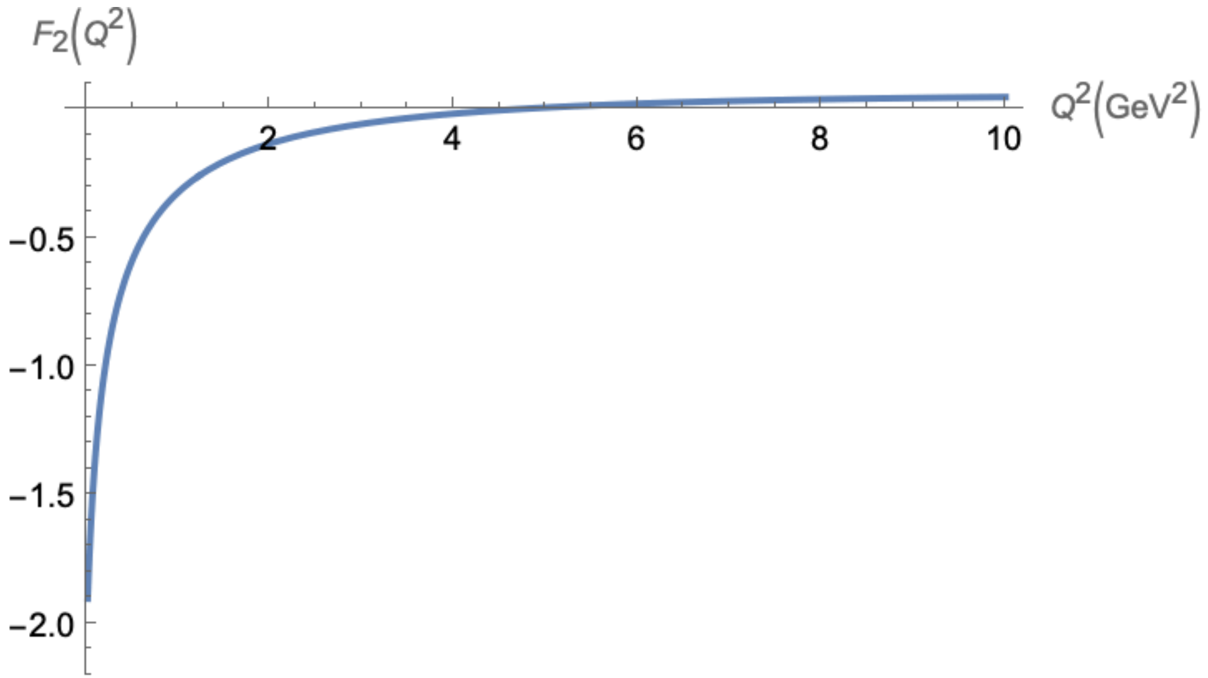
- Admitting the theory doesn't account for what happens above a certain energy





Results—Neutron form factors





Comparing with experiment—electric form factor

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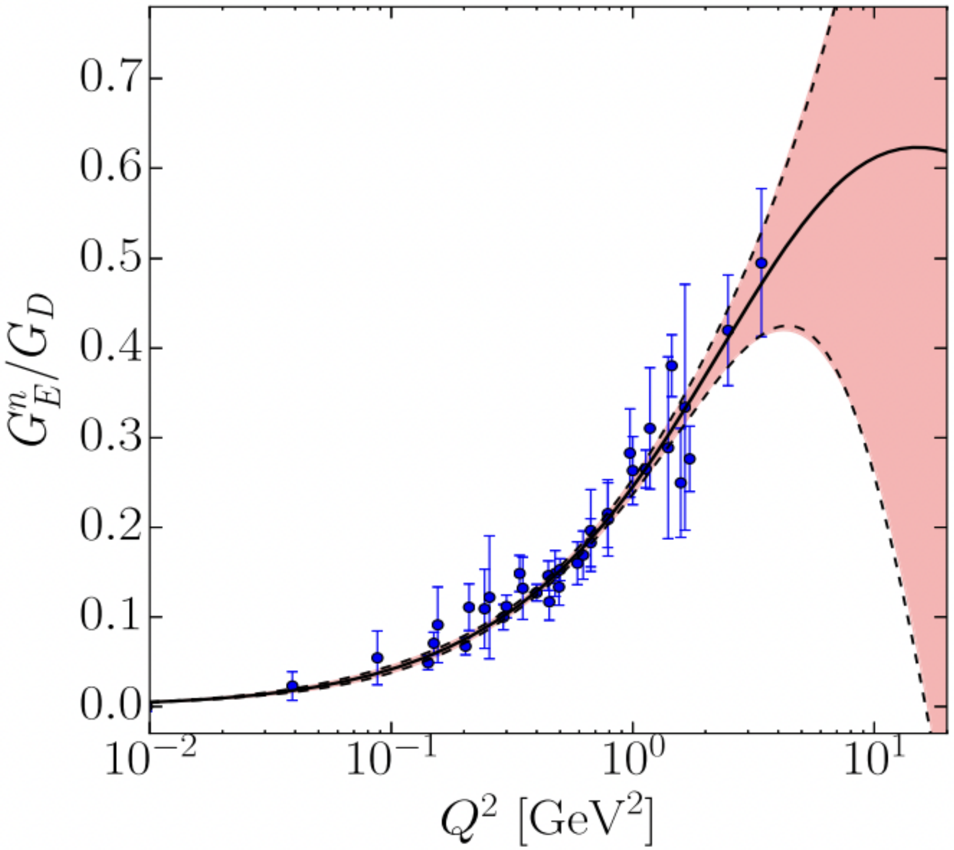

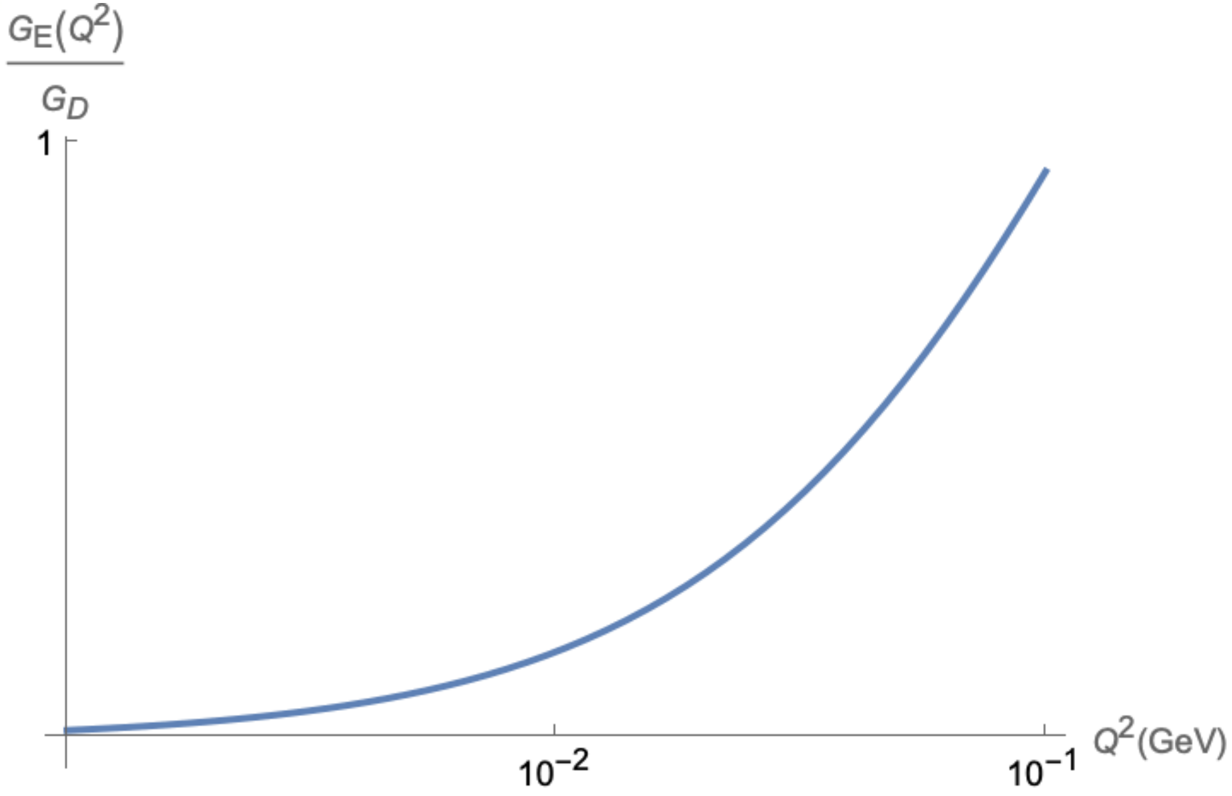

(Preliminary)

Comparing with experiment—magnetic form factor

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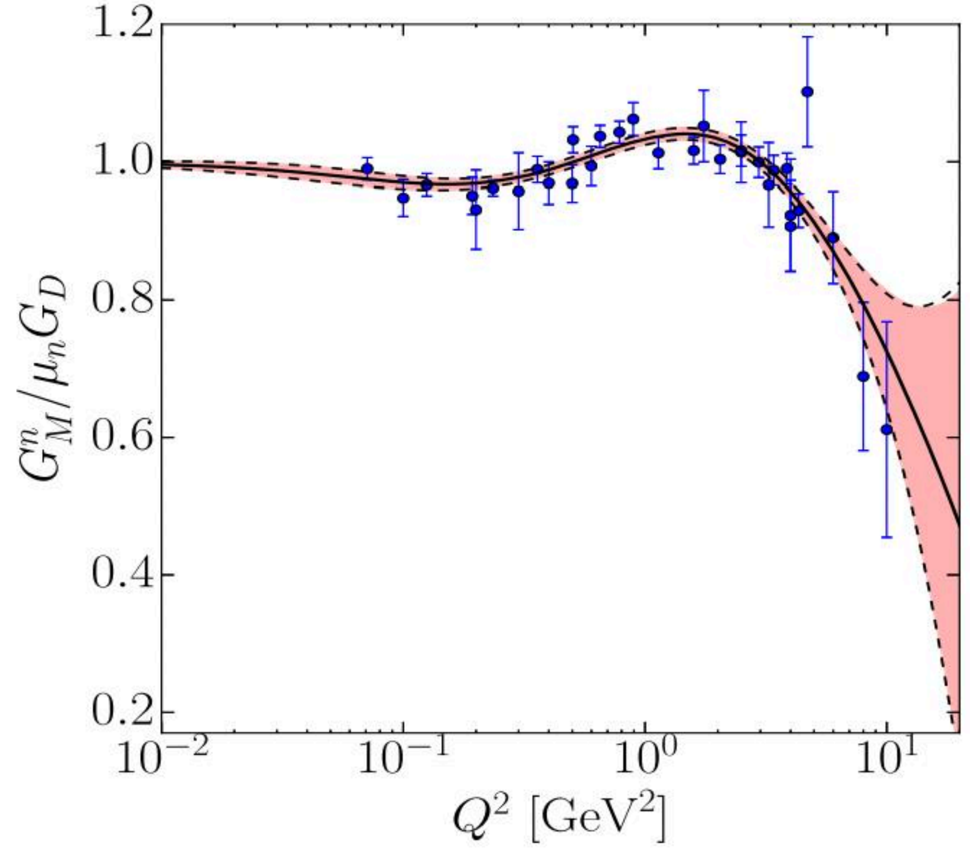
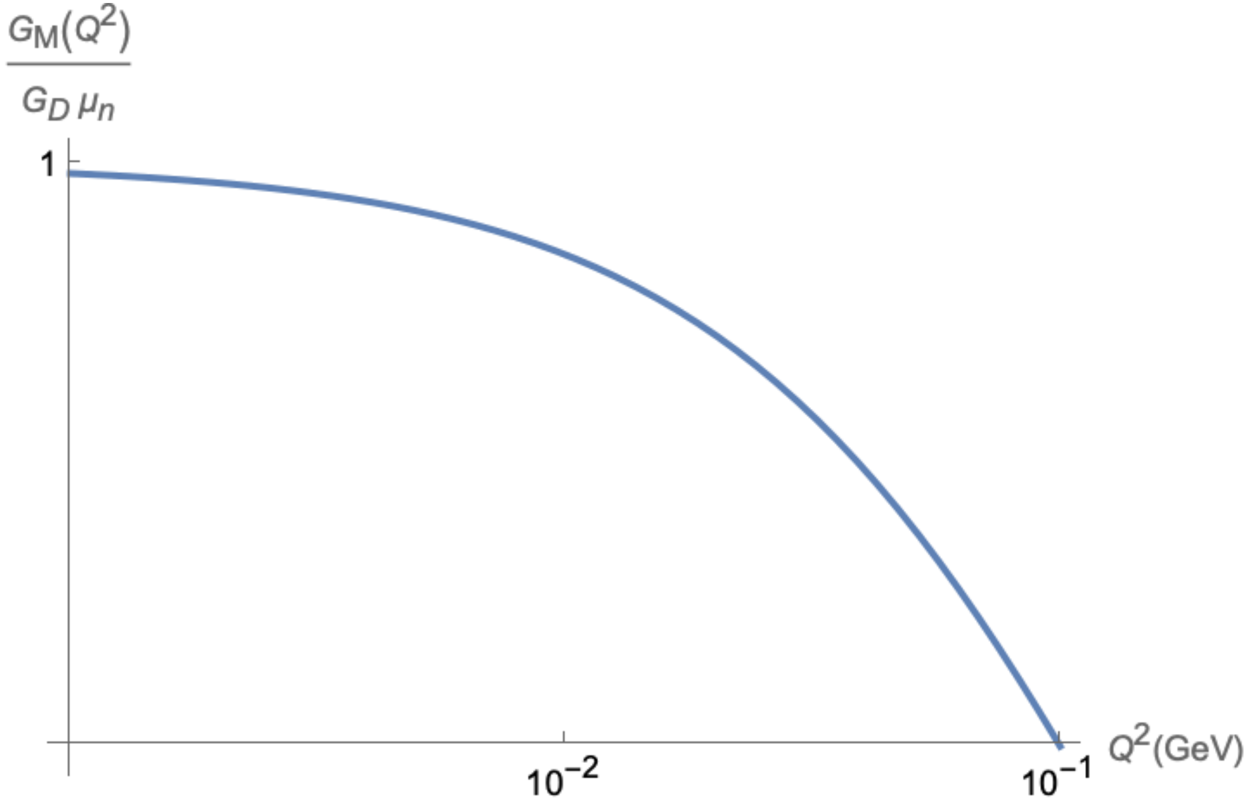
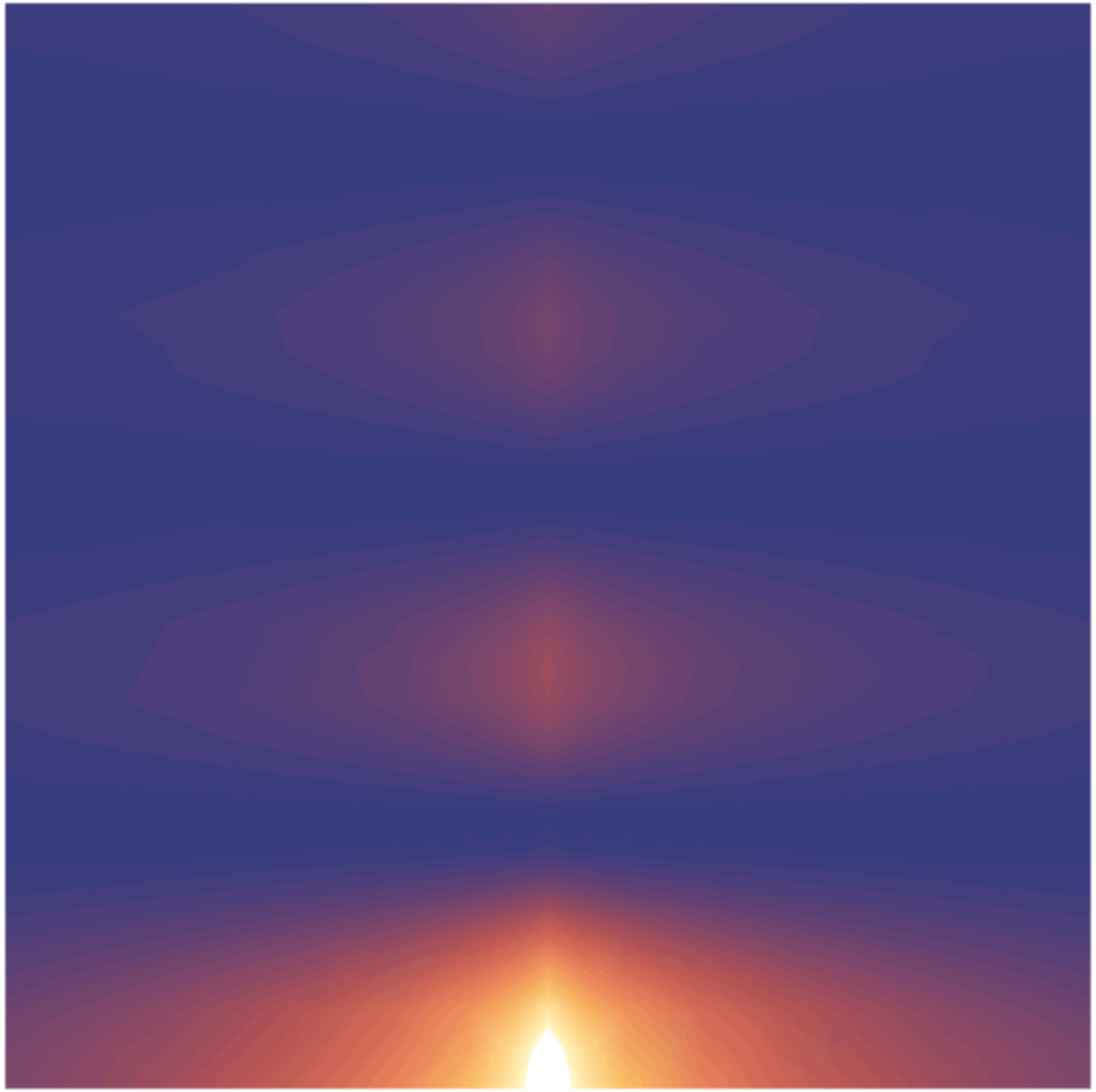




(Preliminary)

Next Steps

- More physically-motivated renormalization
- Extracting spatial information—
  - Light-Front wave functions
- Gravitational form factors



Thank you…

- Professor Miller
- All of you!

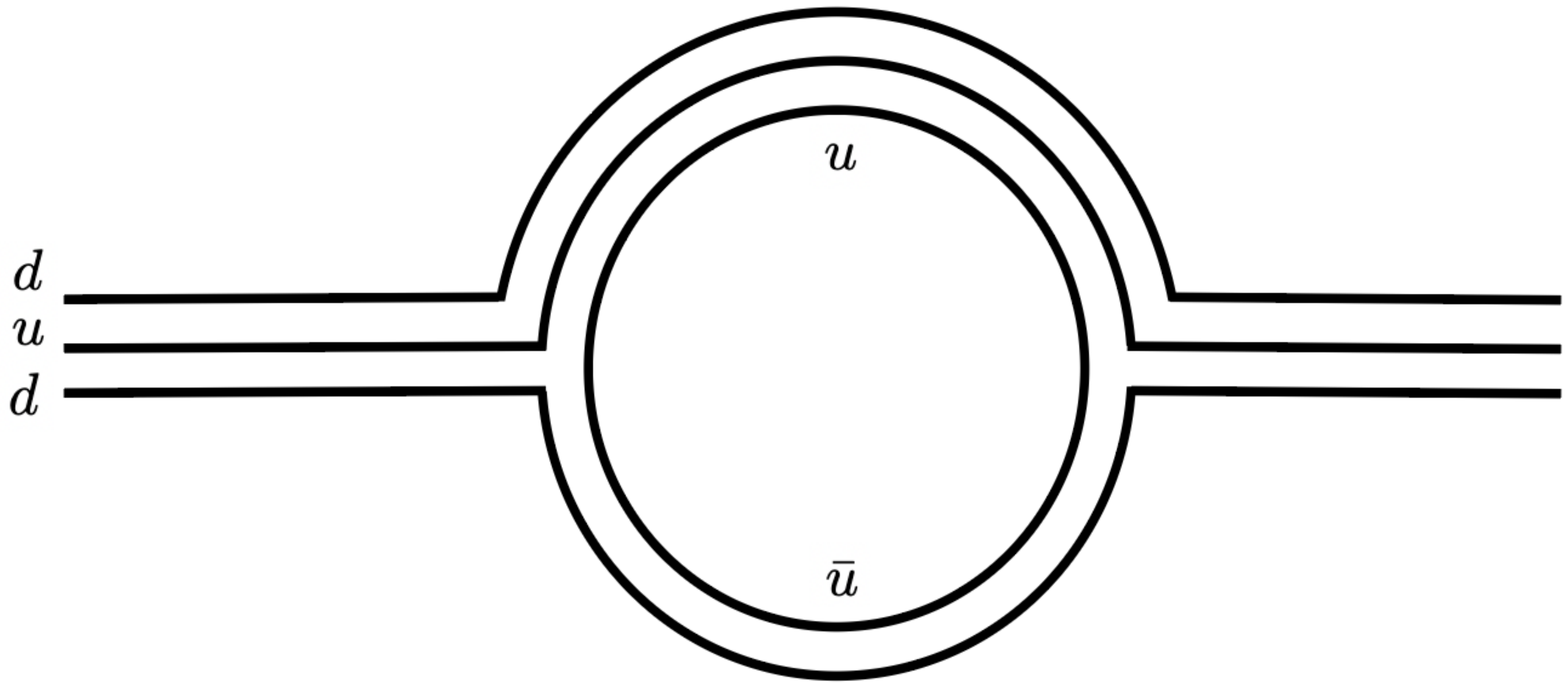
INSTITUTE for NUCLEAR THEORY

Questions?

Backup Slides

The model: $n^0 \to p^+\pi^- \to n^0$

$$\mathcal{L}_{\text{int}} = ig\left(\bar{\psi}_p\gamma^5\psi_n\phi_-^* + \bar{\psi}_n\gamma^5\psi_p\phi_-\right) + \ldots$$



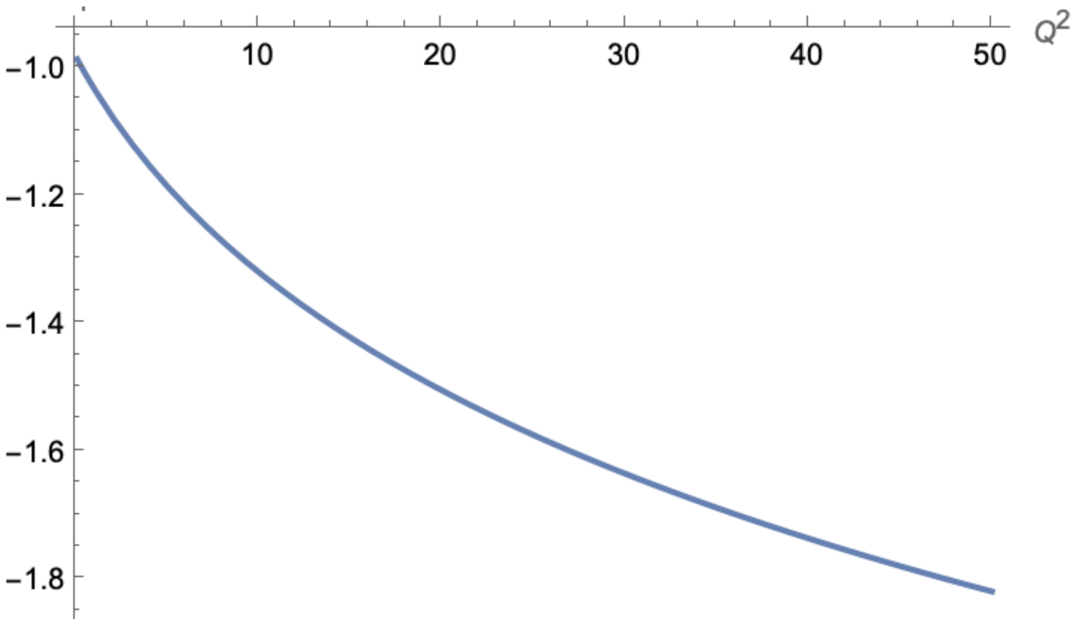
Pauli-Villars Regularization

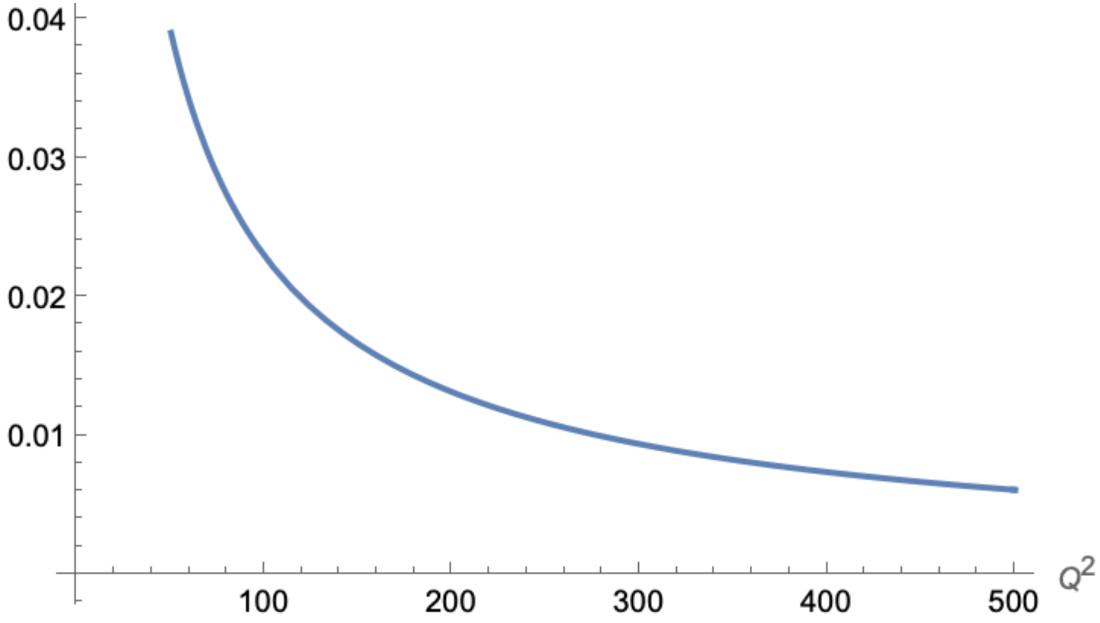
$$Q^2 \ll \Lambda^2$$

$$\dots - \int_0^1 dx \frac{2}{Q}\sqrt{\mathcal{M}^2 + \frac{Q^2x^2}{4}}\tanh^{-1}\left(\frac{Qx}{2\sqrt{\mathcal{M}^2 + \frac{Q^2x^2}{4}}}\right) + \dots$$

$$Q^2 \gg \Lambda^2$$

$$\dots - \frac{\ln(Q)}{Q^2}$$





Light front corrections

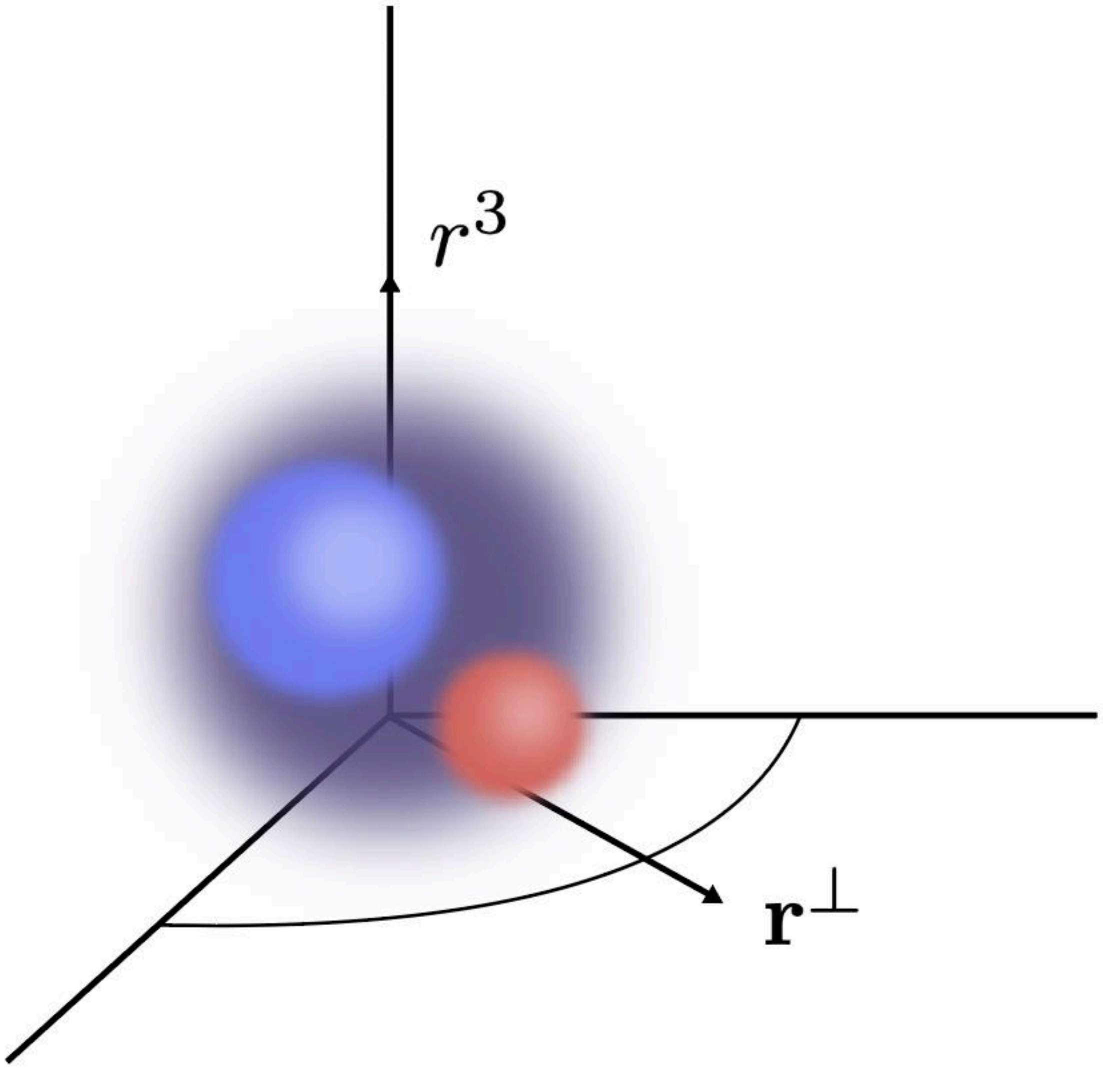
$$\Psi = \Psi(x_0^0, x_0^3, \mathbf{x_0}^{\perp}, r^0, r^3, \mathbf{r}^{\perp})$$

$$x^{\pm} = x^0 \pm x^3$$

$$x^{\pm} \xrightarrow{\text{boost}} \gamma(1+\beta)x^{\pm}$$

$$x^{+} = \text{'time'}$$

$$x^{-}, \mathbf{x}_{\perp} = \text{'space'}$$



Light front corrections

$$\Psi = e^{iP^- x^+/2}\psi(x^-, x_0^\perp)\phi(x^-, r^\perp)$$

$$q^+ \equiv 0$$

$$T_{fi} \propto \delta(P'^- - P^- - q^-)\int dx^- \int d^2x_0^\perp \psi'^\dagger e^{iq^\perp x_0^\perp}\psi \int d^2r^\perp \phi'^\dagger e^{iq^\perp r^\perp}\phi$$

$$F(q) = \int dx^- \int d^2r^\perp \phi'^\dagger(x^-, r^\perp) e^{iq^\perp r^\perp}\phi(x^-, r^\perp)$$

$$\phi_\perp \equiv \int dx^- \phi(x^-, r^\perp)$$

$$F(q) = \int d^2r \phi_\perp'^\dagger(r^\perp) e^{iq^\perp r^\perp}\phi_\perp(r^\perp)$$